THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2021

BASIC MATHEMATICS

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041 BASIC MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is delighted to issue this report on Students' Item Response Analysis (SIRA) in Basic Mathematics for Form Two National Assessment (FTNA) 2021. This report will provide feedback to students, teachers, policy makers and the public on how the students responded to each assessment item. The feedback mainly includes the statistical analysis of the students' performance and descriptive analysis of their responses focusing on the strengths and weaknesses identified in responding to each item. It also entails the recommendations that could be useful in improving the future students' performance in Basic Mathematics subject.

The statistical analysis shows that, the students had weak performance in all questions that were assessed in FTNA 2021. The analysis of the students' responses reveals that, the main reasons for weak performance include: inability to identify the requirements of the questions, failure to formulate correct equations and mathematical expressions or formulae from figures and inability to formulate mathematical statements from word problems. It was further noted that, the students were unable to apply correct laws, properties, conditions, formulae, theorems and mathematical concepts in answering questions. The analysis also reveals that, the students failed to perform units conversion, make proper substitution, sketch figures or diagrams and graphs as well as interpreting information presented in diagrams and graphs.

The National Examinations Council would like to thank the assessors, examination officers and all others who participated in preparing this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report entails the analysis of the item responses of the students who sat for the Basic Mathematics Assessment in FTNA 2021. It mainly focuses on the areas in which the students portrayed their strengths and weaknesses when responding to the assessment items.

The analysis points out that, the number of students who sat for the assessment in FTNA 2021 was 601,721 out of which 117,433 (19.52%) students passed. Comparatively, in FTNA 2020, a total of 600,751 students sat for the assessment, out of which 95,743 (15.94%) passed. This implies that, the performance has increased by 3.58 percent.

The Basic Mathematics paper consisted of ten (10) short answer questions, each carrying 10 marks. The students were required to answer all the questions. The analysis of the students' performance in each item of the assessment questions is summarized in this report. The analysis was based on the national assessment score intervals: 75 – 100, 65 – 74, 45 – 64, 30 – 44 and 0 – 29 to mean *Excellent*, *Very good*, *Good*, *Satisfactory* and *Fail* respectively. The students' performance in each question is considered *Good*, *Average* or *Weak* provided that the percentage of students who scored at least 30 percent falls in the interval: 65 – 100, 30 – 64 or 0 – 29 respectively.

In describing the data which are presented by using Figures and Appendix, three types of colours, that is green, yellow and red were used to represent good, average and weak performance, respectively. Finally, the report offers some recommendations that will help both students and teachers to improve the performance in future assessments.

2.0 ANALYSIS OF THE STUDENT'S PERFORMANCE IN EACH QUESTION

This section summarizes the requirements of each question and data analysis basing on students' performance. It also includes figures showing students' performance based on the criterion of the score intervals: 10 – 6.5, 6.0 – 3.0 and 2.5 – 0 out of 10 allotted marks for each question to represent *good*, *average* and *weak* performances respectively. However, the section summarizes the students' item response analysis in accordance to the strengths and weaknesses they had when answering the assessment items.

The samples of best and worst responses for each question were presented to support the findings developed from students' responses.

2.1 Question 1: Numbers and Approximations

The question comprised parts (a) and (b). In part (a), the students were required to: (i) write the number 498,030 in words, (ii) express the number given in part (a) (i) in standard notation and (iii) write the lowest common multiple of 3, 10 and 15 by using the listing method. In part (b), they were required to: (i) write in numerals, *nine hundred ninety nine million nine hundred ninety nine thousand nine hundred and one* and (ii) determine the number of significant figures in each of the numbers: 400,780 and 0.00606 and hence approximate each number into one significant figure.

The question was attempted by 596,350 (99.0%) students out of whom 443,575 (74.4%) students scored below 3 marks. This shows that the students' performance in this question was weak. Only 4,711 (0.8%) students scored full marks while a total of 226,511 (38.0%) scored zero. Figure 1 shows the summary of the students' performance in question 1.

