THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORTON THE FORM TWO NATIONAL ASSESSMENT(FTNA) 2023

ADDITIONAL MATHEMATICS

THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2023

042 ADDITIONAL MATHEMATICS

Published by ;
The National Examinations Council of Tanzania,
P.O. Box 2624,
Dar es Salaam, Tanzania.

TABLE OF CONTENTS

FOREWORD

The report on Students' Item Response Analysis (SIRA) in Additional Mathematics for Form Two National Assessment (FTNA) 2023 was prepared and issued to the public by the National Examinations Council of Tanzania (NECTA) for the purpose of informing education stakeholders on how the student responded to the assessed competences.

This analysis shows justifications for the students' performance in the Additional Mathematics subject. The factors noted for good performance includes: students being competent with the assessed topics and concepts; correct interpretation of the questions; adhering to the instructions for each question; and clear arithmetics. On the other hand, students who scored low marks faced difficulties in responding to the questions due to insufficient knowledge of the tested concepts.

The analysis also indicates that, the students had average performance in four (4) topics, including *Logic*, *Locus*, *Geometrical Constructions* and *Sets*. Whereas, five (5) topics were well performed, which are *Numbers*, *Symmetry*, *Coordinate geometry*, *Variations*, and *Algebra*. Thus, generally, the performance of the students was good.

The National Examinations Council of Tanzania hopes that this report will encourage education stakeholders to work on the challenges encountered by students when attempting assessment questions so as to take appropriate measures to improve future students' performance in the subject.

The National Examinations Council of Tanzania is thankful to all who participated in one way or another in the production of this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The preparation of the Students' Item Response Analysis (SIRA) in the Additional Mathematics subject aims at providing feedback to teachers, students, and other education stakeholders about the students' performance in the 2023 Form Two National Assessment (FTNA). The setting of the paper was based on the Form Two National Assessment format of 2019 prepared by NECTA. The paper consisted of ten compulsory questions with 10 marks each.

A total of 441 students sat for the FTNA on Additional Mathematics in 2023, compared to, where 398 students who sat for the assessment in 2022. The performance of the students was good. Table 1 provides a summary of the students' general performance in 2022 and 2023.

Table 1: The Students' Performance on Additional Mathematics (FTNA) 2022 and 2023

Year	Students Sat	Passed		Grades				
		No.	%	A	B	C	D	F
2022	398	306	76.88	26	40	142	98	92
2023	441	310	70.29	74	50	104	82	131

The data in Table 1 show that, the students' performance in Additional Mathematics in 2023 was good since 70.29 per cent of students who sat for the assessment passed, where 16.78 per cent got grade A, 11.34 per cent got grade B, 23.58 per cent got grade C, 18.59 per cent got grade D, and 29.70 per cent got grade F. The analysis shows that there is a decrease in students' performance by 6.59 per cent as compared with the 2022 results.

The analysis of students' performance in each question is detailed in Section 2.0 of this report. It provides short descriptions of the requirements of each question and an analysis of the students' responses to the questions. Extracts of both good and poor students' responses on each question are included to illustrate cases presented. The factors that influenced the performance in each question are also illustrated.

In the analysis green, yellow, and red colours are used to represent good, average and poor performance, respectively. Section 3.0 gives the summary of students' performance in each topic and Section 4.0 of this report gives the conclusions and recommendations that are aimed at helping students improve their performance in future assessments in Additional

Mathematics. It is therefore hopeful that the report will be a useful guide to both teachers and students for improving the teaching and learning process and, hence, the students' performance in the forthcoming assessments.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH QUESTION

This section describes the analysis of students' performance on each question. The performance of students on each question is categorized in three groups; weak performance 0–29 per cent, 30–64 per cent and 65–100 per cent showing average and good performance, respectively. Moreover, the performance is categorized by using different colours whereby green, yellow, and red are used to represent good, average and weak performance, respectively.

2.1 Question 1: Numbers

The question consisted of parts (a) (i), (a) (ii), and (b). In part (a) (i), students were required to study the sequence $-3, -2, -5, -7, -12$ and -19, and then state a reason to verify that the sequence is Fibonacci. In part (a) (ii), students were instructed to use the divisibility rule to determine whether the number 9655 is divisible by 3. In part (b), students were required to complete blank spaces in the following pattern of numbers that obeys Pascal's triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
_ _ _ _ _ _
_ _ _ _ _ _ _
_ _ _ _ _ _ _ _

The analysis revealed that out of 441 students who attempted this question, 50 (11.3%) scored marks ranging from 0 to 2.5, 296 (67.2%) students scored marks ranging from 3.0 to 6.0, and 95 (21.5%) students scored marks ranging from 6.5 to 10. Therefore, the students' performance on this question was generally good. The summary of the students' performance in this question is presented in Figure 1.

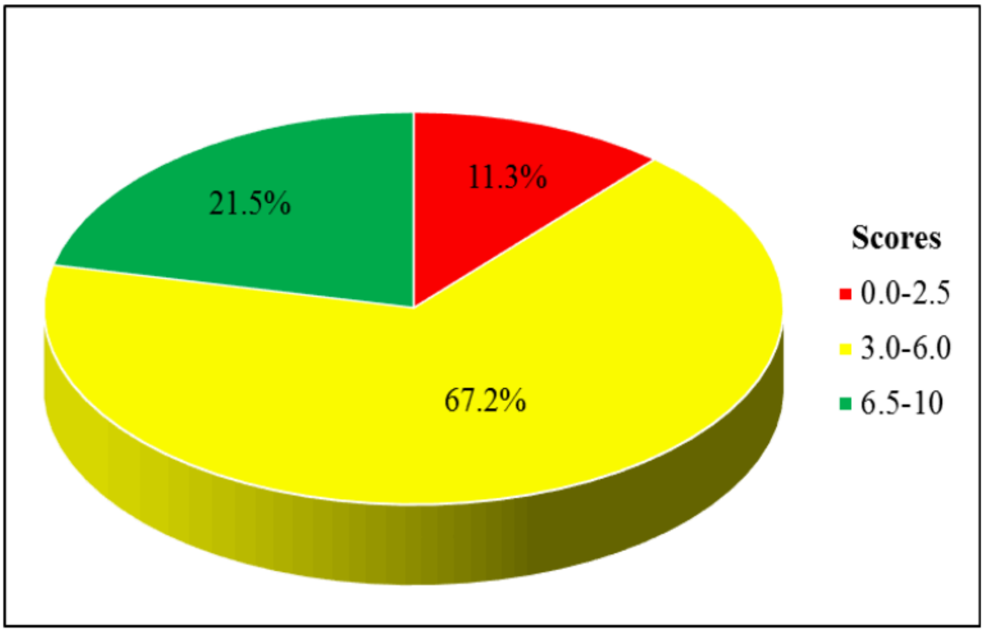
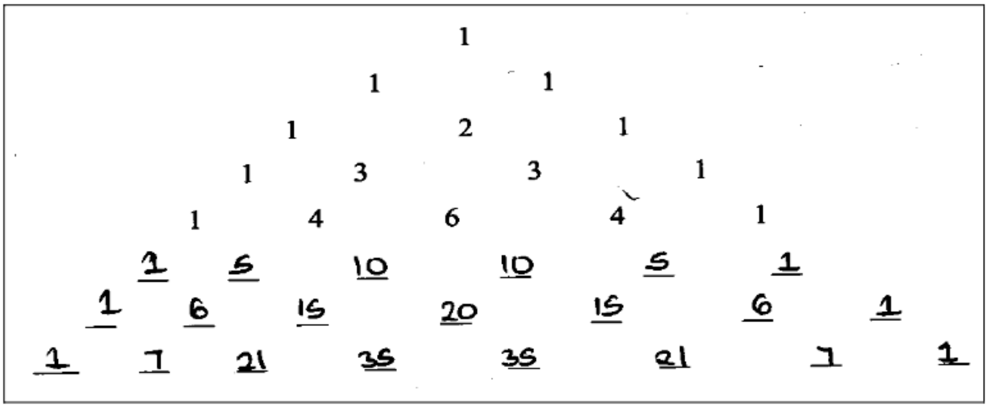


Figure 1: *Students' Performance on Question 1*

In part (a) (i), the analysis of the responses of the students who correctly responded to this question showed that the students computed the sum of the two preceding numbers, that is, $-2+(-3)=-5$, $-5+(-2)=-7$, $-7+(-5)=-12$, and $-12+(-7)=-19$. By studying the terms in the sequence, they realised that the sum of the two preceding consecutive numbers equals the next number. Therefore, they correctly concluded that the sequence is Fibonacci. In part (a) (ii), these students were conversant with the rule for divisibility on 3, that is, a number is divisible by 3 if the sum of its digits is also divisible by 3. These students computed the sum of the digits of the number 9655, that is, $9+6+5+5=25$. Then, they divided 25 by 3 and got $8\frac{1}{3}$, which is not an integer. Thus, they concluded that 25 is not divisible by 3 and consequently, 9655 is not divisible by 3.

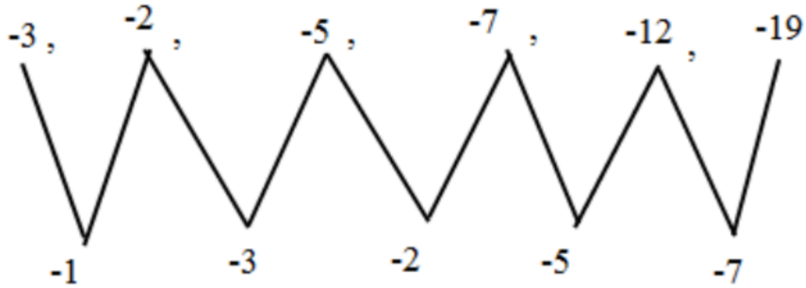
In part (b), the students correctly identified the pattern of numbers that obeys Pascal's triangle, thus completing the first and last blanks of each row by 1. Then, the other blanks in a particular row were completed by writing the sum of two consecutive numbers from the previous row. For example, the second blank from left in the 6^{th} row was completed by writing 5, which is the sum of 1 and 4 found in the 5^{th} row. Hence, they correctly completed the blank spaces in the 6^{th}, 7^{th}, and 8^{th} rows. Extract 1.1 is a sample response from one of the students who correctly attempted this question.



Extract 1.1: A sample of the correct responses to question 1

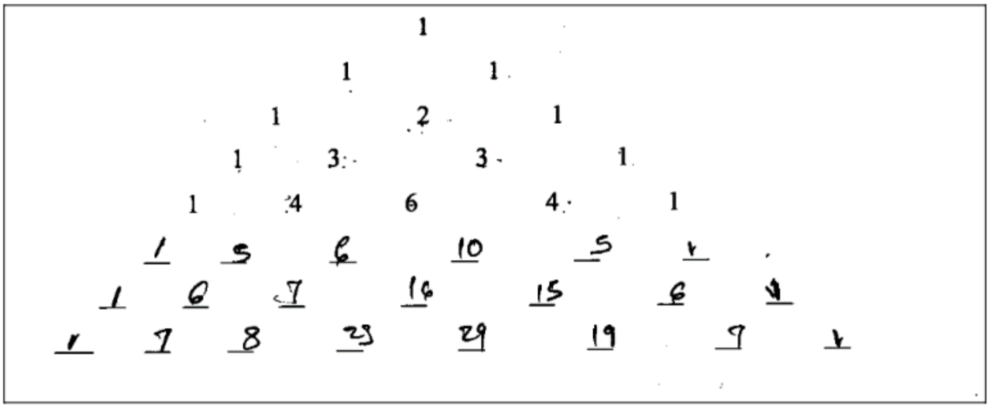
In Extract 1.1, the student applied the concept of Pascal's triangle correctly to complete the blank spaces in the 6^{th}, 7^{th}, and 8^{th} rows.

On the other hand, 95 (11.3%) students scored low marks in this question. In part (a) (i), most students failed to identify the rule governing the terms of the Fibonacci sequence. For example, some students misinterpreted the Fibonacci sequence as any sequence with negative numbers. For instance, some students wrote that, since the sequence -3, -2, -5, -7, -12, and -19 had negative numbers, it was a Fibonacci sequence. The analysis also shows that other students considered any sequence with no common difference to be a Fibonacci sequence; hence, some responded to the question by showing patterns, for instance,



Also, some students wrote, "The sequence is Fibonacci because it has an irregular pattern," and other students wrote, "Since a sequence had not been arranged in ascending order, it is Fibonacci." In part (a) (ii), most of these students applied the appropriate rule for the divisibility of a number on 3. These students considered the last two digits of 9655, indicating that they applied the rule for divisibility of 4 instead of 3. For instance, some students divided 55 by 3 and got 18.3, and finally they concluded that the number is not divisible by 3. Similarly, other students considered the first digit of the number 9655 (that is, 9) is to be divisible by 3. Therefore, the students concluded that the number 9655 is divisible by 3.

In part (b), some students wrongly identified the pattern of numbers that obeys Pascal's triangle, thus incorrectly summed up two consecutive numbers on the particular row and consequently completed the blank spaces with the wrong entries. Extract 1.2 is a sample response from one of the students who faced challenges when responding to the question.



Extract 1.2: A sample of the incorrect responses to question 1

In Extract 1.2, the student wrongly filled in the blank spaces in the required rows of the given pattern of numbers without considering the concept of Pascal's triangle.

2.2 Question 2: Algebra

The question was composed of three parts (a), (b), and (c), which required students to:

(a) simplify the expression $18r-(2r+10)-14r+25$ to its lowest term.

(b) expand completely the following expressions: (i) $3(2c+3)^2-c^2$ and (ii) $2x(x+4y)-x(8x+14y)-2(3+4y)$.

(c) write r in terms of x and y, given that $\frac{x}{y}=\frac{1+r^2}{1-r^2}$.

The analysis of the data depicts that, out of 441 students who responded to this question, 172 students scored marks ranging from 0 to 2.5, 135 students scored marks ranging from 3.0 to 6.0, and 134 students scored marks ranging from 6.5 to 10. Generally, the students' performance on this question was average. Figure 2 provides a summary of the students' performance on this question by percentage.

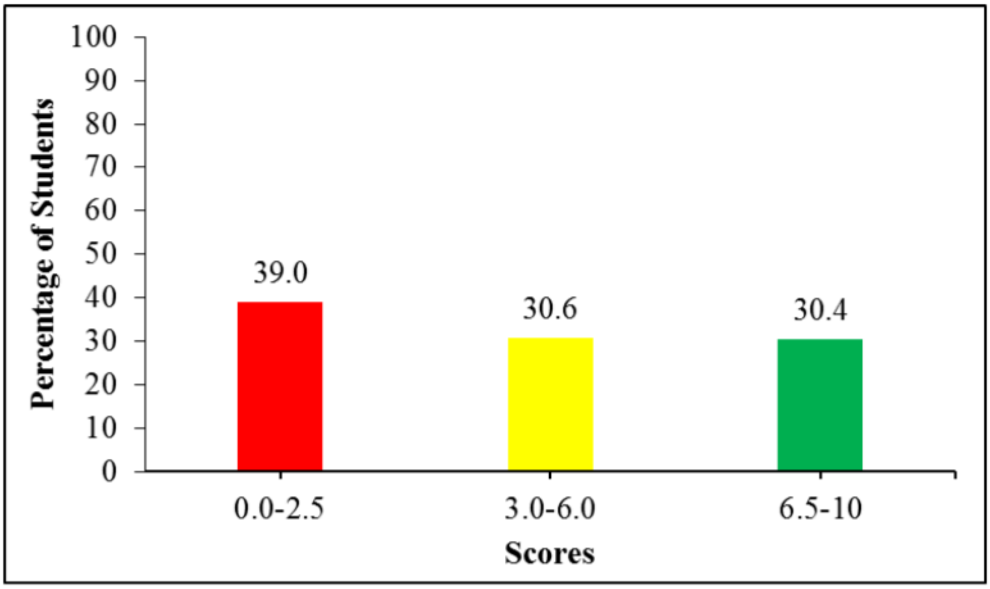


Figure 2: *Students' Performance on Question 2*

In part (a), the analysis shows that the students who managed to score high marks applied the BODMAS rule and performed basic operations correctly. They correctly opened the brackets in the expression $18r-(2r+10)-14r+25$ and got $18r-2r-10-14r+25$, then collected like terms and wrote the expression in the form of $(18r-2r-14r)+(25-10)$ and simplified it into $2r+15$. In part (b) (i), the students realised that $(a+b)^2=(a+b)(a+b)$. Therefore, they correctly rewrote the expression $3(2c+3)^2-c^2$ as $3(2c+3)(2c+3)-c^2$ and expanded it to obtain $3(4c^2+12c+9)-c^2$. Thereafter, the students opened the brackets and got the expression $12c^2+36c+27-c^2$, which was further simplified into $11c^2+36c+27$. Similarly, in part (b) (ii), the students opened the brackets in $2x(x+4y)-x(8x+14y)-2(3+4y)$, and obtained $2x^2+8xy-8x^2-14xy-6-8y$. Then, they rearranged the terms of the expression and got, $(2x^2-8x^2)+(8xy-14xy)-8x-6$, which was further simplified into $-6x^2-6xy-8y-6$.

In part (c), the students correctly performed cross-multiply on $\frac{x}{y} = \frac{1+r^2}{1-r^2}$ and obtained $y(1+r^2) = x(1-r^2)$. Then, they correctly opened the brackets and rearranged the terms of the expressions into $yr^2 + xr^2 = x - y$. Thereafter, the students correctly factored $yr^2 + xr^2$ into $r^2(y+x)$ and thus $yr^2 + xr^2 = x - y$ became $r^2(y+x) = x - y$. The students further divided both sides of $r^2(y+x) = x - y$ by $y + x$ and got $r^2 = \frac{x-y}{x+y}$. Finally, they computed the square root of each side, resulting in $r = \pm\sqrt{\frac{x-y}{x+y}}$. Extract 2.1 is a sample of the correct response from one of the students who attempted the question.

Soln:

$18r - (2r + 10) - 14r + 25$

$18r - 2r - 10 - 14r - 25$

$16r - 14r - 10 + 25$

$2r + 15$

$\therefore \underline{\underline{2r + 15}}$

soln

i) $3(2c+3)^2 - c^2$

$3(2c+3)(2c+3) - c^2$

$3(4c^2+6c+6c+9) - c^2$

$3(4c^2+12c+9) - c^2$

$12c^2+36c+27-c^2$

$\therefore \underline{\underline{11c^2+36c+27}}$

ii) $2x(x+4y) - x(8x+14y) - 2(3+4y$

$2x^2+8xy-8x^2-14xy-6-8y$

$2x^2-8x^2+8xy-14xy-6-8y$

$-6x^2+-6xy-8y-6$

$\therefore \underline{\underline{-6x^2-6xy-8y-6}}$

$\therefore \underline{\underline{-6x^2-8y-6xy-6}}$

Soln

$$\frac{x}{y} = \frac{1+r^2}{1-r^2}$$

$$x - r^2x = y + r^2y$$

$$x - y = r^2y + r^2x$$

$$x - y = r^2(y + x)$$

$$\frac{x-y}{y+x} = \frac{r^2(y+x)}{y+x}$$

$$\sqrt{\frac{x-y}{y+x}} = \sqrt{r^2}$$

$$\therefore r = \sqrt{\frac{x-y}{x+y}}$$

Extract 2.1: A sample of the correct responses to question 2

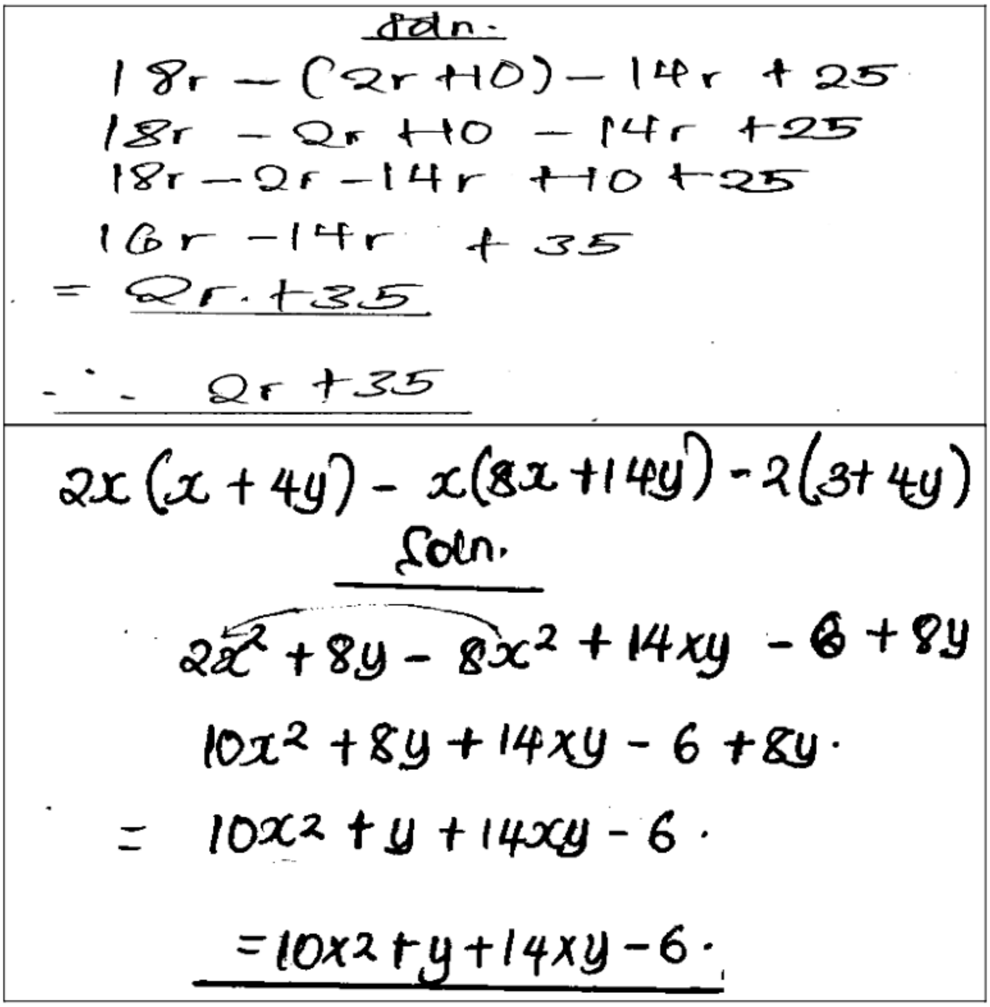
In Extract 2.1, the student correctly applied the BODMAS and basic operations to simplify the given expressions. In part (b), the student managed to expand the given expressions, and in part (c), he or she correctly made r in terms of x and y.

Despite the good performance, 68 (15.4%) students got zero marks. In part (a), most of these students ignored the negative sign when opening brackets. These students rewrote the expression $18r-(2r+10)-14r+25$ as $18r-2r+10-14r+25$ instead of $18r-2r-10-14r+25$ then simplified it into $2r+35$ instead of $2r+15$. Furthermore, some students committed computational errors. For example, some of these students correctly opened the bracket in $18r-(2r+10)-14r+25$ and obtained $18r-2r-10-14r+25$. However, they wrongly simplified it into $37r$.

Similarly, in part (b) (i), most of the students multiplied the term in the brackets by 3 before squaring, and thus, they wrongly rewrote $3(2c+3)^2-c^2$ as $(6c+9)^2-c^2$. These students were supposed to open the brackets by expanding $(2c+3)^2$ into $4c^2+12c+9$, then multiply the resulting expression by 3 and subtracting c^2. In addition, most of the students who got $(6c+9)^2-c^2$ expanded it incorrectly and obtained $36c^2-c^2+9^2$. These students misinterpreted $(a+b)^2$ as a^2+b^2 instead of $(a+b)(a+b)$ or $a^2+2ab+b^2$. Moreover, some students realised that the expression could be expanded by applying the concept of the difference of two squares, that is, $a^2-b^2=(a+b)(a-b)$. However, they failed to convert the given expression into the standard form of the difference of two squares. For example, some students rewrote $3(2c+3)^2-c^2$ as $[3(2c+3)+c][(2c+3)-c]$ instead of $[\sqrt{3}(2c+3)+c][\sqrt{3}(2c+3)-c]$.

Similar to part (a) (i), most students ignored the negative sign when opening the brackets in part (b) (ii). For example, many students wrongly opened brackets in $2x(x+4y)-x(8x+14y)-2(3+4y)$, resulting in $2x^2+8xy-8x^2+14xy-6-8y$ and consequently $-6x^2+22xy-8y-6$. Like wise, other students expanded $2x(x+4y)-x(8x+14y)-2(3+4y)$ into $2x^2+8xy-8x^2-14xy-6+8y$ instead of $-6x^2-6xy-8y-6$. Some other examples of incorrect responses provided by the students due to their inability to apply basic operations were $10x^2+14y^2-6+8y$ and $2x^2-8x^2+6xy-6+8y$.

In part (c), most of the students performed cross-multiply incorrectly. For instance, some students obtained $x-r^2=y+x^2$ instead of $x-xr^2=yr^2+y$ from $\frac{x}{y}=\frac{1+r^2}{1-r^2}$, and therefore, they got an incorrect answer. Similarly, some students applied cross-multiply properly, but they committed errors in factorization. For example, some of these students correctly got $xr^2+yr^2-x+y=0$ from $\frac{x}{y}=\frac{1+r^2}{1-r^2}$ but incorrectly factored it and got $r^2(x+y)-1(x+y)=0$ instead of $r^2(x+y)-(x-y)=0$. Moreover, some students developed incorrect statements from $\frac{x}{y}=\frac{1+r^2}{1+r^2}$, such as $x(1+r^2)+y(1+r^2)$, which was simplified into $x+xr^2+y+yr^2$. Extract 2.2 is a sample of a response from one of the students who faced difficulties in attempting this question.



Soln.

$18r-(2r+10)-14r+25$

$18r-2r+10-14r+25$

$18r-2r-14r+10+25$

$16r-14r+35$

$=2r+35$

$\therefore 2r+35$

$2x(x+4y)-x(8x+14y)-2(3+4y)$

Soln.

$2x^2+8y-8x^2+14xy-6+8y$

$10x^2+8y+14xy-6+8y$

$=10x^2+y+14xy-6$

$=10x^2+y+14xy-6$

Extract 2.2: A sample of the incorrect responses to question 2

In Extract 2.2, in part (a), the student failed to adhere to the effect of the negative sign in opening the brackets, which led to an incorrect answer. In part (b) (ii), the student failed to multiply the variable in the first term and did not recall the negative sign in some steps.

2.3 Question 3: Geometrical Constructions

In this question, the students were informed that the size of an exterior angle of a certain polygon is p and the size of its interior angle is three times the size of the exterior angle. Then, they were required to find: (a) the value of the expression $\frac{6p-16^{\circ}}{2}$; (b) the size of the interior angle; and (c) the sum of the interior angles.

According to the data analysis of the students' performance, it was observed that 441 students attempted the question, out of whom 251 (56.9%) scored from 0 to 2.5 marks, 44 (10.0%) scored from 3.0 to 6.0 marks, and 146 (33.1%) scored marks ranging from 6.5 to 10. The summary of students' performance is presented in Figure 3.

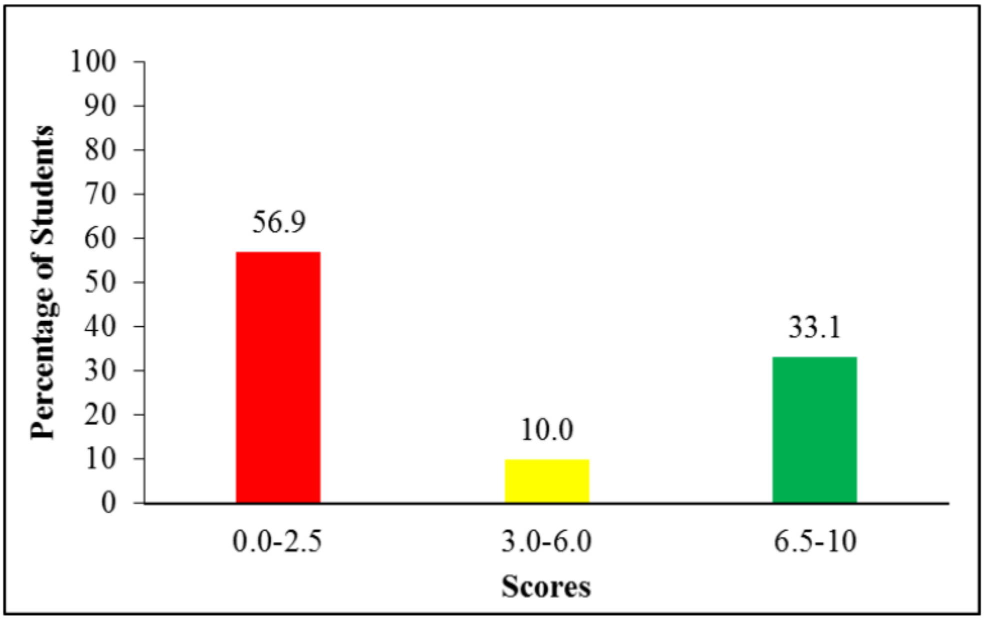


Figure 3: *Students' Performance on Question 3*

The overall performance of the students on this question was average. The students who correctly responded to part (a) recalled the fact that the sum of the exterior angle (p) and interior angle ($3p$) equals 180^0. They developed the equation $p+3p=180^0$ and correctly solved it to get $p=45^0$. Thereafter, they replaced p in $\frac{6p-16^0}{2}$ with 45^0 and correctly worked on the obtained expression, resulting in 127^0. Similar to part (a), many students responded to part (b) by recognizing the fact that the sum of the exterior angle and interior angle equals 180^0. Therefore, they wrote $45^0 + \text{interior angle} = 180^0$ and identified that the size of the interior angle is 135^0. Other students recalled that $p = 45^0$ and the interior angle is $3p$. Therefore, they calculated the product of 3 and 45^0 to determine the size of the interior angle, as $3p = 3\left(45^0\right)$, resulting in 135^0.

