THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2021

ADDITIONAL MATHEMATICS

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042 ADDITIONAL MATHEMATICS

Published by ;
The National Examinations Council of Tanzania,
P.O. Box 2624,
Dar es salaam, Tanzania.

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FOREWORD

The National Examination Council of Tanzania is pleased to issue this report on Students' Item Response Analysis (SIRA) in Additional Mathematics for Form Two National Assessment (FTNA) 2021 in order to inform teachers, parents, policy makers and other education stakeholders on how the students responded to the assessment items. The report will enable the stakeholders to take appropriate measures to enhance the performance of the students and enable them to master the topics, which need more emphasis in teaching and learning.

The analysis of the students' responses was done in order to identify the areas in which the students did well or poorly. It will assist to understand the extent to which the educational system managed or failed to offer the learners during the two years of Secondary Education.

The factors noted for poor performance include the students' inability to: solve equations algebraically; perform factorization of numbers; determine the locus of points moving at equal distance from fixed points and solve simultaneous equations graphically. The factors for good performance include adequate knowledge and skills in concepts on various topics, correct interpretation of the questions as well as abiding to the instructions of the paper. Extracts from the students' responses are used in this report to illustrate poor or good performance that could be used as a practical guide to teachers and students in the teaching and learning process.

The National Examination Council of Tanzania believes that, the education stakeholders will work on the challenges that the students faced when attempting the assessment questions and take appropriate measures to improve the performance in this subject.

The National Examinations Council would like to thank the examination officers and all others who participated in the preparation of this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Students' Item Response Analysis (SIRA) report in Additional Mathematics subject is prepared to provide feedback to stakeholders about the students' performance in Form Two National Assessment (FTNA) 2021. The paper was set basing on the Form Two National Assessment Format of 2017 and the Additional Mathematics syllabus of 2010 for secondary schools. The paper comprised 10 compulsory questions each carrying 10 marks.

In 2021, a total of 484 students sat for the Additional Mathematics Assessment. As compared to the FTNA 2020 results, whereby 516 students sat for Assessment. The statistics represent a decrease in the number of students in this assessment by 6.20 per cent. Table 1 presents the summary of the students' performance in 2020 and 2021.

Table 1: The students' performance in Additional Mathematics (FTNA) 2020 and 2021

Year	Code	Subject Name	Students Sat	Passed		Grades				
				No.	%	A	B	C	D	F
2020	042	Additional Mathematics	516	392	75.97	70	89	137	96	124
2021			484	365	75.41	6	56	197	106	119

Table 1 shows that the students' performance in Additional Mathematics was good as 75.41 per cent who sat for the assessment passed of which 6 (1.2%) students got grade A, 56 (11.6%) got grade B, 197 (40.7%) got grade C, 106 (21.9%) got grade D and 119 (24.6%) got grade F. These results show that students' performance in this subject decreased by 0.56 per cent in comparison with the previous results in 2020.

The percentages of students who passed the assessment in Additional Mathematics in different grades is shown in Figure 1.

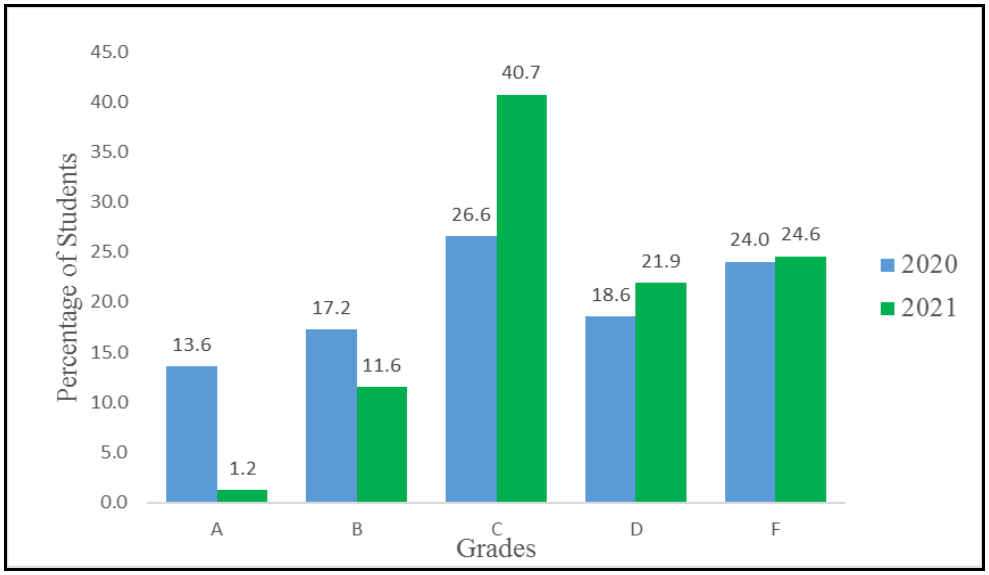


Figure 1: *Distribution of Grades A, B, C, D and F for the 2020 and 2021 Additional Mathematics Assessments*

Figure 1 shows that, the performance in Additional Mathematics in 2021 was low compared to the year 2020, since the percentages of students who passed the assessment with grades A and B were 1.2 and 11.6 respectively. Meanwhile in 2020, 13.6 per cent of the students got grade A while 17.2 per cent got grade B.

The analysis of students' performance in each question is presented in section 2.0.
The section consists of short descriptions about the requirements of questions and the analysis on how the students responded to the questions. Extract for both well and poorly performed questions are included in the analysis for each question. The factors that contributed to good or poor performance in each question are also illustrated by using sample of students' responses. Therefore, the analysis in each question can be used as a practical guide to teachers and students in order to improve teaching and learning process, eventually students' performance in future assessments.

The analysis of students' performance in the topic assessed is also shown in Appendix I where green, yellow and red colours are used to represent good, average and weak performance respectively. Likewise, the comparison of the students' performance in each topic for the FTNA 2020 and 2021 in Additional Mathematics is shown in Appendix II. Finally, the

recommendations are made at the end of this report to help students, teachers and the government to enhance the students' performance in future Additional Mathematics assessment.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE IN EACH QUESTION

This section gives the analysis of the students' performance in each question. The national assessment results are based on the score intervals ranged from 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 to mean excellent, very good, good, satisfactory and fail respectively. The percentage of performance in each question is divided into three categories, including weak performance ranging from 0 – 29 per cent, average performance ranging from 30 - 64 per cent and good performance ranging from 65 - 100 per cent. Also different colours have been used to represent performance of the students, as illustrated by red for weak, yellow for average and green for good performance.

2.1 Question 1: Numbers

The question consisted of three parts (a), (b) and (c). In part (a), the students were given the Numbers 14932, 438454, 1946, 23842, 11748, 254174, 746164, 1914 and were asked to determine which numbers are exactly divisible by 4. In part (b), the students were required to find the next three numbers from the following given pattern 2, 9, 20, 35… In part (c), the students were given the formula $n^{th} = \frac{n(n+1)}{2}$ and were asked to find the 14^{th} term.

The analysis of data shows that, 484 (100%) students attempted this question, out of which 452 (93.4%) students scored marks ranging from 3.0 to 10, whereby 32 (6.6%) students scored marks ranging from 0 to 2.5, while 145 (30.0%) students scored marks ranging from 3.0 to 6.0 and 307 (63.4%) students scored marks ranging from 6.5 to 10. The students' performance summary is presented in Figure 2.

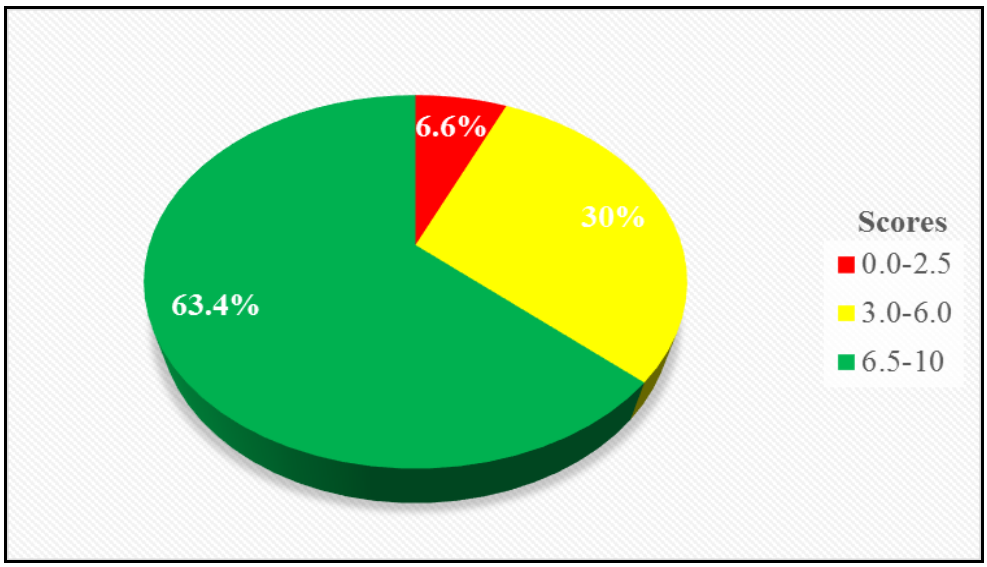


Figure 2: *Students' Performance in Question 1*

Generally, the data analysis shows the students' performance in this question was good.

In part (a), the students who performed well in this question and manage to score full marks, had adequate knowledge and skill on rules of divisibility on natural numbers as they stated correctly that "A Number is exactly divisible by 4 if the number formed by its two last digits is divisible by 4". Then, the students checked the given numbers and noted that only 14932, 11748 and 746164 their last two numbers form the numbers which are exactly divisible by 4. Therefore, they concluded that numbers 14932, 11748 and 746164 are exactly divisible by 4. On other hand, some students applied the technique of division so as to determine the given numbers that are exactly divisible by 4, and realized that the number will be divisible exactly by 4 if it leaves a zero remainder.

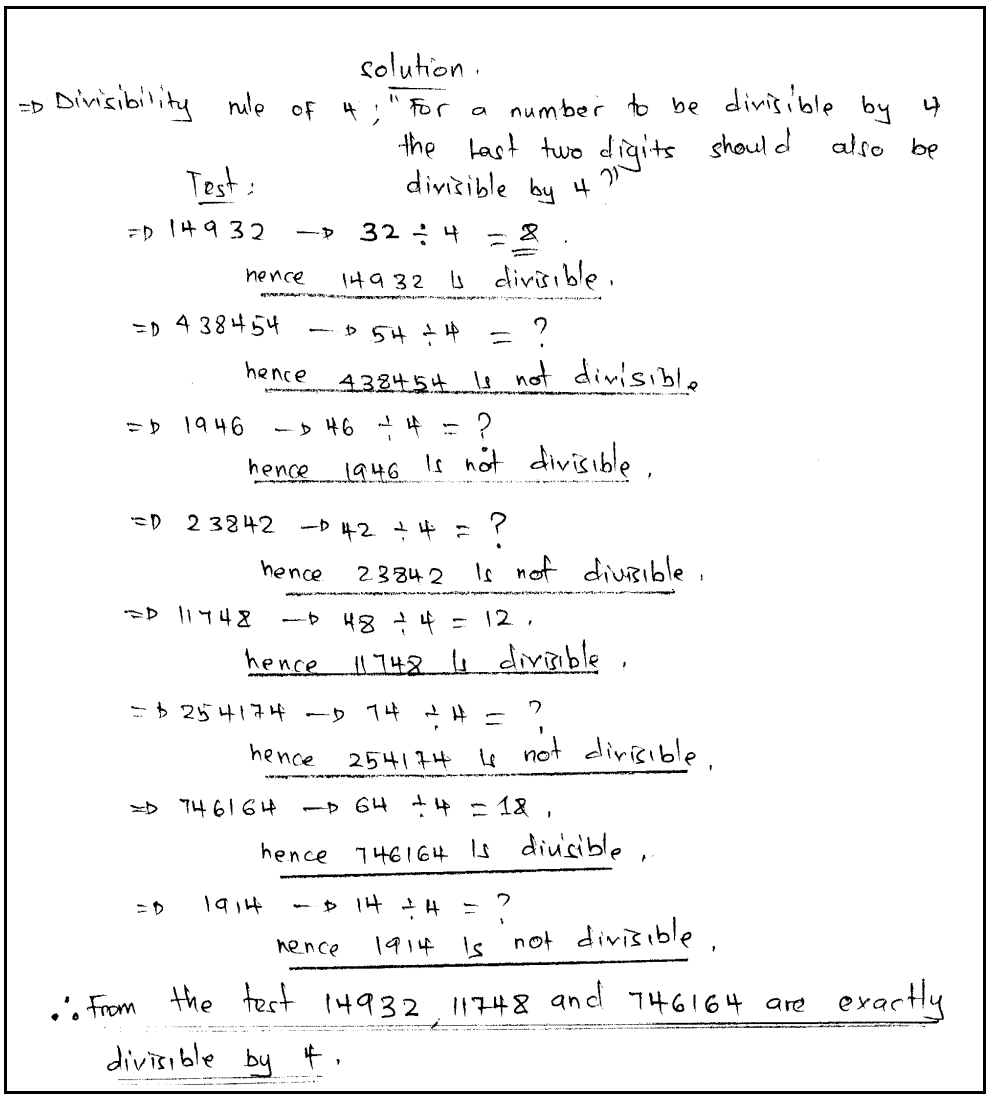
Therefore they divided $\frac{14932}{4}=3733$, $\frac{11748}{4}=2937$ and $\frac{746164}{4}=186541$.

These Numbers 14932, 11748 and 746164 has zero remainder, therefore they are exactly divisible by 4.

In part (b), the students who answered this question correctly used the technique to obtain the next number as the difference between the

consecutive terms are increasing by 4 each time starting with 7. Therefore, they applied the technique as $2_{+7}9_{+11}20_{+15}35_{+19}54_{+23}77_{+27}104$, and obtain the next three numbers as 54, 77 and 104 respectively.

In part (c), the analysis showed that the students were familiar with the sequences whereby they managed to use the given formula $n^{th} = \frac{n(n+1)}{2}$ and substituted correctly the value of $n = 14$ as $14^{th} = \frac{14(14+1}{2}$, when computed correctly they ended up with the 14th term as 105 which was the required answer. Extract 1.1 shows a sample of responses selected from one of the students who performed well in this question.



solution.

⇒ Divisibility rule of 4; "For a number to be divisible by 4 the last two digits should also be divisible by 4"

Test:

⇒ 14932 → 32 ÷ 4 = 8.
hence 14932 is divisible.

⇒ 438454 → 54 ÷ 4 = ?
hence 438454 is not divisible.

⇒ 1946 → 46 ÷ 4 = ?
hence 1946 is not divisible.

⇒ 23842 → 42 ÷ 4 = ?
hence 23842 is not divisible.

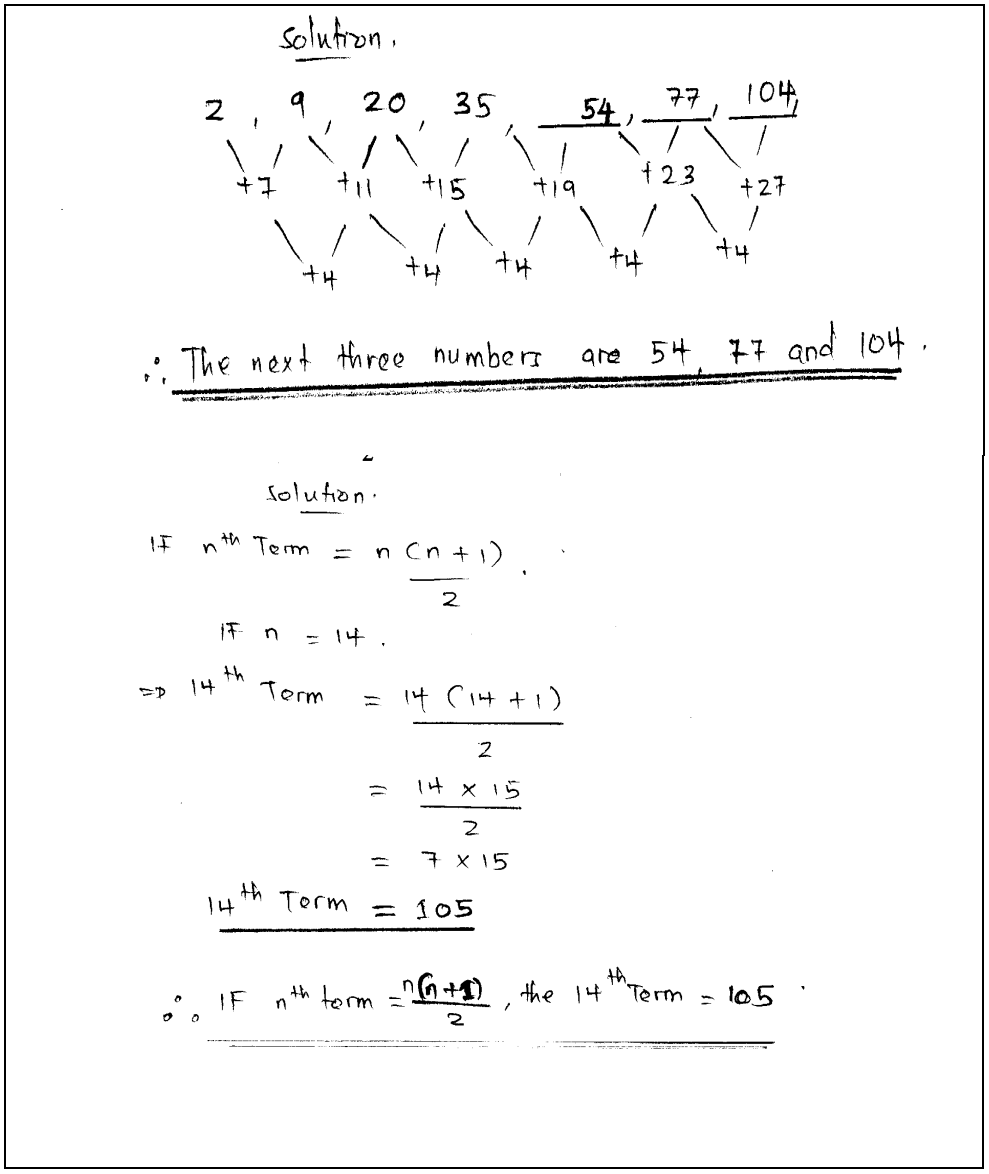
⇒ 11748 → 48 ÷ 4 = 12.
hence 11748 is divisible.

⇒ 254174 → 74 ÷ 4 = ?
hence 254174 is not divisible.

⇒ 746164 → 64 ÷ 4 = 18.
hence 746164 is divisible.

⇒ 1914 → 14 ÷ 4 = ?
hence 1914 is not divisible.

∴ From the test 14932, 11748 and 746164 are exactly divisible by 4.



Extract 1.1: A sample of correct responses in question 1

In extract 1.1, the student applied the rule of divisibility of 4 and obtained numbers which are divisible by 4. Also, the student identified the next three numbers of the given number pattern and lastly determine correctly the 14th term for the given formula.

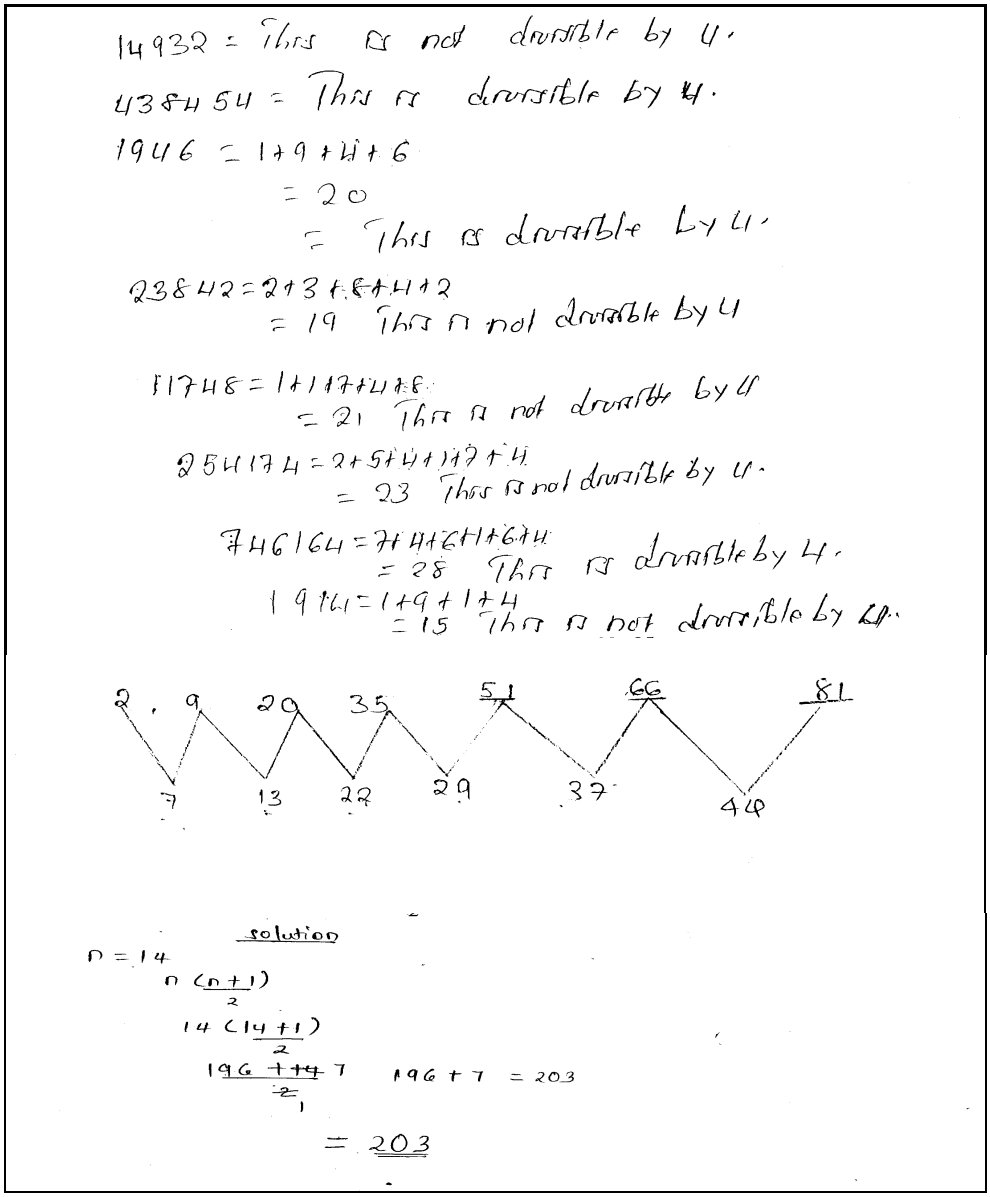
Despite the good performance highlighted, there were 32 (6.6 %) students whose performances were poor. Those students faced the following challenges: -

In part (a), some students failed to recall and apply correctly the divisibility rule, which could be used to test if a number is divisible by four (4). From the given whole numbers, the students took the sum of individual digits making a number and divide it by four (4), in case it is divisible by four (4) then, they concluded that the whole number also could be divisible by 4. From this findings, they disputed the weakness on the concept of numbers. While other students take the sum of the last two digits of the given number and divide the obtained number by 4, showing that they are not familiar with divisibility rules on natural numbers. On the other hand, some students applied the direct division method to test whether the given numbers are exactly divisible by 4, but they failed to perform mathematical operation correctly leading them to end up with wrong solution.