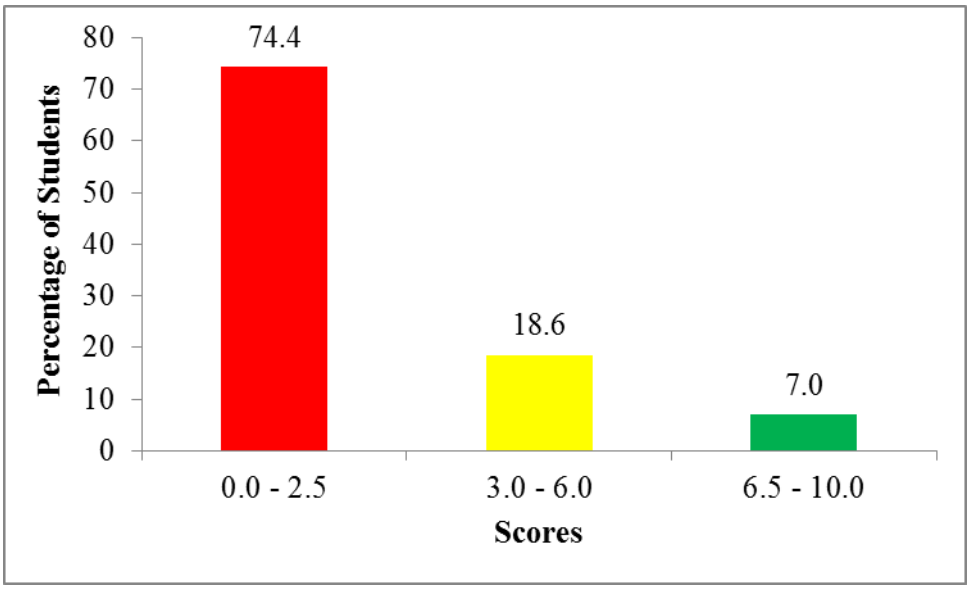
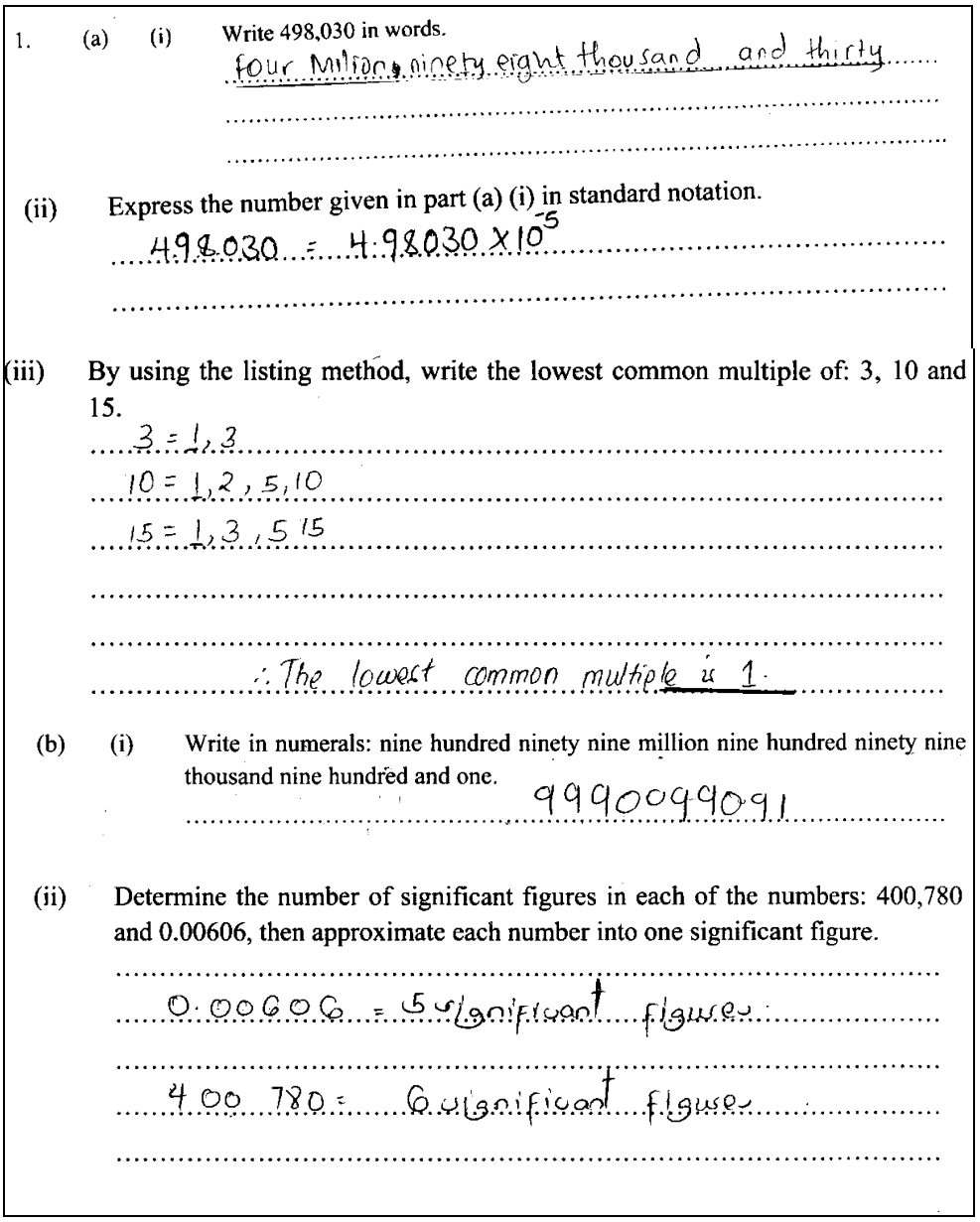