In part (c), the students correctly recalled the formula for calculating the number of sides (n) of a regular polygon, that is, $\mathrm{n} = \dfrac{360^0}{\text{exterior angle}}$. Therefore, they replaced the exterior angle in the formula with 45^0 and correctly worked out to get $n = 8$. Furthermore, the students applied the formula for calculating the sum of interior angles, that is, $(n-2)\times 180^0$, and hence substituted $n = 8$ into the formula and got the sum of interior angles equals 1080^0. Also, some students computed the product of the number of sides (8) and the size of an interior angle (135^0) to get the sum of the interior angles of the polygon, which is 1080^0. Extract 3.1 is a sample of a correct response from one of the students who attempted this question.

Solution

$p + 3p = 180$

$\frac{4p}{4} = \frac{180}{4}$

$p = 45$.

Then

Given $\frac{6p - 16^\circ}{12}$

$= \frac{6(45) - 16}{2}$

$= \frac{270 - 16}{2}$

$= \frac{254}{2}$

$= 127$

∴ The answer is $\underline{127^\circ}$

Solution.
Then.
$E + I = 180$
Then interior angle is three times exterior angle
$I = 3P \cdot (P = 45)$.
$I = 3(45)$
$I = 135^\circ$
∴ The size of interior angle is 135°

Solution
$E = \frac{360}{n}$
$\frac{45}{1} \times \frac{360}{n}$
$\frac{45n}{45} = \frac{360}{45}$
$n = 8$
Then
Sum $= (n-2)180$
$= (8-2)180$
$= (6)180$
$= 1080^\circ$
∴ The Sum of interior angle is 1080°

Extract 3.1: A sample of the correct responses to question 3

In Extract 3.1, the student correctly applied a relation between interior and exterior angles and hence managing to get the value of the given expression in part (a). In part (b), the student correctly calculated the size of the interior angle using the information that the interior angle is three times the exterior angle. In part (c), the student applied the formula $(n-2)\times 180^0$ to determine the sum of the interior angles.

On the other hand, the analysis revealed that some students were not able to correctly recognize the appropriate formula and misinterpreted the word problem. In part (a), some students applied the formula: $\text{interior angle(I)} + \text{exterior angle(E)} = 180^0$, but committed computation errors. Most of these students formulated the correct equation $3p + p = 180^0$ or $4p = 180^0$, but they got incorrect answers, particularly $p = 46^0$. As a result, they got the incorrect value of the expression $\frac{6p-16^0}{2}$, including 130^0. Moreover, some students incorrectly equated the expression to zero, and thus they developed the equation $\frac{6p-16^0}{2} = 0$, and solved it, resulting in $p = \frac{8}{3}$ instead of $p = 45^0$. Futhermore, some student attempted the question by equating the expression to 180^0, that is, $\frac{6p-16^0}{2} = 180^0$, and worked on it, ending up with an incorrect value of p, such as $p = 62.7^0$.

In part (b), many incorrect answers resulted from the mistakes observed in part (a). For example, the students who got $p = 46^0$ resulted in $3p = 138^0$ and hence the incorrect size of the interior angle of 138^0 instead of 135^0. Similarly, the students who got $p = 28.5^0$ resulted to $3p = 75.5^0$. Additionally, other students also used an inappropriate formula, $(n-2)180^0$, which was used to find the sum of interior angles resulting in the incorrect value of an interior angle such as $10,440^0$.

In part (c), the analysis revealed that a number of students committed computation errors. For instance, some students correctly recognized the formula $\text{exterior angle} = \frac{360^0}{n}$, where the size of the exterior angle is 45^0. However, they got an incorrect value of $n = 4.8$ and approximated it to 5. As a result, these students got the incorrect sum of interior angles, particularly 540^0 instead of 1080^0. Furthermore, some students applied the correct formula for the sum of interior angles but used the incorrect value of n. For instance, some of them substituted $n = 135$ in the formula $(n-2)180^0$. Extract 3.2 is a sample of the responses from one of the students who was not able to provide the correct responses.

$(n-2)180°$

$(135°-2)\,180°$

$133° \times 180°$

$13940°$

Size of interior angle $= 13940°$

Extract 3.2: A sample of the incorrect responses to question 3

In Extract 3.2, on part (c) of the question, the student misinterpreted that the size of an interior angle equals the number of sides of a polygon, more over committed computational error that resulted in the wrong answer.

2.4 Question 4: Locus

The question consisted of three parts: (a), (b), and (c). In part (a), the students were required to describe the locus when (i) an orange is falling vertically from a tree at the height of 2 metres from the ground and (ii) the centre of a wheel as a cyclist rides along the road on a horizontal plane. In part (b), they were required to analyse the equation of the locus of point P which is equidistant from the points $L(-2, 2)$ and $M(1, 1\frac{1}{2})$. In part (c), the students were informed that, the locus of point P moves along the plane and intersects the lines whose equations are $m(y-3) = x+1$ and $y = mx$ where, m is a variable. Then, they were required to find the equation of the locus of the point P.

In this question, 247 (56.0%) students scored 0 to 2.5 marks, 160 (36.3%) students scored 3.0 to 6.0 marks, and 34 (7.7%) students scored 6.5 to 10 marks. Generally, students had average performance on this question. The summary of students' performance is presented in Figure 4.

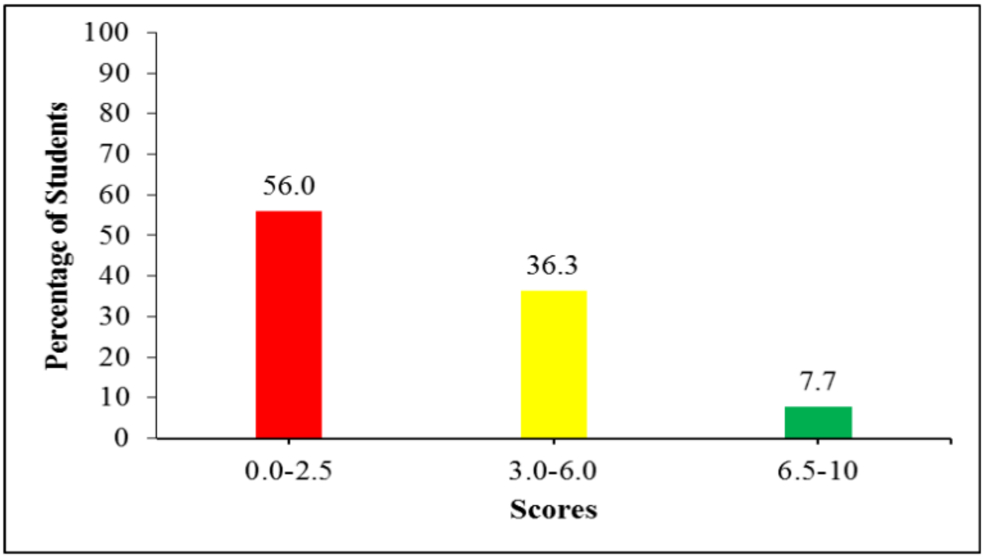


Figure 4: *Students' Performance on Question 4*

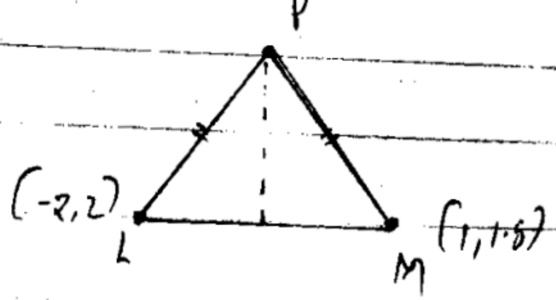
The students who managed to provide correct responses in part (a) (i) correctly described that the locus formed by an orange falling from the tree is a straight vertical line whose length is 2 metres. In part (a) (ii), the students stated that the locus of the centre of a wheel as a cyclist rides along the road on a horizontal plane is a circle. In part (b), the student applied the formula for calculating the distance between two points, $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. They assumed the coordinates of the point P to be (x, y), and therefore they substituted $P(x, y)$ and $L(-2, 2)$ into the formula, resulting in $\overline{PL} = \sqrt{(x - -2)^2 + (y - 2)^2}$. Likewise, they got $\overline{PM} = \sqrt{(x-1)^2 + (y - 1\frac{1}{2})^2}$ after considering $P(x, y)$ and $M(1, 1\frac{1}{2})$. These students also equated $\sqrt{(x - -2)^2 + (y-2)^2}$ to $\sqrt{(x-1)^2 + (y-1\frac{1}{2})^2}$ and simplified it to get $24x - 4y + 19 = 0$, which is the equation of the locus of P.

In part (c), students correctly rewrote $y = mx$, in the form $m = \frac{y}{x}$ and therefore, they replaced m in $m(y-3) = x+1$ with $\frac{y}{x}$ and correctly simplified into $x^2 - y^2 + x + 3y = 0$. Extract 4.1 illustrates the correct solution to this question from one of the students responses.

i) The locus is ~~a~~ straight vertical line of 2 metre

ii) The locus is circle when a wheel as a cyclist rides

Sdr.



From

PL = PM.

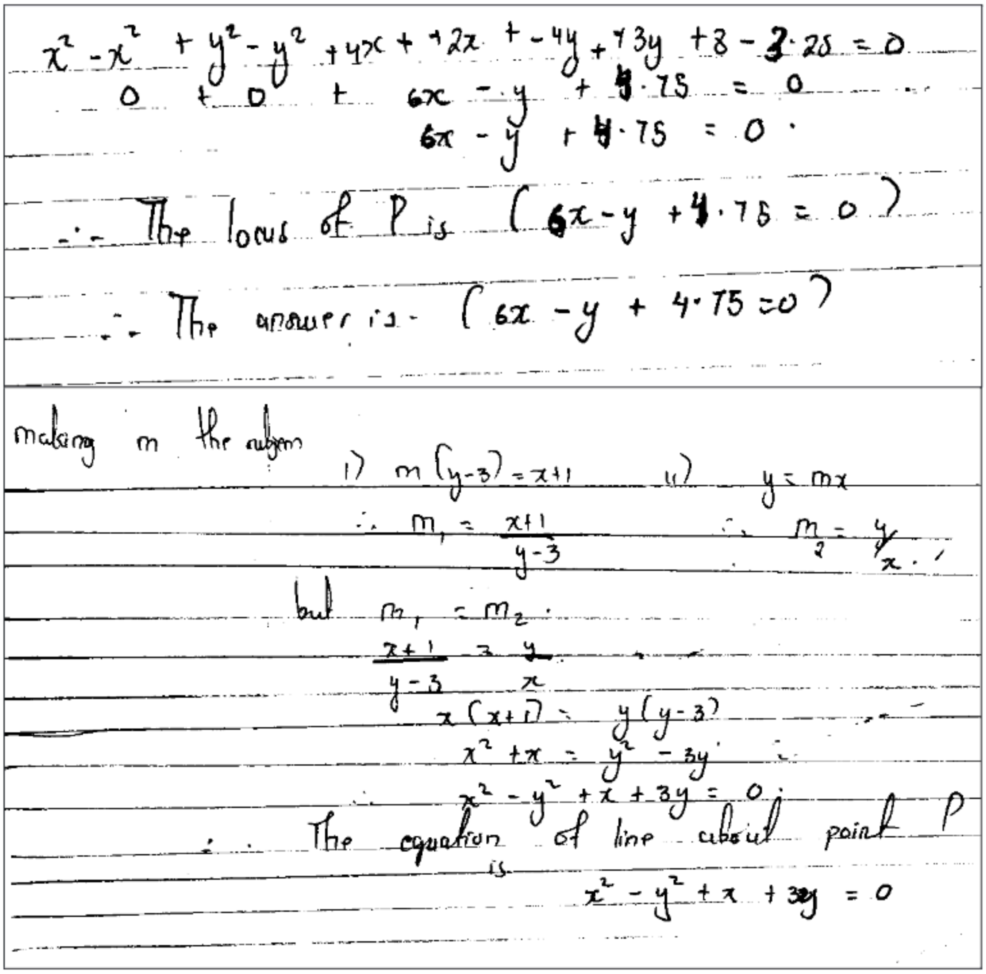
and $\sqrt{(x-x_1)^2 + (y-y_1)^2} = \bar{AB}$

$\left(\sqrt{(x+2)^2 + (y-2)^2}\right)^2 = \left(\sqrt{(x-1)^2 + (y-1.5)^2}\right)^2$

$(x+2)^2 + (y-2)^2 = (x-1)^2 + (y-1.5)^2$ — 1

$x^2 + 4x + 4 + y^2 - 4y + 4 = x^2 - 2x + 1 + y^2 - 3y + 2.25$

$x^2 + y^2 + 4x - 4y + 8 = x^2 + y^2 - 2x - 3y + 3.25.$



$x^2 - x^2 + y^2 - y^2 + 4x + 2x + -4y + 3y + 8 - 3.25 = 0$

$0 + 0 + 6x - y + 4.75 = 0$

$6x - y + 4.75 = 0$

∴ The locus of P is $(6x - y + 4.75 = 0)$

∴ The answer is $(6x - y + 4.75 = 0)$

making m the subject

i) $m(y-3) = x+1$ ii) $y = mx$

$\therefore m_1 = \frac{x+1}{y-3}$ $\therefore m_2 = \frac{y}{x}$

but $m_1 = m_2$

$\frac{x+1}{y-3} = \frac{y}{x}$

$x(x+1) = y(y-3)$

$x^2 + x = y^2 - 3y$

$\therefore x^2 - y^2 + x + 3y = 0$

∴ The equation of line about point P is

$x^2 - y^2 + x + 3y = 0$

Extract 4.1: A sample of the correct responses to question 4

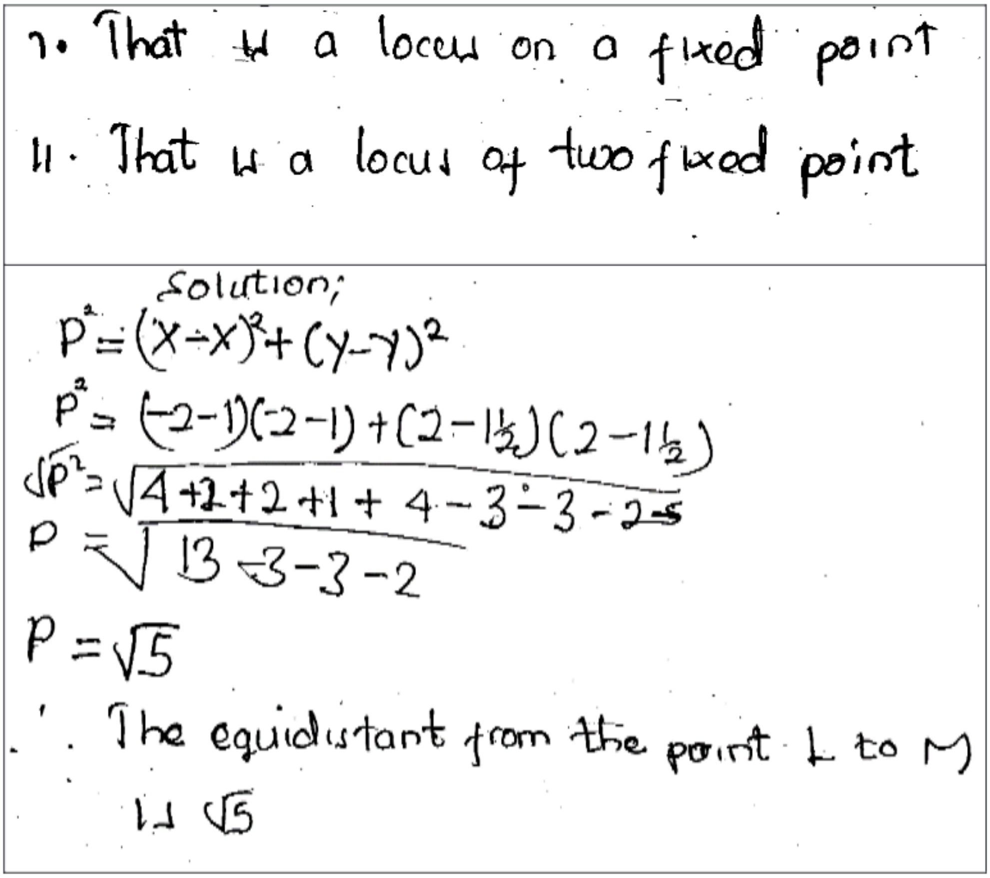
In Extract 4.1, part (a), the student correctly described the locus, and in part (b), he or she applied the distance formula between two points to find the locus of a point. In part (c), the student managed to get the equation of the line of locus of the point P using the fact that the two lines have equal gradients.

As Figure 4 shows, 56.0 per cent of the students scored low marks (0 – 2.5). Those students encountered the following challenges: In part (a) (i), some students described that the locus of an orange falling vertically from a tree at a height of 2 metres from the ground is 4 metres. Likewise, some students said that the locus of an orange falling vertically from a tree at a height of 2 metres from the ground is the one that doesn’t move at a fixed point. Also, other students described that when an orange is falling vertically from a tree at a height of 2 metres from the ground, its locus is about two fixed points obtained as a perpendicular bisector between the two fixed points.

Moreover, in part (a) (ii), some students described that when the centre of a wheel as a cyclist rides along the road on a horizontal plane, its locus is the number of wheels (2), considering the number of wheels of a bicycle. Also, other students wrote that the locus of the centre of a wheel is a horizontal line parallel to the surface, assuming that the locus is a set of points along the road on which the wheel passes. These students were not conversant with the concept of locus at all.

In part (b), most students applied inappropriate formulae. For instance, some students applied the formula for calculating the gradient (m), instead of the distance between the two points. These students replaced (x_1, y_1) and (x_2, y_2) in $m = \frac{y_2 - y_1}{x_2 - x_1}$ with $(-2, 2)$ and $(1, 1\frac{1}{2})$, respectively, and worked out to get $m = \frac{-1}{6}$ and consequently the incorrect equation $y = \frac{-x}{6} + \frac{10}{6}$. Also, some students recalled the incorrect formula. For example, some students wrote $(x_1 + x)^2 + (y_1 + y)^2 = (x_2 + x)^2 (y_2 + y)^2$ in which they replaced (x_1, y_1) and (x_2, y_2) with $(-2, 2)$ and $(1, 1\frac{1}{2})$, respectively, and simplified it into $x^2 + y^2 + 2x - y - 55 = 0$. Moreover, some students misinterpreted the word "equidistant," by assuming that the locus could be a circle centred at $(0, 0)$ whose radius is the distance between L and M. These students computed the square of the distance between $L(-2, 2)$ and $M\left(1, 1\frac{1}{2}\right)$ using the distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ and got $d^2 = \frac{37}{2}$. Thereafter, they replaced r^2 in the general equation of the circle $r^2 = x^2 + y^2$ with $\frac{37}{2}$, resulting in $\left(\frac{37}{2}\right)^2 = x^2 + y^2$.

In part (c), many students strived to find the numerical values of x and y, while these were the variables of the locus under discussion. These students solved equations of the lines $m(y - 3) = x + 1$ and $y = mx$ simultaneously; however, they ended up with incorrect answers. For example, some students wrote $mx - x = 1 + 3m$ and incorrectly solved it, resulting in $x = 2$ and consequently $y = 2m$. Similar to part (a), some students used the general equation of a circle $p^2 = (x-a)^2 + (y-b)^2$ at (a,b). These students replaced (a,b) with $(3,1)$ in $r^2 = (x-a)^2 + (y-b)^2$ to obtain $r^2 = (x-3)^2 + (y-1)^2$ and simplified it into $x^2 + y^2 + 6x + 2y - r^2 = 0$. Extract 4.2 is a sample response from one of the students who faced difficulties when attempting the question.


i. That is a locus on a fixed point

ii. That is a locus of two fixed point

Solution;

$p^2 = (x-x)^2 + (y-y)^2$

$p^2 = (-2-1)(-2-1) + (2-1\frac{1}{2})(2-1\frac{1}{2})$

$\sqrt{p^2} = \sqrt{4+2+2+1+4-3-3-2.5}$

$p = \sqrt{13-3-3-2}$

$p = \sqrt{5}$

∴ The equidistant from the point L to M is $\sqrt{5}$

Extract 4.2: A sample of the incorrect responses to question 4

In Extract 4.2, part (a), the student incorrectly described the locus of the objects asked. In part (b), the student computed the distance between L and M instead of equating the distance between points P and L, and P and M.

2.5 Question 5: Coordinate Geometry

The question consisted of parts (a) and (b). In part (a), the students were required to calculate the value of h given that the points $A(2,5)$, $B(h,-4)$ and $C(1,2)$ are collinear. In part (b), the students were required to determine the equation of a line passing through the point $(-4,-4)$ and parallel to the line whose equation is $2x+6y-9=0$.

The data revealed that 127 (28.8%) students scored 2.5 marks or less, 115 (26.1%) scored marks ranging from 3.0 to 6.0, and 199 (45.1%) students scored 6.5 marks or more. Figure 5 provides a summary of students' performance on this question.

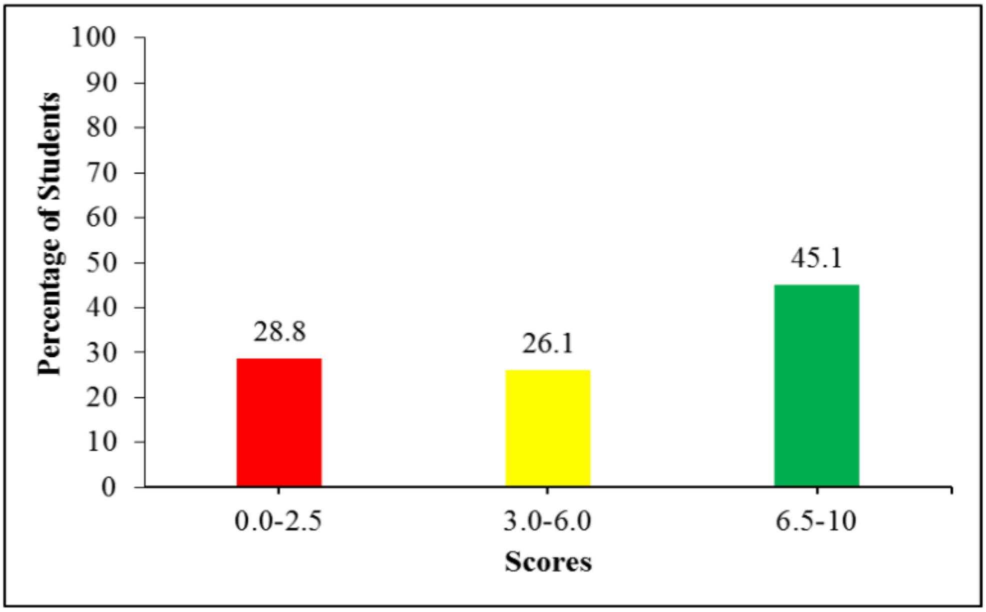


Figure 5: *Students' Performance on Question 5*

About 71.2 per cent equivalent to 314 students, got 3.0 marks or more. Therefore, students' performance on this question was good. In part (a), students were conversant with the fact that collinear points lie on the same straight line and thus their line segments have the same gradient. For this case, most students considered the gradient of line segments AB and AC, AB and BC, or BC and AC to generate an equation which enabled them to determine the value of h. For example, the students who considered the line segments AB and AC correctly applied the formula for calculating the gradient(m) of a straight line, $\mathrm{m}=\frac{y_1-y_2}{x_1-x_2}$, and got $m=3$ and $m=\frac{9}{2-h}$ for the line segments AB and AC, respectively. Then, they equated $\frac{9}{2-h}$ to 3, resulting in the equation $6-3h=9$ and consequently $h=-1$.

In part (b), the students were knowledgeable about the fact that parallel lines have the same gradient. Thus, these students correctly wrote $2x+6y-9=0$ into the standard form $y=-\frac{1}{3}x+\frac{3}{2}$, which enabled them to determine the gradient of the line, $m=-\frac{1}{3}$. Therefore, the students applied the formula $m=\frac{y-y_1}{x-x_1}$, whereas $m=-\frac{1}{3}$ and $(x_1, y_1)=(-4,-4)$, to formulate the equation, that is, $x+3y+16=0$. Extract 5.1 provides a sample of one of the correct solutions to this question.

Solution.

Condition of collinear point $m_1 = m_2 = m_3$

Given that point A (2,5) B, (h, -4) C(1,2)

take point A and C to find m

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-5}{1-2}$$

$$m = \frac{-3}{-1}$$

$$m = 3$$

them from $m_1 = m_2$. take point A and B

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{3}{1} \times \frac{-4-5}{h-2}$$

$$3h - 6 = -4 - 5$$

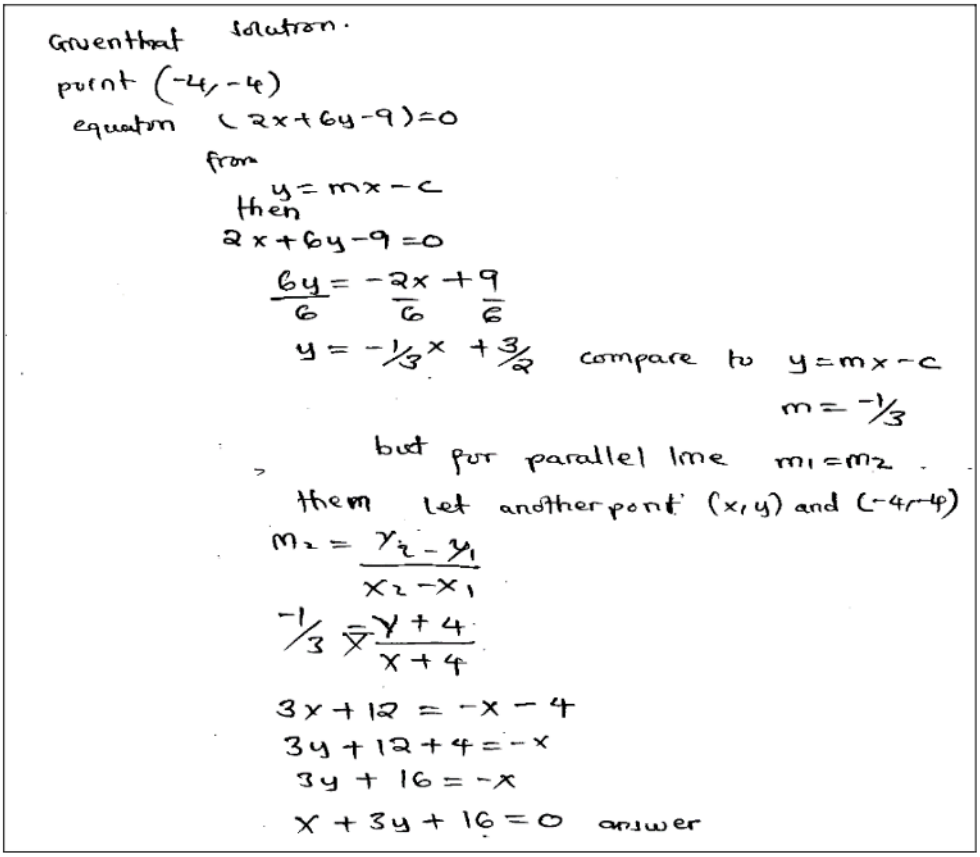
$$3h - 6 = -9$$

$$3h = -9 + 6$$

$$\frac{3h}{3} = \frac{-3}{3}$$

$$h = -1.$$

∴ The value of $h = -1$.


Given that solution.
point $(-4,-4)$
equation $(2x+6y-9)=0$
from
$y=mx-c$
then
$2x+6y-9=0$
$\frac{6y}{6}=\frac{-2x}{6}+\frac{9}{6}$
$y=-\frac{1}{3}x+\frac{3}{2}$ compare to $y=mx-c$
$m=-\frac{1}{3}$
but for parallel line $m_1=m_2$
them let another point (x,y) and $(-4,-4)$
$m_2=\frac{y_2-y_1}{x_2-x_1}$
$-\frac{1}{3}=\frac{y+4}{x+4}$
$3x+12=-x-4$
$3y+12+4=-x$
$3y+16=-x$
$x+3y+16=0$ answer

Extract 5.1: A sample of the correct responses to question 5

In Extract 5.1, part (a), the student formulated the equation by calculating the slopes of the line segments AC and AB and equating them. Then, they correctly solved the equations and obtained $h=-1$. In part (b), the student correctly determined the slope of the lines and the equation of the line passing through $(-4,-4)$.

Despite the good performance, 77 (17.5%) students scored zero. In part (a), most of these students misinterpreted the term collinear. For example, some students wrote $(h,-4)-(2,5)=(1,2)-(2,5)=(1,2)-(h,-4)$ and simplified it to the incorrect values of $h=1$. Also, some students applied an incorrect formula. For example, there were students who wrote $m=\frac{x_2-x_1}{y_2-y_1}$ instead of $m=\frac{y_2-y_1}{x_2-x_1}$. Therefore, these students obtained the wrong equation $3h-6=-10$ after equating the slopes of the line segments AB and AC, which leads to an incorrect value of h, including $h = \frac{-4}{3}$. Moreover, some students committed computational errors. For instance, some students used the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$, and correctly reached $\frac{-9}{h-2} = 3$. However, they committed errors on performing cross-multiply and obtained $3h - 2 = -9$ (instead of $3h - 6 = -9$), which results in $h = \frac{-7}{3}$ instead of $h = -1$.