In part (b), some students failed to recognize the correct pattern that govern the determination of next number as a result these students applied incorrect mathematical operations, which finally resulted to obtain the incorrect next three terms. Other students managed to recognize that the difference between consecutive numbers is increasing by 4 each time starting with 7, but they stipulated poor operation on addition and finally obtain wrong solution. All these indicate that the students lacked skills and knowledge on numbers patterns.

In part (c), students identified that $n=14$ and showed a great skills by substituting it to the given equation as $14^{th}=\frac{14(14+1)}{2}$ when computed and simplified, resulted to $\frac{196+14}{2}$ and finally failed to simplify further this led to obtain incorrect value of n^{th} term = 203 instead of 105, this indicates poor basic operational skills on mathematics. Furthermore, other students expanded the right hand side of the given equation and obtained

$n^{th} = \frac{n^2 + n}{2}$ thereafter applied the technique of cross multiplication which led them to obtain $2n^{th} = n^2 + n$ and equated it with the 14^{th} term then compare it incorrectly resulted to the incorrect term of the 14^{th} as $\frac{2n^{th}}{n^2 + n}$. These students demonstrated that they had insufficient knowledge and skills in numbers operations. Extract 1.2 is sample responses from one of the students who responded in this question incorrectly.



Extract 1.2: A sample of incorrect responses in question 1

In extract 1.2, the student finds the sum of individual digits of a number and divide it by 4 as a technique used to determine if the given numbers are divisible by 4. Also the student finds the next number with no correct pattern of the sequence as well as failed to compute the 14^{th} term of the given formula.

2.2 Question 2: Algebra

The question had three parts (a), (b) and (c). In part (a), the students were required to solve for x if $\left|\frac{x-1}{x+1}\right|=3$. In part (b), the students were required to express h as a subject of the formula given that $s=\frac{wd}{h}(h+d)$ and in part (c), students were required to use graphical method to solve the system of simultaneous equations.

$$\begin{cases} x-y=3 \\ y-2x+5=0 \end{cases}$$

The analysis of data shows that, 484 (100%) students attempted this question, out of which 89 (18.4 %) students scored marks from 0 to 2.5, while 151 (31.2 %) students scored marks from 3.0 to 6.0 and 244 (50.4 %) students scored marks from 6.5 to 10. Therefore, the students' performance in this question was good since 395(81.6%) students scored from 3.0 to 10 marks. The students' performance summary is presented in Figure 3.

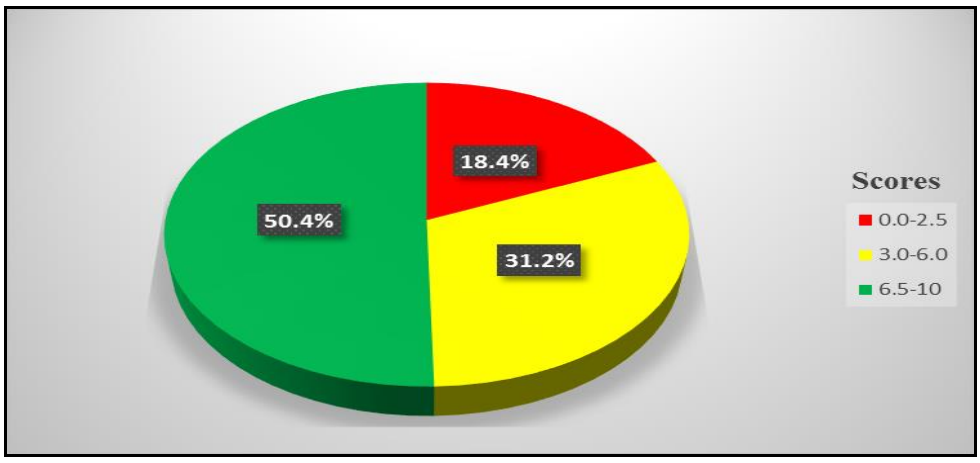


Figure 3: *Students' Performance in Question 2*

The analysis revealed that in part (a), the students who performed well in this question and managed to score full marks had an adequate knowledge and skills on absolute values, as they were able to recognise that the algebra given is absolute value, they introduced the sign $\pm$ and worked on it as $\left|\frac{x-1}{x+1}\right| = \pm\left(\frac{x-1}{x+1}\right)$, simplified and managed to get two equations as $x-1=3x+3$ or $-x+1=3x+3$, later solved the equations and get the value of $x=-2$ or $x=-\frac{1}{2}$.

In part (b), the students were conversant with cross multiplication technique, they responded correctly to the formula given as $s=\frac{wd}{h}(h+d)$, multiplied by h found in the denominator throughout and got $hs=wd(h+d)$, thereafter expanded and managed to get $hs=wdh+wd^2$. Later applied factorization technique and factorized h, finally got $h=\frac{wd^2}{s-wd}$ shows understanding on transposition of formulae.

In part (c), the students who performed well, and got all marks applied x and y intercept method as the approach: Thus for linear equation $x-y=3$

x	0	3
y	-3	0

Likewise for equation $y-2x+5=0$

x	0	$5/2$
y	-5	0

Thereafter, applied the drawing skills and graphical method to draw correctly the system of simultaneous equations $\begin{cases} x-y=3 \\ y-2x+5=0 \end{cases}$. Finally, identified the point of intersection as $(2,-1)$ and conclude that $x=2$ and $y=-1$.

Extract 2.1 is a sample response selected from one of the students who answered correctly this question.

Solution.

$$\left|\frac{x-1}{x+1}\right| = 3$$

Case I. + sign.

$$+\left|\frac{x-1}{x+1}\right| = 3$$

$$+\left(\frac{x-1}{x+1}\right) = 3.$$

$$\frac{x-1}{x+1} = 3$$

$$= \frac{x-1}{x+1} = 3$$

$$= \frac{x-1}{x+1} \times \frac{3}{1}$$

$$3(x+1) = x-1$$

$$3x+3 = x-1$$

$$3x-x = -1-3$$

$$\frac{2x}{2} = \frac{-4}{2}.$$

$$x = -2.$$

Case II. negative sign (-).

$$\left|\frac{x-1}{x+1}\right| = 3.$$

$$-\left(\frac{x-1}{x+1}\right) = 3.$$

$$\frac{-x+1}{x+1} \times \frac{3}{1}$$

$$3(x+1) = -x+1$$

$$3x+3 = -x+1$$

$$3x+x = 1-3$$

$$\frac{+4x}{4} = \frac{-2}{4}$$

$$x = -1/2$$

$\therefore$ $x = -2$ or $x = -1/2$

Solution

Make "h" the subject.

$$s = \frac{wd}{h}(h+d)$$

Multiply "h" both sides.

$$h \times s = \frac{wd}{h}(h+d) \times h$$

$$hs = wd(h+d)$$

$$hs = wdh + wd^2$$

$$hs - wdh = wd^2$$

$$h(s-wd) = wd^2$$

$$\frac{h(s-wd)}{s-wd} = \frac{wd^2}{s-wd}$$

$$h = \frac{wd^2}{s-wd}$$

$$\therefore h = \frac{wd^2}{s-wd}$$

Solution

By graphical method:

Case I:

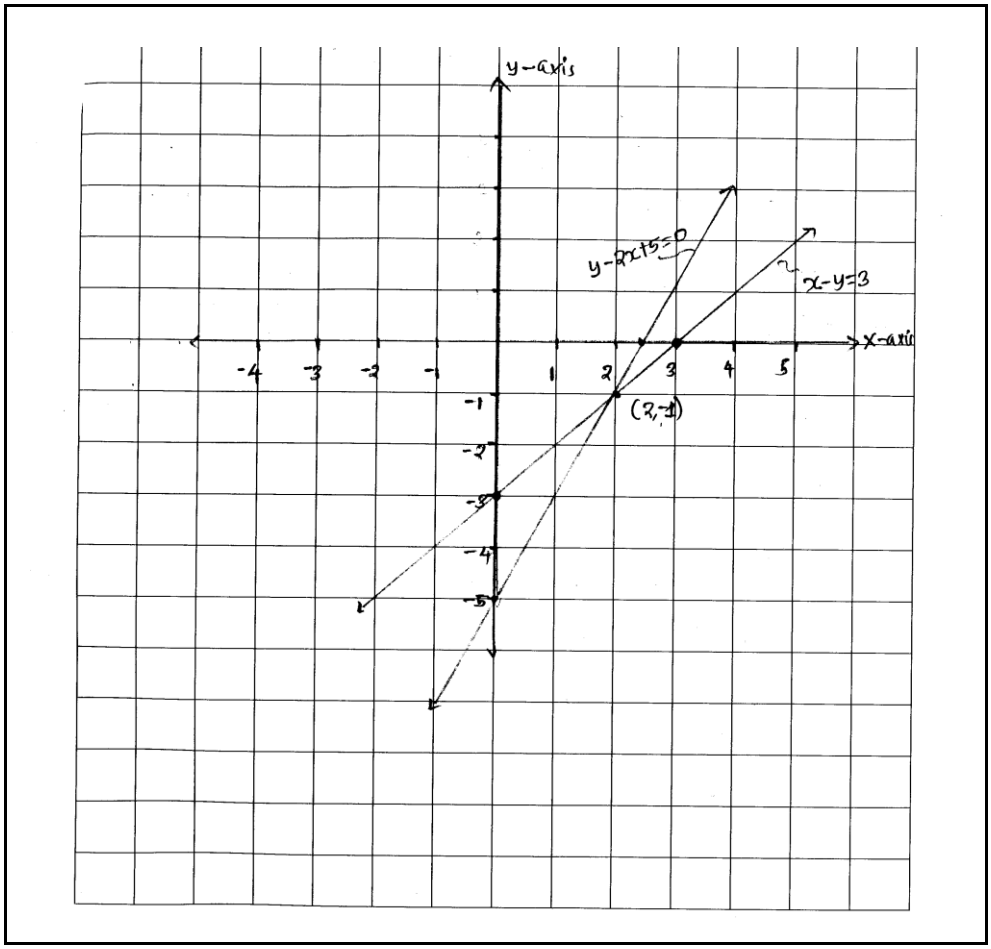
Construct table values for equation: $x - y = 3$ ... (i)

x	0	3
y	-3	0

Case II:

$y - 2x + 5 = 0$.

x	0	5/2
y	-5	0



Extract 2.1: A sample of correct responses in question 2

In extract 2.1, part (a), the student solved the given problem to obtain the values of x using procedures of absolute values. Also in part (b), make h the subject of the formula and lastly in part (c), solved the system of simultaneous equations graphically.

However, 89 (18.4 %) students scored low marks due to lack of knowledge and skills related to the question as follows: In part (a), some students did not realize that, they had to introduce $\pm$. They added the expressions of numerator and of denominator, for instance, $x-1+x+1=3$ simplified to $2x=3$ finally $x=1.5$ instead of $x=-2$ or $-\frac{1}{2}$. Other students, incorrectly equated the expression on denominator = 3 likewise numerator = 3 as separate equations. For example $x-1=3$ and $x+1=3$ simplified to $x=4$

or $x = 2$ instead of $x = -2$ or $-\frac{1}{2}$. Furthermore, other students managed to respond to the question as required but due to lack of mathematical operational skill, failed to introduce negative, as a result they got only one correct value $x = -2$ instead of both $x = -2$ and $-\frac{1}{2}$.

In part (b), some students attempted the question with inadequate knowledge in algebra, such that, instead of multiplying h on both sides of the equation, they factorized h. For instance in the given equation $s = \frac{wd}{h}(h + d)$ they wrote $s = \left[wd(1 + d)\right]h$, thereafter divided by $wd(1 + d)$ on both sides of the expression and obtained incorrect formula $h = \frac{s}{wd + wd^2}$ instead of $h = \frac{wd^2}{s - wd}$. Other students, managed to multiply h on both sides of the equation, that is $h \times s = wd(h + d) \times h$ and obtained a new equation $hs = wd(h + d)$. This solution implies lack of factorization knowledge of which led to divide the given equation by s on both sides and ended up with incorrect formula $h = \frac{wdh + wd^2}{s}$ instead of $h = \frac{wd^2}{s - wd}$. Furthermore, analysis shows that some students responded to the question with insufficient knowledge in algebra. The students multiplied h on both sides, thereafter continued with wrong procedures, for example $s = \frac{wd}{h}(h + d)$ multiplied by h resulted to $hs = wd(h + d)$ then they divided by wd on either side of the expression $\frac{hs}{wd} = \frac{wd}{wd}(h + d)$, finally obtained incorrect h as a subject of the equation given by $h = \frac{hs}{wd} - d$ instead of $h = \frac{wd^2}{s - wd}$.

In part (c), some students lack important knowledge on drawing skills especially on determination of x - intercept and y - intercept. For example, the equation, $x - y = 3$ constructed as

x	0	-3
y	3	0

Instead of

x	0	3
y	-3	0

Also for equation $-2x+y=-5$ constructed as

x	0	-5
y	$\frac{5}{2}$	0

Instead of

x	0	$\frac{5}{2}$
y	-5	0

This incorrect intercepts, led to incorrect graph drawings, as illustrated in extract 2.2. Further analysis shows that, other students ignored the instructions given in the question as they applied wrongly the elimination method to solve the simultaneous equation without even rearranging it. For instance $\begin{cases} x-y=3 \\ y-2x+5=0 \end{cases}$ of which resulted to $x=-2$ and $y=1.5$ instead of $x=2$ and $y=-1$. Extract 2.2 is a sample of incorrect response selected from one of the students.

Given: $\left|\begin{matrix} x+1 \\ x+1 \end{matrix}\right| = 3$

$\pm \left|\begin{matrix} x-1 \\ x+1 \end{matrix}\right| = 3$

$x-1+1 = 3+1$

$x+1-1 = 3-1$

$x = 4$ or $x = 2$

$\therefore \left|\begin{matrix} x-1 \\ x+1 \end{matrix}\right| = 3$ i.e. $x = 4$ or $x = 2$.

Solution

$S = \frac{wd}{h}(h+d)$

$S = \frac{wd(h+d)}{h}$

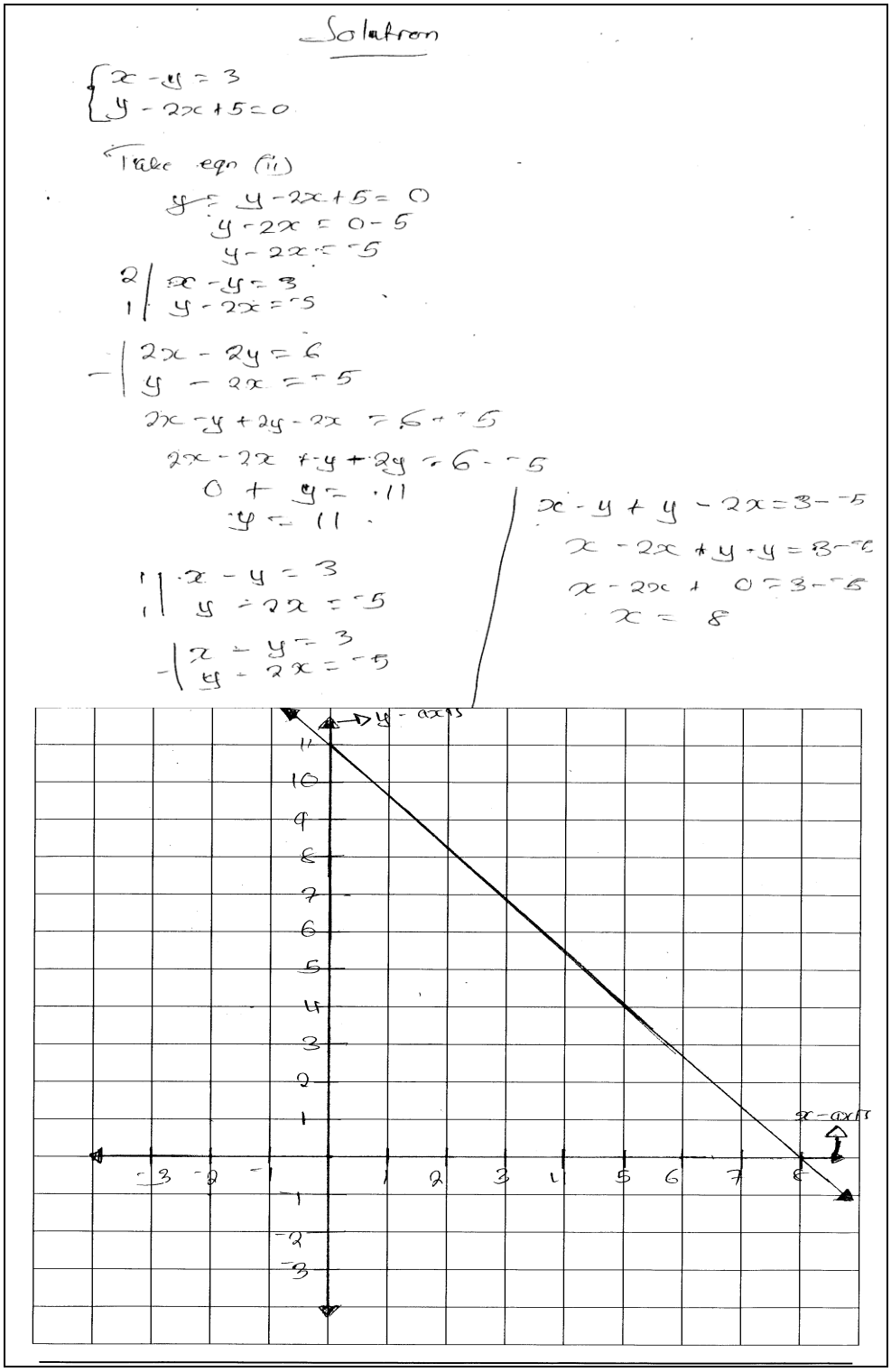
$S = \frac{wdh+wd^2}{h}$

$S = \frac{wdh+wd^2}{h}$

$\frac{wdh+wd^2}{S} = \frac{Sh}{S}$

$\frac{wdh+wd^2}{S} = h$

$S = \frac{wd}{h}(h+d) = \frac{wdh+wd^2}{S} = h$



Extract 2.2: A sample of incorrect responses in question 2

In extract 2.2, part (a), the student solved the given equation without considering fraction and procedures such as using absolute values. In part (b), the student undergoes transposition of the given formula without collecting like terms together and doing factorization. In part (c), the student solved the given system of simultaneous equations without the points needed to draw the graph, of which shows lack of knowledge and skills in solving algebraic problems graphically.

2.3 Question 3: Geometrical Constructions

This question had two parts (a) and (b). In part (a)(i), the students were asked to define the term quadrilateral and in part (a)(ii), the students were required to calculate the number of side of the polygon given that each interior angle of the regular polygon is 150°. In part (b), the students were given the interior angles of a hexagon as 100°, 110°, 120° and 128°, hence required to find the size of the other two angles which are equal.

The analysis of data shows that, 484 (100%) students attempted this question, out of which 97 (20.0%) students scored marks from 0 to 2.5, while 77 (15.9 %) students scored marks from 3.0 to 6.0 and 310 (64.1 %) students scored marks from 6.5 to 10. Therefore, the students' performance in this question was good as 387 (80%) students scored from 3.0 to 10 marks. The summary of the students' performance is presented in Figure 4.

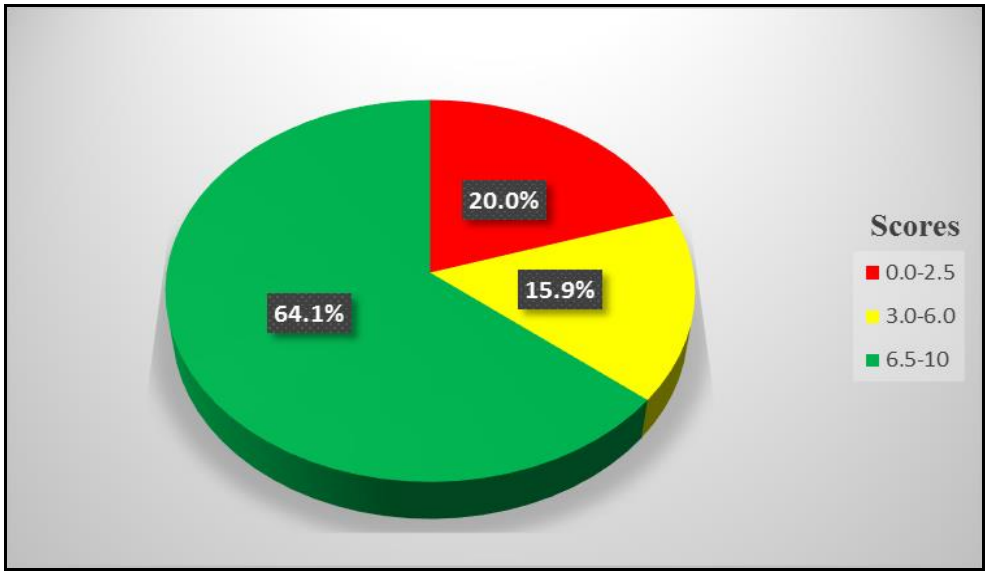


Figure 4: *Students' Performance in Question 3*

In part (a)(i), the analysis revealed that, the students had adequate knowledge and skills on geometry as they managed to recall correctly that quadrilateral is a four (4) sided polygon. Examples of quadrilaterals are rhombus, square, and rectangular figures. In part (a)(ii), the students managed to recall correctly the concept that, $\text{Interior angle} + \text{Exterior angle} = 180^{\circ}$, thereafter substituted the value of interior angle as 150°. That is $150^{\circ} + \text{Exterior angle} = 180^{\circ}$, later simplified and obtain the exterior angle as $180^{\circ} - 150^{\circ} = 30^{\circ}$. After that, they applied correctly the formula to determine the number of sides given by $n = \dfrac{360^{\circ}}{exteriorangle}$ and later computed it as $n = \dfrac{360^{\circ}}{30^{\circ}}$ to get the value of $n = 12$ sides. Not only that but also some students applied the formula, $\mathrm{I} = \dfrac{(n-2)180^{\circ}}{n}$ thereafter substituted the value of the interior angle 150° as $150^{\circ} = \dfrac{(n-2)180^{\circ}}{n}$, then computed and simplified it as $150^{\circ} = (n-2)180^{\circ}$, finally got the number of sides of a regular polygon equals to 12 sides.