Figure 1: *Students' performance in question 1*

The response analysis shows that, the students failed to answer the question correctly due to various reasons. In part (a) (i), the students were unable to recall the concept of base ten numeration in writing numbers into words.

Some students mixed up *thirty* and *thirteen* while others had no knowledge on the concept of the place value of a number. In part (a) (ii), they could not realize that the question required knowledge of exponents, that is when shifting decimal points to the left the exponent must be positive and vice versa. For instance, they wrote incorrect answers like $498030 = 4.9803 \times 10^{-5}$ and $498030 = 4.9803 \times 10^{4}$ instead of $498030 = 4.9803 \times 10^{5}$. In part (a) (iii), students failed to differentiate "multiples" from "factors". Most of them wrote the factors instead of multiples of 3, 10 and 15. For instance, some students wrote the factors of 15 to be 1, 3, 5 and 15 instead of writing the multiples which are: 15, 30, 45, 60, 75 and so on. In some cases, there were students who applied the prime factorization method to find the lowest common multiple instead of using listing method due to failure to adhere to the given instruction.

In part (b) (i), the students showed weaknesses in writing "nine hundred ninety nine million nine hundred ninety nine thousand nine hundred and one" in numerals, that is 999,999,901. The majority wrote incorrect numerals like 99,999,901 which stands for "ninety nine million nine hundred ninety nine thousand nine hundred and one". Others wrote 999,999,910 that stands for "nine hundred ninety nine million nine hundred ninety nine thousand nine hundred and ten" which was contrary to the given statement. In part (b) (ii), some students failed to determine the number of significant figures and make approximations due to lack of awareness on the significance of zero when it is before, between and after non-zero digit number. Most of them wrote that: 400,780 had 3 instead of 5 significant figures while others wrote 6 significant figures. Again, there were students who wrote 0.00606 had 2 instead of 3 significant figures. They also approximated 0.00606 in one decimal place as 0.0 or in one significant figure as 0.0060 instead of 0.006 as required. Again, in approximating 400,780 into one significant figure some students incorrectly wrote 4 instead of 400,000. Extract 1.1 shows a sample of an incorrect response from one of the students.