In part (b), some of the students did not deduce the gradient from the equation $2x + 6y - 9 = 0$. Instead, they used the variable m, the point $(-4, -4)$, and the formula $m = \frac{y - y_1}{x - x_1}$ to formulate the equation $y = m(x + 4) - 4$. Furthermore, some students correctly recognized that parallel lines have the same slope, but they failed to determine the slope of the lines. For example, most of these students failed to rewrite $2x + 6y - 9 = 0$ into the form $y = mx + c$. Instead, they rewrote the equation as $x = -3y + \frac{9}{2}$, and thus, they got the incorrect slope $m = -3$ in particular. It was also observed that some students solved the given equation instead of formulating the equation of the line passing through $(-4, -4)$. However, most of them had an incorrect approach because they applied the quadratic formula $x = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$, taking $a = 2$, $b = 6$, and $c = 9$, resulting in incorrect answers, $x = \frac{3}{2}$ or $x = -\frac{3}{2}$. Extract 5.2 is a sample response from one of the students who faced challenges when attempting the question.

soln;

Point (-4, -4)

Equation to parallel $2x+6y-9=0$

$2x-6y-9=0$

$2x-6y=9$

$\frac{-6y}{-6}=\frac{9+2x}{-6}$

$y=\frac{9}{-6}+\frac{2x}{6}$.

∴ Gradient $=\frac{2}{6}=\frac{1}{3}$

Gradient = (1, 3)

Point $(-4, -4)$

~~Grad~~ Equation = ?

$\frac{1}{3}=\frac{\text{Change in } Y}{\text{Change in } X}$

$\frac{1}{3}=\frac{Y--4}{X--4}$

$\frac{1}{3}=\frac{Y+4}{X+4}$

$1(X+4)=3(Y+4)$

$X+4=3Y+12$

$X-3y=12-4$

$X-3y+8=0$.

∴ The Equation is $X-3y+8=0$.

Extract 5.2: A sample of the incorrect responses to question 5

In Extract 5.2, the student committed computational errors when making y the subject of $2x-6y-9=0$, resulting in incorrect slope, $m=\frac{1}{3}$ instead of $m=-\frac{1}{3}$.

2.6 Question 6: Symmetry

The question comprised parts (a), (b), and (c). In part (a), the students were asked to state the number of lines of symmetry in each shape of the object when Chichi watched Drawing Art on television, as she identified the following shapes of objects: (i) circle, (ii) tree, (iii) flying kite, (iv) cross shape, and (v) rectangular home mat. In part (b), they were required to state the order of rotational symmetry for each of the objects given in the table.

Name of Object	Order of Rotational Symmetry
(i) A rectangular playing card	
(ii) A ten thousand Tanzania shillings	
(iii) A nonagon	
(iv) A pen	
(v) A soccer ball	

In part (c), they were told that Hassan drew different types of vowels in capital letters on a wall for a kindergarten demonstration and were instructed to identify possible symmetrical letters that he drew.

The analysis of the data shows that, out of 441 students who attempted the question, 105 scored marks ranging from 6.5 to 10, 238 scored marks ranging from 3.0 to 6.0, and 98 students scored marks ranging from 0 to 2.5. The students' performance in percentage is summarized in Figure 6.

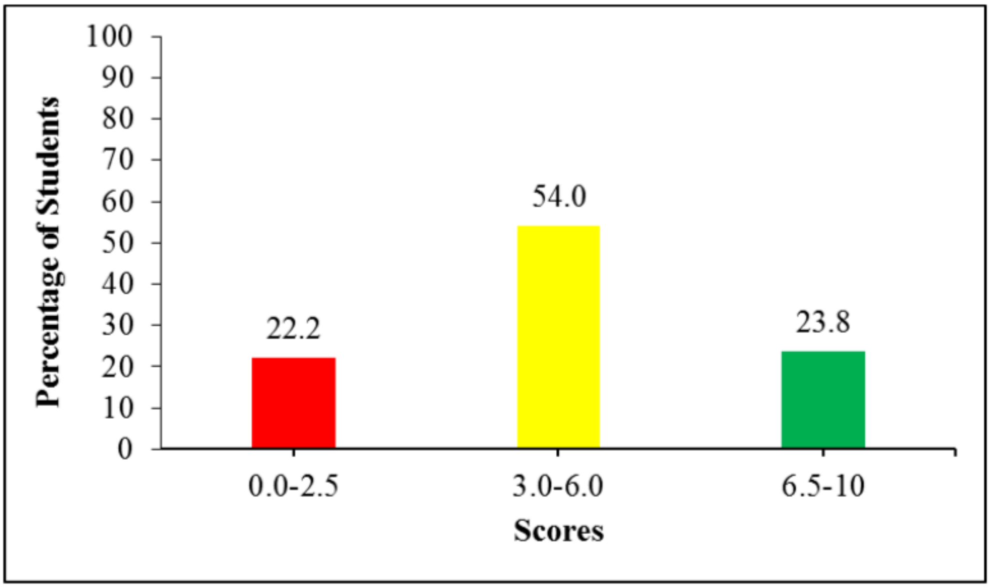


Figure 6: *Students' Performance on Question 6*

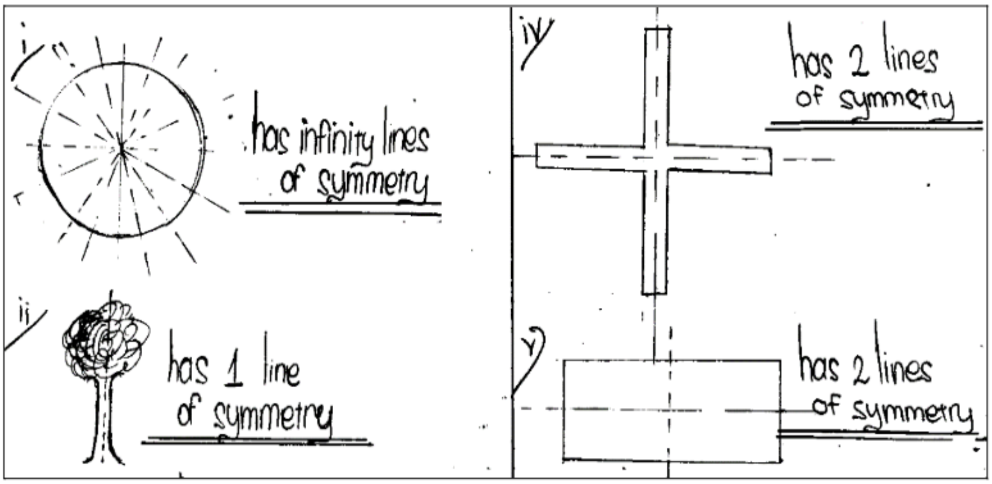
The students' performance on this question was good because 77.8 per cent of students scored from 3.0 to 10 marks. Furthermore, the analysis of the data revealed that 133 (30.2%) students scored 6.0 marks or more. In part (a), the students correctly stated the number of lines of symmetry as follows:

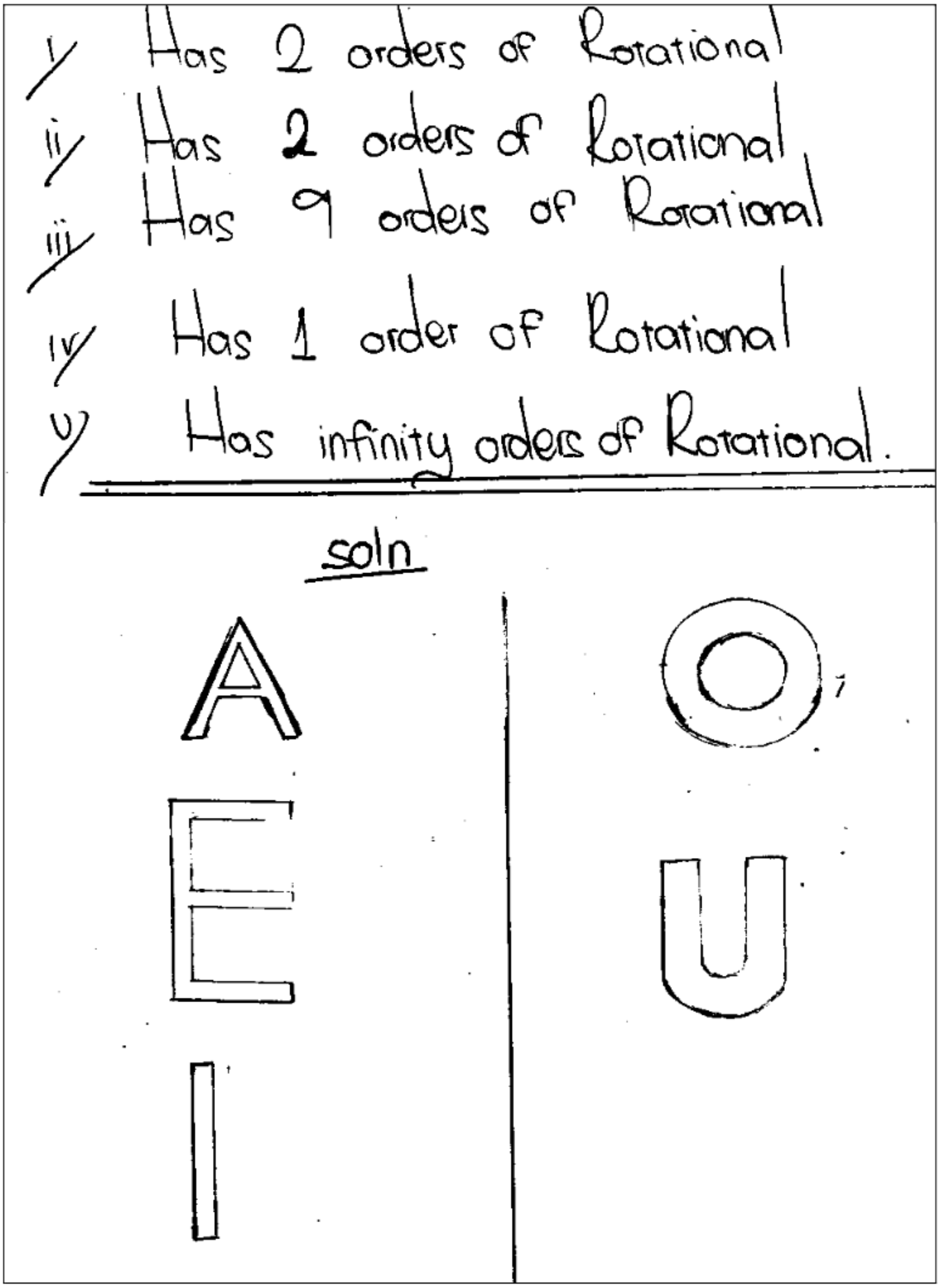
(i) A circle has infinite lines of symmetry.
(ii) A tree has one line of symmetry.
(iii) A flying kite has no line of symmetry.
(iv) A cross shape has two lines of symmetry.
(v) A rectangular home mat has two lines of symmetry.

In part (b), the students correctly stated the order of rotational symmetry as follows:

Name of Object	Order of Rotational Symmetry
(i) A rectangular playing card	Two
(ii) A ten thousand Tanzania shillings	Two
(iii) A nonagon	Nine
(iv) A pen	One
(v) A soccer ball	Infinite

In part (c), the students understood that vowels in capital letters are A, E, I, O and U and all of them are symmetrical letters. Extract 6.1 provides a sample response from one of the students who performed well on this question and managed to score all the marks.





Extract 6.1: A sample of the correct responses to question 6

In Extract 6.1, the student managed to identify the lines of symmetry on each shape of the object in part (a). In part (b), the student stated the correct order of rotational symmetry for each given name of object. In part (c), the student identified all vowels in capital letters as symmetrical letters.

Inspite of the good performance, 22.2 per cent of the students scored 2.5 marks or less. These students faced the following challenges: In part (a), most of these students were not knowledgeable about the concept of symmetry. As a result, they stated the incorrect number of lines of symmetry or order of rotational symmetry. For instance, in part (a) (i), some students stated that a circle has one line of symmetry, while others stated that it has two lines of symmetry, and others stated that it has four lines of symmetry. In part (a) (ii), some students stated that a tree has infinite lines of symmetry, and some stated that it has three lines of symmetry. In part (a) (iii), a few students stated that a flying kite has four lines of symmetry instead of no lines of symmetry. Likewise, in part (a) (iv), some students stated that a cross shape has no line of symmetry, and others stated that, has infinite lines of symmetry instead of two lines of symmetry. In part (a) (v), the students responded that the rectangular home mat has four lines of symmetry, and others said it has eight lines of symmetry rather than two lines of symmetry.

In part (b), the analysis revealed that some students perceived the word rotational symmetry as the sum of the degree measures of the angles of a polygon. For instance, some students stated that the order of rotational symmetry for the rectangular playing card, a nonagon, and a soccer ball is 360^0. Likewise, other incorrect responses provided by some of the students were: a rectangular playing card has one order of rotational symmetry; a ten thousand Tanzania shillings has four orders of rotational symmetry; a pen has no number of rotational symmetry; and a soccer ball has four orders of rotational symmetry.

In part (c), a number of students failed to identify vowels. These students included consonants in their responses: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q and R. Extract 6.2 illustrates a sample response from one of the students who faced challenges while attempting the question.

i) circle soln

– No line of symmetry

ii) Tree

– No line of symmetery

iii) Flying kite

– Four line of symmetry

iv) cross shape

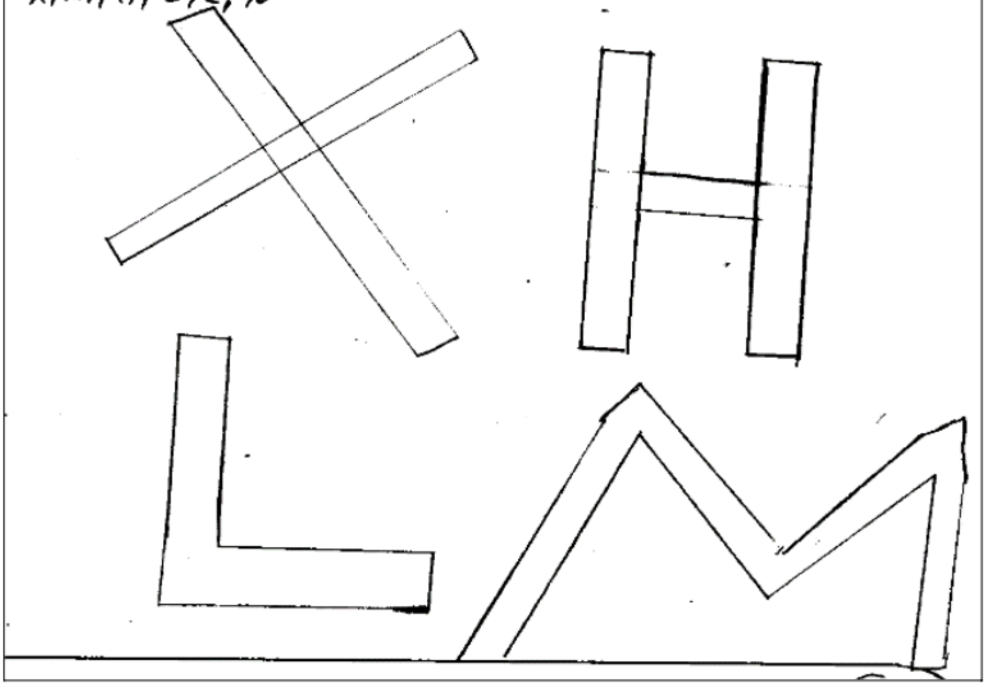
– No line of symmetry

v) Rectangular home mats

– Four line of symmetry

Name of Object	Order of Rotational Symmetry
(i) A rectangular playing card	1 number of symentary
(ii) A ten thousand Tanzania shillings	4 number of symentary
(iii) A nonagon	6 number of symentary
(iv) A pen	No number of symentary
(v) A soccer ball	A number of symentary

X, M, H, O, L, N Solution



Extract 6.2: A sample of the incorrect responses to question 6

In Extract 6.2, part (a), the student provided incorrect number of lines of symmetry. In part (b), the student stated the wrong order of rotational symmetry for the given objects and in part (c), the student provided consonants instead of vowels.

2.7 Question 7: Logic

The question consisted of parts (a), (b) and (c). In part (a), the students were given the statement, "If 6 is an even number, then it is either divisible by 2 or 4." Then, they were required to represent the statement in symbolic form and test its validity by letting p represent "6 is even number," q represent "6 is divisible by 2," and r represent "6 is divisible by 4." In part (b), students were required to copy and complete the following truth table:

p	q	$\sim p \to q$	$q \to p$	$(\sim p \to q) \wedge (q \to p)$

Part (c) instructed the students to write the symbolic form of the statement and draw an electric circuit of "either $2+6=8$ or $6\times 5=11$" using the letters P and Q, given that P stands for "$2+6=8$" and Q stands for "$6\times 5=11$."

The data shows that 193 (43.8%) students scored marks ranging from 0 to 2.5, 140 (31.7%) students scored marks ranging from 3.0 to 6.0, and 108 (24.5%) students scored marks ranging from 6.5 to 10. Therefore, 248 (56.2%) students got 3.0 marks or more, indicating that they performed averagely on this question. The students' performance summary is shown in Figure 7.

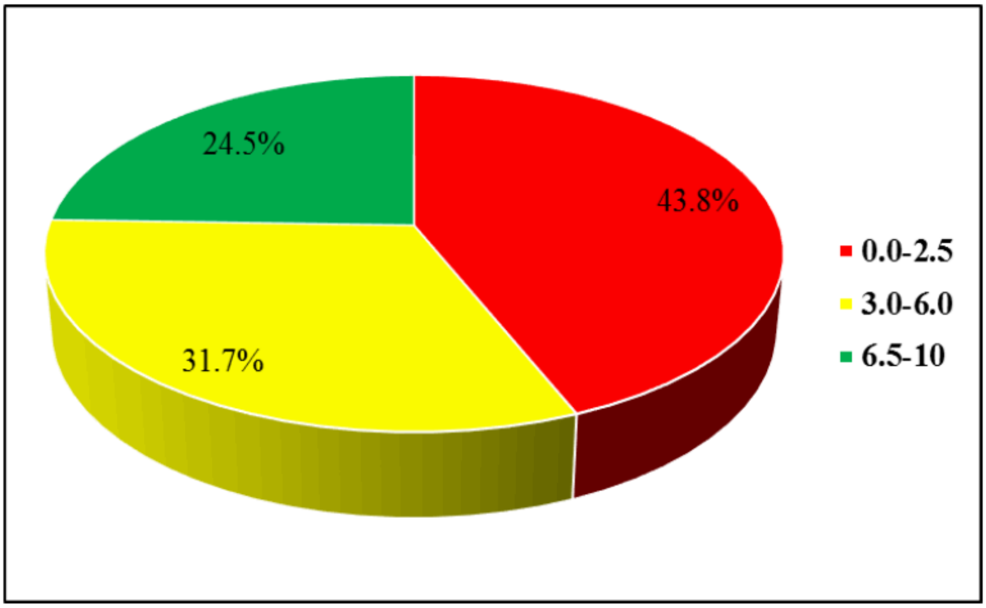


Figure 7: *Students' Performance on Question 7*

Further, the data show that 25 (7.5%) students scored full marks. In part (a), these students correctly wrote the given statement in symbolic form, that is, $p \to (q \vee r)$. Thereafter, they constructed the truth table with the correct truth values for p, q, r and $(q \vee r)$. They also drew and completed the column of $p \rightarrow (q \vee r)$, which enabled them to comment on its validity.

In part (b), the students completed the truth table by correctly performing logical operations, disjunctions, and implication. In part (c), the students wrote the given statement into symbolic form, $p \vee q$, and then they correctly drew the circuit as illustrated in Extract 7.1.

Solution.

p = 6 is even, q = 6 is divisible by 2, r: 6 is divisible by 4

$p \rightarrow (q \vee r)$

Truth table.

p	q	r	q v r	p→(q v r)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

∴ since the last column is not full true it is not tautology

∴ Therefore it is not valid.

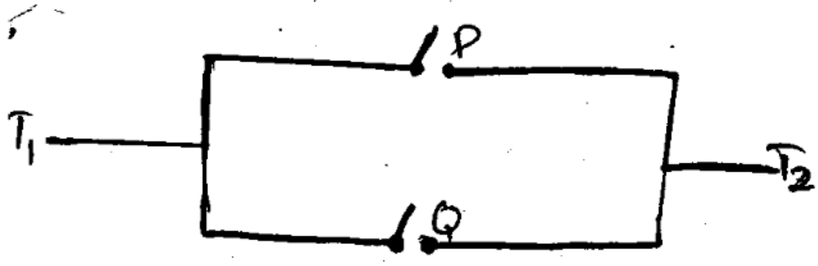
The truth table

p	q	~p	~p→q	q→p	(~p→q)∧(q→p)
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	F	T	F	T	F

Solution.

P stands for 2+6=8
Q stand for 6×5=11
either 2+6=8 or 6×5=11
P ∨ Q

∴



Extract 7.1: A sample of the correct responses to question 7

In Extract 7.1, in part (a), the student transformed the given statement into symbolic form, then constructed the truth table using the correct entries. Thereafter, he/she found the last column not containing all T truth values, implying that it was not valid. In part (b), he/she completed the given truth table correctly using the logical connectives given. And in part (c), the student wrote the statement into symbolic form and drew the electric circuit.

Nevertheless, 104 (23.6%) students scored zero marks. In part (a), most of these students misinterpreted the problem. For example, some students assumed that $p = 6$, $q = 6$, $r = 4$ and incorrectly manipulated some operations, resulting in $r = 2$, $p = 6$, and $q = 3$. These students related the problem to algebra. Further, other students wrote $p \propto \frac{q}{r}$, and then they stated that the symbolic form of the statement is $p = \frac{ky}{r}$. Lastly, write the statement $p = \frac{ky}{r}$. These students wrongly related the problem to the concept of variation. Furthermore, some students assumed that the problem is related to the concept of divisibility. For example, some students commented that 6 is divisible by 2 because it is divisible by 2, and 6 is not divisible by 4 because there are no last two digits.

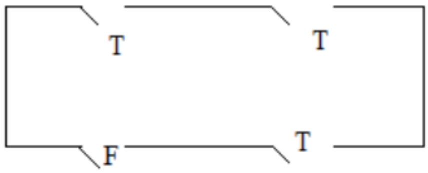
In part (b), some students lacked adequate knowledge of logical operations. For example, some students copied the truth values for q in the columns of $\sim p \rightarrow q$ and $q \rightarrow p$. Moreover, other students responded to the question by finding the number of possible combinations of truth values for a given n number of simple statements using 2^n, that is, $2^n \Rightarrow 2^2 = 4\,\text{rows}$, but failed to complete the truth table correctly. For instance, some students presented the following truth table:

p	q	$\sim p$	□ q	$q \rightarrow p$	$\sim p \rightarrow q$	$(\sim p \rightarrow q) \wedge (q \rightarrow p)$
T	T	F	F	T	T	F
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

From the truth table above, it is observed that, in the third row of the column of $\sim p \rightarrow q$, the students wrote F instead of T.

Furthermore, in part (c), most of the students combined the given simple statements with inappropriate logical connectives. For instance, $p \wedge q$ instead of $p \vee q$, resulting in an incorrect diagram of an electric circuit. These students confused the logical operation "OR" with "AND."

Moreover, some students correctly wrote $p \vee q$, but they failed to construct the appropriate electric circuit.



Extract 7.2 is a sample response from one of the students who faced difficulties.

T	T	p	q	(~q->p)∧(p->q)
T	T	F	T	(~F->T) A (T->F)
F	F	T	F	(~F->F) A (F->T)
F	F	F	F	(~F->F) ∧ (F-F
p	q	~p->q	q->p	(~p->q)∧(q->p)

Extract 7.2: A sample of the incorrect responses to question 7

In Extract 7.2, the student combined the truth values T and F with connectives in the column of $(\sim p \rightarrow q) \wedge (q \rightarrow p)$.

2.8 Question 8: Variations

The question comprised parts (a), (b), and (c). In part (a), students were informed that, the speed L of a certain particle moving on the surface of water is inversely proportional to the cube root of time n and $L = 3$ when $n = 27$. Then, they were required to determine the value of L when $n = 64$. In part (b), students were required to determine the value a when $b = 12$, given that $a \propto \left(b^2 + 3\right)$ and $a = 4$ when $b = 5$. Part (c), stated that: "Suppose p is directly proportional to q^2 and inversely proportional to $\sqrt{r}$ such that $p = 10$ when $q = 6$ and $r = 16$. Find the value of p when $q = 2$ and $r = 64$".

In this question, 142 (32.2%) students scored from 0 to 2.5 marks, 89 (20.2%) scored from 3.0 to 6.0 marks, and 210 (47.6%) students scored from 6.5 to 10 marks (Figure 8).

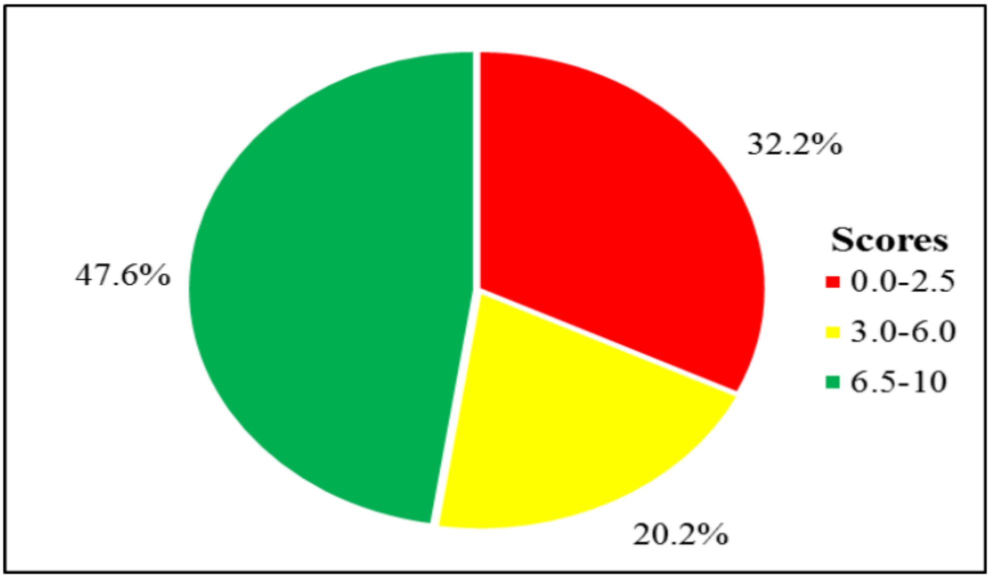
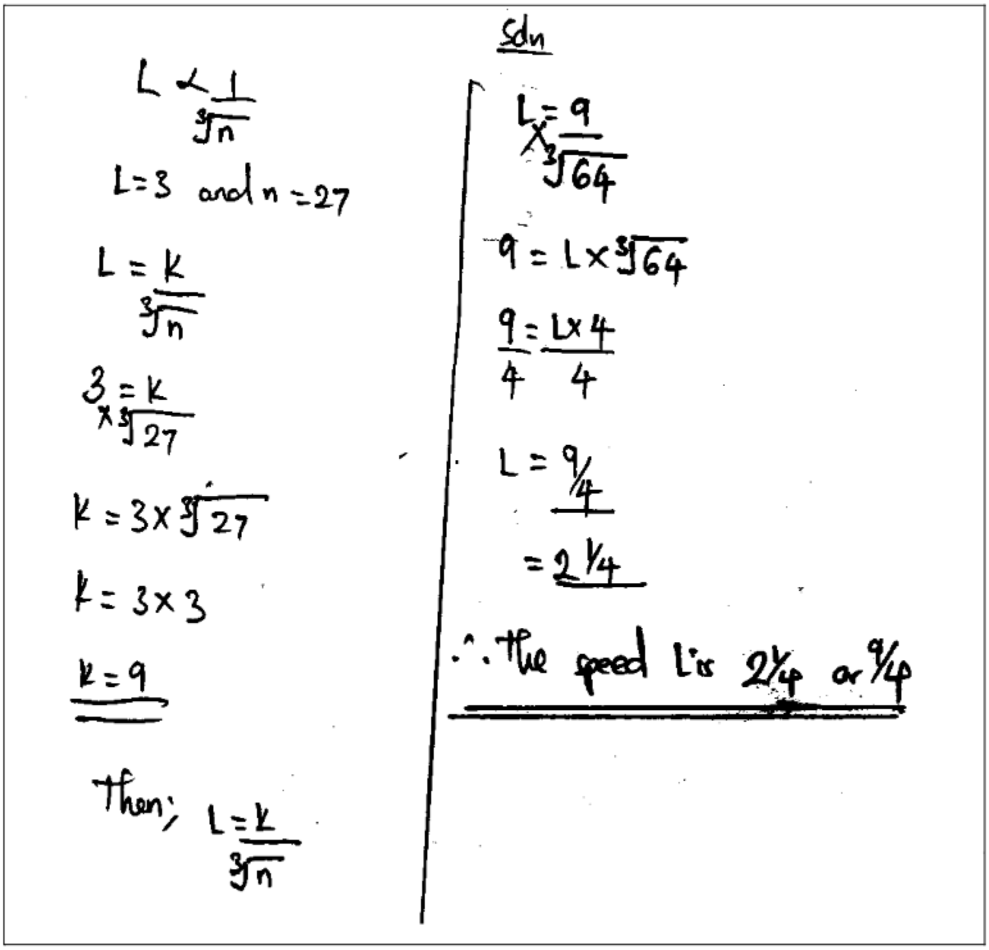


Figure 8: *Students' Performance on Question 8*

The overall performance of the students on this question was good because 299 (67.8%) students scored 3.0 marks or more. In part (a), the students rewrote the statement in symbolic form, $L \propto \frac{1}{\sqrt[3]{n}}$. Then, they introduced the proportionality constant (k) and obtained the equation $L = \frac{k}{\sqrt[3]{n}}$, and consequently interpreted that $L_1\sqrt[3]{n_1} = L_2\sqrt[3]{n_2}$. These students substituted $L_1 = 3$, $n_1 = 27$, and $n_2 = 27$ in $L_1\sqrt[3]{n_1} = L_2\sqrt[3]{n_2}$ and worked out to get $L_2 = \frac{9}{4}$.