In part (b), the students who performed well in this question were able to recall that hexagon is a polygonal figure with six sides (n = 6), thereafter applied correctly the formula for calculating the sum of interior angle = $(n-2)180^{\circ}$ when computed it, resulted to sum of interior angle = $(6-2)180^{\circ}$ and finally obtained the sum of interior angle $= 720^{\circ}$. After all these procedures, they took the summation of the entire given angles as $100^{\circ} + 110^{\circ} + 120^{\circ} + 128^{\circ}$ then they simplified as $458^{\circ} + 2\theta = 720^{\circ}$ finally the size of the remaining angle as $\theta = 131^{\circ}$. Extract 3.1 is a sample response selected from one of the students who answered correctly this question.

Solution:

(i) What is a quadrilateral?

- It a polygon which has four sides and its angles add up to 360° example square, rectangle.

(ii) Data:

each interior angle = 150°

number of sides = ?

Each interior angle = $\frac{(n-2)\,180}{n}$

where n is number of sides

$150^\circ = \frac{(n-2)\,180^\circ}{n}$

$\frac{150^\circ}{1} \times \frac{180n - 360^\circ}{n}$

$150n = 180n - 360^\circ$

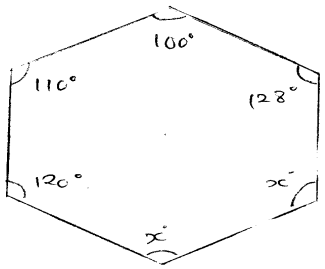
$150n - 180n = -360$

$\frac{-30n}{-30} = \frac{-360}{-30}$

$n = 12$ sides.

∴ number of sides of a regular polygon is 12

Solution



first solve for x

hexagon has 720° sum of ID angles

$100^\circ + 110^\circ + 120^\circ + 128^\circ + x + x = 720^\circ$

$458 + 2x = 720^\circ$

$2x = 720^\circ - 458^\circ$

$\frac{2x}{2} = \frac{262^\circ}{2}$

$x = 131^\circ$

∴ The size size of each angle is 131°

Extract 3.1: A sample of correct responses in question 3

In extract 3.1, part (a)(i), the student defined quadrilateral correctly and in part (a)(ii), the student using a correct formula of interior angle to determine the number of sides of the polygon. In part (b), the student understood the meaning of the hexagon then obtained the size of the two equal angles. It is observed that the student had adequate knowledge and skills in *Geometrical constructions.*

Nevertheless 97 (20.0%) students did not respond well to the question and scored low marks as they encountered the following challenges: In part (a)(i), the student failed to define the term quadrilateral as they confused it with the equilateral triangle as seen in extract 3.2.

Moreover, some of the students recalled the term quadrilateral as the line or object which have the shape of the triangle implies that, the students were not familiar with geometrical constructions. In part (a)(ii), some of the students applied incorrect formula used to determine the size of an interior angle as $\frac{(n+2)}{2}180^\circ$ instead of $\frac{(n-2)}{2}180^\circ$ when calculating the number of sides (n) of a regular polygon, hence its computation and simplification led them to incorrect number of sides of regular polygon as $n=-12$ sides. While others inappropriately used the formula of finding the size of an exterior angle equals to the given interior angle 150°, hence obtained different value of the number of sides, instead of $n=12$ sides. This shows that students were in short of geometrical construction knowledge and skills on how to find number of sides of the polygon.

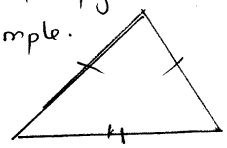
In part (b), some students interchanged the sum of interior angle of hexagon with that of pentagon, where the students took 540° as the sum of interior angles found within hexagon rather than taking 720° when determined the value of the other two equal angles of the hexagon of which resulted to incorrect value of $\theta=90^\circ$.

Other students recalled correctly that hexagon is a polygonal figure with six sides but failed to recall the sum of interior angle is 720°, instead the students come up with 1080° of which led to incorrect answer $x=311^\circ$ (extract 3.2).

solution

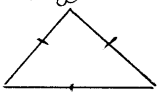
(i) quadrilateral is a figure which three sides which are equal

example.



∴ quadrilateral is a figure /polygon which have three sides which are equal

example is



ii) $(n-2)180^\circ$

$150^\circ = 180^\circ n - 360^\circ$

$360^\circ + 150^\circ = 180^\circ n$

$\frac{510^\circ}{180^\circ} = \frac{180^\circ n}{180^\circ}$

$n = 3$

solution.

$100^\circ + 110^\circ + 120^\circ + 128^\circ + 2x = 1080^\circ$

$210^\circ + 248^\circ + 2x = 1080^\circ$

$458^\circ + 2x = 1080^\circ$

$2x = 1080^\circ - 458$

$\frac{2x}{2} = \frac{622}{2}$

$x = 311$

$\therefore x = 311^\circ$

Extract 3.2: A sample of incorrect responses in question 3

In extract 3.2, part (a)(i), the student failed to defined quadrilateral instead gave the definition of equilateral triangle. In part (a)(ii), the student applied the unrequired formula according to the given data to determine number of sides of the polygon. In part (b), the student failed to understand the sum of interior angles of hexagon and led to obtain incorrect answer.

2.4 Question 4: Locus

This question composed of two parts (a) and (b). In part (a), the students were required to draw the locus of $(x, y): \mathrm{y} = -x^2 + 1$, and state the value of y which makes the locus to be defined. In part (b), the students were asked to find and draw the loci of a moving point whose distance from y – axis is equal to its distance from the x – axis.

The analysis of data shows that, 484 (100%) students attempted this question, out of which 450 (93%) students scored marks ranging from 0 to 2.5, while 34 (7%) students scored marks ranging from 3.0 to 6.0 and no students scored marks from 6.5 to 10. Generally, the students' performance in this question was weak. The summary of students' performance is presented in Figure 5.

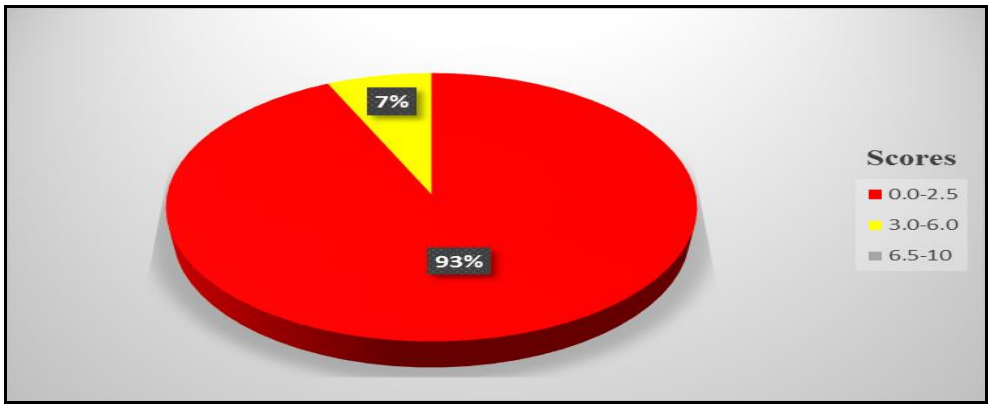


Figure 5: *Students' Performance in Question 4*

The analysis depicts that, students who scored low marks encountered the following challenges: In part (a), some students failed to identify that the coefficient of x^2 is negative hence its effects led them to incorrectly construct the table of value. For example, one of the students constructed it as;

x	-2	-1	0	1	2
$y=-x^2+1$	5	2	1	2	5

Instead of

x	-2	-1	0	1	2
$y=-x^2+1$	-3	0	1	0	-3

The obtained incorrect table of value led students to draw incorrect graph curved upward instead of downward. Other students applied $x-$ intercept and $y-$ intercept method wrongly and obtained values as

x	0	1
$y=-x^2+1$	1	0

Instead of

x	-3	-2	-1	0	1	2	3
$y=-x^2+1$	-8	-3	0	1	0	-3	-8

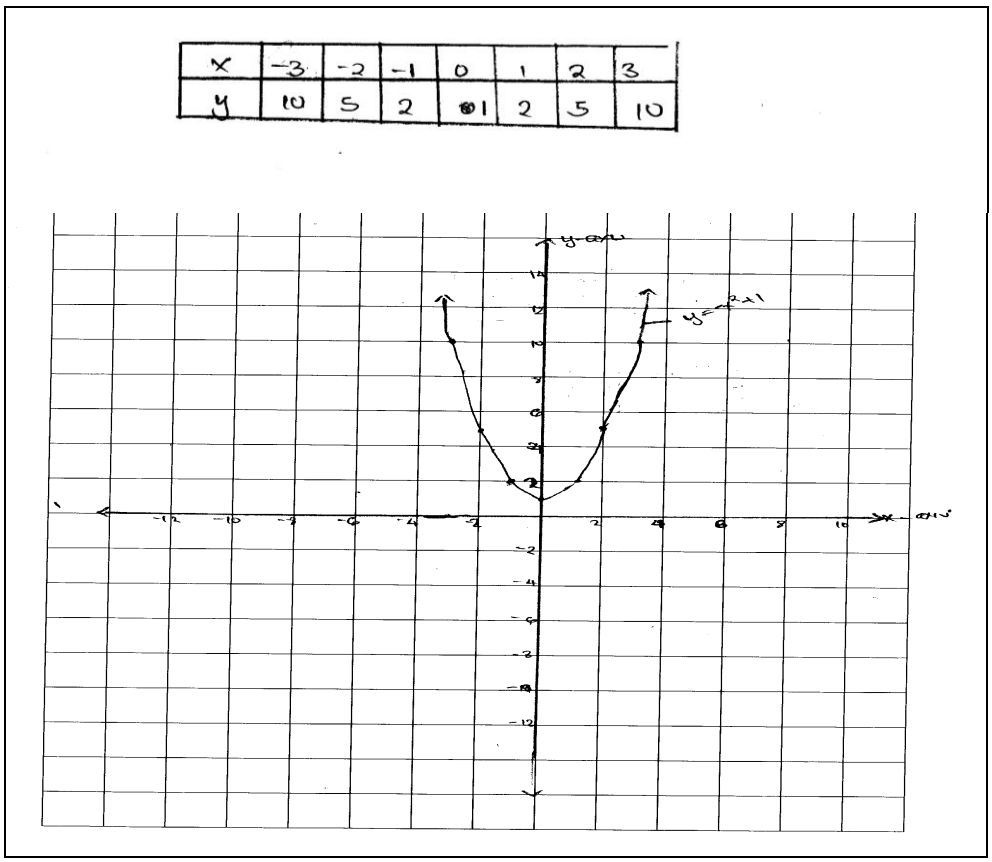
The incorrect table of value led to incorrect graph that look as linear instead of a parabola opening downward. Moreover, when stating the value of y which make the locus to be defined some students whose graph opened upward, they listed 5, 2, 1, 2 and 5. Originated from incorrect table of values constructed earlier. Other students drew a linear graph and shading it on one side, thereafter concluded that the value of y which make the locus to be defined is the shaded region. This approach was contrary to the demand of the question.

In part (b), some students failed to interpret correctly the question especially on loci of a moving point whose distance from $y-$ axis is equal to the distance form $x-$ axis. These students correctly recalled the distance formula as $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, identified incorrect values $A(-2, 2)$, $B(2, 2)$, $P(x, y)$. Later substituted the values in the formula $d=\sqrt{(x-2)^2+(y+2)^2}$, ultimately they wrote AP = BP, led to

$\sqrt{(x-2)^2+(y+2)^2} = \sqrt{(x-2)^2+(y-2)^2}$. Then squared both sides of the expression, to obtain $(x-2)^2+(y+2)^2 = (x-2)^2+(y-2)^2$ finally ended up with incorrect locus given by $8y=0$ instead of $y=x$ or $y=-x$. Also other students recognized the points as (0, 0) and recalled correctly the distance formula as $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ then responded as $d=\sqrt{(x-0)^2+(y-0)^2} = d=\sqrt{(x-0)^2+(y-0)^2}$ Simplified to get

$d=\sqrt{x^2+y^2} = d=\sqrt{x^2+y^2}$

Thereafter square both sides, to obtained $d=x^2+y^2=x^2+y^2$. Finally it yielded $d=2x^2+2y^2$ instead of $y=x$ or $y=-x$. This indicates that students had insufficient knowledge on locus as well as basic operations. Extract 4.1 is a sample response selected from one of the students who responded incorrectly.



x	-3	-2	-1	0	1	2	3
y	10	5	2	1	2	5	10

A(-2,2) B(2,2) P(x,y)

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\overline{AP} = \overline{BP}$

$AP = \sqrt{(x-2)^2 + (y+2)^2}$

$AP = \sqrt{(x-2)^2 + (y+2)^2}$

$BP = \sqrt{(x-2)^2 + (y-2)^2}$

$AP = BP$

$\left(\sqrt{(x-2)^2 + (y+2)^2}\right)^2 = \left(\sqrt{(x-2)^2 + (y-2)^2}\right)^2$

$(x-2)^2 + (y+2)^2 = (x-2)^2 + (y-2)^2$

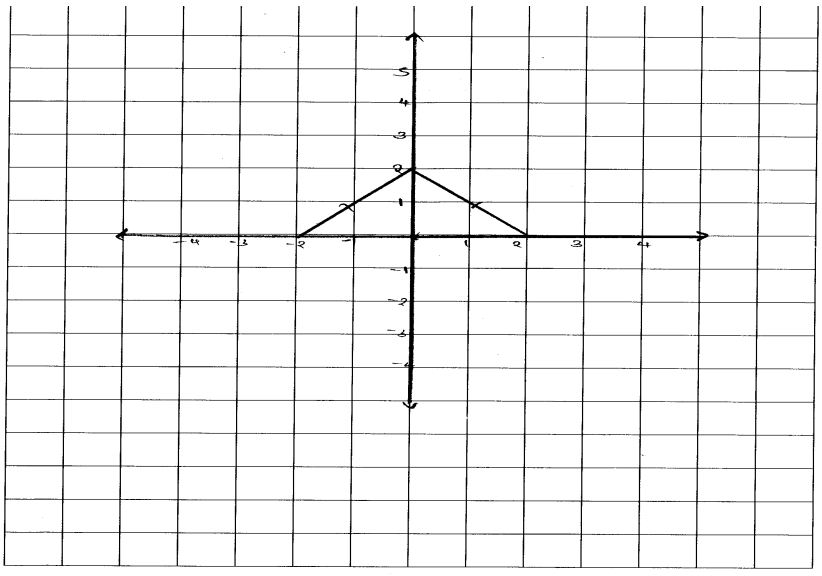
$x^2 - 4x + 4 + y^2 + 4y + 4 = x^2 - 4x + 4 + y^2 - 4y + 4$

$-4x + 4 + 4y + 4 = -4x + 4 - 4y + 4$

$-4x + 4y + 4 - 4 + 4y + 4y + 4 - 4 = 0$

$8y = 0$.

∴ The locus is $8y = 0$.



Extract 4.1: A sample of incorrect responses in question 4

In extract 4.1, part (a), the student failed to obtain the correctly table of values after considering the negative sign within the square of x and not as a coefficient of x, hence incorrectly drew the graph of the locus, as a result failed to define it. In part (b), the student identified wrong points of the locus, hence failed to obtain the correct locus of a moving point and eventually drew the required graph incorrectly.

Despite high percentage of weak performance in the question there were few students who performed well and managed to score full marks. In part (a), they constructed correctly the table of values for $\mathrm{y} = -x^2 + 1$. As

x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8

They applied correctly the constructed table of values to draw the locus of $(x, y) : \mathrm{y} = -x^2 + 1$. Later stated correctly that $y \leq 1$ make the locus to be defined.

In part (b), the students let the moving point be (x, y). Then recognized that distance from $y-$axis = distance from $x-$axis. Implying that,

$$\sqrt{(x-0)^2 + (y-y)^2} = \sqrt{(x-x)^2 + (y-0)^2}$$

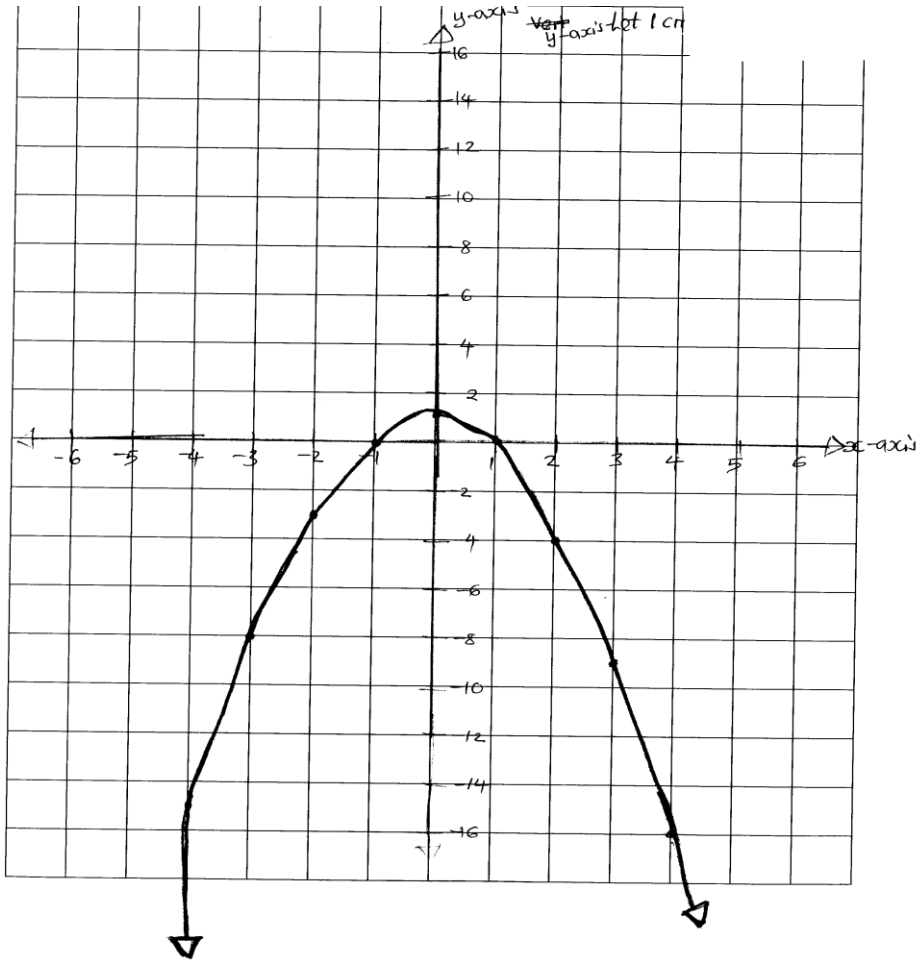
After squaring on both sides and mathematical operations, they obtained $x^2 = y^2$, finally managed to take the square root on both sides and ended up with $y = \pm x$ and concluded that the locus of the moving point is either $y = x$ or $y = -x$. Furthermore, they drew graphs of the two loci, by tabulating correctly values of the two functions $y = x$ and $y = -x$. That is;

x	-3	-2	-1	0	1	2	3
$y = x$	-3	-2	-1	0	1	2	3
$y = -x$	3	2	1	0	-1	-2	-3

Extract 4.2 is a correct response selected from one of the students who attempted the question correctly.

Soln.

x	-4	-3	-2	-1	0	1	2	3	4
$-x^2$	-16	-9	-4	-1	0	-1	-4	-9	-16
+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
y	-15	-8	-3	0	+1	0	-3	-8	15



Soln

Point = (x, y)

Point A = $(0, y)$

Point B = $(x, 0)$

$PA = PB$.