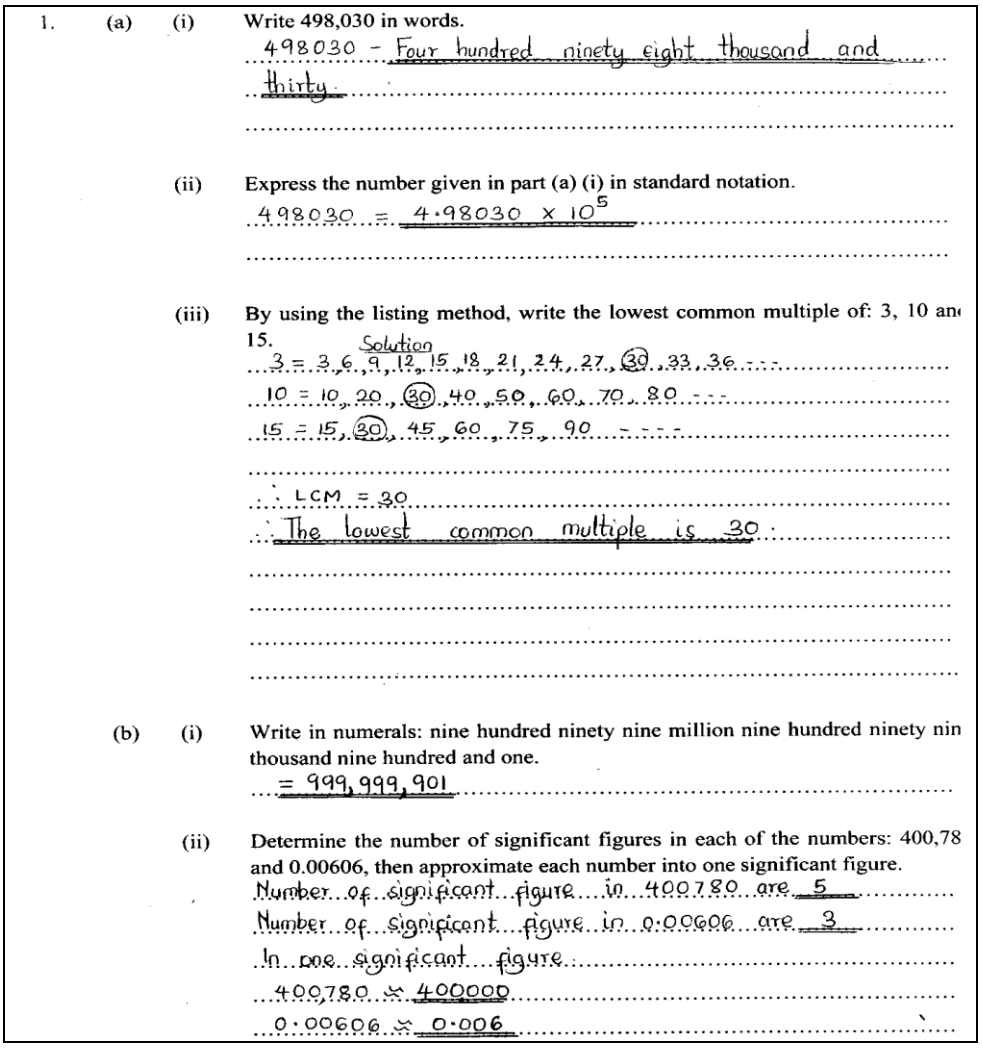


Extract 1.1: A sample of incorrect responses in question 1.

In Extract 1.1, part (a) (i), the student wrote "four million ninety eight thousand and thirty" which stands for 4,098,030 different from the given numeral 498,030. In part (a) (ii), the student expressed 498,030 as 4.9803×10^{-5} instead of 4.9803×10^{5} which is the correct standard notation. In part (a) (iii), the student listed the factors of the given numbers instead of multiples. In part (b) (i), he/she failed to write the correct numeral for the number expressed in words. In part (b) (ii), the student wrote incorrect

number of significant figures indicating lack of knowledge of approximations.

Although the performance was weak, there were the students who answered this question correctly and scored full marks. The students were able to: (a) (i) write correctly the given number 498,030 in words because they had adequate knowledge of identifying the place value of each digit in the number when reading, (ii) express 498,030 in the form $A\times10^{n}$ where $1\leq A<10$ and n is any integer and (iii) identify the lowest common multiple from the multiples of 3, 10 and 15 by listing. They started by listing all multiples of 3, 10, and 15 and hence obtained the lowest common multiple which is 30. In part (b), they were able to write the given number in numerals as 999,999,901 indicating that they had enough knowledge of place values and base ten numerations. They also managed to determine the number of significant figures, that is 400,780 had 5 significant figures, 0.00606 had 3 significant figures and hence approximated 0.00606 and 400,780 into one significant figure, that is $0.00606\approx0.006$ and $400{,}780\approx400{,}000$. This reveals that, the students had enough knowledge and skills on the significance of zero when it is on the left, right and in between the non-zero digits. Extract 1.2 shows a sample of a student's correct response in this question.