In part (b), the students introduced the proportionality constant (k) in $a \propto (b^2 + 3)$ and got $a = k(b^2 + 3)$. Then, they substituted $a = 4$ and $b = 5$ in $a = k(b^2 + 3)$ and correctly solved the resulting equation, ending up with $k = \frac{1}{7}$, implying that $a = \frac{1}{7}(b^2 + 3)$. Finally, the students substituted the value of $b = 12$ in $a = \frac{1}{7}(b^2 + 3)$ and correctly solved the resulting equation to obtain $a = 21$.

Likewise, in part (c), the students rewrote the word problem in symbolic form, $p \propto \frac{q^2}{\sqrt{r}}$. Then, they introduced the proportionality constant (k) and obtained $p = \frac{kq^2}{\sqrt{r}}$. Using this equation as well as $p = 10$, $q = 6$, and $r = 16$, these students correctly solved for k and obtained $k = \frac{10}{9}$. Therefore, these students substituted $k = \frac{10}{9}$, $q = 2$, and $r = 64$ in $p = \frac{kq^2}{\sqrt{r}}$ and correctly solved it, resulting in $p = \frac{5}{9}$. Extract 8.1 provides a sample of one of the correct responses to this question from one of the students.



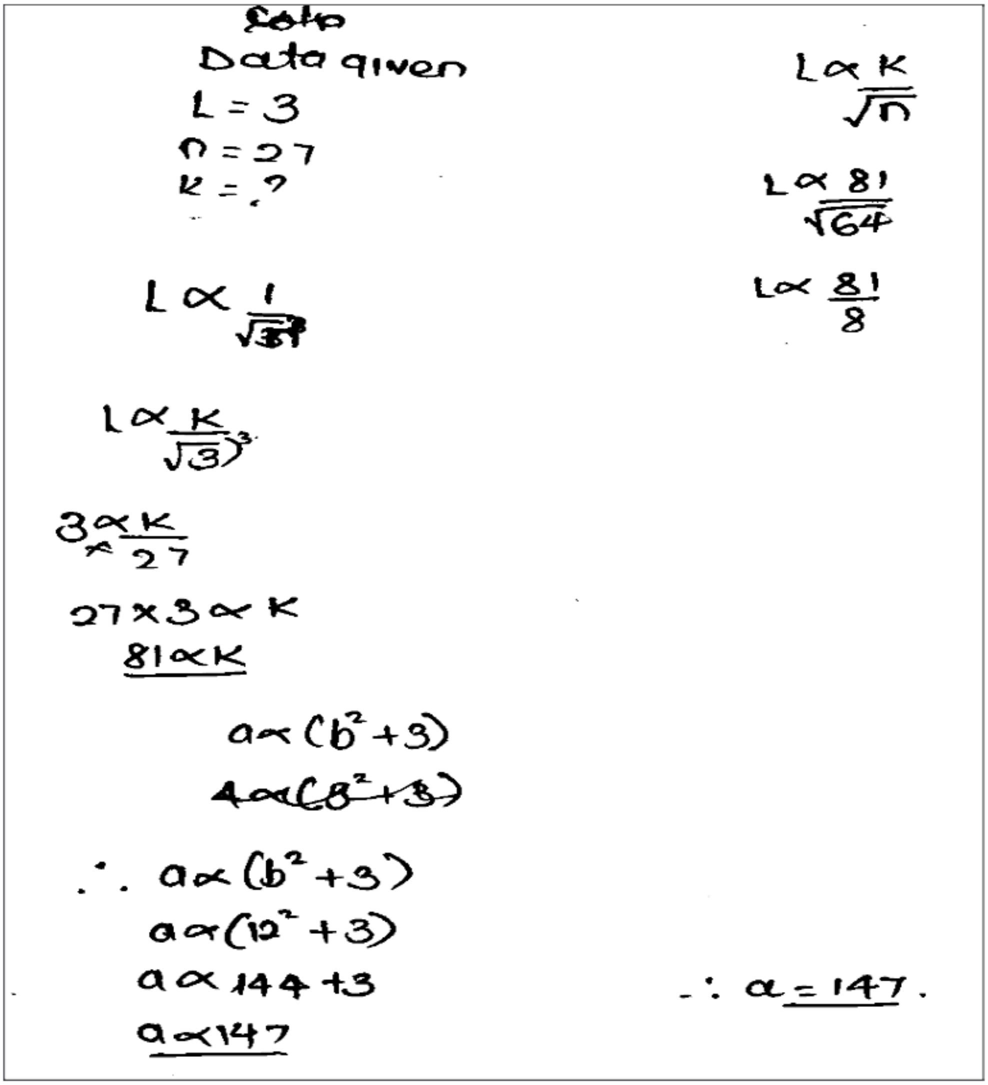
Extract 8.1: A sample of the correct responses to question 8

In Extract 8.1, the student substituted the values of $L=3$ and $n=27$ in $L=\frac{k}{\sqrt[3]{n}}$ to get the value of $k=9$. Then, he/she substituted $k=9$ and $n=64$ in $L=\frac{k}{\sqrt[3]{n}}$ and got $L=\frac{9}{4}$.

Nevertheless, 99 (22.4%) students scored zero due to various difficulties in responding to the question. In part (a), a number of students confused the concept of inversely proportional with directly proportional. These students wrote $L \propto \sqrt{n^3}$, and consequently $L=k\sqrt{n^3}$ which gives $L_2=18$ instead of $L_2=\frac{9}{4}$. Further, few students confused the concept of cube root with that of square root. In this case some students wrote $L \propto \frac{1}{\sqrt{n}}$, resulting to incorrect responses, including $L=\frac{k}{\sqrt{n}}$ and $L_2=\frac{81}{8}$.

In part (b), a number of students did not apply the knowledge of variation. Instead, they directly substituted the values into $a \propto (b^2+3)$. As a result, these students ended up with the meaningless statement $4 \propto 28$. Moreover, some students wrote $4 \times (5^2+3)$ and worked out to get 403. These students wrongly interpreted the proportionality sign as a multiplication sign. In addition, some students committed computation errors. For example, some students correctly got $4=k(5^2+3)$; however, they opened the brackets inappropriately and obtained $4=25k+3$ instead of $4=25k+3k$, and hence, they got incorrect answers, $k=\frac{1}{25}$ and $a=\frac{147}{25}$ in particular.

As in part (b), most students responded to part (c) by writing $p \propto q^2\sqrt{r}$ instead of $p \propto \frac{q^2}{\sqrt{r}}$. Therefore, they got $k=14\frac{2}{5}$ and $p=460$ instead of $k=\frac{10}{9}$ and $p=\frac{5}{9}$, respectively. Moreover, some students wrote $p^2+q^2=\sqrt{r}$ and substituted $p=10$, $q=6$, and $r=16$, resulting in a meaningless statement of $136=4$. While other students committed computational errors. For instance, some of them correctly got $p = \frac{kq^2}{\sqrt{r}}$, but, they wrote $10 = \frac{k \times 6}{\sqrt{16}}$ instead of $10 = \frac{k \times 6^2}{\sqrt{16}}$. Therefore, they obtained incorrect answers, including $k = \frac{20}{3}$ and $p = \frac{10}{3}$. Extract 8.2 provides a sample response from one of the students who faced challenges in responding to the question.


Soln
Data given
$L = 3$
$n = 27$
$k = ?$

$L \propto \frac{1}{\sqrt{3?}}$

$L \propto \frac{K}{\sqrt{3})^3}$

$3 \propto \frac{K}{27}$

$27 \times 3 \propto K$
$81 \propto K$

$a \propto (b^2+3)$
$4 \propto (8^2+3)$

$\therefore a \propto (b^2+3)$
$a \propto (12^2+3)$
$a \propto 144+3$
$a \propto 147$

$L \propto \frac{K}{\sqrt{n}}$

$L \propto \frac{81}{\sqrt{64}}$

$L \propto \frac{81}{8}$

$\therefore a = 147.$

Extract 8.2: A sample of the incorrect responses to question 8

In Extract 8.2, the student substituted directly the values in the model without introducing a constant k in order to get the value of L and a.

2.9 Question 9: Algebra

In this question, students were informed that "The sales records of a certain fuel filling station were as follows; the total sales of six litres of diesel and five litres of petrol were Tsh. 6000, while the sales of seven litres of diesel and five litres of petrol were Tsh. 6800." Then, they were required to use the elimination method to find the price of a litre of diesel and a litre of petrol.

The data analysis revealed that 441 students responded to the question. Among them, 321 students scored marks ranging from 3.0 to 10 while 120 students scored marks ranging from 0 to 2.5. Therefore, the overall performance of the students was good. Figure 9 provides a summary of the students' performance on this question.

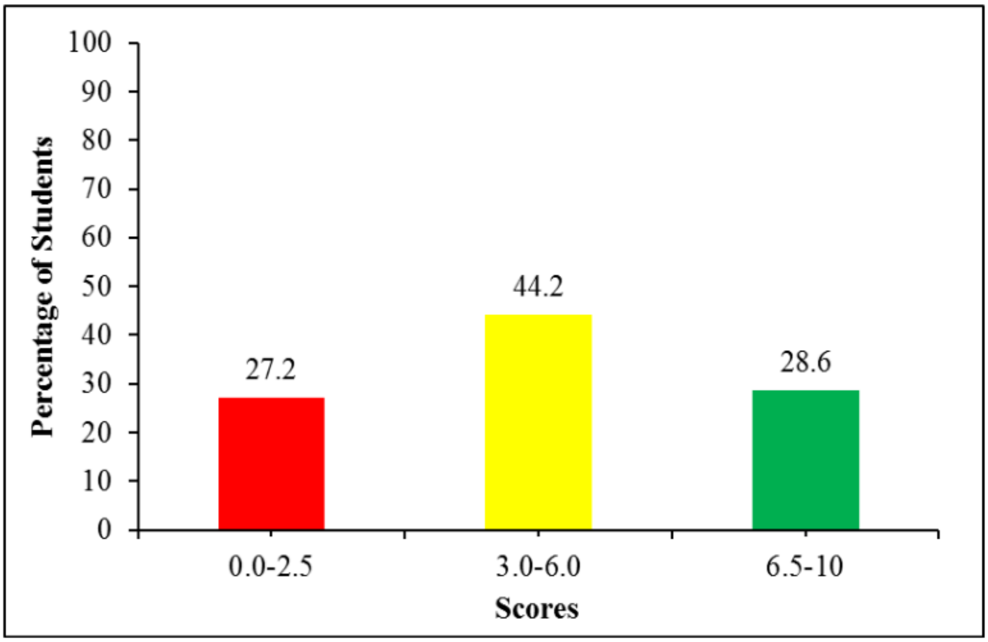


Figure 9: *Students' Performance on Question 9*

About 1.4 per cent of the students who attempted this question scored all ten marks. Most of those students assumed that x and y were the prices of one litre of diesel and one litre of petrol, respectively. Thus, they correctly represented the word problem using the equations $6x+5y=6000$ and $7x+5y=6800$. Then, they applied the elimination method to solve for x and y. The students obtained $x=800$ when eliminating y, and similarly they obtained $y=240$ when eliminating x.

Therefore, the students concluded that the price of a litre of diesel is Tsh. 800/= and petrol is Tsh. 240/=. A sample of the correct response from one of the students is provided in Extract 9.1.

Solution.

Let x be ~~litres of~~ Price of litres of diesel and y be Price of litres of petrol.

We eliminate y

$$-\begin{cases} 6x+5y=6000. \\ 7x+5y=6800. \end{cases}$$

$$6x-7x+5y-5y=6000-6800.$$

$$\frac{-x}{-1}=\frac{-800}{-1}$$

$$x=800.$$

We eliminate x.

$$\begin{matrix} 7 \\ 6 \end{matrix}\begin{cases} 6x+5y=6000. \\ 7x+5y=6800 \end{cases}$$

$$-\begin{cases} 42x+35y=42000. \\ 42x+30y=40800 \end{cases}$$

$$42x-42x+35y-30y=42000-40800$$

$$\frac{5y}{5}=\frac{1200}{5}$$

$$y=240$$

∴ The price of one litre of diesel was 800 and Price of one litre of petrol was 240

Extract 9.1: A sample of the correct responses to question 9

In Extract 9.1, the student correctly formulated and solved the equations using the elimination method and interpreted the answers $x = 800$ and $y = 240$, concluding that the price of one litre of diesel is Tsh. 800/= and that of petrol is Tsh. 240/=.

As Figure 9 shows, 27.2 per cent of the students scored low marks (0 – 2.5). The analysis shows that many students interpreted the word problem incorrectly. For instance, some students wrote the incorrect equations $6x - 5y = 6000$ and $7x - 5y = 6800$. These students assumed that 6000 is the difference between the cost of 6 litres of diesel and 5 litres of petrol, and 6800 is the difference between the cost of 7 litres of diesel and 5 litres of petrol. These students misinterpreted the word total as a difference instead of a sum. Then, by using elimination method they obtained $x = 400$ and $y = 720$. Furthermore, some students formulated the equation involving only one variable. For example, some students wrote $7 + 5x = 6800$ and consequently $x = 1358.6$. Extract 9.2 provides a sample of a response chosen from one of the students who responded to the question incorrectly.

Solution.

Let x → is diesel

let y → is petrol

$6x - 5y = 6000$

$7x - 5y = 6800$

$$6x - 7x = 6000 - 6800$$

$$\frac{+2x}{+2x} = \frac{+800}{+2}$$

$$x = 400$$

$\therefore$ Diesel is the 400

$$6x - 5y = 6000$$

$$6 \times 400 - 5y = 6000$$

$$2400 - 5y = 6000$$

$$2400 - 6000 = 5y$$

$$\frac{3600}{5} = \frac{5y}{5}$$

$$y = 720$$

$$(x, y) = (400, 720)$$

Extract 9.2: A sample of the incorrect responses to question 9

In Extract 9.2, the students considered the total cost of litres of diesel and petrol as a difference instead of a sum which led to incorrect answers for one litre of diesel and petrol.

2.10 Question 10: Sets

In this question, the students were given the universal set μ and subsets D, P, and S such that :

$\mu = \{x : x \text{ is an intager } 3 \leq x < 18\}$

$D = \{x : x \text{ is an odd number}\}$

$P = \{x : x \text{ is prime number}\}$

$S = \{x : x \text{ is a perfect square}\}$.

Then, they were required to:

(a) list the elements of each set.

(b) represent these sets in a Venn diagram.

(c) find (i) $P \cap D$ (ii) $(P \cup D \cup S)'$.

The data shows that 288 (65.3%) students scored 2.5 marks or less, 88 (20.0%) students scored marks ranging from 3.0 to 6.0, and 65 (14.7%) students scored 6.5 marks or more. The summary of the students' performance is shown in Figure 10.

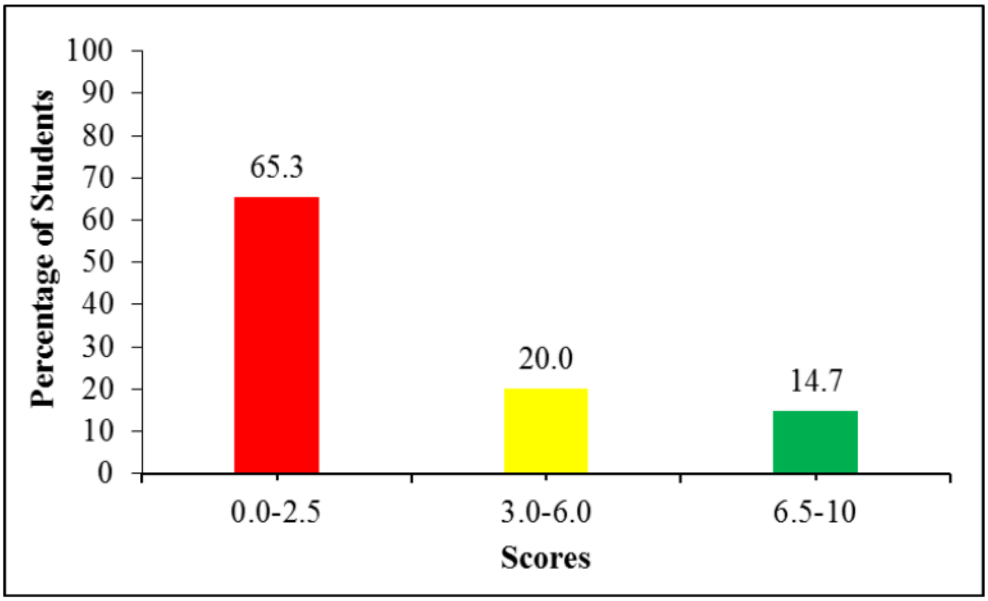


Figure 10: *Students' Performance in Question 10*

Furthermore, the data shows that 153 (34.7%) students scored 3.0 marks or more. Therefore, the general performance of the students in this question was average. In part (a), the students correctly listed all the elements of each set as follows:

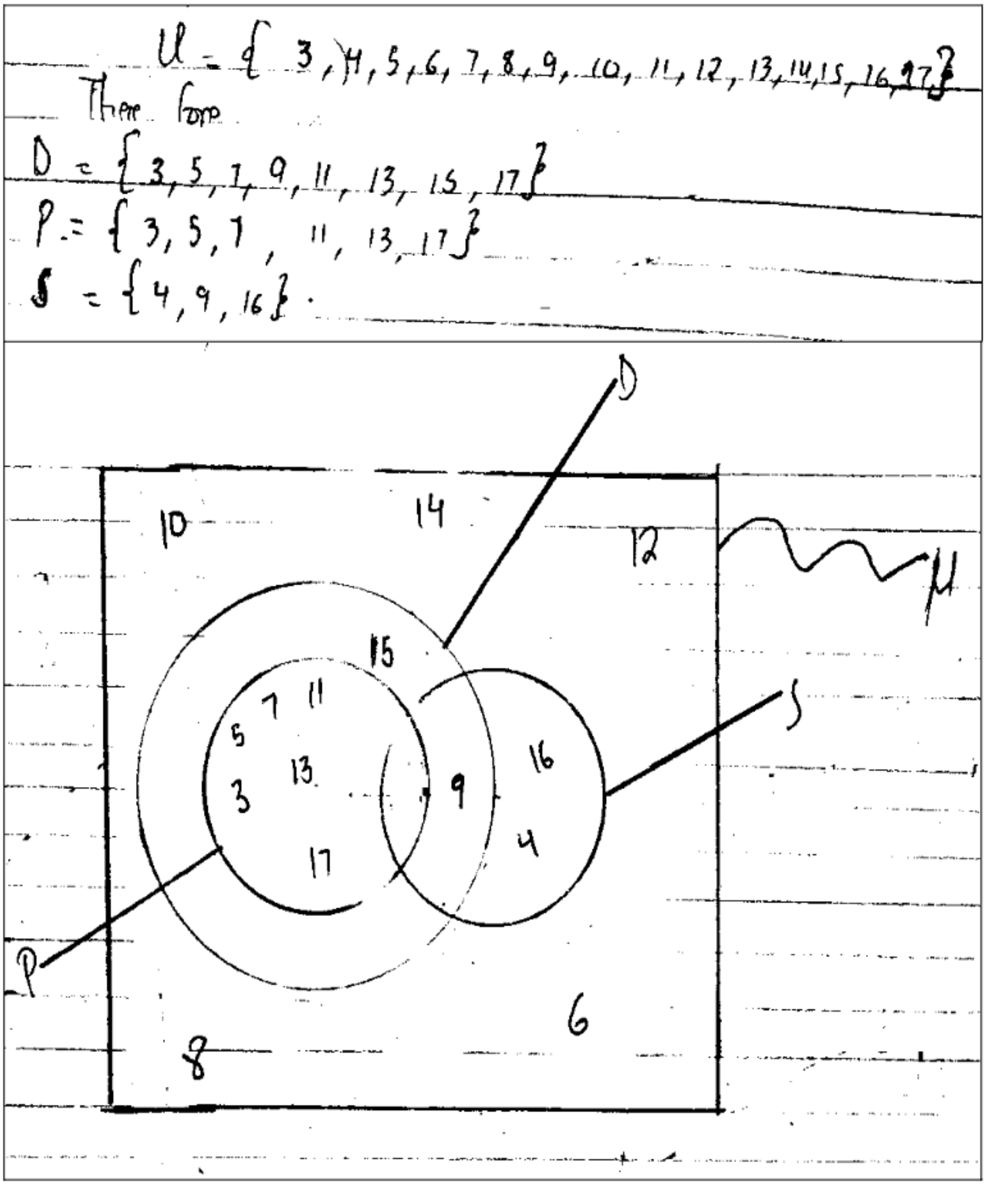
$\mu = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$

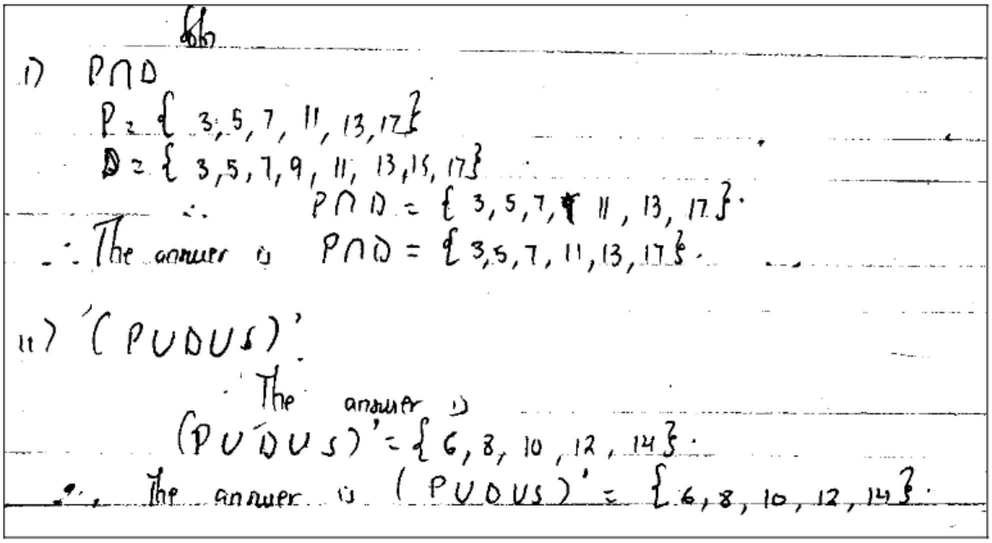
$D = \{3, 5, 7, 9, 11, 13, 15, 17\}$

$P = \{3, 5, 7, 11, 13, 17\}$

$S = \{4, 9, 16\}$

In part (b), the students drew the Venn diagram displaying all the elements in their respective regions. Likewise, in part (c) (i), the students identified the common elements found in both P and D to obtain $P \cap D = \{3, 5, 7, 11, 13, 17\}$, and in part (c) (ii), the students listed the elements that were not found in either of the sets P, D, or S and got $(P \cup D \cup S)' = \{6, 8, 10, 12, 14\}$. Extract 10.1 illustrates a sample of one of the students who responded correctly to this question.




6b
i) $P \cap D$
$P = \{3, 5, 7, 11, 13, 17\}$
$D = \{3, 5, 7, 9, 11, 13, 15, 17\}$
$\therefore$ $P \cap D = \{3, 5, 7, 11, 13, 17\}$.
$\therefore$ The answer is $P \cap D = \{3, 5, 7, 11, 13, 17\}$.

ii) $(P \cup D \cup S)'$
The answer is
$(P \cup D \cup S)' = \{6, 8, 10, 12, 14\}$.
$\therefore$ The answer is $(P \cup D \cup S)' = \{6, 8, 10, 12, 14\}$.

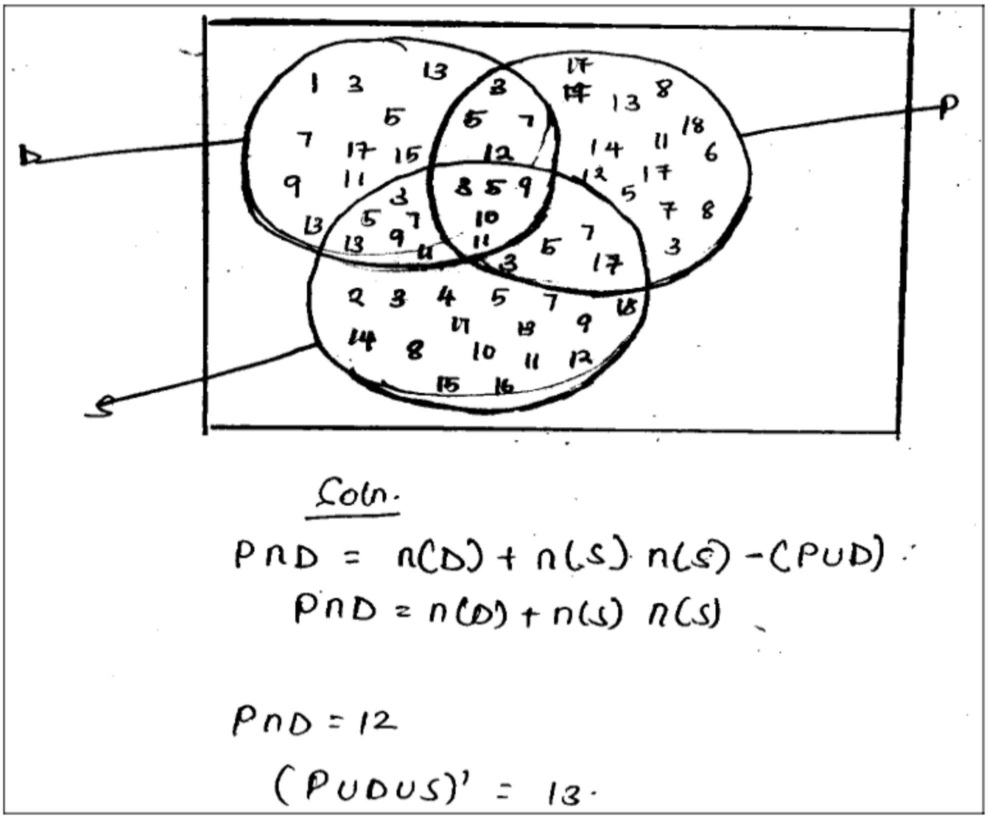
Extract 10.1: A sample of the correct responses to question 10

In Extract 10.1, the student listed all elements of each set in part (a). In part (b), the student drew a Venn diagram showing the elements in each region. Also in part (c), the student correctly listed the elements of $P \cap D$ as well as $(P \cup D \cup S)'$.

However, 65.3 per cent of the students scored low marks due to various challenges. In part (a), the students lacked knowledge about integers, odd, or prime numbers. For example, some students wrote: $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$, $D = \{1, 3, 5, 7, 9, 11, 13, 17\}$, $S = \{1, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$ and $P = \{1, 9, 15\}$, the set that includes all elements that are not prime numbers because the numbers are divisible by some numbers different from 1 and itself. In addition, some of the students were not familiar with set notation because they responded to the question by listing the elements without enclosing them in brackets. For example, some of the students wrote; $P = 3, 5, 7, 11, 13, 17$.

In part (b), some students wrote the incorrect entries in the regions of the Venn diagram. Poor performance in part (c) was highly attributted to the incorrect answers obtained in part (a). For instance, some of the responses provided by the students in part (c) were $P \cap D = \{1, 3, 5, 7, 11, 13, 17\}$ and $(P \cup D \cup S)' = \{1, 3, 5, 7\}$. Extract 10.2

provides a sample response from one of the students who were not able to respond to the question correctly.



Extract 10.2: A sample of the incorrect responses to question 10

In Extract 10.2, part (b), the student drew an incorrect Venn diagram and entered elements of the universal set in every region of P, D, and S. In part (c), the student provided the formula to determine the intersection of two sets of elements and gave incorrect answers for $(P \cap D)$ and $(P \cup D \cup S)$.

3.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH TOPIC

The Additional Mathematics paper was composed of 10 questions from nine (9) topics namely *Numbers*, *Algebra*, *Geometrical Constructions*, *Locus*, *Coordinate Geometry*, *Symmetry*, *Logic*, *Variations* and *Sets*. The data analysis on students' performance shows that among these topics, five (5) were well performed and four (4) were averagely performed. Students had good performance in *Numbers* (88.7%), *Symmetry* (77.8%), *Coordinate Geometry* (71.2%), *Variations* (67.8%) and *Algebra* (66.9%). Students' good performance on these topics was attributed to the competence demonstrated by the students in interpreting the questions, adhering to the instructions, and performing arithmetics correctly.

Furthermore, the students had average performance on four (4) topics, which are *Logic* (56.2%), *Locus* (44.0%), *Geometrical Constructions* (43.1%) and *Sets* (34.7%). The students' average performance on those topics was attributed to inappropriate use of formulae, misinterpretation of some special sets of numbers, failure to draw Venn diagrams, misinterpretation of the requirements of the questions, failure to perform arithmetic and algebraic computations, and interpreting loci as well as logical connectives. The students' performance on each topic is presented in Appendix I.