$(x, y)(0, y) = (x, y)(x, 0)$

Formula:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

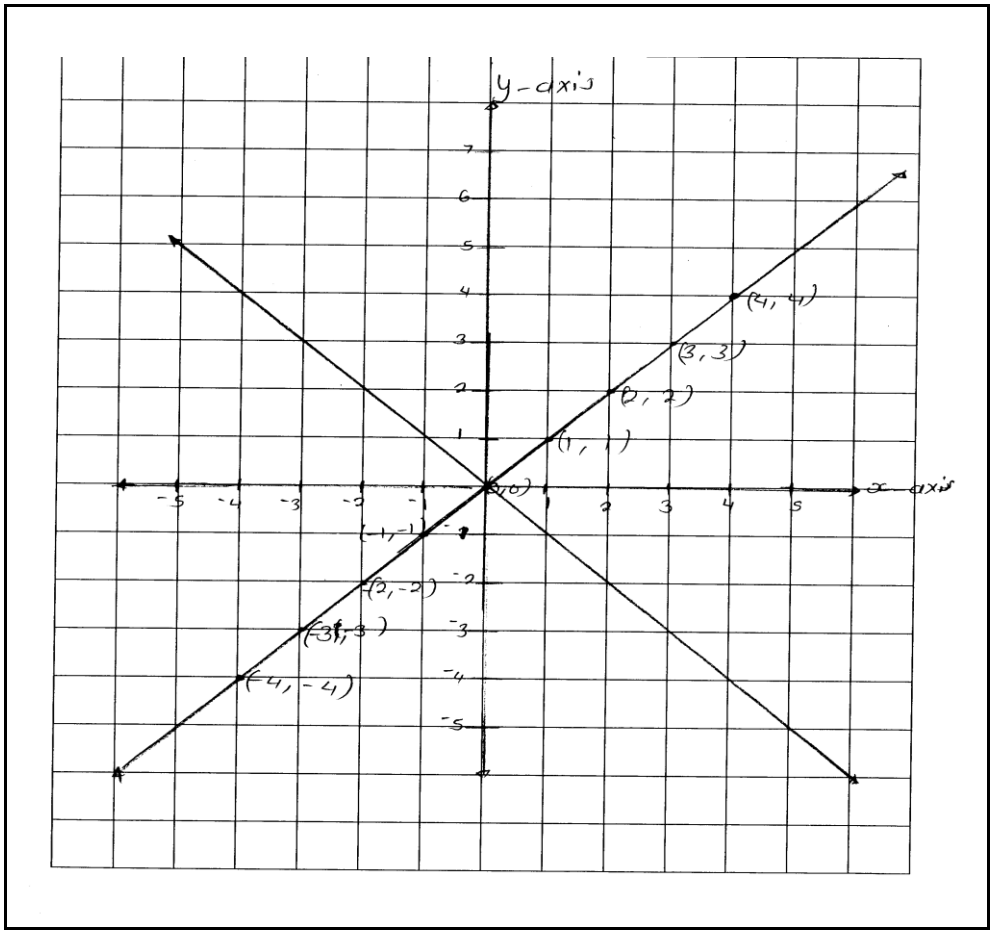
$$\left(\sqrt{(x-0)^2 + (y-y)^2}\right)^2 = \left(\sqrt{(x-x)^2 + (y-0)^2}\right)^2$$

$$x^2 + 0 = y^2$$

$$\sqrt{x^2} = \sqrt{y^2} \quad \text{--- (1)}$$

$$y = \pm x$$

x	-3	-2	-1	0	1	2	3
$y = x$	-3	-2	-1	0	1	2	3
$y = -x$	3	2	1	0	-1	-2	-3



Extract 4.2: A sample of correct responses in question 4

In extract 4.2, part (a), the student managed to draw the locus of the given equation by first constructing correctly a table of values used to draw the locus. In part (b), the student identified the three points used to obtain the locus and lastly draw correctly the locus of a moving point (x, y).

2.5 Question 5: Coordinate Geometry

This question composed of two parts (a) and (b). In part (a), the students were asked to find the system of simultaneous equations satisfying the given graph and in part (b), the students were asked to:

(i) find the value of k so that $(1, k+2), (4, 3k)$, $(10, 6k)$ are collinear.

(ii) find k so that $ky + 3x = 2$ is parallel to $y = \frac{1}{2}x - 4$.

The analysis of data shows that, 484 (100%) students attempted this question, out of which 338 (69.8%) students scored marks ranging from 0 to 2.5, while 132 (27.3%) students scored marks ranging from 3.0 to 6.0 and 14 (2.9%) students scored marks ranging from 6.5 to 10. Therefore, the students' performance in this question was average. Figure 6 illustrates the students' performance in this question.

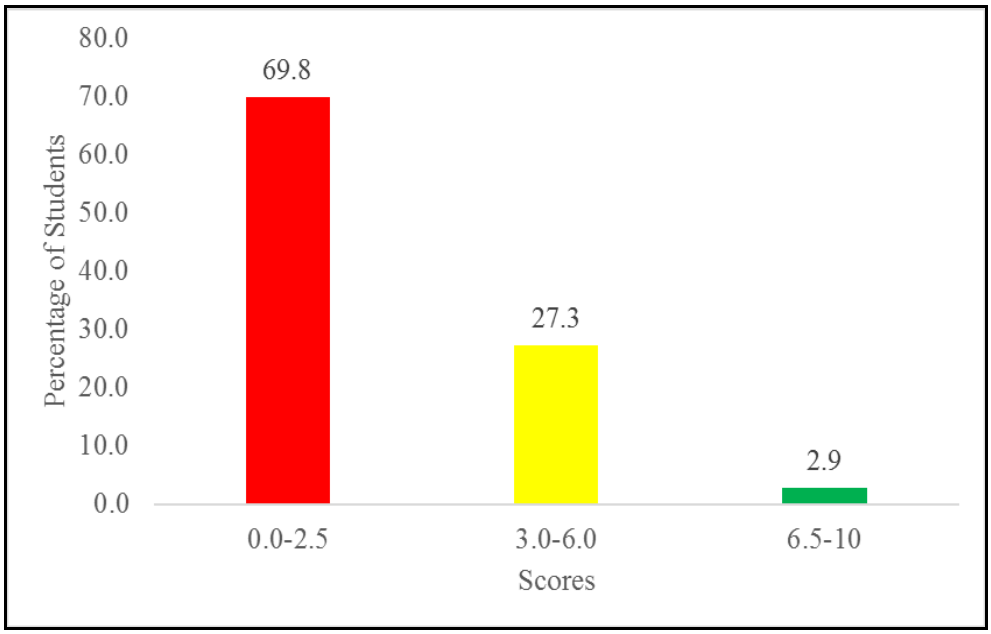


Figure 6: *Students' Performance in Question 5*

In part (a), according to the analysis the students who responded correctly to this question and managed to score full marks, expressed good understanding on coordinate geometry concepts, such that clearly identified the line labelled L_1 passes thought points $(0,-2)$ and $(3,1)$ while the line labelled L_2 passes through the point $(3,1)$ and $(4,-1)$. The two lines intersect at the point $(3,1)$. Finally worked out for slope of L_1 and L_2 as follows:

The slope of L_1

$$m_1 = \frac{-2-1}{0-3} = \frac{-3}{-3} = 1 \text{ and}$$

The slope of L_2

$m_2 = \frac{1-(-1)}{3-4} = \frac{2}{-1} = -2$

They supposed $P(x, y)$ be a variable point on line L_1 and line L_2, so that recalled correctly the formula for equation as $\frac{y-y_1}{x-x_1} = m$, and substituted the identified point in the formula, that is $L_1 : \frac{y--(2)}{x-0} = 1$ and simplified to $y - x = -2$, also $L_2 : \frac{y-1}{x-3} = -2$ simplified to $2x + y = 7$. Ultimately, concluded that the system of simultaneous equations satisfying the given graph is $\begin{cases} x - y = 2 \\ 2x + y = 7 \end{cases}$

In part (b)(i), the students were familiar with the meaning of collinear points, they stated correctly that collinear points are points which lie on the same line. Thereafter, recalled the formula for slope $(m) = \frac{y_2 - y_1}{x_2 - x_1}$, then worked out for slope (m_1) of $(1, k+2)$ and $(4, 3k)$ thus $(m_1) = \frac{3k-(k+2)}{3}$ simplified to $\frac{2k-2}{3}$ . . . . (i).

While slope (m_2) of $(4, 3k)$ and $(10, 6k)$, is found to be:

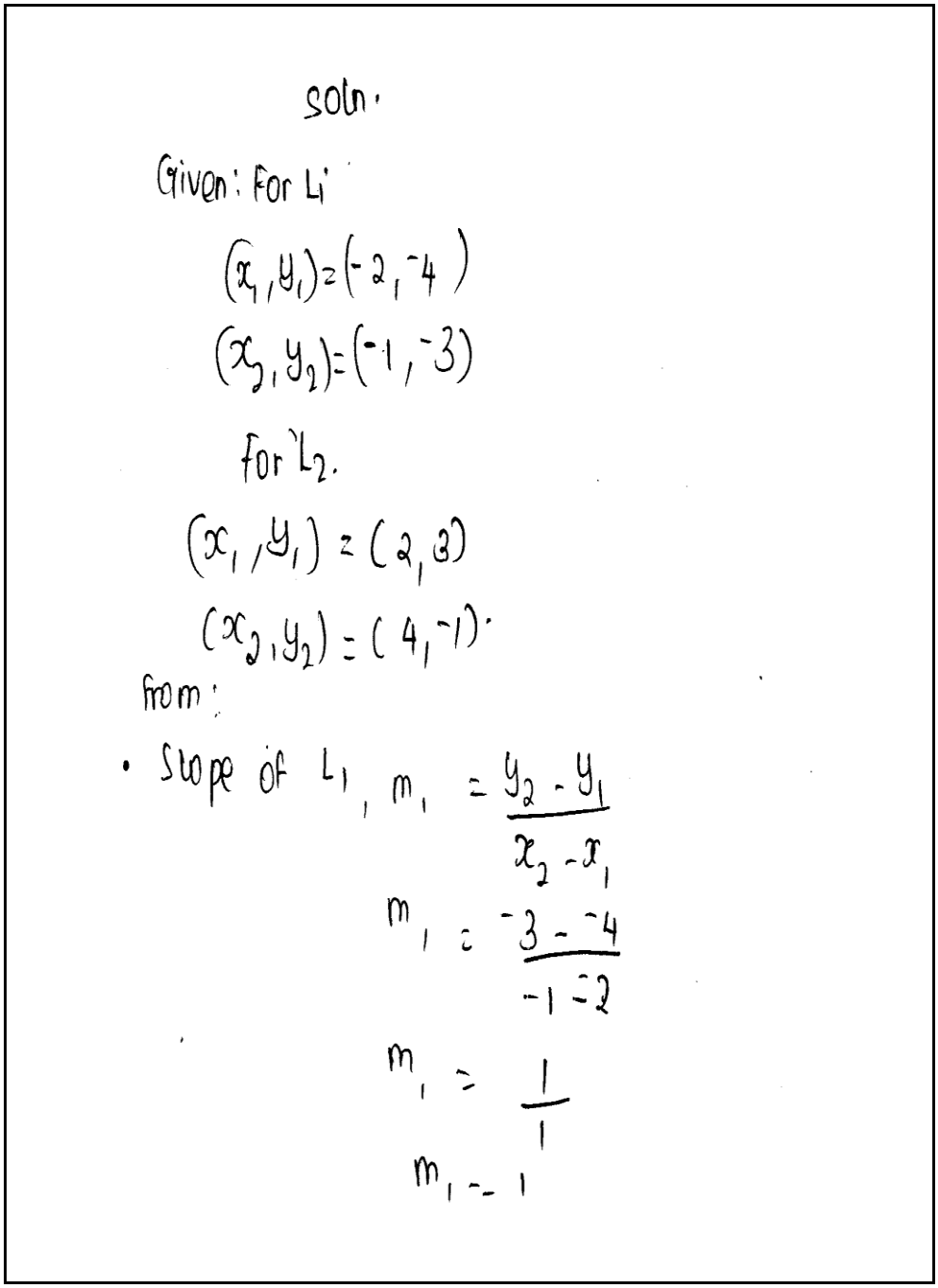
$m_2 = \frac{6k-3k}{10-4}$ Simplified to $\frac{k}{2}$ . . . . (ii)

Thereafter, they applied the parallel lines concept $m_1 = m_2$, equated the two slope as $\frac{2k-2}{3} = \frac{k}{2}$ reduced to $4k - 4 = 3k$, finally obtained $k = 4$.

(ii) The students managed to determine the slope of $ky + 3x = 2$ by expressing y as the subject of the equation, thus $y = \frac{-3}{k}x + \frac{2}{k}$ . . . (i)

While the slope of the line $y = \frac{1}{2}x - 4$ was identified as $\frac{1}{2}$, later retrieved well the concept of parallel lines, that is $m_1 = m_2$ then equated the two

slopes as $\frac{-3}{k}=\frac{1}{2}$, simplified correctly to obtain $k=-6$. Extract 5.1 is a correct response selected from one of the students who attempted this question correctly.


soln.

Given: For L_1

$(x_1, y_1) = (-2, -4)$

$(x_2, y_2) = (-1, -3)$

For L_2.

$(x_1, y_1) = (2, 3)$

$(x_2, y_2) = (4, -1)$.

from:

- Slope of L_1, $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{-3 - -4}{-1 - -2}$

$m_1 = \frac{1}{1}$

$m_1 = 1$

• Slope of L_2, $m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_2 = \frac{-1 - 3}{4 - 2}$$

$$m_2 = \frac{-4}{2}$$

$$m_2 = -2.$$

Ⓐ equation for L_1

$$y = m(x - x_1) + y_1$$
$$y = 1(x - -2) + -4$$
$$y = 1(x + 2) - 4$$
$$y = x + 2 - 4$$
$$y = x - 2. \quad ---(i)$$

Equation of L_2.

$$y = m(x - x_1) + y_1$$
$$y = -2(x - 2) + 3$$
$$y = -2x + 4 + 3$$
$$y = -2x + 7 \quad ---(ii)$$

From eqn (i)

$$y = x - 2.$$
$$2 = x - y \quad ----(i)$$

From eqn (ii)

$$y = -2x + 7$$
$$y + 2x = 7 \quad ---(ii)$$

$$\begin{cases} x - y = 2 & --(i) \\ 2x + y = 7 & --(ii) \end{cases}$$

∴ The required ~~set~~ system of simultaneous equation is $\begin{cases} x - y = 2 \\ 2x + y = 7 \end{cases}$

(b) (i) Find the value of k so that $(1, k+2)$, $(4, 3k)$ and $(10, 6k)$ are collinear.

(ii) Find k so that line $ky+3x=2$ is parallel to line $y=\frac{1}{2}x-4$.

Soln

5. b) i) Given: A(1, K+2)

B(4, 3K)

C(10, 6K)

Req: To find the value of K.

From:

- Slope of $\bar{AB}$, $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_1 = \frac{3K - (K+2)}{4 - 1}$$

$$m_1 = \frac{3K - K - 2}{3}$$

$$m_1 = \frac{2K - 2}{3}$$

- Slope of $\bar{BC}$, $m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_2 = \frac{6K - 3K}{10 - 4}$$

$$m_2 = \frac{3K}{6}$$

- Slope of $\bar{AC}$, $m_3 = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_3 = \frac{6K - (K+2)}{10 - 1}$$

$$m_3 = \frac{6K - K - 2}{9}$$

$$m_3 = \frac{5K - 2}{9}$$

$m_1 = m_2 = m_3$ (Are collinear)

$$\frac{2K-2}{3} = \frac{3K}{6} = \frac{5K-2}{9}$$

$$\frac{2K-2}{3} \times \frac{3K}{6}$$

$$9K = 6(2K-2)$$

$$9K = 12K - 12$$

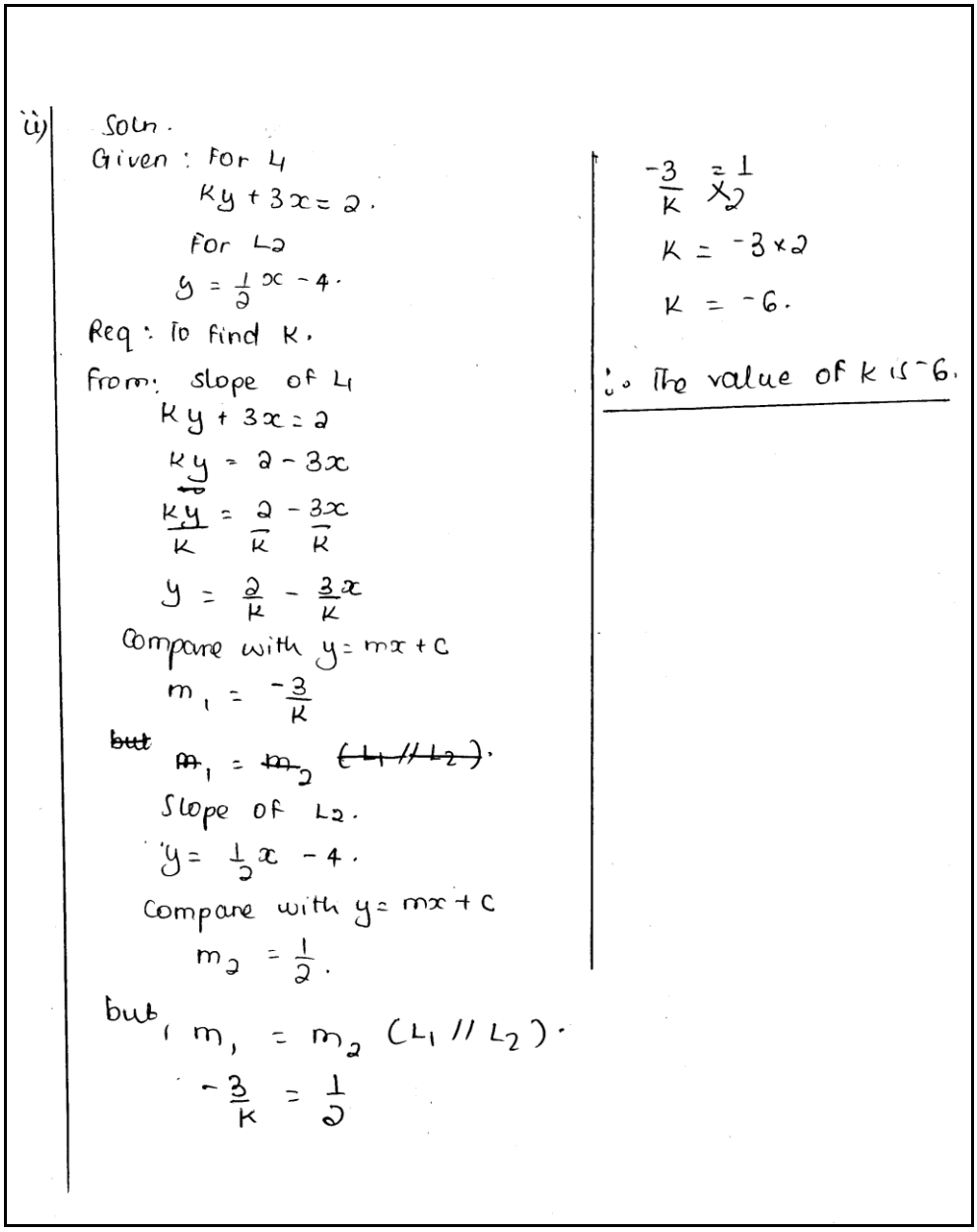
$$9K - 12K = -12$$

$$-3K = -12$$

$$\frac{-3K}{-3} = \frac{-12}{-3}$$

$$K = 4.$$

∴ The value of K is 4



Extract 5.1: A sample of correct responses in question 5

In extract 5.1, part (a), the student determined the points through which the two lines passes and used it to obtain slopes of the lines, at last the student found the equations of the lines by using the obtained slopes. In part (b)(i), the student applied the concept of the points which are collinear (having the same slope) to determine the value of k. In part (b)(ii), the student rewrite the given equation in the form of $y = mx + c$ to obtain slope for each line of the equation then equating the two slopes to solve the value of k.

In spite of a substantial number of students who had a good performance, however 69.8% of students who sat for the assessment scored low marks, because of the following reasons:

In part (a), some students failed to identify the correct points which lie on the lines.

For instance, they selected the points $(2, 2)$ and $(3.5, 5)$ later worked on Slope $(m_1) = \frac{5-2}{3.5-2}$ resulted to incorrect slope $(m_1) = 2$, instead of slope being L_1: $m_1 = 1$ and slope of L_2: $m_2 = -2$. Thereafter applied the incorrect slope $(m_1) = 2$ to find the equation L_1: $\frac{y-2}{x-2} = 2$, of which simplified to incorrect equation $y = 2x - 2$, like wise for the equation L_2: $\frac{y-5}{x-3.5} = 2$ computed incorrectly as $y = -2x + 12$, and finally concluded that the required equations are $\begin{cases} 2x - y = 2 \\ 2x + y = 12 \end{cases}$ Instead of $\begin{cases} x - y = 2 \\ 2x - y = 7 \end{cases}$

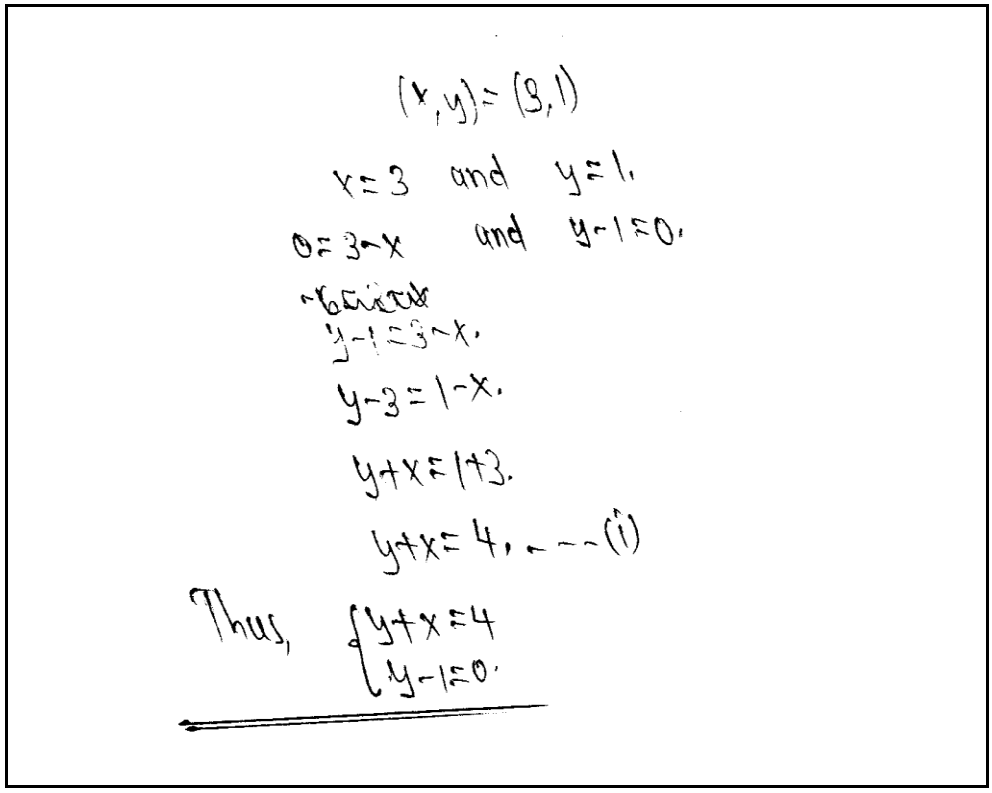
While, other students identified correctly the point of intersection as $x = 3$ and $y = 1$, but they failed to show how to determine the two equations of the lines, for example one of the students wrote the incorrect equation as $\begin{cases} x + y = 4 \\ 3x - 2y = 7 \end{cases}$ just by guessing after obtaining the correct point of intersection.