1. (a) (i) Write 498,030 in words.

498030 - Four hundred ninety eight thousand and thirty.

(ii) Express the number given in part (a) (i) in standard notation.

$498030 = 4{\cdot}98030 \times 10^5$

(iii) By using the listing method, write the lowest common multiple of: 3, 10 an 15.

Solution

3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, (30), 33, 36 ...

10 = 10, 20, (30), 40, 50, 60, 70, 80 ...

15 = 15, (30), 45, 60, 75, 90 ...

∴ LCM = 30

The lowest common multiple is 30.

(b) (i) Write in numerals: nine hundred ninety nine million nine hundred ninety nin thousand nine hundred and one.

= 999,999,901

(ii) Determine the number of significant figures in each of the numbers: 400,78 and 0.00606, then approximate each number into one significant figure.

Number of significant figure in 400780 are 5

Number of significant figure in 0·00606 are 3

In one significant figure

400780 ≈ 400000

0·00606 ≈ 0·006

Extract 1.2: A sample of the correct responses in question 1.

In Extract 1.2, the student was able to write 498030 in words correctly in part (a) (i). In part (a) (ii), the students managed to express 498,030 in correct standard form and in part (a) (iii), he/she determined the lowest common multiple of 3, 10, and 15 by listing as required. In part (b) (i), he/she was able to write the given number in numerals correctly and in (b) (ii), he/she succeeded to determine the number of significant figures as well as approximating the given number correct to one significant figure.

2.2 Question 2: Fractions, Decimals and Percentages

The question had parts (a) and (b). In part (a) (i), the students were required to write the fractions $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$ and $\frac{1}{2}$ in order of magnitude starting with the

smallest fraction. In part (a) (ii), they were required to find the product of the fractions given in part (a) (i). In part (b), the students were required to subtract $0.0\dot{2}$ of Tsh. 270,000 from 36% of Tsh. 50,000.

The data analysis shows that, the question was answered by 588,306 (97.6%) students and among them, 72,714 (74.2%) students scored below 3 marks. This indicates that the students' performance in this question was weak. It was further noted that, only 19,826 (3.4%) students scored all 10 marks while a total of 327,812 (55.7%) scored zero. Figure 2 summarizes the students' performance in question 2.