Further analysis shows that there is an increase in performance of the students in four (4) topics of *Numbers* (33.5%), *Locus* (17.9%), *Coordinate Geometry* (9.4%) and *Variations* (8.3%) for 2023 as compared to 2022. Likewise, there are five (5) topics that show a decrease in students' performance as compared to 2022. These topics are *Logic* (2.8%), *Algebra* (3.7%), *Symmetry* (9.7%), *Geometrical Constructions* (11.9%) and *Sets* (25.1%). Appendix II provides a comparison of students' performance per topic for 2022 and 2023.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The general performance on the Additional Mathematics assessment in 2023 was good, as 310 (70.29%) students passed. Generally, the strengths and weaknesses of the students shown in this report are expected to be valuable information for education stakeholders. Moreover, the recommendations presented in this report are expected to help improve students' performance on future Additional Mathematics assessments.

4.2 Recommendations

In order to improve the performance of the students in this subject, the following recommendations are made:

(a) Group discussion techniques should be used during teaching and learning; In the groups, students get opportunity to feel interactive; the number of groups should be small to enable everyone to participate.

As well students should read more Mathematics books and do enough exercises in order to improve their competence.

(b) Teachers should provide enough exercises to learners and emphasizes drawing Venn diagrams and solving related problems involving three sets. This will improve students' competence in the topic of Set which was not well performed.

(c) Teachers should be given seminars and workshops so that they may improve their teaching strategies and competence in some difficult areas.

(d) Teachers should implant confidence among the students to approach various problems and be able to consult them whenever they do not understand.

Appendix I

Students' Performance on Each Topic in 2023

S/N	Topic	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Numbers	1	88.7	Good
2.	Symmetry	6	77.8	Good
3.	Coordinate Geometry	5	71.2	Good
4.	Variations	8	67.8	Good
5.	Algebra	2 & 9	66.9	Good
6.	Logic	7	56.2	Average
7.	Locus	4	44.0	Average
8.	Geometrical Constructions	3	43.1	Average
9.	Sets	10	34.7	Average

Appendix Ii

Comparison of Students' Performance in 2022 and 2023

S/N	Topic	2022			2023		
		Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Numbers	1	55.2	Average	1	88.7	Good
2.	Symmetry	6	87.5	Good	6	77.8	Good
3.	Coordinate Geometry	5 & 9	61.8	Average	5	71.2	Good
4.	Variations	8	59.5	Average	8	67.8	Good
5.	Algebra	2	70.6	Good	2 & 9	66.9	Good
6.	Logic	7	59.0	Average	7	56.2	Average
7.	Locus	4	26.1	Weak	4	44.0	Average
8.	Geometrical Constructions	3	55.0	Average	3	43.1	Average
9.	Sets	10	59.8	Average	10	34.7	Average

THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2023

042 ADDITIONAL MATHEMATICS

Published by ;
The National Examinations Council of Tanzania,
P.O. Box 2624,
Dar es Salaam, Tanzania.

TABLE OF CONTENTS

FOREWORD

The report on Students' Item Response Analysis (SIRA) in Additional Mathematics for Form Two National Assessment (FTNA) 2023 was prepared and issued to the public by the National Examinations Council of Tanzania (NECTA) for the purpose of informing education stakeholders on how the student responded to the assessed competences.

This analysis shows justifications for the students' performance in the Additional Mathematics subject. The factors noted for good performance includes: students being competent with the assessed topics and concepts; correct interpretation of the questions; adhering to the instructions for each question; and clear arithmetics. On the other hand, students who scored low marks faced difficulties in responding to the questions due to insufficient knowledge of the tested concepts.

The analysis also indicates that, the students had average performance in four (4) topics, including *Logic*, *Locus*, *Geometrical Constructions* and *Sets*. Whereas, five (5) topics were well performed, which are *Numbers*, *Symmetry*, *Coordinate geometry*, *Variations*, and *Algebra*. Thus, generally, the performance of the students was good.

The National Examinations Council of Tanzania hopes that this report will encourage education stakeholders to work on the challenges encountered by students when attempting assessment questions so as to take appropriate measures to improve future students' performance in the subject.

The National Examinations Council of Tanzania is thankful to all who participated in one way or another in the production of this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The preparation of the Students' Item Response Analysis (SIRA) in the Additional Mathematics subject aims at providing feedback to teachers, students, and other education stakeholders about the students' performance in the 2023 Form Two National Assessment (FTNA). The setting of the paper was based on the Form Two National Assessment format of 2019 prepared by NECTA. The paper consisted of ten compulsory questions with 10 marks each.

A total of 441 students sat for the FTNA on Additional Mathematics in 2023, compared to, where 398 students who sat for the assessment in 2022. The performance of the students was good. Table 1 provides a summary of the students' general performance in 2022 and 2023.

Table 1: The Students' Performance on Additional Mathematics (FTNA) 2022 and 2023

Year	Students Sat	Passed		Grades				
		No.	%	A	B	C	D	F
2022	398	306	76.88	26	40	142	98	92
2023	441	310	70.29	74	50	104	82	131

The data in Table 1 show that, the students' performance in Additional Mathematics in 2023 was good since 70.29 per cent of students who sat for the assessment passed, where 16.78 per cent got grade A, 11.34 per cent got grade B, 23.58 per cent got grade C, 18.59 per cent got grade D, and 29.70 per cent got grade F. The analysis shows that there is a decrease in students' performance by 6.59 per cent as compared with the 2022 results.

The analysis of students' performance in each question is detailed in Section 2.0 of this report. It provides short descriptions of the requirements of each question and an analysis of the students' responses to the questions. Extracts of both good and poor students' responses on each question are included to illustrate cases presented. The factors that influenced the performance in each question are also illustrated.

In the analysis green, yellow, and red colours are used to represent good, average and poor performance, respectively. Section 3.0 gives the summary of students' performance in each topic and Section 4.0 of this report gives the conclusions and recommendations that are aimed at helping students improve their performance in future assessments in Additional

Mathematics. It is therefore hopeful that the report will be a useful guide to both teachers and students for improving the teaching and learning process and, hence, the students' performance in the forthcoming assessments.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH QUESTION

This section describes the analysis of students' performance on each question. The performance of students on each question is categorized in three groups; weak performance 0–29 per cent, 30–64 per cent and 65–100 per cent showing average and good performance, respectively. Moreover, the performance is categorized by using different colours whereby green, yellow, and red are used to represent good, average and weak performance, respectively.

2.1 Question 1: Numbers

The question consisted of parts (a) (i), (a) (ii), and (b). In part (a) (i), students were required to study the sequence $-3, -2, -5, -7, -12$ and -19, and then state a reason to verify that the sequence is Fibonacci. In part (a) (ii), students were instructed to use the divisibility rule to determine whether the number 9655 is divisible by 3. In part (b), students were required to complete blank spaces in the following pattern of numbers that obeys Pascal's triangle.

$$\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & _ & & _ & & _ & & _ & & _ & & _ & & \\
 & _ & & _ & & _ & & _ & & _ & & _ & & _ & \\
_ & & _ & & _ & & _ & & _ & & _ & & _ & & _
\end{array}$$

The analysis revealed that out of 441 students who attempted this question, 50 (11.3%) scored marks ranging from 0 to 2.5, 296 (67.2%) students scored marks ranging from 3.0 to 6.0, and 95 (21.5%) students scored marks ranging from 6.5 to 10. Therefore, the students' performance on this question was generally good. The summary of the students' performance in this question is presented in Figure 1.

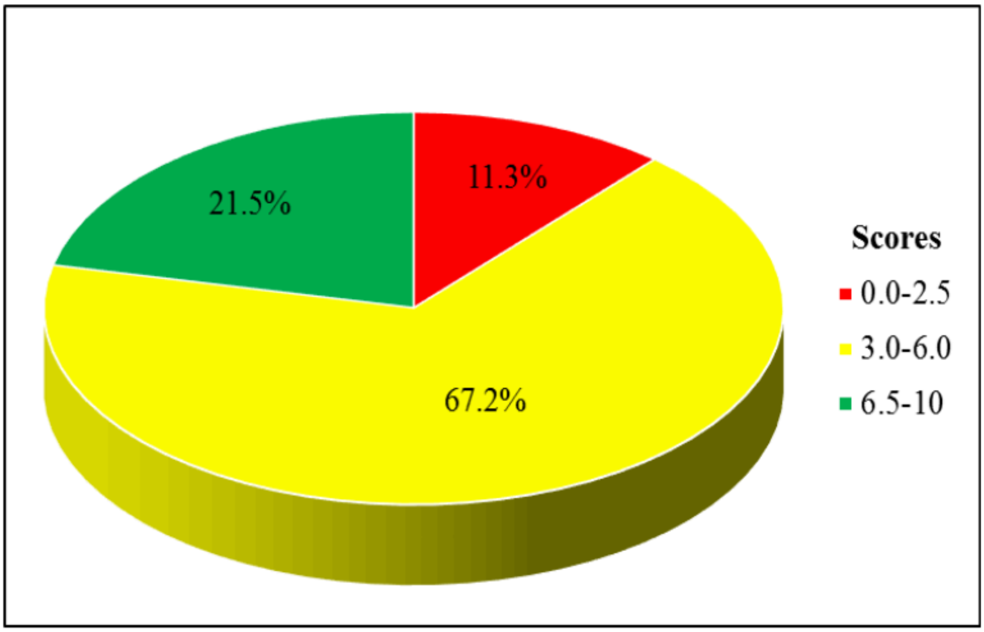
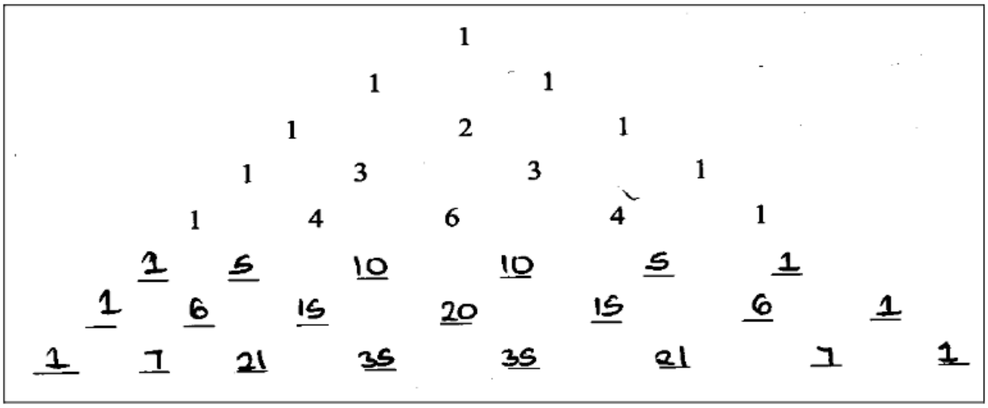


Figure 1: *Students' Performance on Question 1*

In part (a) (i), the analysis of the responses of the students who correctly responded to this question showed that the students computed the sum of the two preceding numbers, that is, $-2+(-3)=-5$, $-5+(-2)=-7$, $-7+(-5)=-12$, and $-12+(-7)=-19$. By studying the terms in the sequence, they realised that the sum of the two preceding consecutive numbers equals the next number. Therefore, they correctly concluded that the sequence is Fibonacci. In part (a) (ii), these students were conversant with the rule for divisibility on 3, that is, a number is divisible by 3 if the sum of its digits is also divisible by 3. These students computed the sum of the digits of the number 9655, that is, $9+6+5+5=25$. Then, they divided 25 by 3 and got $8\frac{1}{3}$, which is not an integer. Thus, they concluded that 25 is not divisible by 3 and consequently, 9655 is not divisible by 3.

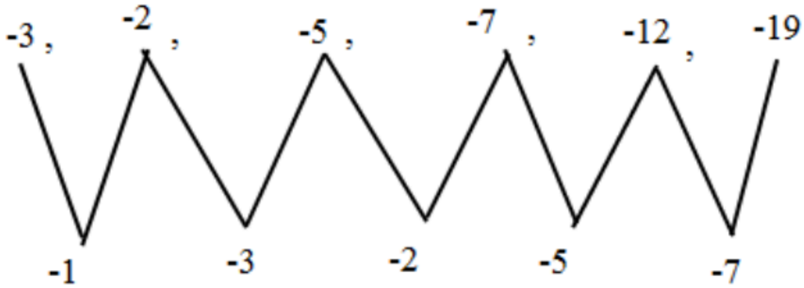
In part (b), the students correctly identified the pattern of numbers that obeys Pascal's triangle, thus completing the first and last blanks of each row by 1. Then, the other blanks in a particular row were completed by writing the sum of two consecutive numbers from the previous row. For example, the second blank from left in the 6^{th} row was completed by writing 5, which is the sum of 1 and 4 found in the 5^{th} row. Hence, they correctly completed the blank spaces in the 6^{th}, 7^{th}, and 8^{th} rows. Extract 1.1 is a sample response from one of the students who correctly attempted this question.



Extract 1.1: A sample of the correct responses to question 1

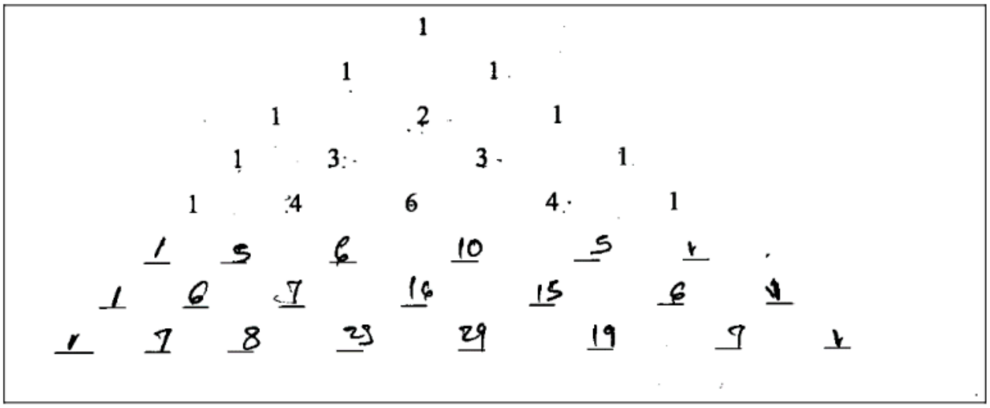
In Extract 1.1, the student applied the concept of Pascal's triangle correctly to complete the blank spaces in the 6th, 7th, and 8th rows.

On the other hand, 95 (11.3%) students scored low marks in this question. In part (a) (i), most students failed to identify the rule governing the terms of the Fibonacci sequence. For example, some students misinterpreted the Fibonacci sequence as any sequence with negative numbers. For instance, some students wrote that, since the sequence -3, -2, -5, -7, -12, and -19 had negative numbers, it was a Fibonacci sequence. The analysis also shows that other students considered any sequence with no common difference to be a Fibonacci sequence; hence, some responded to the question by showing patterns, for instance,



Also, some students wrote, "The sequence is Fibonacci because it has an irregular pattern," and other students wrote, "Since a sequence had not been arranged in ascending order, it is Fibonacci." In part (a) (ii), most of these students applied the appropriate rule for the divisibility of a number on 3. These students considered the last two digits of 9655, indicating that they applied the rule for divisibility of 4 instead of 3. For instance, some students divided 55 by 3 and got 18.3, and finally they concluded that the number is not divisible by 3. Similarly, other students considered the first digit of the number 9655 (that is, 9) is to be divisible by 3. Therefore, the students concluded that the number 9655 is divisible by 3.

In part (b), some students wrongly identified the pattern of numbers that obeys Pascal's triangle, thus incorrectly summed up two consecutive numbers on the particular row and consequently completed the blank spaces with the wrong entries. Extract 1.2 is a sample response from one of the students who faced challenges when responding to the question.



Extract 1.2: A sample of the incorrect responses to question 1

In Extract 1.2, the student wrongly filled in the blank spaces in the required rows of the given pattern of numbers without considering the concept of Pascal's triangle.

2.2 Question 2: Algebra

The question was composed of three parts (a), (b), and (c), which required students to:

(a) simplify the expression $18r-(2r+10)-14r+25$ to its lowest term.

(b) expand completely the following expressions: (i) $3(2c+3)^2-c^2$ and (ii) $2x(x+4y)-x(8x+14y)-2(3+4y)$.

(c) write r in terms of x and y, given that $\frac{x}{y}=\frac{1+r^2}{1-r^2}$.

The analysis of the data depicts that, out of 441 students who responded to this question, 172 students scored marks ranging from 0 to 2.5, 135 students scored marks ranging from 3.0 to 6.0, and 134 students scored marks ranging from 6.5 to 10. Generally, the students' performance on this question was average. Figure 2 provides a summary of the students' performance on this question by percentage.

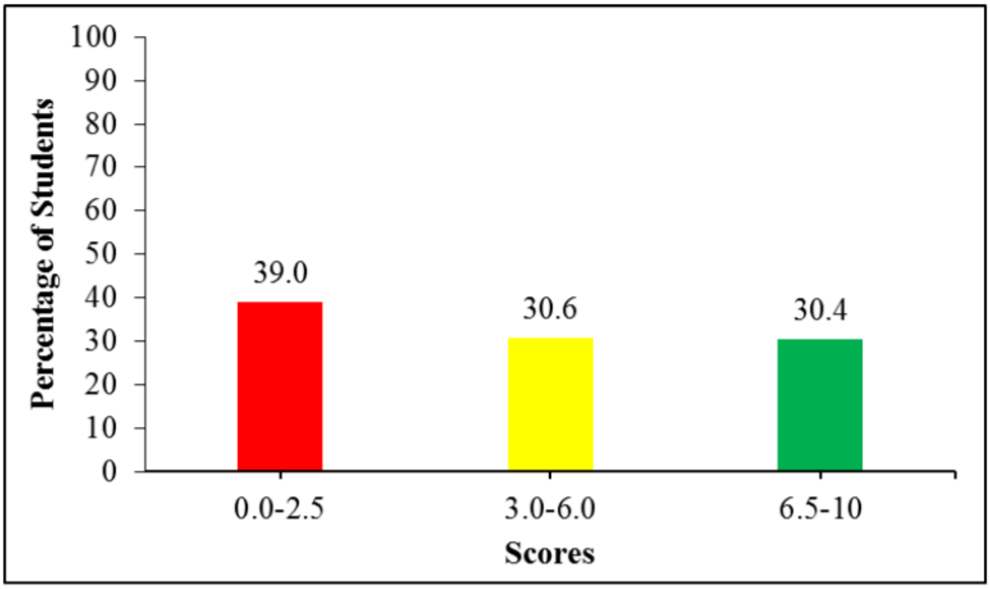


Figure 2: *Students' Performance on Question 2*

In part (a), the analysis shows that the students who managed to score high marks applied the BODMAS rule and performed basic operations correctly. They correctly opened the brackets in the expression $18r-(2r+10)-14r+25$ and got $18r-2r-10-14r+25$, then collected like terms and wrote the expression in the form of $(18r-2r-14r)+(25-10)$ and simplified it into $2r+15$. In part (b) (i), the students realised that $(a+b)^2=(a+b)(a+b)$. Therefore, they correctly rewrote the expression $3(2c+3)^2-c^2$ as $3(2c+3)(2c+3)-c^2$ and expanded it to obtain $3(4c^2+12c+9)-c^2$. Thereafter, the students opened the brackets and got the expression $12c^2+36c+27-c^2$, which was further simplified into $11c^2+36c+27$. Similarly, in part (b) (ii), the students opened the brackets in $2x(x+4y)-x(8x+14y)-2(3+4y)$, and obtained $2x^2+8xy-8x^2-14xy-6-8y$. Then, they rearranged the terms of the expression and got, $(2x^2-8x^2)+(8xy-14xy)-8x-6$, which was further simplified into $-6x^2-6xy-8y-6$.

In part (c), the students correctly performed cross-multiply on $\frac{x}{y} = \frac{1+r^2}{1-r^2}$ and obtained $y(1+r^2) = x(1-r^2)$. Then, they correctly opened the brackets and rearranged the terms of the expressions into $yr^2 + xr^2 = x - y$. Thereafter, the students correctly factored $yr^2 + xr^2$ into $r^2(y+x)$ and thus $yr^2 + xr^2 = x - y$ became $r^2(y+x) = x - y$. The students further divided both sides of $r^2(y+x) = x - y$ by $y + x$ and got $r^2 = \frac{x-y}{x+y}$. Finally, they computed the square root of each side, resulting in $r = \pm\sqrt{\frac{x-y}{x+y}}$. Extract 2.1 is a sample of the correct response from one of the students who attempted the question.

Soln:

$$18r - (2r + 10) - 14r + 25$$

$$18r - 2r - 10 - 14r - 25$$

$$16r - 14r - 10 + 25$$

$$2r + 15$$

$$\therefore \underline{\underline{2r + 15}}$$

$\underline{\text{soln}}$

i) $3(2c+3)^2 - c^2$

$3(2c+3)(2c+3) - c^2$

$3(4c^2+6c+6c+9) - c^2$

$3(4c^2+12c+9) - c^2$

$12c^2+36c+27-c^2$

$\therefore \underline{\underline{11c^2+36c+27}}$

ii) $2x(x+4y) - x(8x+14y) - 2(3+4y$

$2x^2+8xy-8x^2-14xy-6-8y$

$2x^2-8x^2+8xy-14xy-6-8y$

$-6x^2+-6xy-8y-6$

$\therefore \underline{\underline{6x^2-6xy-8y-6}}$

$\therefore \underline{\underline{-6x^2-8y-6xy-6}}$

Soln

$$\frac{x}{y} = \frac{1+r^2}{1-r^2}$$

$$x - r^2x = y + r^2y$$

$$x - y = r^2y + r^2x$$

$$x - y = r^2(y+x)$$

$$\frac{x-y}{y+x} = \frac{r^2(y+x)}{y+x}$$

$$\sqrt{\frac{x-y}{y+x}} = \sqrt{r^2}$$

$$\therefore r = \sqrt{\frac{x-y}{x+y}}$$

Extract 2.1: A sample of the correct responses to question 2

In Extract 2.1, the student correctly applied the BODMAS and basic operations to simplify the given expressions. In part (b), the student managed to expand the given expressions, and in part (c), he or she correctly made r in terms of x and y.

Despite the good performance, 68 (15.4%) students got zero marks. In part (a), most of these students ignored the negative sign when opening brackets. These students rewrote the expression $18r-(2r+10)-14r+25$ as $18r-2r+10-14r+25$ instead of $18r-2r-10-14r+25$ then simplified it into $2r+35$ instead of $2r+15$. Furthermore, some students committed computational errors. For example, some of these students correctly opened the bracket in $18r-(2r+10)-14r+25$ and obtained $18r-2r-10-14r+25$. However, they wrongly simplified it into $37r$.

Similarly, in part (b) (i), most of the students multiplied the term in the brackets by 3 before squaring, and thus, they wrongly rewrote $3(2c+3)^2 - c^2$ as $(6c+9)^2 - c^2$. These students were supposed to open the brackets by expanding $(2c+3)^2$ into $4c^2+12c+9$, then multiply the resulting expression by 3 and subtracting c^2. In addition, most of the students who got $(6c+9)^2 - c^2$ expanded it incorrectly and obtained $36c^2 - c^2 + 9^2$. These students misinterpreted $(a+b)^2$ as a^2+b^2 instead of $(a+b)(a+b)$ or $a^2+2ab+b^2$. Moreover, some students realised that the expression could be expanded by applying the concept of the difference of two squares, that is, $a^2-b^2=(a+b)(a-b)$. However, they failed to convert the given expression into the standard form of the difference of two squares. For example, some students rewrote $3(2c+3)^2 - c^2$ as $\left[3(2c+3)+c\right]\left[(2c+3)-c\right]$ instead of $\left[\sqrt{3}(2c+3)+c\right]\left[\sqrt{3}(2c+3)-c\right]$.

Similar to part (a) (i), most students ignored the negative sign when opening the brackets in part (b) (ii). For example, many students wrongly opened brackets in $2x(x+4y)-x(8x+14y)-2(3+4y)$, resulting in $2x^2+8xy-8x^2+14xy-6-8y$ and consequently $-6x^2+22xy-8y-6$. Like wise, other students expanded $2x(x+4y)-x(8x+14y)-2(3+4y)$ into $2x^2+8xy-8x^2-14xy-6+8y$ instead of $-6x^2-6xy-8y-6$. Some other examples of incorrect responses provided by the students due to their inability to apply basic operations were $10x^2+14y^2-6+8y$ and $2x^2-8x^2+6xy-6+8y$.

In part (c), most of the students performed cross-multiply incorrectly. For instance, some students obtained $x-r^2=y+x^2$ instead of $x-xr^2=yr^2+y$ from $\frac{x}{y}=\frac{1+r^2}{1-r^2}$, and therefore, they got an incorrect answer. Similarly, some students applied cross-multiply properly, but they committed errors in factorization. For example, some of these students correctly got $xr^2+yr^2-x+y=0$ from $\frac{x}{y}=\frac{1+r^2}{1-r^2}$ but incorrectly factored it and got $r^2(x+y)-1(x+y)=0$ instead of $r^2(x+y)-(x-y)=0$. Moreover, some students developed incorrect statements from $\frac{x}{y}=\frac{1+r^2}{1+r^2}$, such as $x(1+r^2)+y(1+r^2)$, which was simplified into $x+xr^2+y+yr^2$. Extract 2.2 is a sample of a response from one of the students who faced difficulties in attempting this question.

Soln.

$18r-(2r+10)-14r+25$

$18r-2r+10-14r+25$

$18r-2r-14r+10+25$

$16r-14r+35$

$=2r+35$

$\therefore 2r+35$

$2x(x+4y)-x(8x+14y)-2(3+4y)$

Soln.

$2x^2+8y-8x^2+14xy-6+8y$

$10x^2+8y+14xy-6+8y$

$=10x^2+y+14xy-6$

$=10x^2+y+14xy-6$

Extract 2.2: A sample of the incorrect responses to question 2

In Extract 2.2, in part (a), the student failed to adhere to the effect of the negative sign in opening the brackets, which led to an incorrect answer. In part (b) (ii), the student failed to multiply the variable in the first term and did not recall the negative sign in some steps.

2.3 Question 3: Geometrical Constructions

In this question, the students were informed that the size of an exterior angle of a certain polygon is p and the size of its interior angle is three times the size of the exterior angle. Then, they were required to find: (a) the value of the expression $\frac{6p-16^{\circ}}{2}$; (b) the size of the interior angle; and (c) the sum of the interior angles.

According to the data analysis of the students' performance, it was observed that 441 students attempted the question, out of whom 251 (56.9%) scored from 0 to 2.5 marks, 44 (10.0%) scored from 3.0 to 6.0 marks, and 146 (33.1%) scored marks ranging from 6.5 to 10. The summary of students' performance is presented in Figure 3.

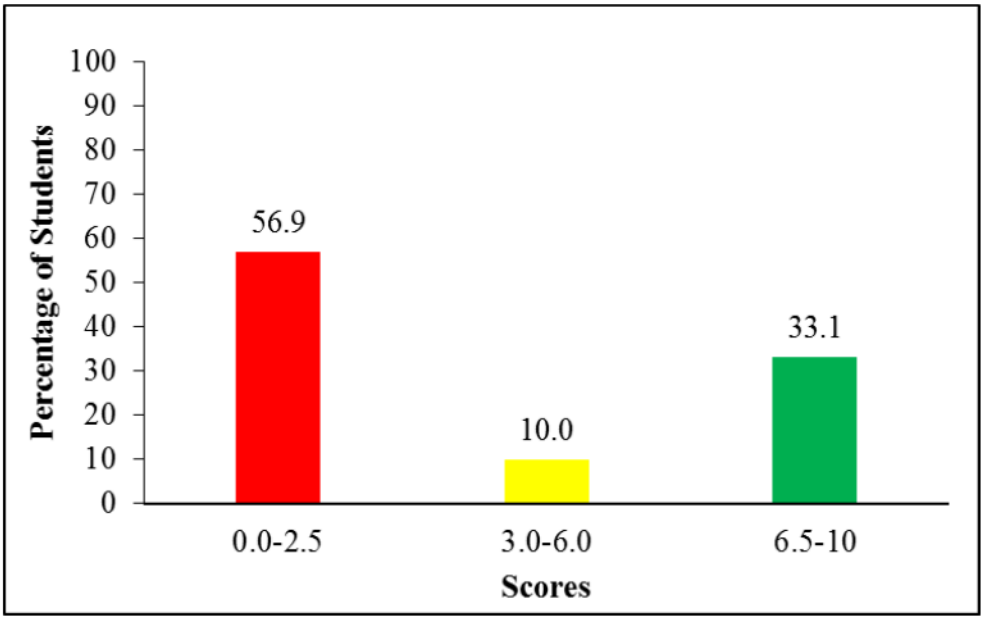
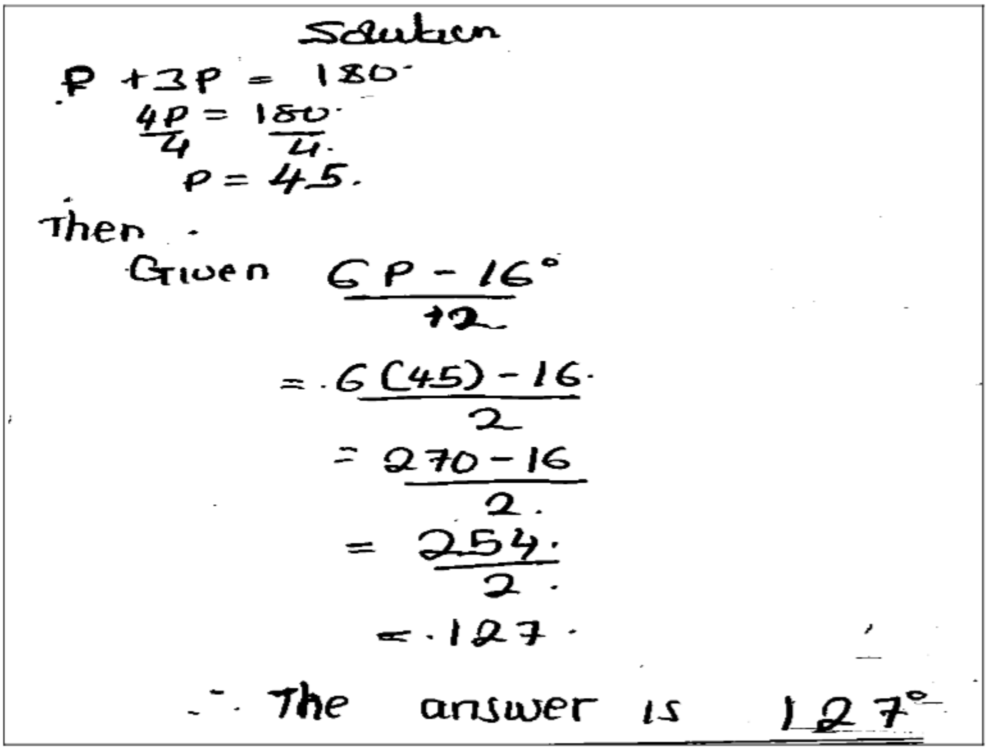


Figure 3: *Students' Performance on Question 3*

The overall performance of the students on this question was average. The students who correctly responded to part (a) recalled the fact that the sum of the exterior angle (p) and interior angle ($3p$) equals 180^{0}. They developed the equation $p+3p=180^{0}$ and correctly solved it to get $p=45^{0}$. Thereafter, they replaced p in $\frac{6p-16^{0}}{2}$ with 45^{0} and correctly worked on the obtained expression, resulting in 127^{0}. Similar to part (a), many students responded to part (b) by recognizing the fact that the sum of the exterior angle and interior angle equals 180^0. Therefore, they wrote $45^0 + \text{interior angle} = 180^0$ and identified that the size of the interior angle is 135^0. Other students recalled that $p = 45^0$ and the interior angle is $3p$. Therefore, they calculated the product of 3 and 45^0 to determine the size of the interior angle, as $3p = 3(45^0)$, resulting in 135^0.