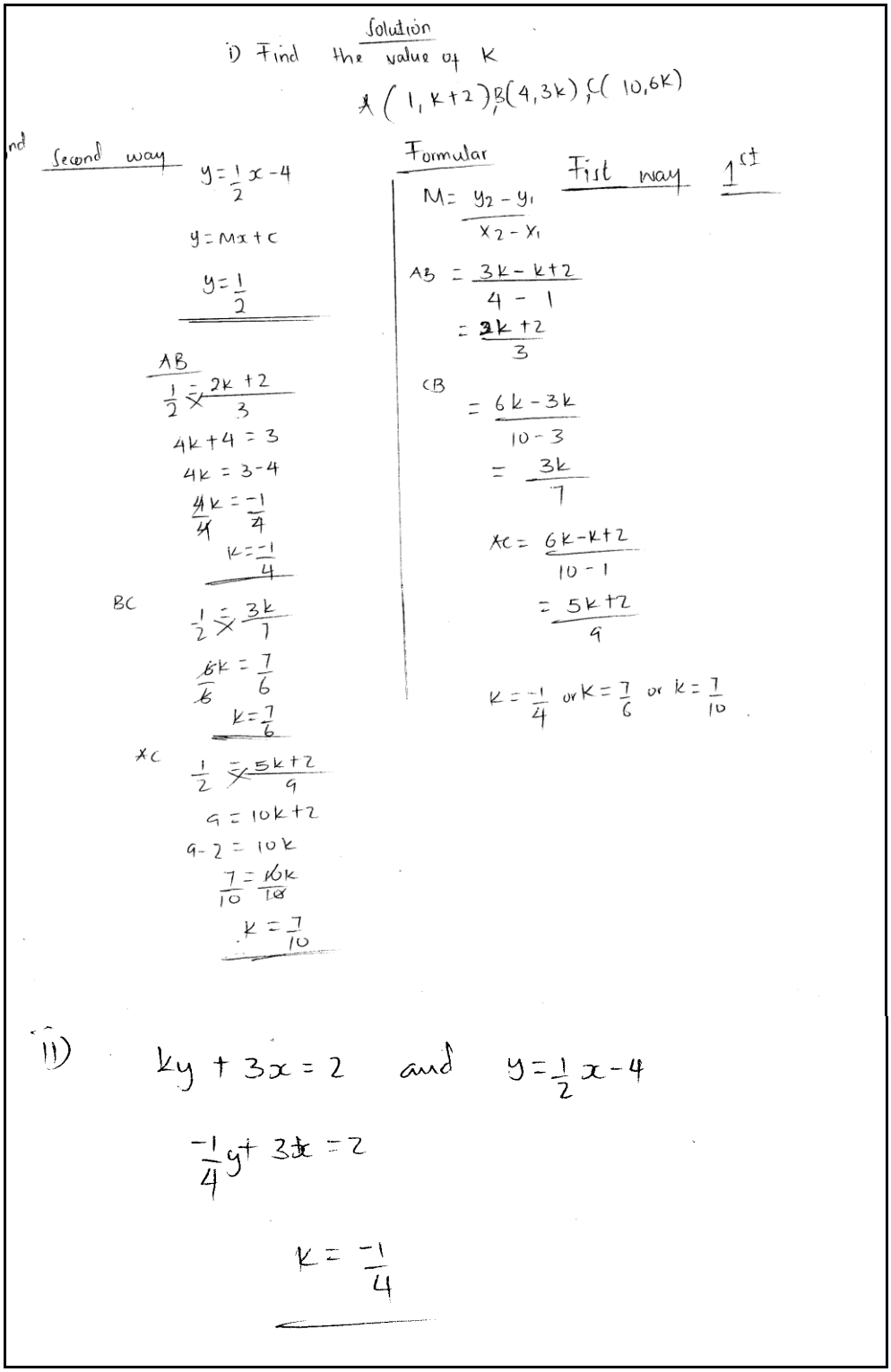
In part (b)(i), it is revealed that, inadequate knowledge and skills on coordinate geometry especially on collinear points, the students added the points instead of finding and comparing the slopes of the collinear points. That is $(1, k+2) + (4, 3k+1) + (10, 6k)$ summed up to 10 + 16 + 3k which simplified further to 2 + 3k instead of k = 4.

Other students had knowledge and skills in determining slope of a line given by $(m) = \frac{y_2 - y_1}{x_2 - x_1}$, however responded wrongly by neglecting in closing brackets for $k + 2$. Hence obtained, $\frac{3k - k + 2}{4 - 1} = \frac{6k - 3k}{10 - 4} = \frac{6k - k + 2}{10 - 1}$ which

simplified to incorrect equation $30k-27k=-12$ resulting into $k=-4$ instead of $k=4$

(ii) Some students failed to respond correctly due to interchanging the conditions for parallel lines and perpendicular line as $m_1=m_2$ and $m_1\times m_2=-1$ respectively then calculated m_2 as $\frac{1}{2}\times m_2=-1$, resulted to $m_2=-2$. Finally, compared the slope $-2x=\frac{3x}{k}$ simplified incorrectly to $k=\frac{3}{2}$ instead of $k=4$. Furthermore, other students determined incorrectly $m_1=ky+3x=2$ and $m_2=\frac{1}{2}x-4$ then equated the two equations as $ky+3x=2=y=\frac{1}{2}x-4$, in the process failed to proceed any more. Extract 5.2 is a sample of incorrect response selected from one of the poor performed students in question 5.





Extract 5.2: A sample of incorrect responses in question 5

In extract 5.2, part (a), the student identified only one point through which the lines pass and tried to find the equations of the lines, of which led to incorrect equations of the lines. In part (b)(i), the student used the equation $y = \frac{1}{2}x - 4$ to obtain the slope of collinear points to find the required value of k. In part (b)(ii), the student find the value of k by creating unrequired equation and compare with the equation $ky + 3x = 2$.

2.6 Question 6: Symmetry

This question comprised of parts (a), (b) and (c). In part (a), the students were given $A(1, 2)$, $B(3, 1)$, $C(-3, 2)$ and $D(3, -1)$ are some of the vertices of a six sided polygon, then they were asked to draw the complete figure on $x-y$ plane so that $x-$axis is the only line of symmetry. In part (b)(i), the students were given the letters **O, N, T, V** and were asked to determine which one of the given letters has no axis of symmetry while in part (b)(ii), the students were asked to draw the line of symmetry on the given figure. In part (c)(i), the students were asked to copy the given shape and complete it so that the dotted line to become line of symmetry. In part (c)(ii), the students were asked to add one line to the given diagram so that the resulting figure will have rotational symmetry but no line of symmetry.

The analysis of data shows that, all students about 484 (100%) attempted this question. Among them, 61(12.6%) students scored marks ranging from 0 to 2.5, while 310 (64.0%) students scored marks ranging from 3.0 to 6.0 and 113 (23.4%) students scored marks ranging from 6.5 to 10. The students' performance summary is presented in Figure 7.

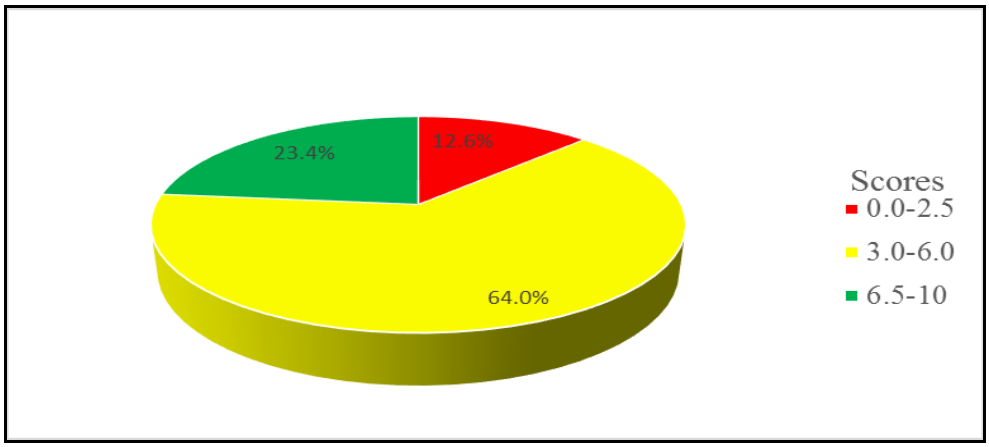
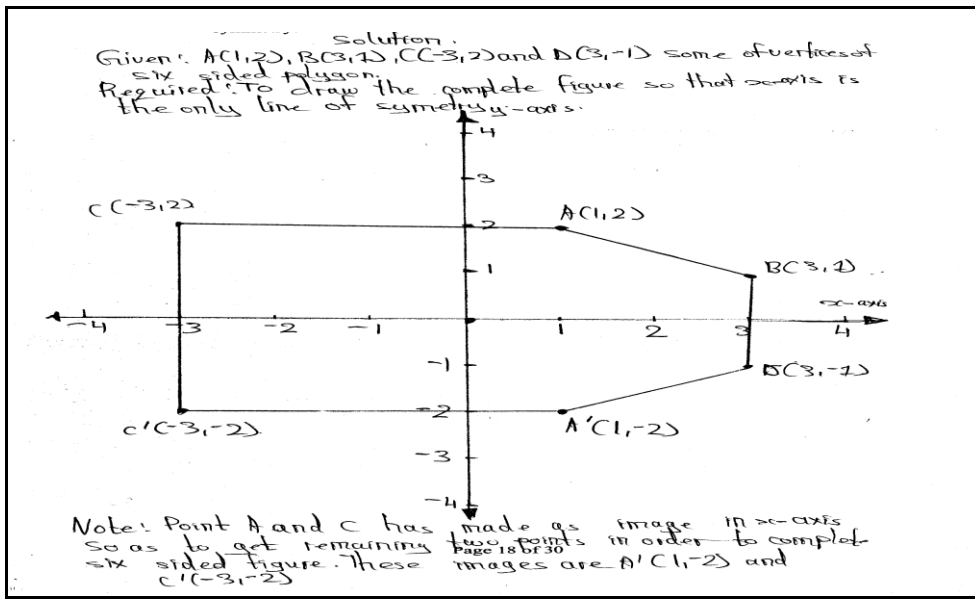


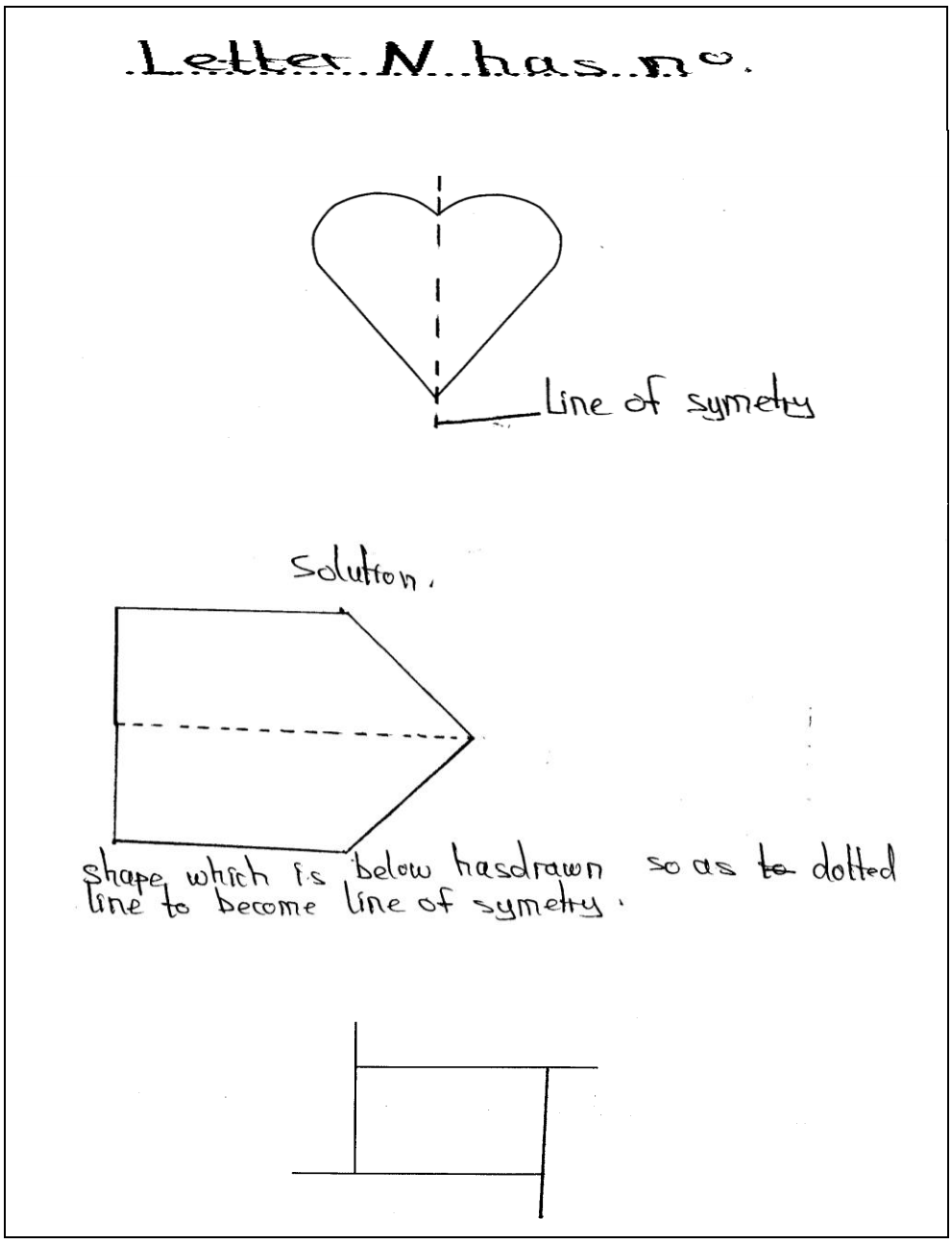
Figure 7: *Students' Performance in Question 6*

From the above summary of performance, it is observed that, the students' performance in this question was good as 87.4% students who sat for the assessment scored from 3 to 10 marks. In part (a), the students who performed this question correctly had adequate knowledge and skills on symmetry concept. The students drew correctly the given vertices, $A(1, 2)$, $B(3, 1)$, $C(-3, 2)$ and $D(3, -1)$ on the $x-y$ plane as well as completing the obtained figure to get a six sided polygon, just by tracing the image of the given points by considering reflection on $x-$ axis which led them to obtain A', B' and C' as $(1, -2)$, $(3, -1)$ and $(-3, -2)$ respectively.

In part (b)(i), the students applied the knowledge and skills of axis of symmetry on both given letters and finally realized that only letter **N** has no axis of symmetry, simply because it can't be divided it into two halves and obtain equal parts. In part (b)(ii), the students managed to draw correctly the line of symmetry by dividing the figure into two equal parts and indicated it by a dotted line as illustrated in extract 6.1.

In part (c)(i), the students demonstrated a good understanding in drawing whereby they managed to copy the given shape of the figure as well as completing it such that the dotted line become the line of symmetry. In part (c)(ii), the students added correctly one line closing the open end of the figure resulting it to have rotational symmetry of order 2 without any line of symmetry. Extract 6.1 is a correct response selected from one of the students who attempted this question correctly.





Extract 6.1: A sample of correct responses in question 6

In extract 6.1, part (a), the student draw an $xy-$plane, then located the given vertices and its image in order to draw the complete figure such that $x-$axis is the only axis of symmetry. In part (b)(i), the student identified correctly that, the letter **N** has no axis of symmetry. In part (b)(ii), the student drawn correctly line of symmetry on the given figure. In part (c)(i), the student abides with the requirement of the question by copying and

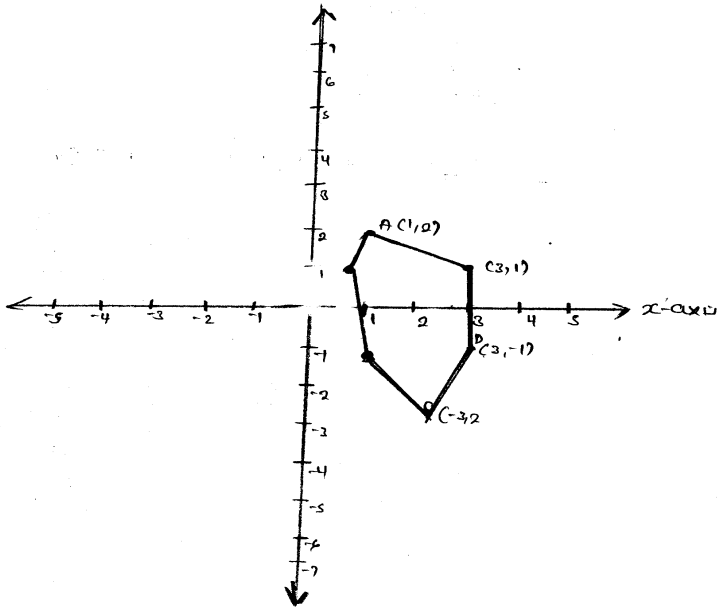
completed the given figure correctly. In part (c)(ii), the student added one line to the diagram given correctly so that the resulting figure has rotational symmetry but no line of symmetry. Analysis shows that the student has adequate knowledge and skills on symmetry.

However, 61(12.6%) of the students failed to respond to the question accordingly. The reasons behind their failure were associated with little knowledge on the concepts of symmetry hence, in part (a), some students were able to draw $xy-plane$ and locate correctly the given points, but failed to connect the adjacent points in order to meet the requirements of the question. Despite the fact that, they indicated the horizontal and vertical movement of the given points yet end up with incorrect answer. Furthermore, other students failed to abide with the requirement of the question, where by instead of locating the given points on $xy-$plane and connecting the adjacent points to obtain the required figure, they applied the formula for calculating the gradient using the given points and finally resulted to the slope AB, BC, AC, AD and CD as $-\frac{1}{2}$, $-\frac{1}{6}$, 0, $-\frac{3}{2}$, and $-\frac{1}{2}$ respectively.

In part (b)(i), the students seemed to lack clear knowledge and skills on the concepts of symmetry, whereby they failed to recognize that letter **O** has infinity line of symmetry. Nevertheless, other students interchanged the line of symmetry with the rotational symmetry, whereby instead of determining the letter which has no axis of symmetry, they stated the term rotational symmetry. In part (b)(ii), some students encountered some difficult in drawing the line of symmetry to the given figure as a result, they drew multiple lines.

In part (c)(i), some students lacked knowledge and skills on symmetry as they failed to understand that, for a line to be considered as the axis of symmetry it must divide the figure into two equal parts that is to say, one side will be the reflection of the other. In part (c)(ii), other students demonstrated some weakness on drawing skills, rotational symmetry and axis of symmetry because they failed to draw and add one line that could make the presented figure to have rotational symmetry without line of symmetry. Extract 6.2 is a sample response selected from one of the students who answered the question incorrectly.

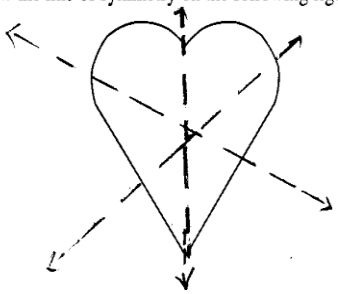
Solution.



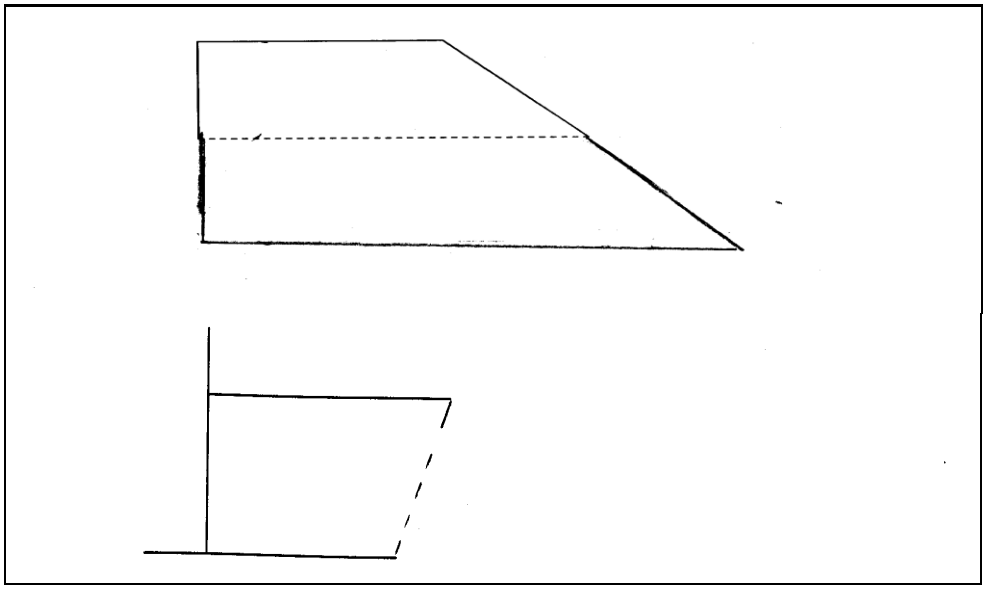
The other two points are (-1,-1) and (0.5,0.5)

Letter T

(ii) Draw the line of symmetry on the following figure:



There are three line of symmetry.



Extract 6.2: A sample of incorrect responses in question 6

In extract 6.2, part (a), the student failed to locate some given points as well as their images such that, instead of locating $C(-3,2)$ had located point C at $(3,1)$ and finally incorrect figure was drawn. In part (b)(i), the student failed to identify the letter with no axis of symmetry. In part (b)(ii), the student had drawn two more lines which are not lines of symmetry on the given figure. In part (c)(i), the student copied and completed the figure, but the dotted line is not the line of symmetry. In part (c)(ii), the student added a line but failed to obtain the required figure.

2.7 Question 7: Logic

The question is composed of two parts (a) and (b). In part (a)(i), the question stated as follows: let P be 'He is happy' and q be 'He is rich'. The students were required to write the statement 'He is rich but not happy' in symbolic form and construct the corresponding truth table. (ii), They were required to draw an electrical network for the statement $(p \wedge q) \vee r$. In part (b)(i), the students were asked to use truth table to verify the equivalence of the following logical statement $(p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$. In part (b)(ii), they were required to test the validity of the argument: "If I like Mathematics, then I will study. Either I study or I fail. Therefore, if I fail then I do not like Mathematics.

The data analysis shows that, 484 (100%) students attempted the question, out of which 179 (37.0%) students scored marks ranging from 0 to 2.5, while 201(41.5%) students scored marks ranging from 3.0 to 6.0 and 104 (21.5%) students scored marks ranging from 6.5 to 10. The students' performance summary is presented in Figure 8.

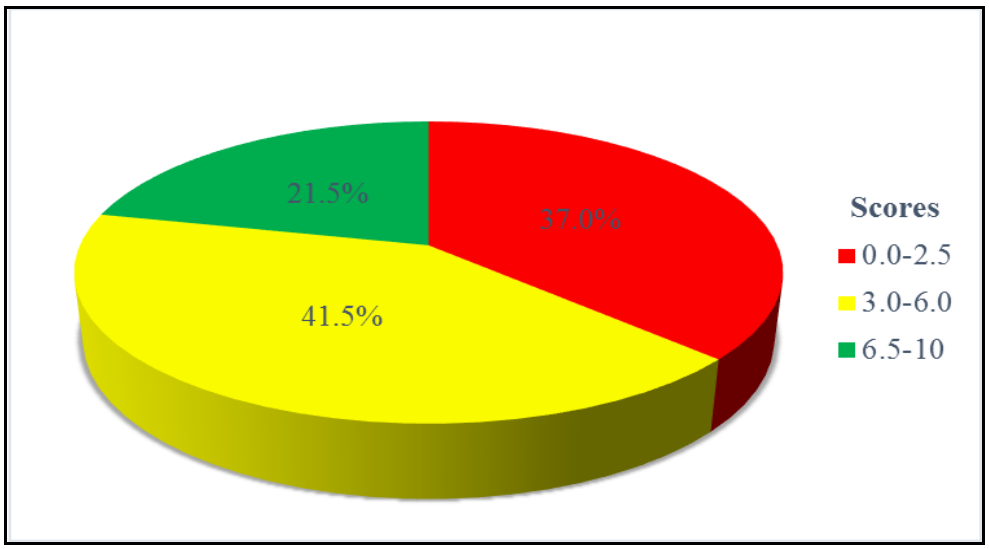


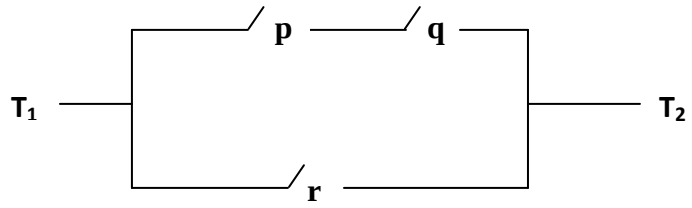
Figure 8: *Students' Performance in Question 7*

Therefore, the analysis shows that, the students' performance in this question was average because little number of students had knowledge and skills in Logic hence in part (a)(i), some students managed to identify the correct connective required as $\wedge$ meaning conjunction, also recognized that the statement 'He is rich but not happy' having a negation of p. Hence, finally wrote correctly the statement in symbolic form as $q \wedge \sim p$. Furthermore, the students applied the logic knowledge and managed to construct correctly the truth table for $q \wedge \sim p$ with four columns as illustrated underneath.

p	q	$\sim p$	$q \wedge \sim p$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

In part (a)(ii), The students demonstrated high degree of understanding connectives as conjunction $\wedge$ implying that the switches are in series,

likewise disjunction symbol $\vee$ implying that the switches are in parallel. Thereafter drew correctly an electrical circuit for the statement $(p \wedge q) \vee r$, where $(p \wedge q)$ are in series and r is parallel as illustrated in the following figure.



The Electrical Network for $(p \wedge q) \vee r$

In part (b)(i), the students managed to identify the compound statement $(p \to q) \wedge (q \to p)$ as a condition and $p \leftrightarrow q$ as bi condition. Thereafter recalled well the required columns and applied truth table, then correctly constructed the table with six columns and four rows and finally concluded that, since column five and six have the same truth-values then $(p \to q) \wedge (q \to p) \equiv p \leftrightarrow q$.

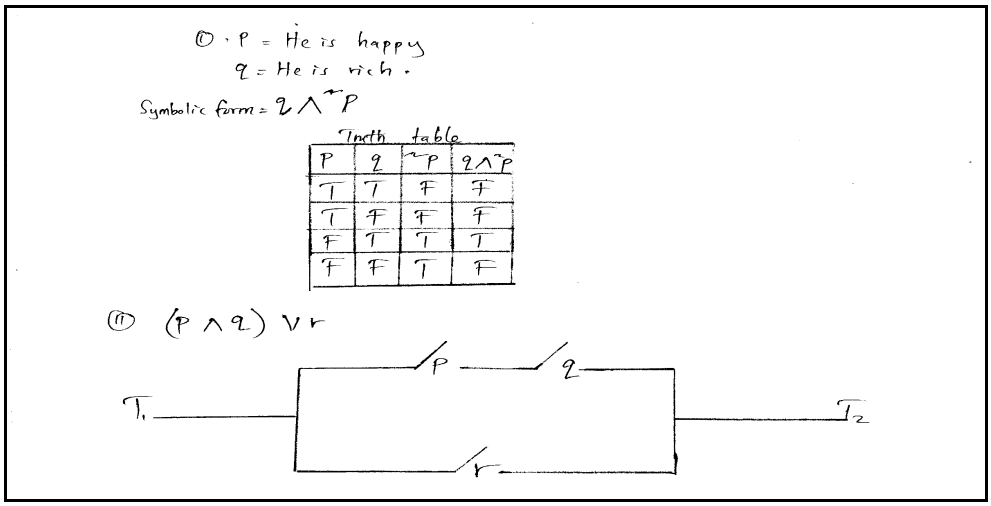
p	q	$p \to q$	$q \to p$	$3 \wedge 4$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T
1	2	3	4	5	6

In part (b)(ii), the students succeeded to identify correctly connectives for conjunction as $\wedge$ likewise for disjunction as $\vee$ and for conditional statement as $\to$, also recognized that the statement 'I do not like Mathematics' is a negation of the statement 'I like Mathematics'. Moreover, they assigned letters to the statements such as p stands for 'I like Mathematics', q stands for 'I will study' and r stands for 'I fail'. Later,

they wrote correctly the compound statement as, $(p \to q) \wedge (q \vee r) \to (r \to \sim p)$. In the process of problem solving, they constructed a truth table with nine columns and eight rows. Finally, they concluded that since the 9th columns do not contain all T, the statement is not tautology, hence is not valid as shown below:

p	q	r	$\sim p$	$p \to q$	$q \vee r$	$5 \wedge 6$	$r \to \sim p$	$7 \to 8$
T	T	T	F	T	T	T	F	F
T	T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F	T
T	F	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	T	T
1	2	3	4	5	6	7	8	9

Extract 7.1 is a sample of response selected from one of the students who responded correctly to this question.



(i) $(p \to q) \wedge (q \to p) \equiv p \leftrightarrow q$

Truth table 1

p	q	$p \to q$ (A)	$q \to p$ (B)	$A \wedge B$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Truth table 2

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Hence shown.

$(p \to q) \wedge (q \to p) \equiv p \leftrightarrow q$

b (ii) Let
p = I like mathematics
q = I will study
r = I fail

in symbolic form

$((p \to q) \wedge (q \vee r)) \to (r \to \sim p)$

p	q	r	~p	$p \to q$ (A)	$q \vee r$ (B)	$r \to \sim p$ (C)	$A \wedge B$	$D \to C$ (D)
T	T	T	F	T	T	F	T	F
T	T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F	T
T	F	F	F	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	T	F	T

Since the last column has F the statement is not tautology, therefore the argument is invalid.

Extract 7.1: A sample of correct responses in question 7

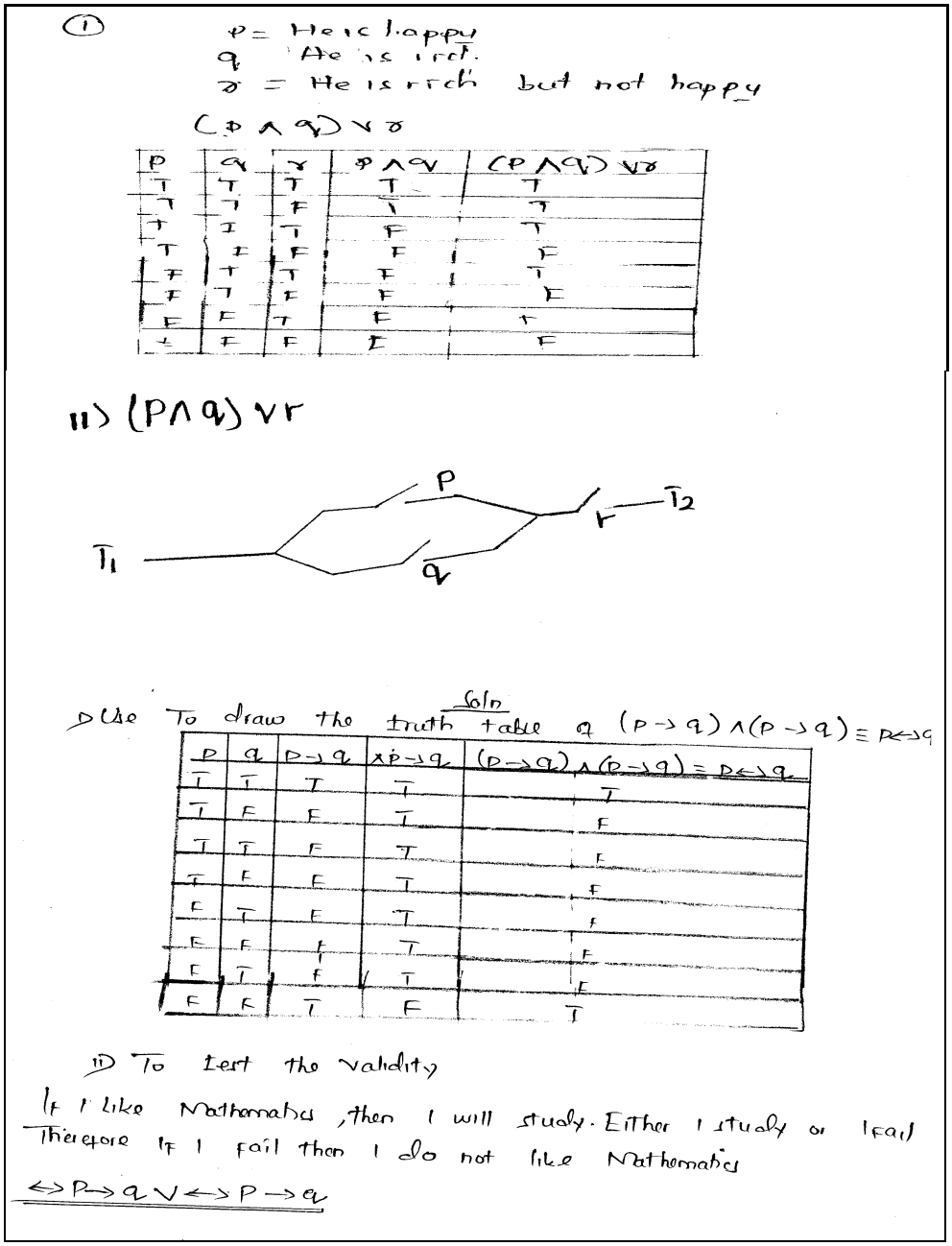
In extract 7.1, part (a)(i), the student symbolized the given statement correctly and constructed the truth table required. In part (a)(ii), the student drawn correctly the required circuit. In part (b)(i), the student constructed the truth table of the given statement which used to verify its equivalence. In part (b)(ii), the student symbolized the given statement, then tested the validity for it by using a truth table.

However, 179 (37.0%) students failed to answer the question accordingly due to insufficient knowledge and skills in logic, for instance some did not realize that the statement 'not happy' is the negation of happy. They wrote the logical statement without a negation symbol as, $p \wedge q$ instead of $q \wedge \sim p$, implying that the students incorrectly interpreted the statement "he is rich but not happy". Likewise some students created a logical statement as $(p \wedge q) \vee r$ such that p stands for 'He is happy', q stands for 'He is rich' and r stands for 'He is rich but not happy'. Other students used connectives, which were not required such as $\sim (p \wedge q)$ instead of $q \wedge \sim p$. Meanwhile some students wrote a statement in logical symbol incorrectly, as $p \vee \sim q$ while they were supposed to write, $q \wedge \sim p$. Moreover, the incorrectly logical symbols formulated by some students, led to incorrect truth tables, for instance, some of the truth tables portrayed columns $(p \wedge q), (p \vee \sim q), \sim (p \wedge q), q \rightarrow \sim p$.

In part (a)(ii), some students failed to distinguish switch in series and switch in parallel in drawing electrical circuits for the statement $(p \wedge q) \vee r$. The electrical circuit drawn was for the statement $(p \wedge r) \vee q$, meaning that p and r are two switches in series and q is the one in parallel, of which was contrary to the requirement of the question. While other students drew an electrical circuit of $(p \vee q) \wedge r$, all the attempts by the students revealed insufficient knowledge and skills in logic especially on electrical circuits.

In part (b)(i), some students failed to recognize the number of truth table's rows required in verifying the equivalence of the logical statement, they constructed truth table of 10 rows instead of six columns and 4 rows. Moreover, other students realized the columns and rows correctly, but failed to insert the truth-values required in each columns and rows as illustrated in their procedures. In part (b)(ii), other students misinterpreted the logical

statement, as they wrote $p \rightarrow q \rightarrow \sim p$ along with $\left[(p \rightarrow q) \vee r\right] \rightarrow \sim p$ instead of $[(p \rightarrow q) \wedge (q \vee r)] \rightarrow (r \rightarrow \sim p)$, of which end up with a formulation of incorrect logical statement. Eventually students failed to test the validity. Extract 7.2 is a sample response selected from one of the students who responded incorrectly.



Extract 7.2: A sample of incorrect responses in question 7

In extract 7.2, part (a)(i), the student symbolized the given statement but failed to link the correct symbol, instead of $q \wedge \sim p$, had written $(p \wedge q) \vee \mathrm{r}$, hence eventually failed to construct correctly the truth table for the given statement. In part (a)(ii), the student drawn an electric circuit for the statement $(p \vee q) \wedge \mathrm{r}$ instead of $(p \wedge q) \vee \mathrm{r}$. In part (b)(i), the student drawn a truth table for a proposition of three statements instead of two statements. In part (b)(ii), the student copied the question and write incorrect symbols for the proposition and didn't test its validity.

2.8 Question 8: Variations

The question is composed of two parts, (a) and (b). In part (a), the students were required to calculate the value of y when $x = 5$, given that y is directly proportional to the square of x and $y = 98$ when $x = 7$ and in part (b), they were given that, y varies directly as x and inversely as z. If x varies inversely as y^2, prove that z^2 varies directly as x^3.

The data analysis shows that, all students (484) attempted this question, whereby 122 (25.2%) students scored marks ranging from 0 to 2.5, while 345 (71.3%) students scored marks ranging from 3.0 to 6.0 and 17 (3.5%) students scored marks ranging from 6.5 to 10. Therefore, the students' performance in this question was good. The students' performance summary is presented in Figure 9.

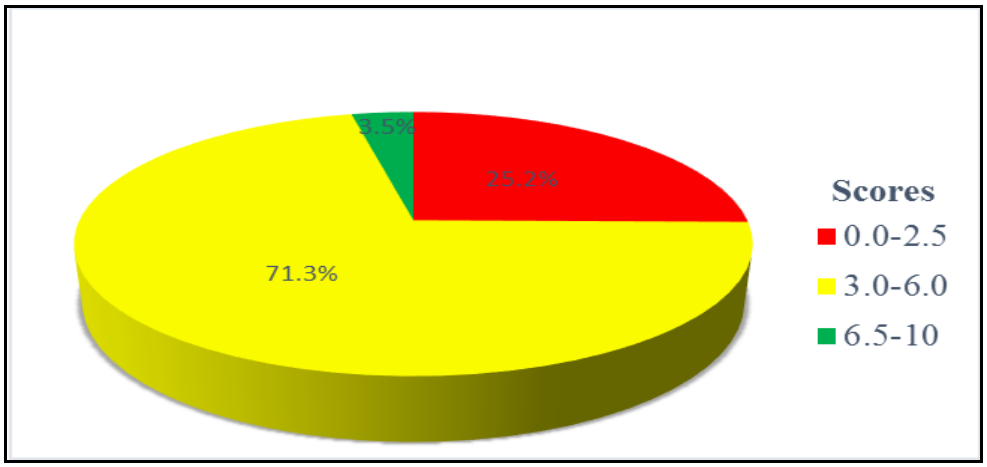
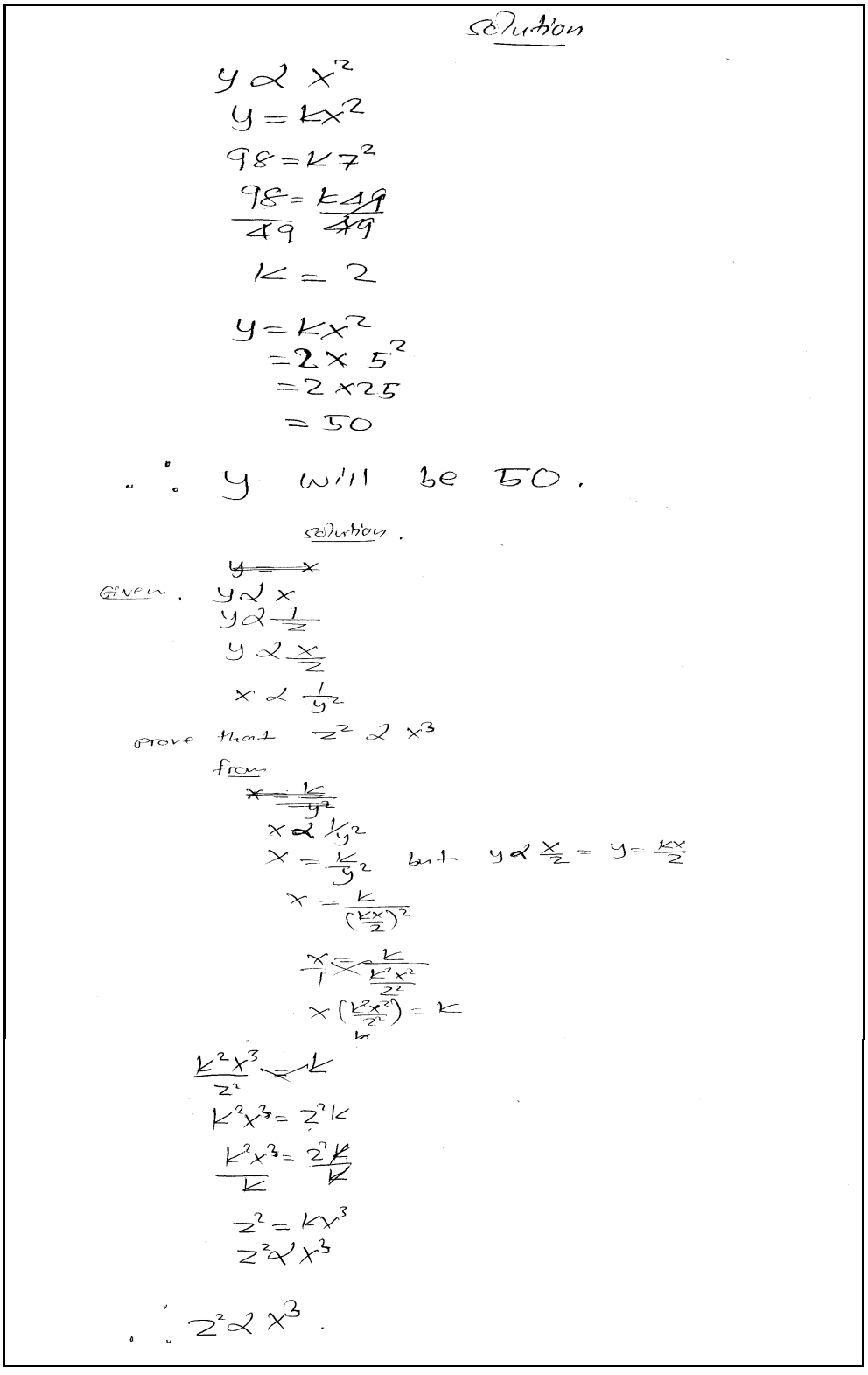


Figure 9: *Students' Performance in Question 8*

In part (a), the students who performed the question correctly managed to transform the given statement into mathematical form as $y \propto x^2$, thereafter introduced the constant k of proportionality as $y = kx^2$ and substituted the given value of x and y as 7 and 98 respectively. The process led to $98 = 7^2 k$, then they simplified it by making k the subject of the equation and ended up with $k = \frac{98}{49} = 2$. Finally they substituted $k = 2$ and $x = 5$ into $y = kx^2$ and obtained $y = 50$.