In part (c), the students correctly recalled the formula for calculating the number of sides (n) of a regular polygon, that is, $\text{n} = \frac{360^0}{\text{exterior angle}}$. Therefore, they replaced the exterior angle in the formula with 45^0 and correctly worked out to get $n = 8$. Furthermore, the students applied the formula for calculating the sum of interior angles, that is, $(n-2)\times 180^0$, and hence substituted $n = 8$ into the formula and got the sum of interior angles equals 1080^0. Also, some students computed the product of the number of sides (8) and the size of an interior angle (135^0) to get the sum of the interior angles of the polygon, which is 1080^0. Extract 3.1 is a sample of a correct response from one of the students who attempted this question.



Solution

$p + 3p = 180$

$\frac{4p}{4} = \frac{180}{4}$

$p = 45.$

Then.

Given $\frac{6p - 16^\circ}{2}$

$= \frac{6(45) - 16}{2}$

$= \frac{270 - 16}{2}$

$= \frac{254}{2}$

$= 127$

$\therefore$ The answer is 127°

Solution.

Then.

$E + I = 180$

Then interior angle is three times exterior angle

$I = 3P$. $(P = 45)$.

$I = 3(45)$

$I = 135^\circ$

∴ The size of interior angle is 135°

Solution

$E = \frac{360}{n}$

$\frac{45}{1} = \frac{360}{n}$

$\frac{45n}{45} = \frac{360}{45}$

$n = 8$

Then

$Sum = (n-2)180$

$= (8-2)180$

$= (6)180$

$= 1080^\circ$

∴ The Sum of interior angle is 1080°

Extract 3.1: A sample of the correct responses to question 3

In Extract 3.1, the student correctly applied a relation between interior and exterior angles and hence managing to get the value of the given expression in part (a). In part (b), the student correctly calculated the size of the interior angle using the information that the interior angle is three times the exterior angle. In part (c), the student applied the formula $(n-2)\times 180^0$ to determine the sum of the interior angles.

On the other hand, the analysis revealed that some students were not able to correctly recognize the appropriate formula and misinterpreted the word problem. In part (a), some students applied the formula: $\text{interior angle(I)} + \text{exterior angle(E)} = 180^0$, but committed computation errors. Most of these students formulated the correct equation $3p + p = 180^0$ or $4p = 180^0$, but they got incorrect answers, particularly $p = 46^0$. As a result, they got the incorrect value of the expression $\frac{6p-16^0}{2}$, including 130^0. Moreover, some students incorrectly equated the expression to zero, and thus they developed the equation $\frac{6p-16^0}{2} = 0$, and solved it, resulting in $p = \frac{8}{3}$ instead of $p = 45^0$. Futhermore, some student attempted the question by equating the expression to 180^0, that is, $\frac{6p-16^0}{2} = 180^0$, and worked on it, ending up with an incorrect value of p, such as $p = 62.7^0$.

In part (b), many incorrect answers resulted from the mistakes observed in part (a). For example, the students who got $p = 46^0$ resulted in $3p = 138^0$ and hence the incorrect size of the interior angle of 138^0 instead of 135^0. Similarly, the students who got $p = 28.5^0$ resulted to $3p = 75.5^0$. Additionally, other students also used an inappropriate formula, $(n-2)180^0$, which was used to find the sum of interior angles resulting in the incorrect value of an interior angle such as $10,440^0$.

In part (c), the analysis revealed that a number of students committed computation errors. For instance, some students correctly recognized the formula $\text{exterior angle} = \frac{360^0}{n}$, where the size of the exterior angle is 45^0. However, they got an incorrect value of $n = 4.8$ and approximated it to 5. As a result, these students got the incorrect sum of interior angles, particularly 540^0 instead of 1080^0. Furthermore, some students applied the correct formula for the sum of interior angles but used the incorrect value of n. For instance, some of them substituted $n = 135$ in the formula $(n-2)180^0$. Extract 3.2 is a sample of the responses from one of the students who was not able to provide the correct responses.

$(n-2)180°$

$(135°-2)\ 180°$

$133° \times 180°$

$13940°$

Size of interior angle $= 13940°$

Extract 3.2: A sample of the incorrect responses to question 3

In Extract 3.2, on part (c) of the question, the student misinterpreted that the size of an interior angle equals the number of sides of a polygon, more over committed computational error that resulted in the wrong answer.

2.4 Question 4: Locus

The question consisted of three parts: (a), (b), and (c). In part (a), the students were required to describe the locus when (i) an orange is falling vertically from a tree at the height of 2 metres from the ground and (ii) the centre of a wheel as a cyclist rides along the road on a horizontal plane. In part (b), they were required to analyse the equation of the locus of point P which is equidistant from the points $L(-2, 2)$ and $M(1, 1\frac{1}{2})$. In part (c), the students were informed that, the locus of point P moves along the plane and intersects the lines whose equations are $m(y-3)=x+1$ and $y=mx$ where, m is a variable. Then, they were required to find the equation of the locus of the point P.

In this question, 247 (56.0%) students scored 0 to 2.5 marks, 160 (36.3%) students scored 3.0 to 6.0 marks, and 34 (7.7%) students scored 6.5 to 10 marks. Generally, students had average performance on this question. The summary of students' performance is presented in Figure 4.

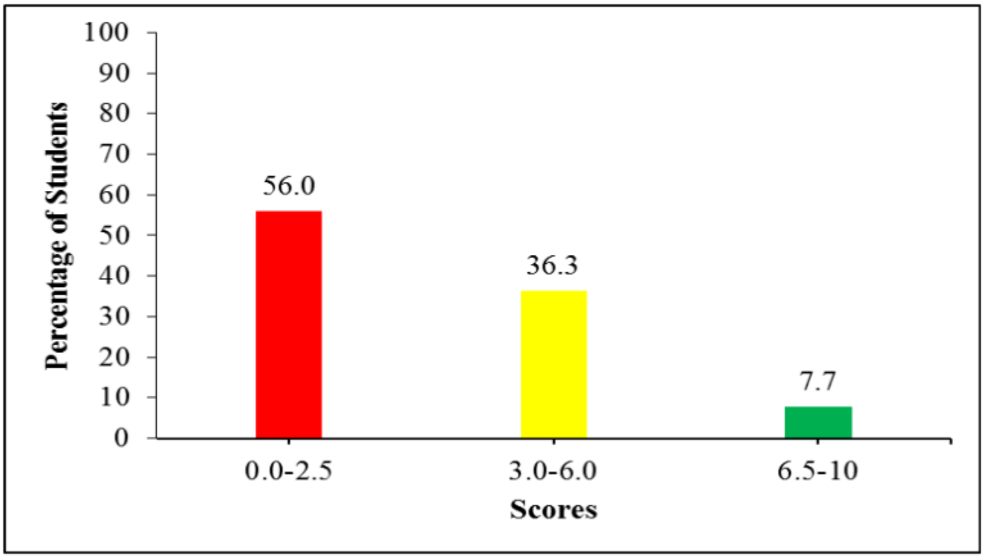


Figure 4: *Students' Performance on Question 4*

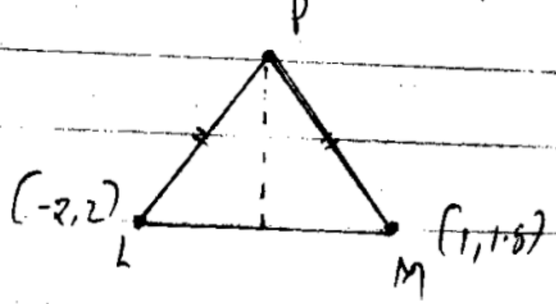
The students who managed to provide correct responses in part (a) (i) correctly described that the locus formed by an orange falling from the tree is a straight vertical line whose length is 2 metres. In part (a) (ii), the students stated that the locus of the centre of a wheel as a cyclist rides along the road on a horizontal plane is a circle. In part (b), the student applied the formula for calculating the distance between two points, $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. They assumed the coordinates of the point P to be (x, y), and therefore they substituted $P(x, y)$ and $L(-2, 2)$ into the formula, resulting in $\overline{PL}=\sqrt{(x--2)^2+(y-2)^2}$. Likewise, they got $\overline{PM}=\sqrt{(x-1)^2+(y-1\frac{1}{2})^2}$ after considering $P(x, y)$ and $M(1, 1\frac{1}{2})$. These students also equated $\sqrt{(x--2)^2+(y-2)^2}$ to $\sqrt{(x-1)^2+(y-1\frac{1}{2})^2}$ and simplified it to get $24x-4y+19=0$, which is the equation of the locus of P.

In part (c), students correctly rewrote $y=mx$, in the form $m=\frac{y}{x}$ and therefore, they replaced m in $m(y-3)=x+1$ with $\frac{y}{x}$ and correctly simplified into $x^2-y^2+x+3y=0$. Extract 4.1 illustrates the correct solution to this question from one of the students responses.

i) The locus is ~~a~~ straight vertical line of 2 metre

ii) The locus is circle when a wheel as a cyclist rides

Sol.



From

PL = PM.

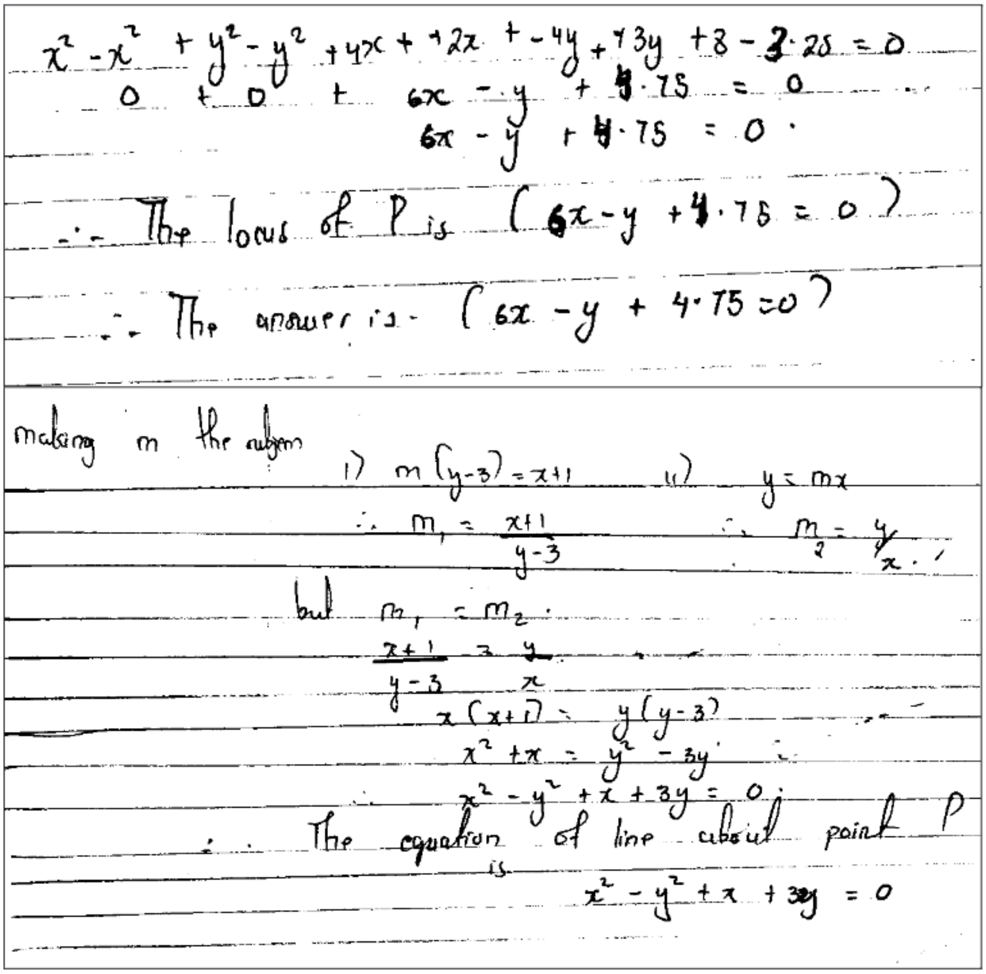
and $\sqrt{(x-x_1)^2 + (y-y_1)^2} = \bar{AB}$

$\left(\sqrt{(x+2)^2 + (y-2)^2}\right)^2 = \left(\sqrt{(x-1)^2 + (y-1.5)^2}\right)^2$

$(x+2)^2 + (y-2)^2 = (x-1)^2 + (y-1.5)^2$ —— 1

$x^2 + 4x + 4 + y^2 - 4y + 4 = x^2 - 2x + 1 + y^2 - 3y + 2.25$

$x^2 + y^2 + 4x - 4y + 8 = x^2 + y^2 - 2x - 3y + 3.25.$



$x^2 - x^2 + y^2 - y^2 + 4x + 2x + -4y + 3y + 8 - 3.25 = 0$

$0 + 0 + 6x - y + 4.75 = 0$

$6x - y + 4.75 = 0$

$\therefore$ The locus of P is $(6x - y + 4.75 = 0)$

$\therefore$ The answer is $(6x - y + 4.75 = 0)$

making m the subject

i) $m(y-3) = x+1$ ii) $y = mx$

$\therefore m_1 = \frac{x+1}{y-3}$ $\therefore m_2 = \frac{y}{x}$

but $m_1 = m_2$

$\frac{x+1}{y-3} = \frac{y}{x}$

$x(x+1) = y(y-3)$

$x^2 + x = y^2 - 3y$

$\therefore x^2 - y^2 + x + 3y = 0$

$\therefore$ The equation of line about point P is

$x^2 - y^2 + x + 3y = 0$

Extract 4.1: A sample of the correct responses to question 4

In Extract 4.1, part (a), the student correctly described the locus, and in part (b), he or she applied the distance formula between two points to find the locus of a point. In part (c), the student managed to get the equation of the line of locus of the point P using the fact that the two lines have equal gradients.

As Figure 4 shows, 56.0 per cent of the students scored low marks (0 – 2.5). Those students encountered the following challenges: In part (a) (i), some students described that the locus of an orange falling vertically from a tree at a height of 2 metres from the ground is 4 metres. Likewise, some students said that the locus of an orange falling vertically from a tree at a height of 2 metres from the ground is the one that doesn’t move at a fixed point. Also, other students described that when an orange is falling vertically from a tree at a height of 2 metres from the ground, its locus is about two fixed points obtained as a perpendicular bisector between the two fixed points.

Moreover, in part (a) (ii), some students described that when the centre of a wheel as a cyclist rides along the road on a horizontal plane, its locus is the number of wheels (2), considering the number of wheels of a bicycle. Also, other students wrote that the locus of the centre of a wheel is a horizontal line parallel to the surface, assuming that the locus is a set of points along the road on which the wheel passes. These students were not conversant with the concept of locus at all.

In part (b), most students applied inappropriate formulae. For instance, some students applied the formula for calculating the gradient (m), instead of the distance between the two points. These students replaced (x_1, y_1) and (x_2, y_2) in $m = \frac{y_2 - y_1}{x_2 - x_1}$ with $(-2, 2)$ and $(1, 1\frac{1}{2})$, respectively, and worked out to get $m = \frac{-1}{6}$ and consequently the incorrect equation $y = \frac{-x}{6} + \frac{10}{6}$. Also, some students recalled the incorrect formula. For example, some students wrote $(x_1 + x)^2 + (y_1 + y)^2 = (x_2 + x)^2 (y_2 + y)^2$ in which they replaced (x_1, y_1) and (x_2, y_2) with $(-2, 2)$ and $(1, 1\frac{1}{2})$, respectively, and simplified it into $x^2 + y^2 + 2x - y - 55 = 0$. Moreover, some students misinterpreted the word "equidistant," by assuming that the locus could be a circle centred at $(0, 0)$ whose radius is the distance between L and M. These students computed the square of the distance between $L(-2, 2)$ and $M\left(1, 1\frac{1}{2}\right)$ using the distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ and got $d^2 = \frac{37}{2}$. Thereafter, they replaced r^2 in the general equation of the circle $r^2 = x^2 + y^2$ with $\frac{37}{2}$, resulting in $\left(\frac{37}{2}\right)^2 = x^2 + y^2$.

In part (c), many students strived to find the numerical values of x and y, while these were the variables of the locus under discussion. These students solved equations of the lines $m(y - 3) = x + 1$ and $y = mx$ simultaneously; however, they ended up with incorrect answers. For example, some students wrote $mx - x = 1 + 3m$ and incorrectly solved it, resulting in $x = 2$ and consequently $y = 2m$. Similar to part (a), some students used the general equation of a circle $p^2 = (x-a)^2 + (y-b)^2$ at (a,b). These students replaced (a,b) with $(3,1)$ in $r^2 = (x-a)^2 + (y-b)^2$ to obtain $r^2 = (x-3)^2 + (y-1)^2$ and simplified it into $x^2 + y^2 + 6x + 2y - r^2 = 0$. Extract 4.2 is a sample response from one of the students who faced difficulties when attempting the question.

i. That is a locus on a fixed point

ii. That is a locus of two fixed point

Solution;

$p^2 = (x-x)^2 + (y-y)^2$

$p^2 = (-2-1)(-2-1) + (2-1\frac{1}{2})(2-1\frac{1}{2})$

$\sqrt{p^2} = \sqrt{4+2+2+1+4-3-3-2.5}$

$p = \sqrt{13-3-3-2}$

$p = \sqrt{5}$

∴ The equidistant from the point L to M is $\sqrt{5}$

Extract 4.2: A sample of the incorrect responses to question 4

In Extract 4.2, part (a), the student incorrectly described the locus of the objects asked. In part (b), the student computed the distance between L and M instead of equating the distance between points P and L, and P and M.

2.5 Question 5: Coordinate Geometry

The question consisted of parts (a) and (b). In part (a), the students were required to calculate the value of h given that the points $A(2,5)$, $B(h,-4)$ and $C(1,2)$ are collinear. In part (b), the students were required to determine the equation of a line passing through the point $(-4,-4)$ and parallel to the line whose equation is $2x+6y-9=0$.

The data revealed that 127 (28.8%) students scored 2.5 marks or less, 115 (26.1%) scored marks ranging from 3.0 to 6.0, and 199 (45.1%) students scored 6.5 marks or more. Figure 5 provides a summary of students' performance on this question.

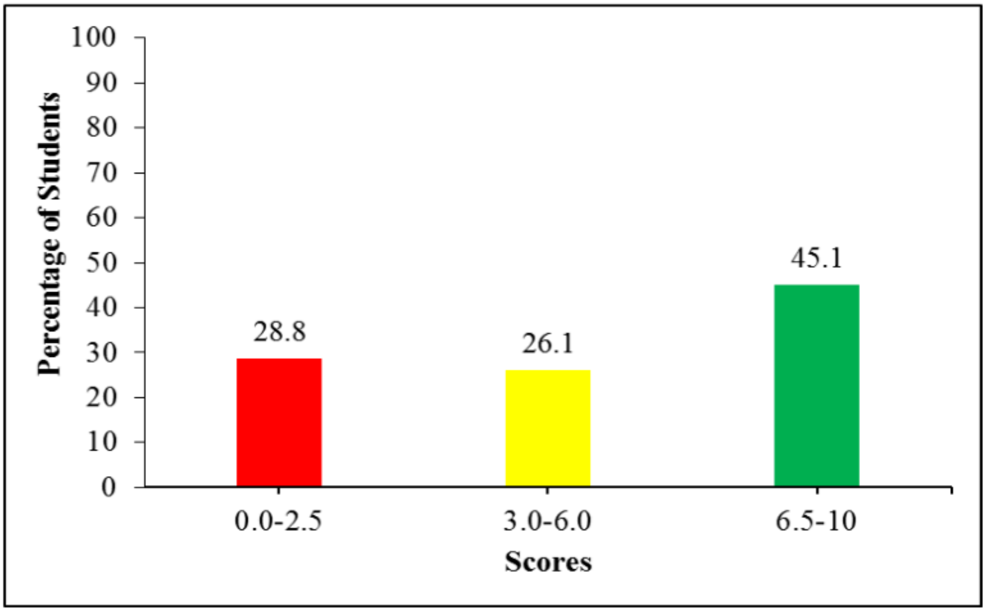


Figure 5: *Students' Performance on Question 5*

About 71.2 per cent equivalent to 314 students, got 3.0 marks or more. Therefore, students' performance on this question was good. In part (a), students were conversant with the fact that collinear points lie on the same straight line and thus their line segments have the same gradient. For this case, most students considered the gradient of line segments AB and AC, AB and BC, or BC and AC to generate an equation which enabled them to determine the value of h. For example, the students who considered the line segments AB and AC correctly applied the formula for calculating the gradient(m) of a straight line, $m = \frac{y_1 - y_2}{x_1 - x_2}$, and got $m = 3$ and $m = \frac{9}{2-h}$ for the line segments AB and AC, respectively. Then, they equated $\frac{9}{2-h}$ to 3, resulting in the equation $6 - 3h = 9$ and consequently $h = -1$.

In part (b), the students were knowledgeable about the fact that parallel lines have the same gradient. Thus, these students correctly wrote $2x + 6y - 9 = 0$ into the standard form $y = -\frac{1}{3}x + \frac{3}{2}$, which enabled them to determine the gradient of the line, $m = -\frac{1}{3}$. Therefore, the students applied the formula $m = \frac{y - y_1}{x - x_1}$, whereas $m = -\frac{1}{3}$ and $(x_1, y_1) = (-4, -4)$, to formulate the equation, that is, $x + 3y + 16 = 0$. Extract 5.1 provides a sample of one of the correct solutions to this question.

Solution.
Condition of collinear point $m_1 = m_2 = m_3$
Given that point A (2,5) B, (h, −4) C(1,2)
take point A and C to find m

$m = \frac{Y_2 - Y_1}{X_2 - X_1}$

$m = \frac{2 - 5}{1 - 2}$

$m = \frac{-3}{-1}$

$m = 3$

them from $m_1 = m_2$. take point A and B

$m_2 = \frac{Y_2 - Y_1}{X_2 - X_1}$

$\frac{3}{1} \times \frac{-4 - 5}{h - 2}$

$3h - 6 = -4 - 5$

$3h - 6 = -9$

$3h = -9 + 6$

$\frac{3h}{3} = \frac{-3}{3}$

$h = -1$.

∴ The value of $h = -1$.

Giventhat solution.

point $(-4,-4)$

equation $(2x+6y-9)=0$

from

then $y = mx - c$

$2x+6y-9=0$

$\frac{6y}{6} = \frac{-2x}{6} + \frac{9}{6}$

$y = -\frac{1}{3}x + \frac{3}{2}$ compare to $y = mx - c$

$m = -\frac{1}{3}$

but for parallel line $m_1 = m_2$.

them let anotherpoint (x,y) and $(-4,-4)$

$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$-\frac{1}{3} = \frac{y+4}{x+4}$

$3x + 12 = -x - 4$

$3y + 12 + 4 = -x$

$3y + 16 = -x$

$x + 3y + 16 = 0$ answer

Extract 5.1: A sample of the correct responses to question 5

In Extract 5.1, part (a), the student formulated the equation by calculating the slopes of the line segments AC and AB and equating them. Then, they correctly solved the equations and obtained $h=-1$. In part (b), the student correctly determined the slope of the lines and the equation of the line passing through $(-4,-4)$.

Despite the good performance, 77 (17.5%) students scored zero. In part (a), most of these students misinterpreted the term collinear. For example, some students wrote $(h,-4)-(2,5)=(1,2)-(2,5)=(1,2)-(h,-4)$ and simplified it to the incorrect values of $h=1$. Also, some students applied an incorrect formula. For example, there were students who wrote $m=\frac{x_2-x_1}{y_2-y_1}$ instead of $m=\frac{y_2-y_1}{x_2-x_1}$. Therefore, these students obtained the wrong equation $3h-6=-10$ after equating the slopes of the line segments AB and AC, which leads to an incorrect value of h, including $h = \frac{-4}{3}$. Moreover, some students committed computational errors. For instance, some students used the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$, and correctly reached $\frac{-9}{h-2} = 3$. However, they committed errors on performing cross-multiply and obtained $3h - 2 = -9$ (instead of $3h - 6 = -9$), which results in $h = \frac{-7}{3}$ instead of $h = -1$.

In part (b), some of the students did not deduce the gradient from the equation $2x + 6y - 9 = 0$. Instead, they used the variable m, the point $(-4, -4)$, and the formula $m = \frac{y - y_1}{x - x_1}$ to formulate the equation $y = m(x + 4) - 4$. Furthermore, some students correctly recognized that parallel lines have the same slope, but they failed to determine the slope of the lines. For example, most of these students failed to rewrite $2x + 6y - 9 = 0$ into the form $y = mx + c$. Instead, they rewrote the equation as $x = -3y + \frac{9}{2}$, and thus, they got the incorrect slope $m = -3$ in particular. It was also observed that some students solved the given equation instead of formulating the equation of the line passing through $(-4, -4)$. However, most of them had an incorrect approach because they applied the quadratic formula $x = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$, taking $a = 2$, $b = 6$, and $c = 9$, resulting in incorrect answers, $x = \frac{3}{2}$ or $x = -\frac{3}{2}$. Extract 5.2 is a sample response from one of the students who faced challenges when attempting the question.

soln;

Point (−4, −4)

Equation to parallel $2x+6y-9=0$

$2x-6y-9=0$

$2x-6y=9$

~~$+6y$~~ $=9+2x$

~~$+6$~~ -6 $+6$

$y=\frac{9}{-6}+\frac{2x}{6}$.

∴ Gradient $=\frac{2}{6}$ (cancelled to $\frac{1}{3}$)

Gradient = (1, 3)

Point $(-4, -4)$ (x, y)

~~Grad~~ Equation = ?

$\frac{1}{3}=\frac{\text{Change in } Y}{\text{Change in } X}$

$\frac{1}{3}=\frac{Y--4}{X--4}$

$\frac{1}{3}=\frac{Y+4}{X+4}$

$1(X+4)=3(Y+4)$

$X+4=3Y+12$

$X-3y=12-4$

$X-3y+8=0$.

∴ The Equation is $X-3y+8=0$.

Extract 5.2: A sample of the incorrect responses to question 5

In Extract 5.2, the student committed computational errors when making y the subject of $2x-6y-9=0$, resulting in incorrect slope, $m=\frac{1}{3}$ instead of $m=-\frac{1}{3}$.

2.6 Question 6: Symmetry

The question comprised parts (a), (b), and (c). In part (a), the students were asked to state the number of lines of symmetry in each shape of the object when Chichi watched Drawing Art on television, as she identified the following shapes of objects: (i) circle, (ii) tree, (iii) flying kite, (iv) cross shape, and (v) rectangular home mat. In part (b), they were required to state the order of rotational symmetry for each of the objects given in the table.

Name of Object	Order of Rotational Symmetry
(i) A rectangular playing card	
(ii) A ten thousand Tanzania shillings	
(iii) A nonagon	
(iv) A pen	
(v) A soccer ball	

In part (c), they were told that Hassan drew different types of vowels in capital letters on a wall for a kindergarten demonstration and were instructed to identify possible symmetrical letters that he drew.

The analysis of the data shows that, out of 441 students who attempted the question, 105 scored marks ranging from 6.5 to 10, 238 scored marks ranging from 3.0 to 6.0, and 98 students scored marks ranging from 0 to 2.5. The students' performance in percentage is summarized in Figure 6.