In part (b), the students interpreted and translated correctly, the given statements into Mathematical model as $y \propto \frac{x}{z}$ and $x \propto \frac{1}{y^2}$. Furthermore, they managed to introduce the constant k to obtain $y = \frac{kx}{z}$ and $x = \frac{k}{y^2}$. Later, they demonstrated good knowledge and skills in transposition of the formula as they managed to make y the subject from $x = \frac{k}{y^2}$ and obtained $k = xy^2$, further computations and simplifications resulted to the correct equation as $y = \sqrt{\frac{k}{x}}$. Moreover, the students equated the two equations to obtained $\frac{kx}{z} = \sqrt{\frac{k}{x}}$, thereafter applied the technique of squaring on either side of the two equations and got $\frac{k^2x^2}{z^2} = \frac{k}{x}$ and finally they make z the subject of the equation to obtained $z^2 = x^3 k$ which imply that $z^2 \propto x^3$. Extract 8.1 is a sample response selected from a student who answered correctly this question.



Extract 8.1: A sample of correct responses in question 8

In extract 8.1, part (a), the student decoded the given statement into mathematical symbol then introduced the constant k to obtain an equation which then solved to obtain the value of y. In part (b), the student transformed the given statements into mathematical symbol of proportionality then introduced the constant k. Thereafter simplified the two equations and prove the relation given.

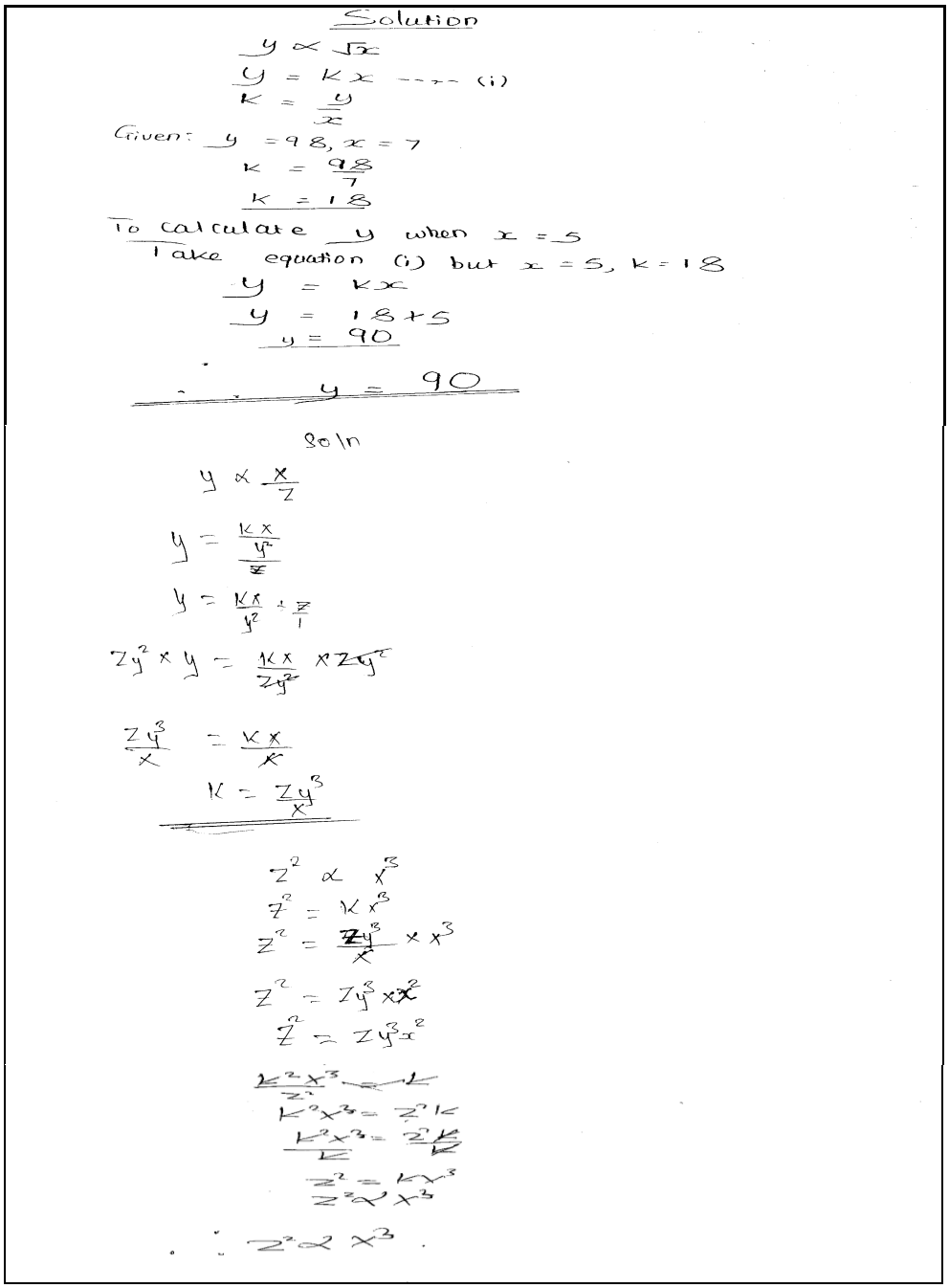
Contrarily, 122 (25.2%) students scored low marks on the question because of the following challenges:

Some students seemed to have insufficient knowledge and skills in variations concepts such that they demonstrated and translated incorrectly the given variation statement into Mathematical form. These students confused between square root and square concepts that led them to obtain incorrect variations expression $y \propto \sqrt{x}$ instead of, $y \propto x^2$ and thereafter introduced a constant k and obtained $y = kx$ and proceeded with further computation and obtained $k = \frac{y}{x}$ then substituted $y = 98$ and $x = 7$ which led them to obtain incorrect value of $k = 18$ instead of $k = 2$. Furthermore, they substituted the obtained value of $k = 18$ and $x = 5$, and further simplifications resulted to incorrect value of $y = 90$.

In part (b), other students failed to recognize that, the given variation is joint variation, however they considered it to be direct variation as they were transforming the given statement into Mathematical form as $y \propto xz$ instead of $y \propto \frac{x}{z}$. Then they introduced constant k of proportionality as $y = kxz$ and later they wrote k as the subject of the equation given by $k = \frac{y}{xz}$. Lastly, they computed it incorrectly and obtained $k = \frac{y^2}{x^3 z^2}$ while the question instructed them to show that $z^2 \propto x^3$.

The data analysis also revealed that, other students demonstrated a good understanding on variation concepts as they managed to change correctly the given statements into Mathematical form and introduced constant k on each variation expression and obtained $y = \frac{kx}{z}$ and $x = \frac{k}{y^2}$. However, they

failed to realize that for the second equation they were supposed to make y the subject of the equation so that, the two equations would have the same identity. Extract 8.2 is a sample response selected from one of the students who failed to answer this question correctly.



Extract 8.2: A sample of incorrect responses in question 8

In extract 8.2, part (a), the student related the mathematical symbol of the given statement incorrectly as $y \propto \sqrt{x}$ instead of, $y \propto x^2$, hence obtained a wrong value of y. In part (b), the student when introduced the constant k, divide the r.h.s with y which was not right also failed to show steps to prove the given relation.

2.9 Question 9: Sets

The question is composed of three parts, (a), (b) and (c). In part (a), the question was: In analysing the food preference of 80 teaching staff members it was observed that 43 members selected carrots, 30 selected potatoes and 43 selected tomatoes, 29 members selected carrots and tomatoes, 21 selected carrots and potatoes, 25 selected potatoes and tomatoes and 19 selected all three vegetables. The students were asked: (i) To show this information in a Venn diagram. (ii) How many members selected carrots only or tomatoes only?

In part, (b) the students were given that, if A, B and C are the sets and $n(A) = 20, n(B) = 24, n(C) = 28, n(A \cap B) = 12, n(A \cap C) = 13, n(B \cap C)$ and $n(A \cap B \cap C) = 10$, they were asked to find $n(A \cup B \cup C)$

In part, (c) The students were given that,

If A= {all prime factors of 30}

B= {all prime factors of 70}

C= {all prime factors of 42}

They were required to list the elements of A, B, C, and show their relationship in a Venn diagrams.

The data analysis shows that, 484 (100%) students attempted this question, out of which 178 (36.8%) students scored marks ranging from 0 to 2.5, while 223 (46.1%) students scored marks ranging from 3.0 to 6.0 and 83 (17.1%) students scored marks ranging from 6.5 to 10. Therefore, the students' performance in this question was average as far as 63.2% of the students sat for the assessment scored 3.0 to 10 marks. The students' performance summary is presented in Figure 10.

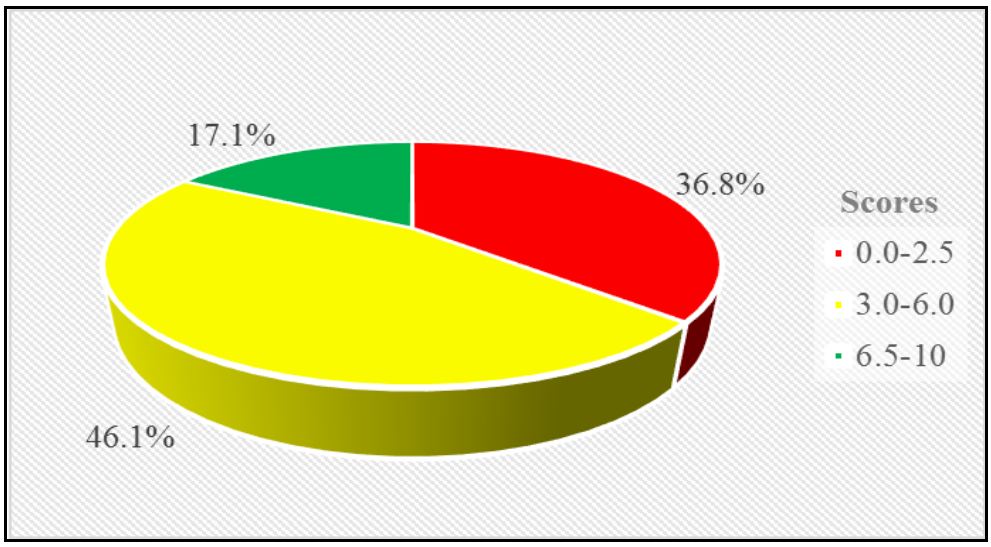
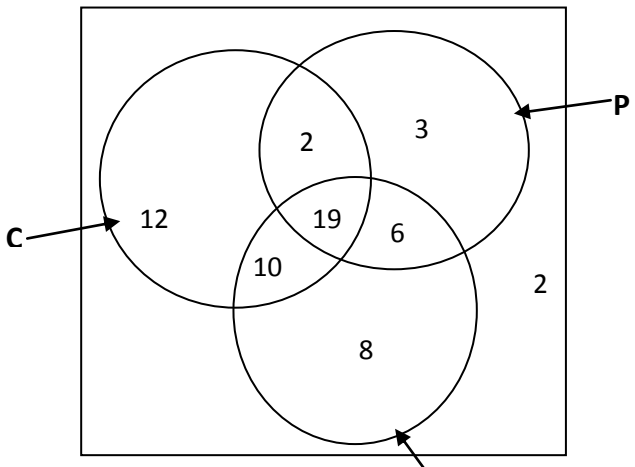


Figure 10: *Students' Performance in Question 9*

In part (a)(i), the students who responded correctly to this question were competent enough to let C be carrots, P be potatoes and T be Tomatoes, Later on identified $n(C)=43,\ n(P)=30,\ n(T)=43,\ n(C\cap T)=29,$ $n(C\cap P)=21$ in addition, they managed to identify $n(T\cap P)=25$ and $(C\cap T\cap P)=19$. Thereafter, they drew a Venn diagram with three ovals overlapping each other as illustrated in the following Venn diagram and in Extract 9.1 of the correct response selected from one of the competent student.



Venn diagram

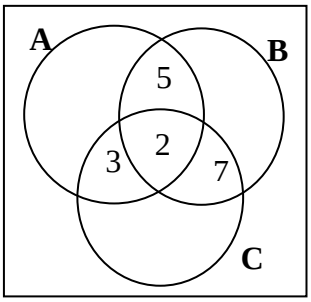
(ii) From the responses in part (a)(i), the students determined members selected carrots only as 12, and tomatoes only 8. Finally added the two to obtain 20 teaching staff members, as required answer.

In part (b), the students recalled correctly the formula for union of sets as;

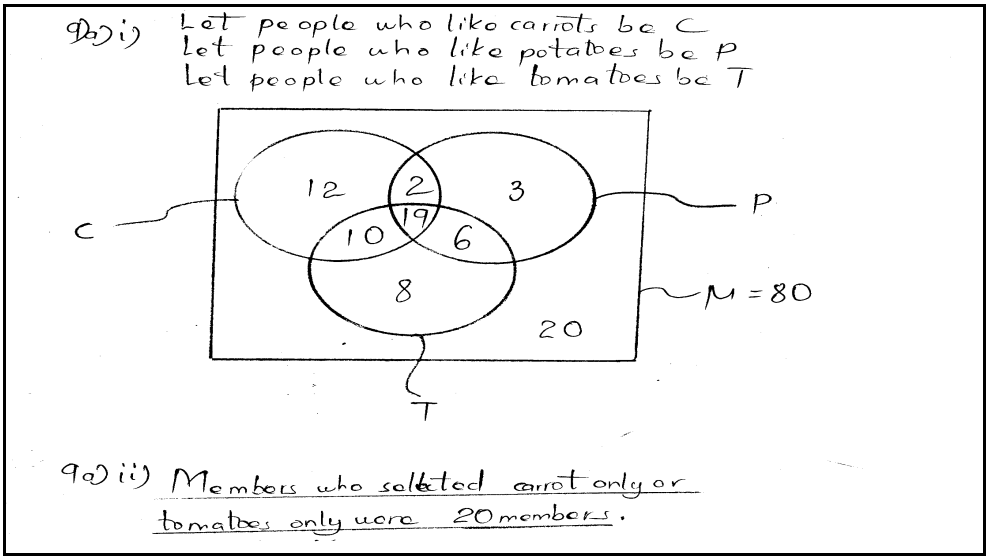
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$ and substituted all the given values of the sets as

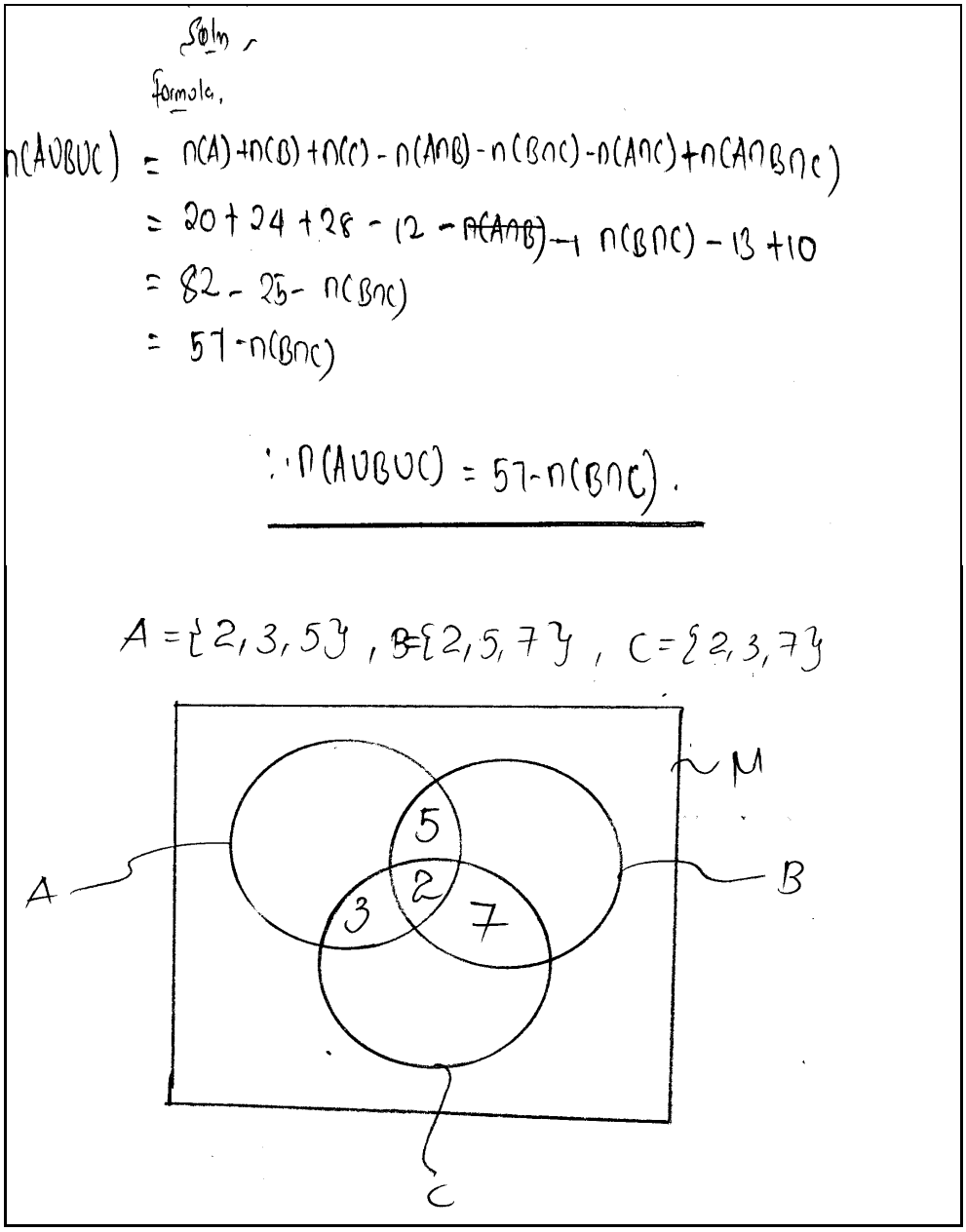
$n(A \cup B \cup C) = 20 + 24 + 28 - 12 - 13 - n(B \cap C) + 10$. Then simplified to $57 - n(B \cap C)$ which was the correct answer.

In part (c), The students managed to list the elements $A = \{2, 3, 5\}$, $B = \{2, 5, 7\}$ and $C = \{2, 3, 7\}$ of which indicated an adequate knowledge and skills in sets operations. Then, applied the Venn diagram to show the relationship. Extract 9.1 illustrates how the students responded correctly to the question.



Venn diagram





Extract 9.1: A sample of correct responses in question 9

In extract 9.1, part (a)(i), the student shows the given information correctly in a Venn diagram. In part (a)(ii), the student linked the given information in Venn diagram to obtain the members selected carrots or tomatoes correctly. In part (b), the student wrote the correct formula for union of sets and hence obtained the correct answer. In part (c), the student listed correctly the elements of the given sets and managed to show its relationship in a Venn diagram.

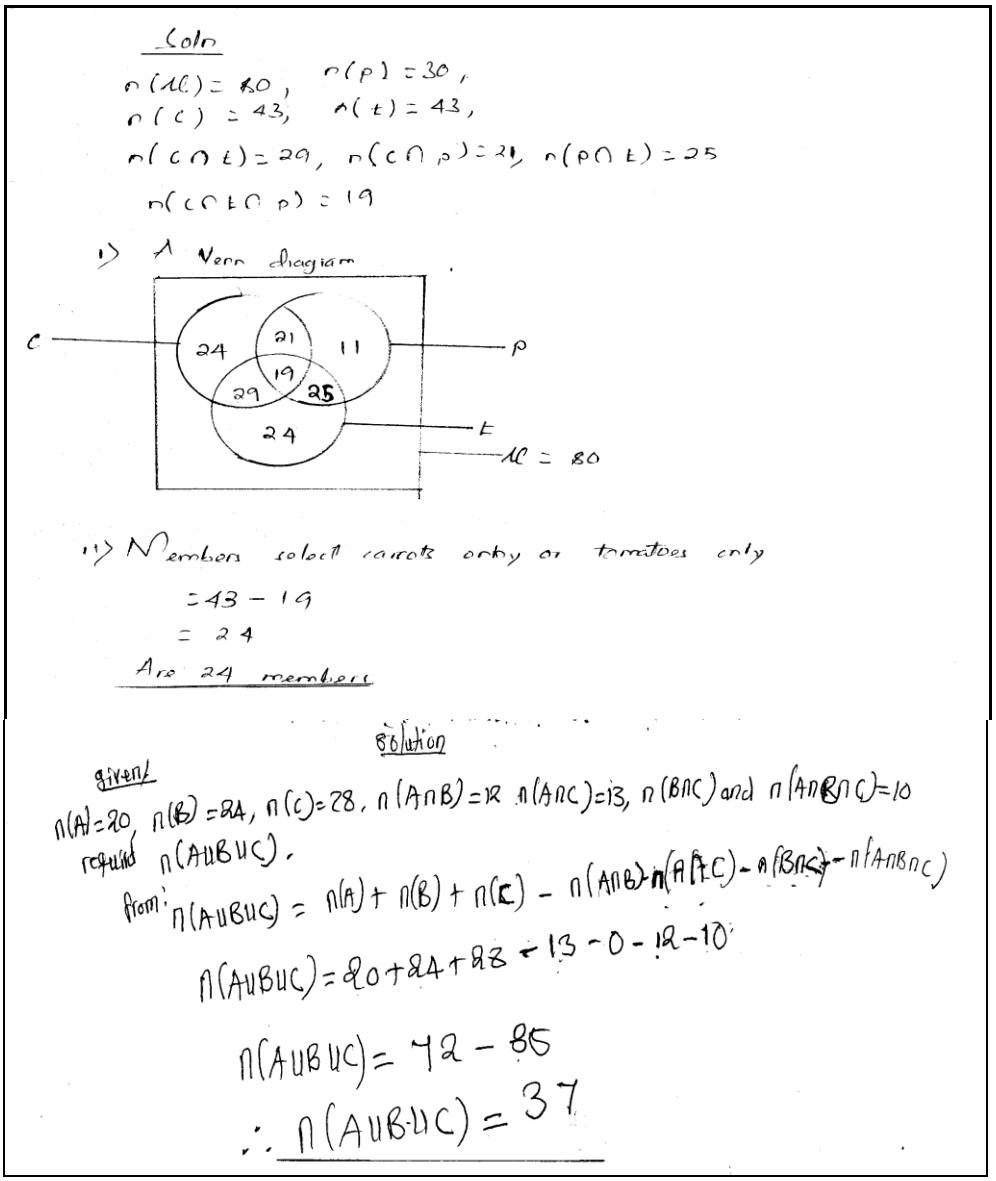
Despite the students' good performance in the question, 36.8% of the students who attempted the question scored below the average performance.

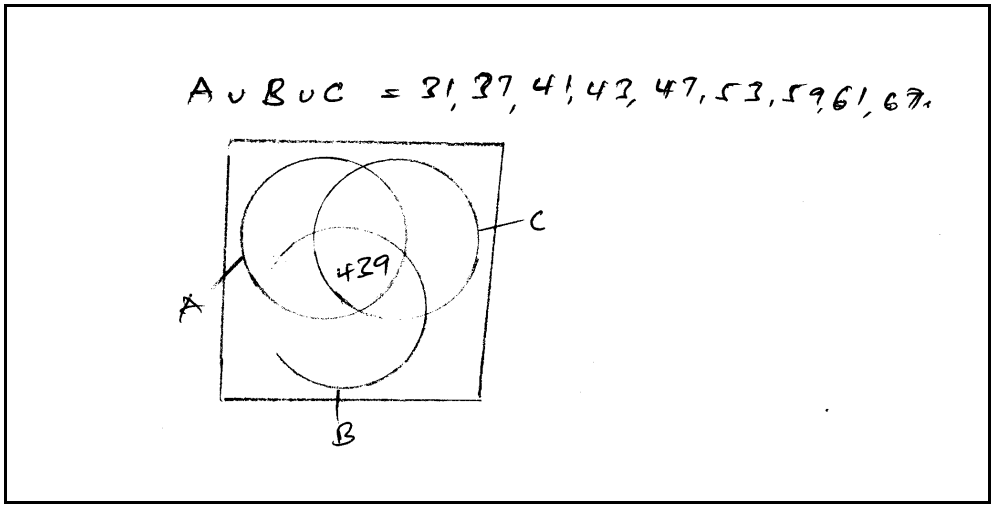
In part (a)(i), some students managed to draw Venn diagram as they were required, but they failed to determine the number of some elements of the union of sets as well as number of intersection of these sets. They entered the sets as were given without subtracting the intersection of all three sets. All these indicated insufficient knowledge and skills in sets operations. Other students, supposed correctly C to be carrots, P to be potatoes, however failed to display the information in the Venn diagram, as a result incorrect values entered. In part (ii), the failure in displaying information in the Venn diagram led them to obtain incorrect number of members selected carrots only or tomatoes only. For example other students found carrots only by adding $29+21-43=7$ and tomatoes only as $29+25+9=43$ and ended up with $43=63$. Furthermore, other students got correctly the number of teaching staff members who selected carrots only as 12 and those selected tomatoes only as 8, but they failed to add the two to obtain the required answer 20.

In part (b), some students failed to realize that $n(B\cap C)$ was not given, yet in responding to the question they correctly recalled the formula for $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$, however when substituting the given number of elements of sets in the formula, got incorrect $n(A\cup B\cup C)$. For instance $n(A\cup B\cup C)=20+24+28-12-13-10+10$ and simplified to 72 instead of $57-n(B\cap C)$. Also other students failed to recall correctly the formula for $n(A\cup B\cup C)$ instead they wrote $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)-n(A\cap B\cap C)$ and substituted the number of elements of sets incorrectly and finally got 47 instead of $57-n(B\cap C)$.

In part (c), some students failed to identify correctly the prime factors, they listed the elements of A, B and C of which is contrary to the requirement of the question, for instance they listed the elements of A, B and C as $A=\{2,3,5,6,15,30\}$, $B=\{2,5,7,10,35,70\}$ and $C=\{2,3,6,7,21,42\}$. Moreover, other students perceived wrongly prime factors as prime factorization as they responded to the question by applying prime

factorization method. For example $30 = 2 \times 3 \times 5$, $70 = 2 \times 5 \times 7$, $42 = 2^4 \times 3$. Some students went further, listed all prime factors, in between them. Thus, $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$, $B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41\}$ and $C = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41\}$. All these indicated insufficient knowledge and skills in sets. In presenting the relationship in a Venn diagram, they faced challenges in listing prime factors, of which led the students drew incorrect diagram. Extract 9.2 is a sample response selected from one of the students who encountered challenges in attempting the question.





Extract 9.2: A sample of incorrect responses in question 9

In extract 9.2, part (a)(i), the student failed to translate correctly the given information in Venn diagram, especially the intersections information. Hence provide the incorrect Venn diagram. In part (a)(ii), the student got the wrong answer due to in accuracy information committed in (i). In part (b), the student gave the wrong formula for the $n(A \cup B \cup C)$. In part (c), the student failed to list the required elements of sets as a result gave the incorrect Venn diagram.

2.10 Question 10: Variations

The question composed of three parts (a), (b) and (c). In part (a) (i), the students were given that, if $R \propto \frac{1}{T}$ and $T = 8$ and R=4, thereafter, they were asked to find the relationship between R and T, while in part (a) (ii), the students were given the information that, x varies directly as y and inversely as the square root of z, and when $x = 300$, $y = 65$ and $z = 25$. They were required to calculate the value of x when $y = 468$ and $z = 144$. In part (b), the students were given the information that, a motor cyclist estimates that, her annual expenditure is inversely proportional to the distance covered. The cost when she covers $800km$ is *Tsh* 1,000,000/=. Then, they were asked to find the cost when she covers $1500km$. In part (c), the students were asked that, if 18 men can dig a trench in 4 days, how many men will dig the same trench for 9 days?

The data analysis reveals that, 484 (100%) students attempted the question, out of which 158 (32.6%) students scored marks ranging from 0 to 2.5,

while 138 (28.5%) students scored marks ranging from 3.0 to 6.0 and 188 (38.8%) students scored marks ranging from 6.5 to 10. The students' performance summary is presented in Figure 11.

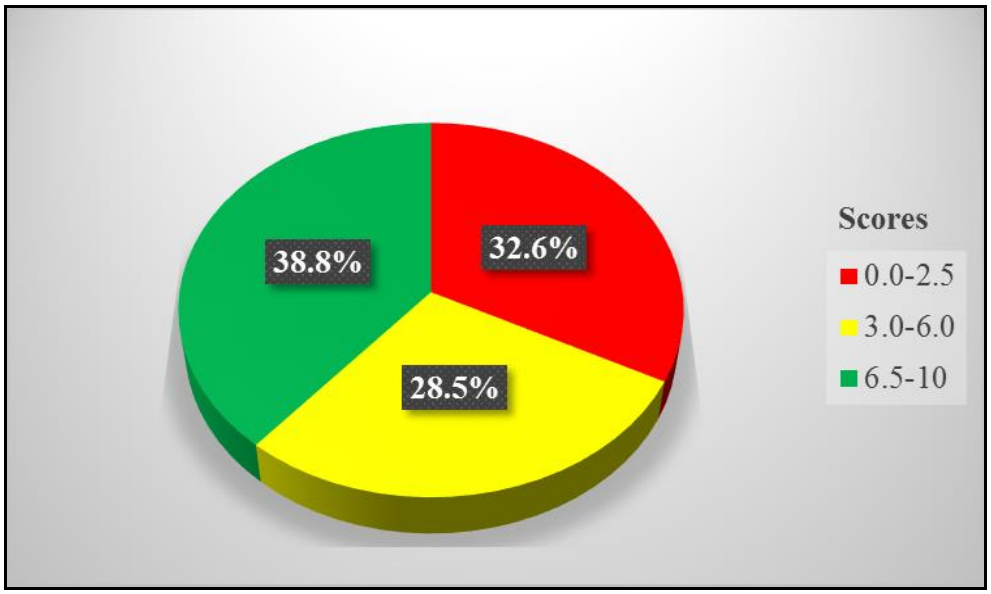


Figure 11: *Students' Performance in Question 10*

Therefore, the analysis in this question shows that the students' performance was good.

The analysis revealed further that, in part (a)(i), students who answered the question correctly, had clear knowledge and understanding in variations concepts, as they managed to introduce the constant k of proportionality to the given variation, and changed to equation $R = \frac{K}{T}$, then they substituted the values of $T = 8$ and R=4 to the equation and simplified it to obtain $k = 32$. Thereafter substituted the value of k into $R = \frac{K}{T}$ which resulted to $R = \frac{32}{T}$ as per the equation reflects in the relationship between R and T. In part (a)(ii), the students translated correctly the given statement into mathematical form given by, $x \propto \frac{y}{\sqrt{z}}$ and introduced the constant k to obtain $x = \frac{ky}{\sqrt{z}}$. Furthermore, they substituted the values $x = 300$, $y = 65$

and $z=25$, then computed and simplified it to $300=\frac{65k}{\sqrt{25}}$ of which brought $k=\frac{300}{13}$. Thereafter, substituted the value of $k=\frac{300}{13}$, $y=468$ and $z=144$ into $x=\frac{ky}{\sqrt{z}}$ as $\frac{300\times 468}{13\times\sqrt{144}}$ and computed it resulting to the value of $x=900$.

In part (b), the students demonstrated that they had sufficient knowledge and skills in variations as they were capable to transform the given statement into mathematical form as $a\propto\frac{1}{d}$ and later introduced constant k to obtain the equation $a=\frac{k}{d}$, thereafter, the students managed to make k the subject of the equation, which resulted to $k=ad$ and substituted $a=1,000,000$ and $d=800$, which led to $k=1000000\times 800$ and obtained the value of $k=800000000$. Finally substituted the obtained value of k and $d=1500$ to $a=\frac{k}{d}$ as $a=\frac{800000000}{1500}$, lastly they simplify it to obtain the value of annual expenditure $a=533,333.3$, that is the cost is Tshs 533,333.30.

In part (c), the students managed to make suppositions by $m=$number of men and $d=$number of days. Then, transformed the given statement into mathematical form as they recognized that, men(m) is inversely proportional to the number of days(d) used to accomplish the work as $m\propto\frac{1}{d}$, thereafter introduced the constant of proportionality k to obtain $m=\frac{k}{d}$. Later they wrote k in terms of m and d to obtain $k=md$ in which when $m=18$ and $d=4$, they substituted to the formulated equation to obtain $k=72$. Finally, $m=\frac{72}{9}$ and obtained the number of men used to accomplish the work is 8 men. Extract 10.1 is a sample response selected from one of the students who performed correctly the question.

Solution

i) $R \propto \frac{1}{T}$

$R = \frac{k}{T}$

$4 = \frac{k}{8}$

$k = 32.$

$R = \frac{32}{T}$

∴ The relationship between R and T is $R = \frac{32}{T}$

ii) $x \propto \frac{y}{\sqrt{z}}$

$x = \frac{ky}{\sqrt{z}}$

$300 = \frac{k65}{\sqrt{25}}$

$300 = \frac{k65}{5}$

$\frac{300}{13} = \frac{k13}{13}$

$k = \frac{300}{13}.$

$x = \frac{300y}{13\sqrt{z}}$

$= \frac{300 \times 468}{13\sqrt{144}}$

$= \frac{300 \times 468}{13 \times 12}$

$= \frac{11700}{13}$

$= 900$

∴ $x = 900$

Solution.

$\text{Cost} \propto \frac{1}{\text{distance covered}}$

$\text{Cost} = \frac{k}{\text{distance covered}}$

$\frac{1{,}000{,}000}{1} = \frac{k}{800\text{km}}$

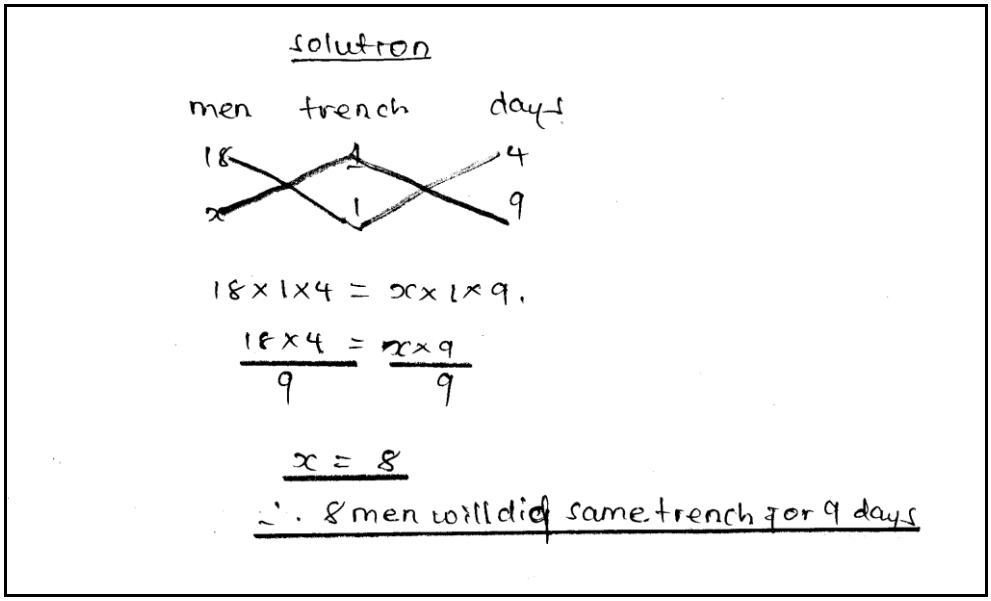
$k = 600{,}000{,}000$

$\text{Cost} = \frac{800{,}000{,}000}{\text{distance covered}}$

$= \frac{800{,}000{,}000}{1500}$

$= 533{,}333/=$

∴ The cost = 533,333/= approximately



Extract 10.1: A sample of correct responses in question 10

In extract 10.1, part (a)(i), the student found the relationship required between R and T by introducing the proportionality constant and simplifying the equation. In part (a)(ii), the student calculated the value of x immediately after transforming the given relation into mathematical symbol and introduced the proportionality constant then solved the obtained equation. In part (b), the student applied the concept of variations correctly to obtained the correct answer needed. In part (c), the student used the concept of variation correctly to obtain the correct answer.

Nevertheless, 32.6% of the students encountered some challenges when responding to this question, such that in part (a)(i), some students managed to introduce the constant k of proportionality to the given question as $R = \frac{K}{T}$. Thereafter, substituted correctly the given values of $T = 8$ and R=4 into $R = \frac{K}{T}$ as, $4 = \frac{K}{8}$ and simplified it to obtain $k = 32$ and ended up only with this value, as if they were asked to find the value of a constant k only. This observation indicates that the students failed to adhere to the requirements of the question. While other students substituted the given values of $T = 8$ and R=4 directly to the variation expression, which led them to obtain incorrect solution. In part (a)(ii), some students demonstrated correctly the transformation of the given statement into mathematical form

as $x \propto \frac{y}{\sqrt{z}}$. Later, introduced the constant k to obtain $x = \frac{ky}{\sqrt{z}}$ after that the students computed it correctly by making k the subject of the formula and obtained $k = \frac{x\sqrt{z}}{y}$. However, they failed to proceed further as they equated it as $k_1 = k_2$ which resulted to $\frac{x_1\sqrt{z_1}}{y_1} = \frac{x_2\sqrt{z_2}}{y_2}$ and substituted incorrectly the given values as $\frac{30 \times \sqrt{144}}{65} = \frac{x \times \sqrt{144}}{468}$ and simplified more to obtain $x = 23$.

In part (b), students failed to interpret the demand of the question as they equated the given value and thereafter, applied the cross multiplication techniques as, $\begin{matrix} 800\ km = 1,000,000\ tsh \\ 1500\ km = ? \end{matrix}$, thereafter simplified it to obtain $x = \frac{15,000,000}{8}$ which led to incorrect value of the required amount 1,875,000.

In part (c), several students seemed to confuse the term inversely proportional and directly proportional. The students computed the given question for directly related instead of attempting it as inversely related given by $m \propto d$ instead of $m \propto \frac{1}{d}$. Later, introduced constant to obtain $m = kd$, when computed and simplified, resulted into incorrect solutions. Extract 10.2 shows a sample response selected from incorrect solutions.

soln.

10 @ (i) $R \propto \frac{1}{T}$ and $T = 8$ when $R = 4$. The relationship between R and T

$R \propto \frac{1}{T}$

$R \propto \frac{1}{8}$

$4 \propto \frac{1}{8}$

By crossing multiplication.

$4 \times 8 = 1$

$\frac{32}{1} = \frac{1}{1}$

$= 32$

The relationship between R and T is 32.

(ii) $x \propto y$ and inversely as z^2

When $x = 300$, $y = 65$ and $z = 25$

Required; to find value of x when $y = 468$ and $z = 144$.

$x \propto 468$ and 144^2

$x \propto 468 \sim 10736$

$x \propto 10736 - 468$

$x \propto 468$ and inversely as 144^2

$x \propto 10268$.

∴ The value of x is 10268.

soln.

10 (b) Given: cost when she covers 800km is Tshs 1000000/-

Required; to find cost when she covers 1500km

800km = 1,000,000/=

1500km = ? x

$\frac{800km\ x}{800km\ x} = \frac{1500km \times 1,000,000}{800km}$

$\frac{800km\ x}{800km\ x} = \frac{1500 \times 1000}{8}$

$x = \frac{1500000}{8}$

$x = 187,5000$

∴ The cost is Tshs. 1,875,000 when she covers 1500km.

soln.

let m = be men &
d = be days

$m \propto d$ 18 men 4

$m \propto kd$? X

$x =$

$\frac{4\ days\ x}{4\ days\ x} = \frac{18 men \times 9\ days}{4\ days}$

$x = \frac{16\ 2}{4}$

$x = \frac{81}{4}$

$x = 20.25$

$x = 20.25 \sim 20$

∴ Men will dig the same trench for 9 days are 20

Extract 10.2: A sample of incorrect responses in question 10

In extract 10.2, part (a)(i), the student failed to find the relationship between R and T, hence obtained 32 instead of $R=\frac{32}{T}$. In part (a)(ii), the student failed to translate the given information, hence obtained incorrect value of x. In part (b), the student had a wrong translation of the given information by considering the cost to be proportional to the distance covered. In part (c), the student had a wrong translation of the given information by considering the number of men at work to be proportional to the number of days, hence got the incorrect answer.

3.0 ANALYSIS OF THE STUDENTS' PERFORMANCE IN EACH TOPIC

The Additional Mathematics paper composed of ten (10) questions that were set from nine (9) topics namely *Numbers*, *Algebra*, *Geometrical constructions*, *Locus*, *Coordinate geometry*, *Symmetry*, *Logic*, *Variations* and *Sets*. The analysis of students' performance in a topic perspective in the 2021 assessment indicated that the five (5) topics were well performed, three (3) topics were averagely performed and only one (1) topic was poorly performed as statistically presented in appendix I.

The topics which had good performance were *Numbers* (93.4%), *Symmetry* (87.4%), *Algebra* (81.6%), *Geometrical Constructions* (80.0%), and *Variations* (71.1%). The students' good performance in these topics was attributed to adequate knowledge and skills in the topics, appropriate competencies to use concepts in responding to the questions as well as the correct and right interpretation of the questions' requirements.

On the other hand, there were some topics that attained the average performance, specifically *Coordinate Geometry* (30.2%), *Logic* (63.0%) and *Sets* (63.2%). The average performance in these topics was mainly attributed to students' moderate knowledge and skills in the conceptualization of the question's contents.

Further analysis reveals that, a topic which had poor performance was *Locus* (07.0%). The weak performance was greatly attributed to the wrong interpretation of the need of the question and inadequate knowledge and skills in describing locus of a point. The analysis of the students' performance for each topic is presented in Appendix I.

The analysis of students' performance in a topic perspective in 2020 revealed that, six (6) topics were well performed and three (3) topics were

averagely performed. Primarily, the data analysis of students' performance per topic reveals that for the year 2021 there is an increase in performance in *Algebra* compared to the year 2020, however in the topics of *Logic* and *Sets* the student' performance has slightly decreased, like wise in *Locus* the performance has decreased from average to poor performance as indicated in appendix II.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The analysis of questions as well as the topics shows that, the overall performance in Additional Mathematics has decreased by 0.56 per cent as compared to the performance of 2020. The reasons justifying the weak performance in this subject are attributed to many aspects, including but are not limited to inability to identify the requirement of the question, inadequate knowledge and skills in various magnitudes, poor drawing skills, poor computational skills, provision of answers that were not related to the demands of the questions and use of incorrect concepts /formula for solving the questions.

Generally, the report has revealed the main areas where they portrayed their strength and weak potentials. It is expected that; the analysis will be valuable to educational stakeholders with respect to their position as well as the recommendations made in this report will help to improve the performance of students in future Additional Mathematics assessments.

4.2 Recommendations

In order to improve the students' performance in future it is recommended that:

(i) Students should use various learning materials such as books, journals and internet in order to improve their competence in Additional Mathematics.

(ii) Students should form discussion groups and subject clubs and participate effectively in solving questions.

(iii) Students should concentrate more on the concepts of the Locus when teachers lead them during the learning process.

(iv) Teachers should assess students regularly so as to identify their weakness and assist them according to their learning ability.

(v) The Ministry of Education, Science and Technology should facilitate in-service training for teachers especially on competence-based assessment in order to update their knowledge.

APPENDIX I: Analysis of Students' Performance in Each Topic

S/N	Topics	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Numbers	1	93.4%	Good
2.	Symmetry	6	87.4%	Good
3.	Algebra	2	81.6%	Good
4.	Geometrical Constructions	3	80%	Good
5.	Variations	8 & 10	71.1%	Good
6.	Sets	9	63.2%	Average
7.	Logic	7	63%	Average
8.	Coordinate Geometry	5	30.2%	Average
9.	Locus	4	07%	Poor

APPENDIX II: Comparison of Students' Performance in Each Topic for the year 2020 and 2021

S/N	Topics	2020			2021		
		Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks	Question Number	Percentage of Students who Scored an Average of 30% or Above	Remarks
1.	Numbers	1	74.3	Good	1	93.4%	Good
2.	Algebra	2 &10	62.3	Average	2	81.6%	Good
3.	Symmetry	6	89.0	Good	6	87.4%	Good
4.	Geometrical Constructions	3	76.8	Good	3	80%	Good
5.	Variations	8	79.3	Good	8 & 10	71.1%	Good
6.	Coordinate Geometry	5	44.5	Average	5	30.2%	Average
7.	Sets	9	80.7	Good	9	63.2%	Average
8.	Logic	7	68.6	Good	7	63%	Average
9.	Locus	4	48.6	Average	4	07%	Poor