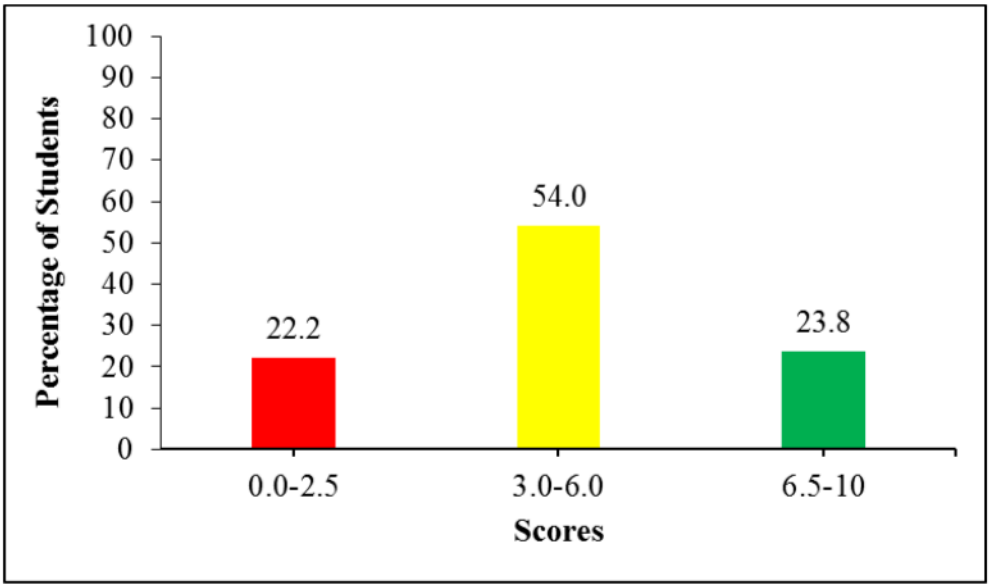


Figure 6: *Students' Performance on Question 6*

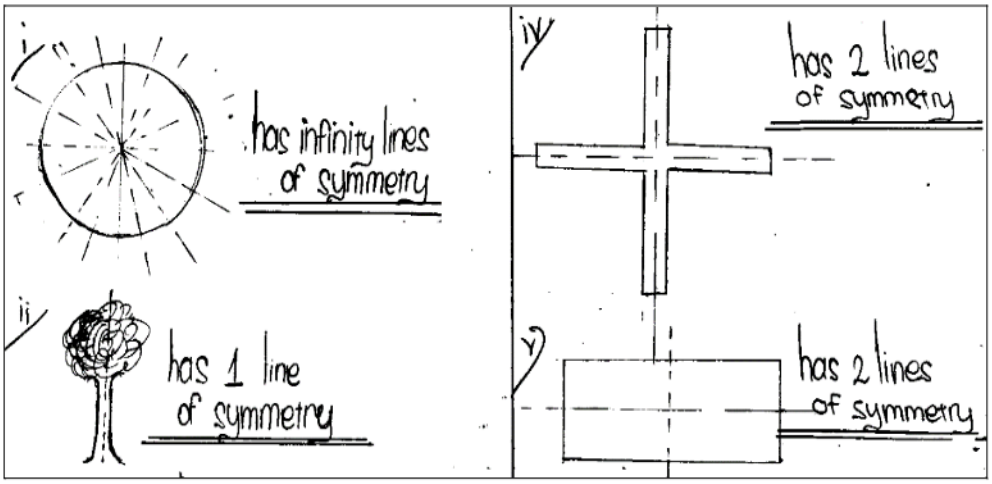
The students' performance on this question was good because 77.8 per cent of students scored from 3.0 to 10 marks. Furthermore, the analysis of the data revealed that 133 (30.2%) students scored 6.0 marks or more. In part (a), the students correctly stated the number of lines of symmetry as follows:

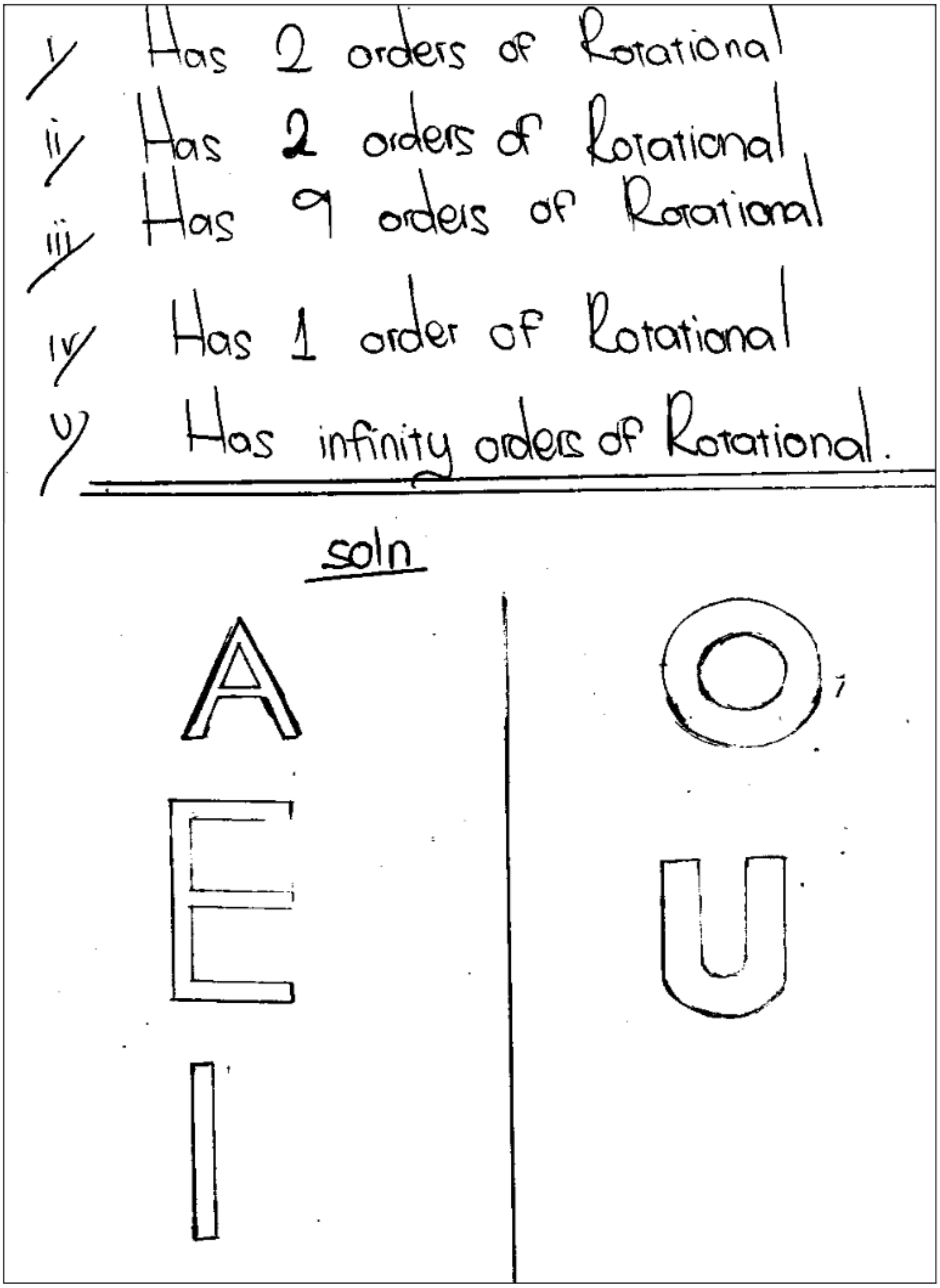
(i) A circle has infinite lines of symmetry.
(ii) A tree has one line of symmetry.
(iii) A flying kite has no line of symmetry.
(iv) A cross shape has two lines of symmetry.
(v) A rectangular home mat has two lines of symmetry.

In part (b), the students correctly stated the order of rotational symmetry as follows:

Name of Object	Order of Rotational Symmetry
(i) A rectangular playing card	Two
(ii) A ten thousand Tanzania shillings	Two
(iii) A nonagon	Nine
(iv) A pen	One
(v) A soccer ball	Infinite

In part (c), the students understood that vowels in capital letters are A, E, I, O and U and all of them are symmetrical letters. Extract 6.1 provides a sample response from one of the students who performed well on this question and managed to score all the marks.





i) Has 2 orders of Rotational
ii) Has 2 orders of Rotational
iii) Has 9 orders of Rotational
iv) Has 1 order of Rotational
v) Has infinity orders of Rotational.

soln

A | O
E | U
I |

Extract 6.1: A sample of the correct responses to question 6

In Extract 6.1, the student managed to identify the lines of symmetry on each shape of the object in part (a). In part (b), the student stated the correct order of rotational symmetry for each given name of object. In part (c), the student identified all vowels in capital letters as symmetrical letters.

Inspite of the good performance, 22.2 per cent of the students scored 2.5 marks or less. These students faced the following challenges: In part (a), most of these students were not knowledgeable about the concept of symmetry. As a result, they stated the incorrect number of lines of symmetry or order of rotational symmetry. For instance, in part (a) (i), some students stated that a circle has one line of symmetry, while others stated that it has two lines of symmetry, and others stated that it has four lines of symmetry. In part (a) (ii), some students stated that a tree has infinite lines of symmetry, and some stated that it has three lines of symmetry. In part (a) (iii), a few students stated that a flying kite has four lines of symmetry instead of no lines of symmetry. Likewise, in part (a) (iv), some students stated that a cross shape has no line of symmetry, and others stated that, has infinite lines of symmetry instead of two lines of symmetry. In part (a) (v), the students responded that the rectangular home mat has four lines of symmetry, and others said it has eight lines of symmetry rather than two lines of symmetry.

In part (b), the analysis revealed that some students perceived the word rotational symmetry as the sum of the degree measures of the angles of a polygon. For instance, some students stated that the order of rotational symmetry for the rectangular playing card, a nonagon, and a soccer ball is 360^0. Likewise, other incorrect responses provided by some of the students were: a rectangular playing card has one order of rotational symmetry; a ten thousand Tanzania shillings has four orders of rotational symmetry; a pen has no number of rotational symmetry; and a soccer ball has four orders of rotational symmetry.

In part (c), a number of students failed to identify vowels. These students included consonants in their responses: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q and R. Extract 6.2 illustrates a sample response from one of the students who faced challenges while attempting the question.

i) circle soln

- No line of symmetry

ii) Tree

- No line of symmetry

iii) Flying kite

- Four line of symmetry

iv) cross shape

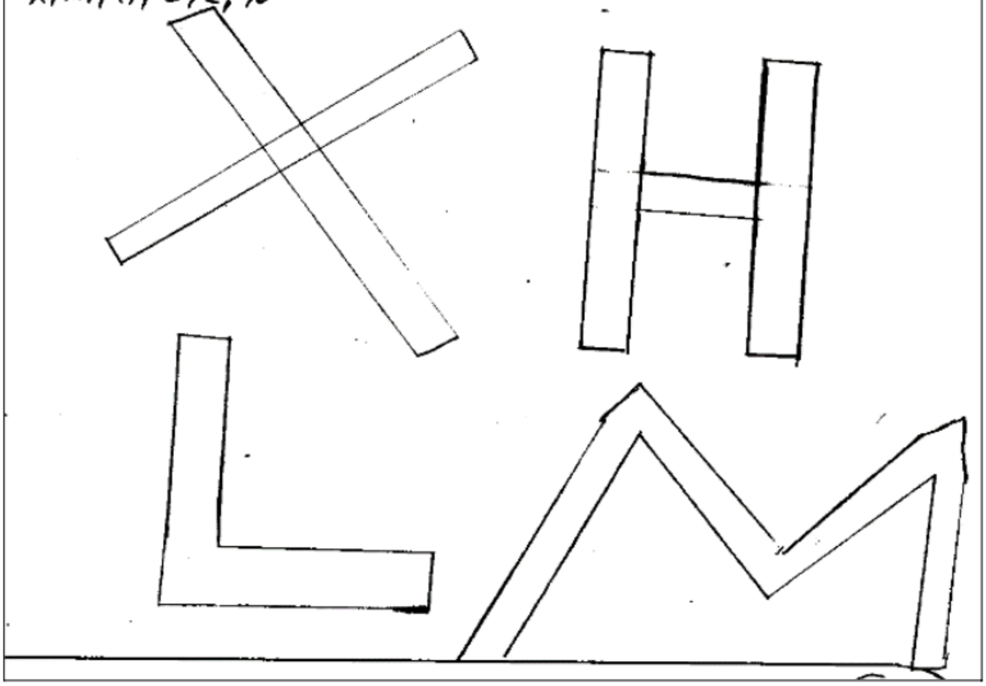
- No line of symmetry

v) Rectangular home mat

- Four line of symmetry

Name of Object	Order of Rotational Symmetry
(i) A rectangular playing card	1 number of symentary
(ii) A ten thousand Tanzania shillings	4 number of symentary
(iii) A nonagon	6 number of symentary
(iv) A pen	No number of symentary
(v) A soccer ball	1 number of symentary

X, M, H, O, L, N   Solution



Extract 6.2: A sample of the incorrect responses to question 6

In Extract 6.2, part (a), the student provided incorrect number of lines of symmetry. In part (b), the student stated the wrong order of rotational symmetry for the given objects and in part (c), the student provided consonants instead of vowels.

2.7 Question 7: Logic

The question consisted of parts (a), (b) and (c). In part (a), the students were given the statement, "If 6 is an even number, then it is either divisible by 2 or 4." Then, they were required to represent the statement in symbolic form and test its validity by letting p represent "6 is even number," q represent "6 is divisible by 2," and r represent "6 is divisible by 4." In part (b), students were required to copy and complete the following truth table:

p	q	$\sim p \to q$	$q \to p$	$(\sim p \to q) \wedge (q \to p)$

Part (c) instructed the students to write the symbolic form of the statement and draw an electric circuit of "either $2+6=8$ or $6\times5=11$" using the letters P and Q, given that P stands for "$2+6=8$" and Q stands for "$6\times5=11$."

The data shows that 193 (43.8%) students scored marks ranging from 0 to 2.5, 140 (31.7%) students scored marks ranging from 3.0 to 6.0, and 108 (24.5%) students scored marks ranging from 6.5 to 10. Therefore, 248 (56.2%) students got 3.0 marks or more, indicating that they performed averagely on this question. The students' performance summary is shown in Figure 7.

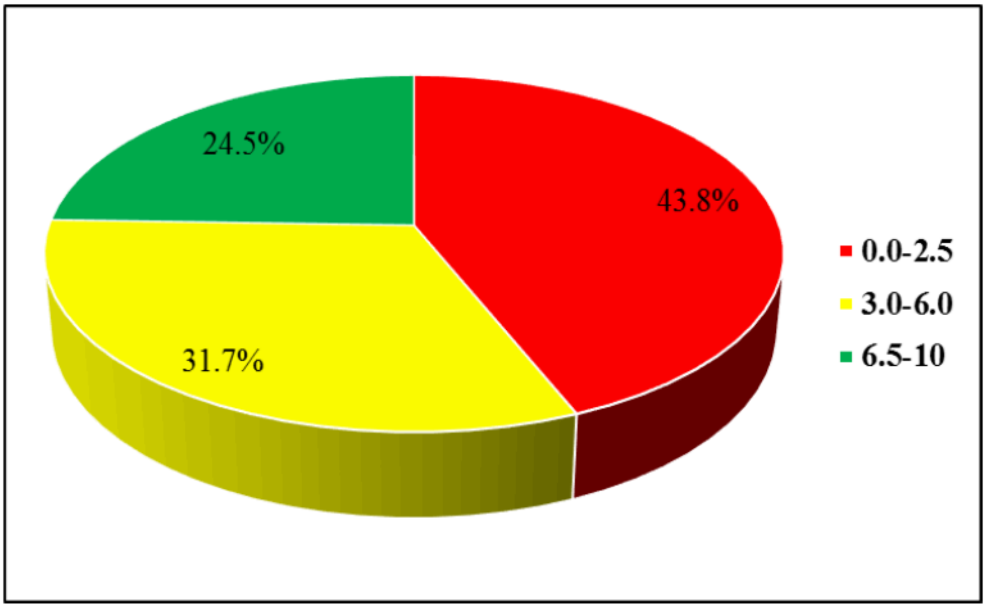


Figure 7: *Students' Performance on Question 7*

Further, the data show that 25 (7.5%) students scored full marks. In part (a), these students correctly wrote the given statement in symbolic form, that is, $p \to (q \vee r)$. Thereafter, they constructed the truth table with the correct truth values for p, q, r and $(q \vee r)$. They also drew and completed the column of $p \rightarrow (q \vee r)$, which enabled them to comment on its validity.

In part (b), the students completed the truth table by correctly performing logical operations, disjunctions, and implication. In part (c), the students wrote the given statement into symbolic form, $p \vee q$, and then they correctly drew the circuit as illustrated in Extract 7.1.

Solution:

P = 6 is even, q = 6 is divisible by 2, r 6 is divisible by 4

$p \rightarrow (q \vee r)$

Truth table.

p	q	r	q v r	p→(q v r)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

∴ since the last column is not full true it is not tautology

∴ Therefore it is not valid.

The truth table

p	q	$\sim p$	$\sim p \to q$	$q \to p$	$(\sim p \to q) \wedge (q \to p)$
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	F	T	F	T	F

Solution.

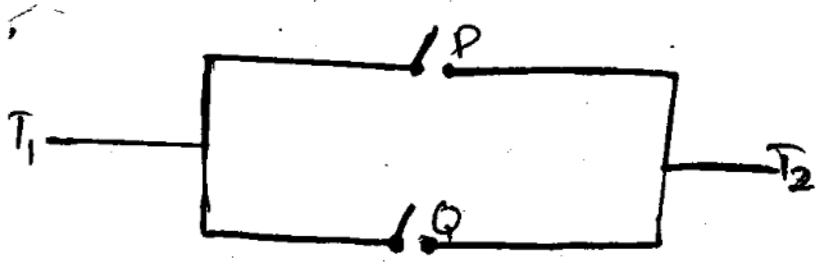
P stands for $2+6=8$

Q stand for $6 \times 5 = 11$

either $2+6=8$ or $6 \times 5 = 11$

$P \vee Q$

∴



Extract 7.1: A sample of the correct responses to question 7

In Extract 7.1, in part (a), the student transformed the given statement into symbolic form, then constructed the truth table using the correct entries. Thereafter, he/she found the last column not containing all T truth values, implying that it was not valid. In part (b), he/she completed the given truth table correctly using the logical connectives given. And in part (c), the student wrote the statement into symbolic form and drew the electric circuit.

Nevertheless, 104 (23.6%) students scored zero marks. In part (a), most of these students misinterpreted the problem. For example, some students assumed that $p=6$, $q=6$, $r=4$ and incorrectly manipulated some operations, resulting in $r=2$, $p=6$, and $q=3$. These students related the problem to algebra. Further, other students wrote $p \propto \frac{q}{r}$, and then they stated that the symbolic form of the statement is $p=\frac{ky}{r}$. Lastly, write the statement $p=\frac{ky}{r}$. These students wrongly related the problem to the concept of variation. Furthermore, some students assumed that the problem is related to the concept of divisibility. For example, some students commented that 6 is divisible by 2 because it is divisible by 2, and 6 is not divisible by 4 because there are no last two digits.

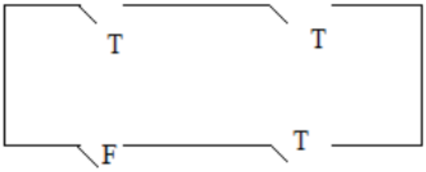
In part (b), some students lacked adequate knowledge of logical operations. For example, some students copied the truth values for q in the columns of $\sim p \rightarrow q$ and $q \rightarrow p$. Moreover, other students responded to the question by finding the number of possible combinations of truth values for a given n number of simple statements using 2^n, that is, $2^n \Rightarrow 2^2 = 4\,\text{rows}$, but failed to complete the truth table correctly. For instance, some students presented the following truth table:

p	q	$\sim p$	□ q	$q \rightarrow p$	$\sim p \rightarrow q$	$(\sim p \rightarrow q) \wedge (q \rightarrow p)$
T	T	F	F	T	T	F
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

From the truth table above, it is observed that, in the third row of the column of $\sim p \rightarrow q$, the students wrote F instead of T.

Furthermore, in part (c), most of the students combined the given simple statements with inappropriate logical connectives. For instance, $p \wedge q$ instead of $p \vee q$, resulting in an incorrect diagram of an electric circuit. These students confused the logical operation “OR” with “AND.”

Moreover, some students correctly wrote $p \vee q$, but they failed to construct the appropriate electric circuit.



Extract 7.2 is a sample response from one of the students who faced difficulties.

T	T	P	q	(~q→p)∧(p→q)
T	T	F	T	(~F→T)∧(T→F)
F	F	T	F	(~F→F)∧(F→T)
F	F	F	F	(~F→F)∧(F-F
p	q	~p→q	q→p	(~p→q)∧(q→p)

Extract 7.2: A sample of the incorrect responses to question 7

In Extract 7.2, the student combined the truth values T and F with connectives in the column of $(\sim p \to q) \wedge (q \to p)$.

2.8 Question 8: Variations

The question comprised parts (a), (b), and (c). In part (a), students were informed that, the speed L of a certain particle moving on the surface of water is inversely proportional to the cube root of time n and $L = 3$ when $n = 27$. Then, they were required to determine the value of L when $n = 64$. In part (b), students were required to determine the value a when $b = 12$, given that $a \propto \left(b^2 + 3\right)$ and $a = 4$ when $b = 5$. Part (c), stated that: "Suppose p is directly proportional to q^2 and inversely proportional to $\sqrt{r}$ such that $p = 10$ when $q = 6$ and $r = 16$. Find the value of p when $q = 2$ and $r = 64$".

In this question, 142 (32.2%) students scored from 0 to 2.5 marks, 89 (20.2%) scored from 3.0 to 6.0 marks, and 210 (47.6%) students scored from 6.5 to 10 marks (Figure 8).

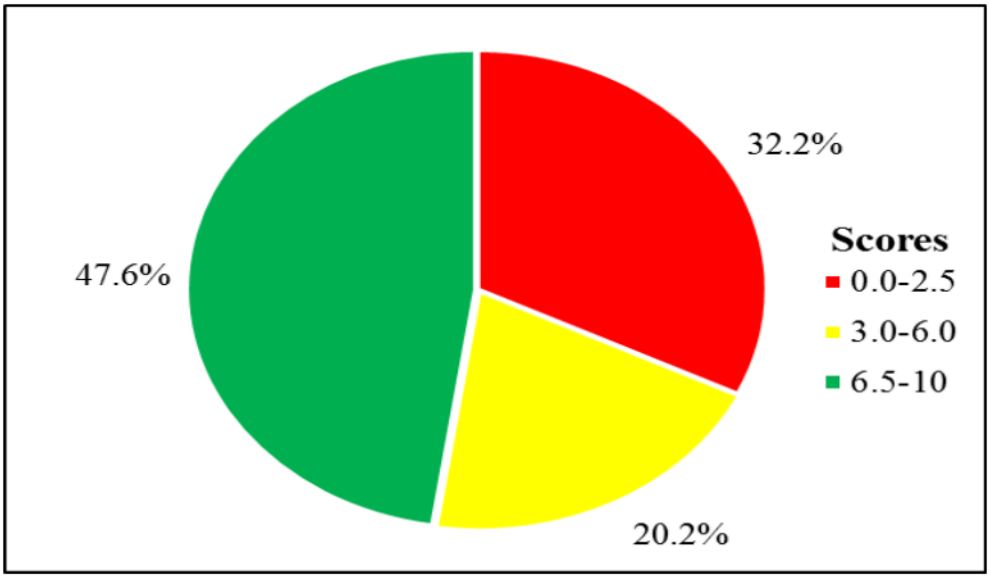
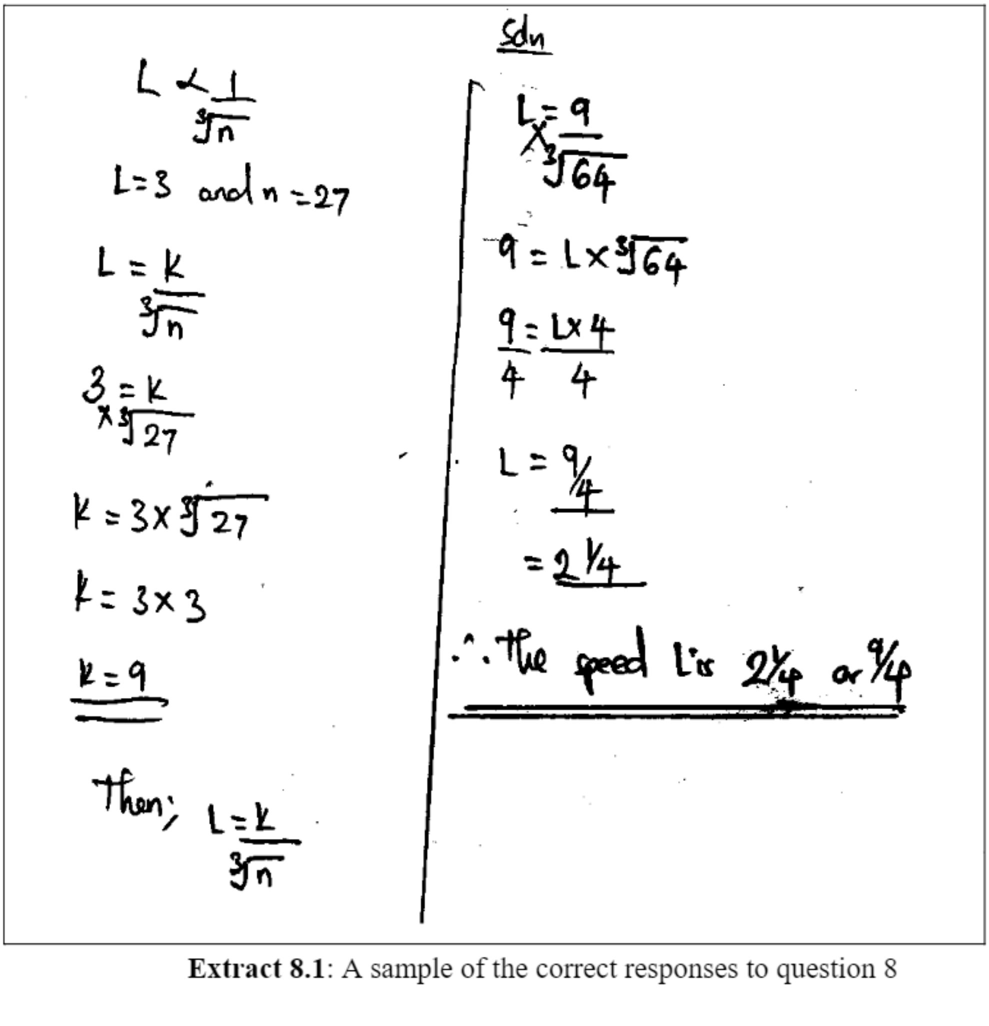


Figure 8: *Students' Performance on Question 8*

The overall performance of the students on this question was good because 299 (67.8%) students scored 3.0 marks or more. In part (a), the students rewrote the statement in symbolic form, $L \propto \frac{1}{\sqrt[3]{n}}$. Then, they introduced the proportionality constant (k) and obtained the equation $L = \frac{k}{\sqrt[3]{n}}$, and consequently interpreted that $L_1\sqrt[3]{n_1} = L_2\sqrt[3]{n_2}$. These students substituted $L_1 = 3$, $n_1 = 27$, and $n_2 = 27$ in $L_1\sqrt[3]{n_1} = L_2\sqrt[3]{n_2}$ and worked out to get $L_2 = \frac{9}{4}$.

In part (b), the students introduced the proportionality constant (k) in $a \propto (b^2 + 3)$ and got $a = k(b^2 + 3)$. Then, they substituted $a = 4$ and $b = 5$ in $a = k(b^2 + 3)$ and correctly solved the resulting equation, ending up with $k = \frac{1}{7}$, implying that $a = \frac{1}{7}(b^2 + 3)$. Finally, the students substituted the value of $b = 12$ in $a = \frac{1}{7}(b^2 + 3)$ and correctly solved the resulting equation to obtain $a = 21$.

Likewise, in part (c), the students rewrote the word problem in symbolic form, $p \propto \frac{q^2}{\sqrt{r}}$. Then, they introduced the proportionality constant (k) and obtained $p = \frac{kq^2}{\sqrt{r}}$. Using this equation as well as $p = 10$, $q = 6$, and $r = 16$, these students correctly solved for k and obtained $k = \frac{10}{9}$. Therefore, these students substituted $k = \frac{10}{9}$, $q = 2$, and $r = 64$ in $p = \frac{kq^2}{\sqrt{r}}$ and correctly solved it, resulting in $p = \frac{5}{9}$. Extract 8.1 provides a sample of one of the correct responses to this question from one of the students.



$L \propto \frac{1}{\sqrt[3]{n}}$

$L = 3$ and $n = 27$

$L = \frac{k}{\sqrt[3]{n}}$

$3 = \frac{k}{\sqrt[3]{27}}$

$k = 3 \times \sqrt[3]{27}$

$k = 3 \times 3$

$k = 9$

Then; $L = \frac{k}{\sqrt[3]{n}}$

Soln

$L = \frac{9}{\sqrt[3]{64}}$

$9 = L \times \sqrt[3]{64}$

$\frac{9}{4} = \frac{L \times 4}{4}$

$L = 9/4$

$= 2\,1/4$

∴ the speed L is 2¼ or 9/4

Extract 8.1: A sample of the correct responses to question 8

In Extract 8.1, the student substituted the values of $L=3$ and $n=27$ in $L=\frac{k}{\sqrt[3]{n}}$ to get the value of $k=9$. Then, he/she substituted $k=9$ and $n=64$ in $L=\frac{k}{\sqrt[3]{n}}$ and got $L=\frac{9}{4}$.

Nevertheless, 99 (22.4%) students scored zero due to various difficulties in responding to the question. In part (a), a number of students confused the concept of inversely proportional with directly proportional. These students wrote $L \propto \sqrt{n^3}$, and consequently $L=k\sqrt{n^3}$ which gives $L_2=18$ instead of $L_2=\frac{9}{4}$. Further, few students confused the concept of cube root with that of square root. In this case some students wrote $L \propto \frac{1}{\sqrt{n}}$, resulting to incorrect responses, including $L=\frac{k}{\sqrt{n}}$ and $L_2=\frac{81}{8}$.

In part (b), a number of students did not apply the knowledge of variation. Instead, they directly substituted the values into $a \propto \left(b^2+3\right)$. As a result, these students ended up with the meaningless statement $4 \propto 28$. Moreover, some students wrote $4 \times \left(5^2+3\right)$ and worked out to get 403. These students wrongly interpreted the proportionality sign as a multiplication sign. In addition, some students committed computation errors. For example, some students correctly got $4=k\left(5^2+3\right)$; however, they opened the brackets inappropriately and obtained $4=25k+3$ instead of $4=25k+3k$, and hence, they got incorrect answers, $k=\frac{1}{25}$ and $a=\frac{147}{25}$ in particular.

As in part (b), most students responded to part (c) by writing $p \propto q^2\sqrt{r}$ instead of $p \propto \frac{q^2}{\sqrt{r}}$. Therefore, they got $k=14\frac{2}{5}$ and $p=460$ instead of $k=\frac{10}{9}$ and $p=\frac{5}{9}$, respectively. Moreover, some students wrote $p^2+q^2=\sqrt{r}$ and substituted $p=10$, $q=6$, and $r=16$, resulting in a meaningless statement of $136=4$. While other students committed computational errors. For instance, some of them correctly got $p=\frac{kq^2}{\sqrt{r}}$, but, they wrote $10=\frac{k\times 6}{\sqrt{16}}$ instead of $10=\frac{k\times 6^2}{\sqrt{16}}$. Therefore, they obtained incorrect answers, including $k=\frac{20}{3}$ and $p=\frac{10}{3}$. Extract 8.2 provides a sample response from one of the students who faced challenges in responding to the question.

Soln

Data given

$L = 3$

$n = 27$

$k = ?$

$L \propto \frac{1}{\sqrt{3}}$

$L \propto \frac{K}{\sqrt{3})^3}$

$3 \propto \frac{K}{27}$

$27 \times 3 \propto K$

$81 \propto K$

$a \propto (b^2+3)$

$4 \propto (8^2+3)$

$\therefore a \propto (b^2+3)$

$a \propto (12^2+3)$

$a \propto 144+3$

$a \propto 147$

$L \propto \frac{K}{\sqrt{n}}$

$L \propto \frac{81}{\sqrt{64}}$

$L \propto \frac{81}{8}$

$\therefore a = 147.$

Extract 8.2: A sample of the incorrect responses to question 8

In Extract 8.2, the student substituted directly the values in the model without introducing a constant k in order to get the value of L and a.

2.9 Question 9: Algebra

In this question, students were informed that "The sales records of a certain fuel filling station were as follows; the total sales of six litres of diesel and five litres of petrol were Tsh. 6000, while the sales of seven litres of diesel and five litres of petrol were Tsh. 6800." Then, they were required to use the elimination method to find the price of a litre of diesel and a litre of petrol.

The data analysis revealed that 441 students responded to the question. Among them, 321 students scored marks ranging from 3.0 to 10 while 120 students scored marks ranging from 0 to 2.5. Therefore, the overall performance of the students was good. Figure 9 provides a summary of the students' performance on this question.

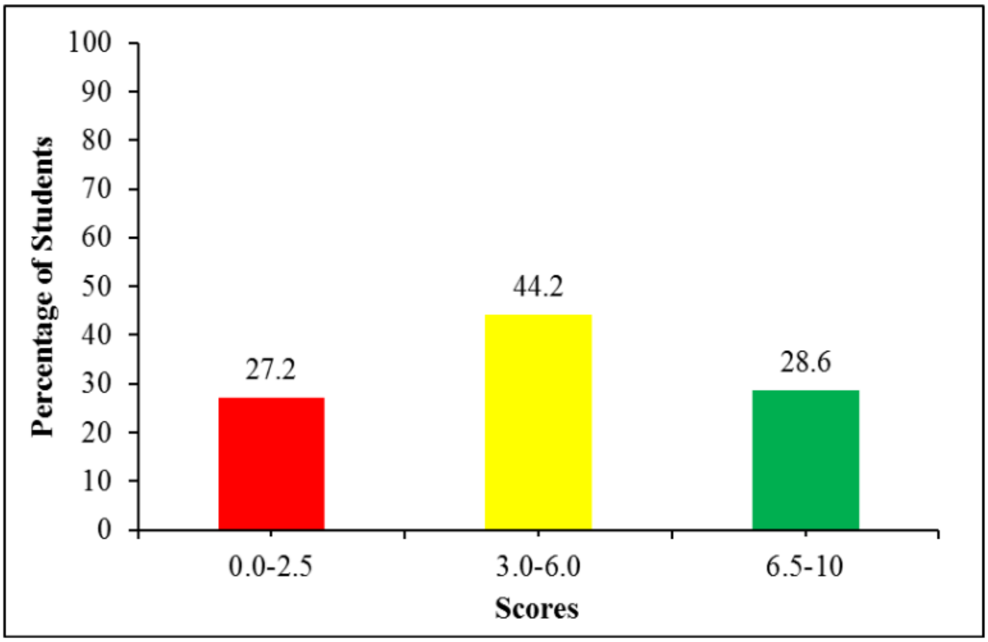


Figure 9: *Students' Performance on Question 9*

About 1.4 per cent of the students who attempted this question scored all ten marks. Most of those students assumed that x and y were the prices of one litre of diesel and one litre of petrol, respectively. Thus, they correctly represented the word problem using the equations $6x+5y=6000$ and $7x+5y=6800$. Then, they applied the elimination method to solve for x and y. The students obtained $x=800$ when eliminating y, and similarly they obtained $y=240$ when eliminating x.

Therefore, the students concluded that the price of a litre of diesel is Tsh. 800/= and petrol is Tsh. 240/=. A sample of the correct response from one of the students is provided in Extract 9.1.

Solution.

Let x be ~~litres of~~ Price of litres of diesel and y be Price of litres of petrol.

We eliminate y

$$-\begin{cases} 6x+5y=6000 \\ 7x+5y=6800 \end{cases}$$

$$6x-7x+5y-5y=6000-6800$$

$$\frac{-x}{-1}=\frac{-800}{-1}$$

$$x=800.$$

We eliminate x.

$$\begin{matrix} 7 \\ 6 \end{matrix}\begin{cases} 6x+5y=6000 \\ 7x+5y=6800 \end{cases}$$

$$-\begin{cases} 42x+35y=42000 \\ 42x+30y=40800 \end{cases}$$

$$42x-42x+35y-30y=42000-40800$$

$$\frac{5y}{5}=\frac{1200}{5}$$

$$y=240$$

$\therefore$ The price of one litre of diesel was 800 and Price of one litre of petrol was 240

Extract 9.1: A sample of the correct responses to question 9

In Extract 9.1, the student correctly formulated and solved the equations using the elimination method and interpreted the answers $x = 800$ and $y = 240$, concluding that the price of one litre of diesel is Tsh. 800/= and that of petrol is Tsh. 240/=.

As Figure 9 shows, 27.2 per cent of the students scored low marks (0 – 2.5). The analysis shows that many students interpreted the word problem incorrectly. For instance, some students wrote the incorrect equations $6x - 5y = 6000$ and $7x - 5y = 6800$. These students assumed that 6000 is the difference between the cost of 6 litres of diesel and 5 litres of petrol, and 6800 is the difference between the cost of 7 litres of diesel and 5 litres of petrol. These students misinterpreted the word total as a difference instead of a sum. Then, by using elimination method they obtained $x = 400$ and $y = 720$. Furthermore, some students formulated the equation involving only one variable. For example, some students wrote $7 + 5x = 6800$ and consequently $x = 1358.6$. Extract 9.2 provides a sample of a response chosen from one of the students who responded to the question incorrectly.

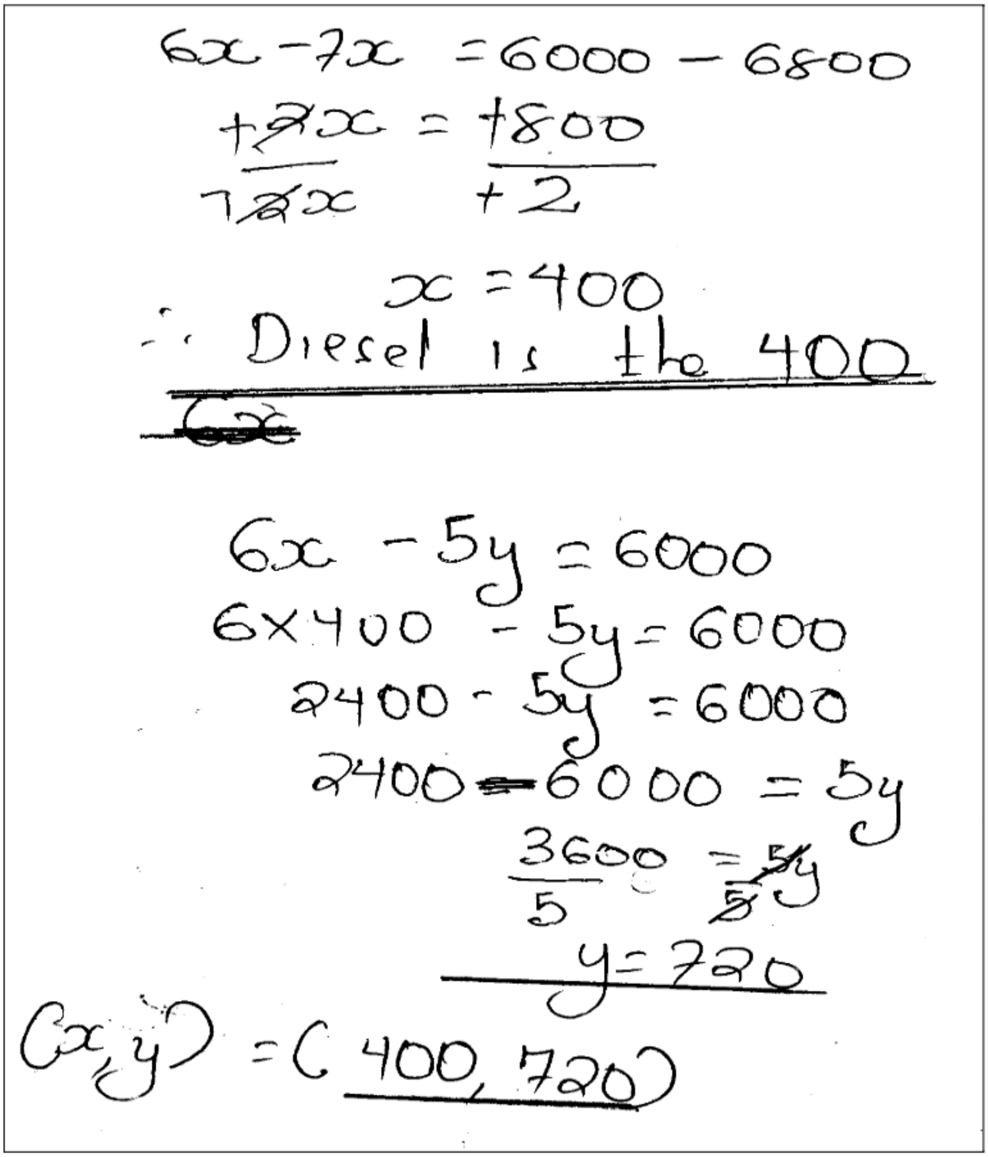
Solution.

Let x → is diesel

let y → is petrol

$6x - 5y = 6000$

$7x - 5y = 6800$


$6x - 7x = 6000 - 6800$

$\frac{+7x}{7x} = \frac{+800}{+2}$

$x = 400$

∴ Diesel is the 400

$6x - 5y = 6000$

$6 \times 400 - 5y = 6000$

$2400 - 5y = 6000$

$2400 - 6000 = 5y$

$\frac{3600}{5} = \frac{5y}{5}$

$y = 720$

$(x, y) = (400, 720)$

Extract 9.2: A sample of the incorrect responses to question 9

In Extract 9.2, the students considered the total cost of litres of diesel and petrol as a difference instead of a sum which led to incorrect answers for one litre of diesel and petrol.

2.10 Question 10: Sets

In this question, the students were given the universal set μ and subsets D, P, and S such that :

$\mu = \{x : x \text{ is an intager } 3 \le x < 18\}$

$D = \{x : x \text{ is an odd number}\}$

$P = \{x : x \text{ is prime number}\}$

$S = \{x : x \text{ is a perfect square}\}$.

Then, they were required to:

(a) list the elements of each set.

(b) represent these sets in a Venn diagram.

(c) find (i) $P \cap D$ (ii) $(P \cup D \cup S)'$.

The data shows that 288 (65.3%) students scored 2.5 marks or less, 88 (20.0%) students scored marks ranging from 3.0 to 6.0, and 65 (14.7%) students scored 6.5 marks or more. The summary of the students' performance is shown in Figure 10.

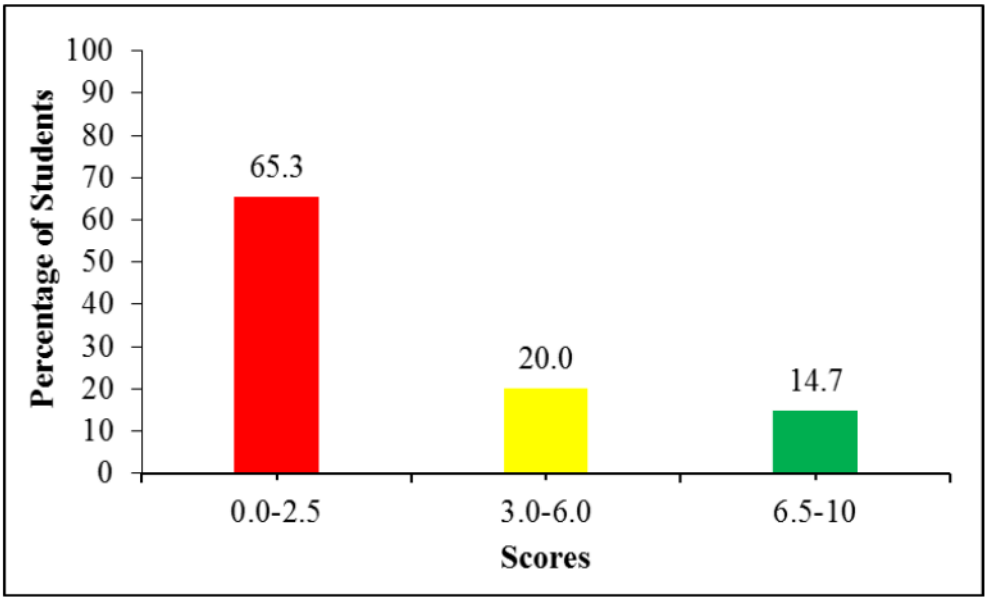


Figure 10: *Students' Performance in Question 10*

Furthermore, the data shows that 153 (34.7%) students scored 3.0 marks or more. Therefore, the general performance of the students in this question was average. In part (a), the students correctly listed all the elements of each set as follows:

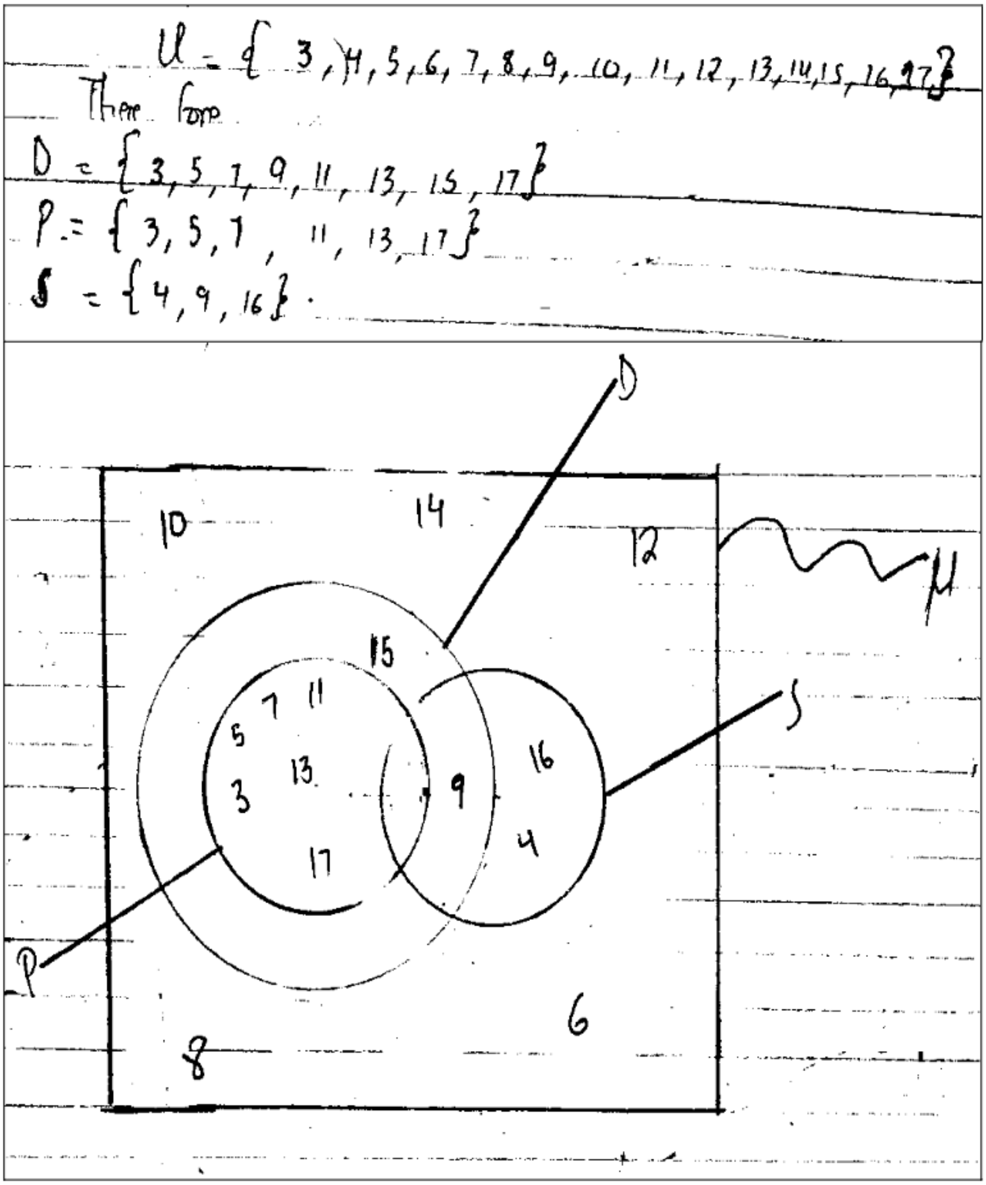
$\mu = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$

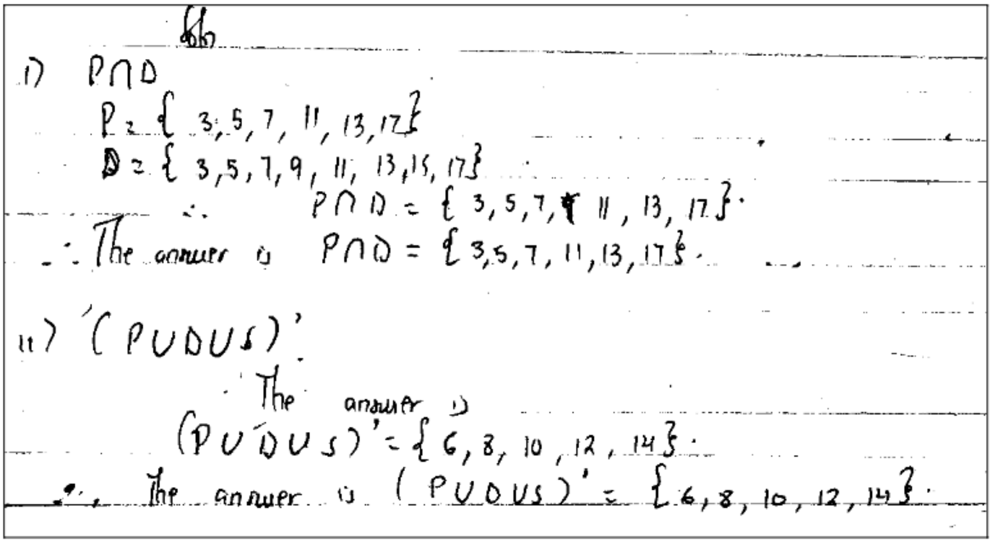
$D = \{3, 5, 7, 9, 11, 13, 15, 17\}$

$P = \{3, 5, 7, 11, 13, 17\}$

$S = \{4, 9, 16\}$

In part (b), the students drew the Venn diagram displaying all the elements in their respective regions. Likewise, in part (c) (i), the students identified the common elements found in both P and D to obtain $P \cap D = \{3, 5, 7, 11, 13, 17\}$, and in part (c) (ii), the students listed the elements that were not found in either of the sets P, D, or S and got $(P \cup D \cup S)' = \{6, 8, 10, 12, 14\}$. Extract 10.1 illustrates a sample of one of the students who responded correctly to this question.





i) $P \cap D$
$P = \{3, 5, 7, 11, 13, 17\}$
$D = \{3, 5, 7, 9, 11, 13, 15, 17\}$
$\therefore$ $P \cap D = \{3, 5, 7, 11, 13, 17\}$.
$\therefore$ The answer is $P \cap D = \{3, 5, 7, 11, 13, 17\}$.

ii) $(P \cup D \cup S)'$
The answer is
$(P \cup D \cup S)' = \{6, 8, 10, 12, 14\}$.
$\therefore$ The answer is $(P \cup D \cup S)' = \{6, 8, 10, 12, 14\}$.

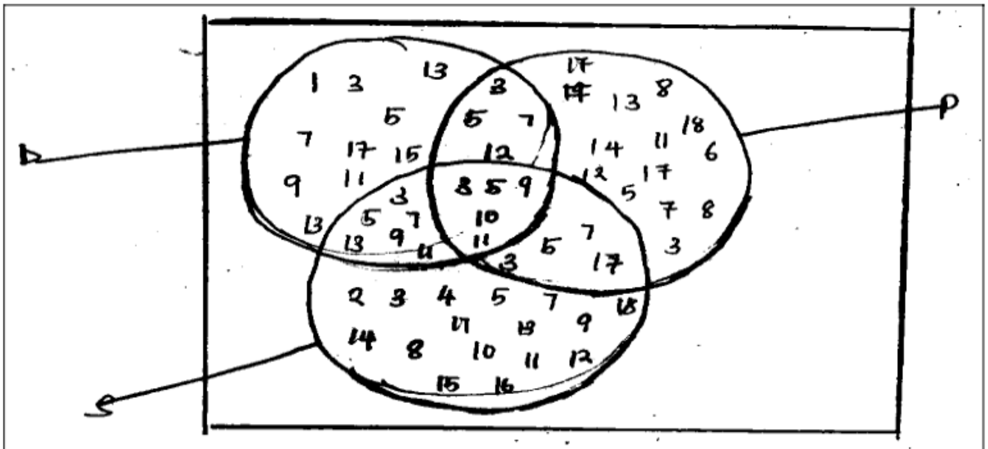
Extract 10.1: A sample of the correct responses to question 10

In Extract 10.1, the student listed all elements of each set in part (a). In part (b), the student drew a Venn diagram showing the elements in each region. Also in part (c), the student correctly listed the elements of $P \cap D$ as well as $(P \cup D \cup S)'$.

However, 65.3 per cent of the students scored low marks due to various challenges. In part (a), the students lacked knowledge about integers, odd, or prime numbers. For example, some students wrote: $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$, $D = \{1, 3, 5, 7, 9, 11, 13, 17\}$, $S = \{1, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$ and $P = \{1, 9, 15\}$, the set that includes all elements that are not prime numbers because the numbers are divisible by some numbers different from 1 and itself. In addition, some of the students were not familiar with set notation because they responded to the question by listing the elements without enclosing them in brackets. For example, some of the students wrote; $P = 3, 5, 7, 11, 13, 17$.

In part (b), some students wrote the incorrect entries in the regions of the Venn diagram. Poor performance in part (c) was highly attributted to the incorrect answers obtained in part (a). For instance, some of the responses provided by the students in part (c) were $P \cap D = \{1, 3, 5, 7, 11, 13, 17\}$ and $(P \cup D \cup S)' = \{1, 3, 5, 7\}$. Extract 10.2

provides a sample response from one of the students who were not able to respond to the question correctly.



Soln.

$P \cap D = n(D) + n(S) \cdot n(S) - (P \cup D)$

$P \cap D = n(D) + n(S) \; n(S)$

$P \cap D = 12$

$(P \cup D \cup S)' = 13.$

Extract 10.2: A sample of the incorrect responses to question 10

In Extract 10.2, part (b), the student drew an incorrect Venn diagram and entered elements of the universal set in every region of P, D, and S. In part (c), the student provided the formula to determine the intersection of two sets of elements and gave incorrect answers for $(P \cap D)$ and $(P \cup D \cup S)$.

3.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH TOPIC

The Additional Mathematics paper was composed of 10 questions from nine (9) topics namely *Numbers*, *Algebra*, *Geometrical Constructions*, *Locus*, *Coordinate Geometry*, *Symmetry*, *Logic*, *Variations* and *Sets*. The data analysis on students' performance shows that among these topics, five (5) were well performed and four (4) were averagely performed. Students had good performance in *Numbers* (88.7%), *Symmetry* (77.8%), *Coordinate Geometry* (71.2%), *Variations* (67.8%) and *Algebra* (66.9%). Students' good performance on these topics was attributed to the competence demonstrated by the students in interpreting the questions, adhering to the instructions, and performing arithmetics correctly.

Furthermore, the students had average performance on four (4) topics, which are *Logic* (56.2%), *Locus* (44.0%), *Geometrical Constructions* (43.1%) and *Sets* (34.7%). The students' average performance on those topics was attributed to inappropriate use of formulae, misinterpretation of some special sets of numbers, failure to draw Venn diagrams, misinterpretation of the requirements of the questions, failure to perform arithmetic and algebraic computations, and interpreting loci as well as logical connectives. The students' performance on each topic is presented in Appendix I.

Further analysis shows that there is an increase in performance of the students in four (4) topics of *Numbers* (33.5%), *Locus* (17.9%), *Coordinate Geometry* (9.4%) and *Variations* (8.3%) for 2023 as compared to 2022. Likewise, there are five (5) topics that show a decrease in students' performance as compared to 2022. These topics are *Logic* (2.8%), *Algebra* (3.7%), *Symmetry* (9.7%), *Geometrical Constructions* (11.9%) and *Sets* (25.1%). Appendix II provides a comparison of students' performance per topic for 2022 and 2023.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The general performance on the Additional Mathematics assessment in 2023 was good, as 310 (70.29%) students passed. Generally, the strengths and weaknesses of the students shown in this report are expected to be valuable information for education stakeholders. Moreover, the recommendations presented in this report are expected to help improve students' performance on future Additional Mathematics assessments.

4.2 Recommendations

In order to improve the performance of the students in this subject, the following recommendations are made:

(a) Group discussion techniques should be used during teaching and learning; In the groups, students get opportunity to feel interactive; the number of groups should be small to enable everyone to participate.

As well students should read more Mathematics books and do enough exercises in order to improve their competence.

(b) Teachers should provide enough exercises to learners and emphasizes drawing Venn diagrams and solving related problems involving three sets. This will improve students' competence in the topic of Set which was not well performed.

(c) Teachers should be given seminars and workshops so that they may improve their teaching strategies and competence in some difficult areas.

(d) Teachers should implant confidence among the students to approach various problems and be able to consult them whenever they do not understand.

Appendix I

Students' Performance on Each Topic in 2023

S/N	Topic	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Numbers	1	88.7	Good
2.	Symmetry	6	77.8	Good
3.	Coordinate Geometry	5	71.2	Good
4.	Variations	8	67.8	Good
5.	Algebra	2 & 9	66.9	Good
6.	Logic	7	56.2	Average
7.	Locus	4	44.0	Average
8.	Geometrical Constructions	3	43.1	Average
9.	Sets	10	34.7	Average

Appendix Ii

Comparison of Students' Performance in 2022 and 2023

S/N	Topic	2022			2023		
		Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Numbers	1	55.2	Average	1	88.7	Good
2.	Symmetry	6	87.5	Good	6	77.8	Good
3.	Coordinate Geometry	5 & 9	61.8	Average	5	71.2	Good
4.	Variations	8	59.5	Average	8	67.8	Good
5.	Algebra	2	70.6	Good	2 & 9	66.9	Good
6.	Logic	7	59.0	Average	7	56.2	Average
7.	Locus	4	26.1	Weak	4	44.0	Average
8.	Geometrical Constructions	3	55.0	Average	3	43.1	Average
9.	Sets	10	59.8	Average	10	34.7	Average