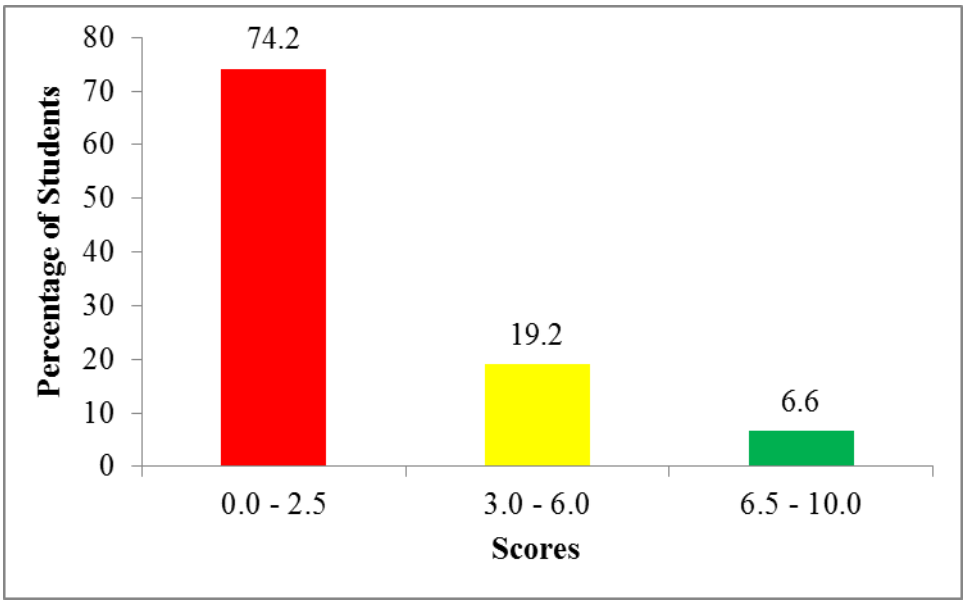
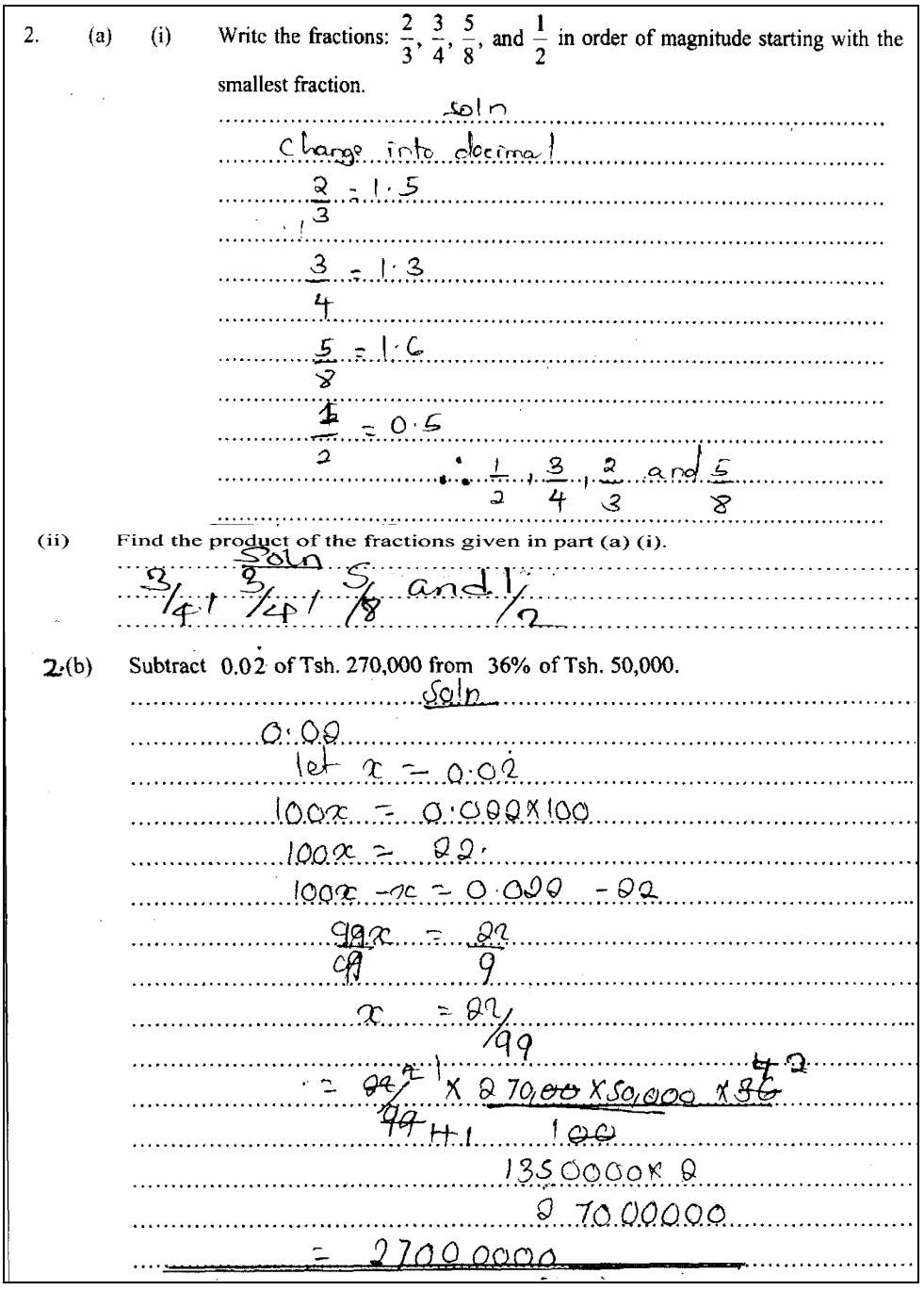


Figure 2: *Students' performance in question 2*

The analysis reveals that the majority of students were not able to answer this question correctly; as a result, they scored less than 3 marks. In part (a) (i), the students failed to understand how accurately they could write and arrange fractions in order of magnitude. Some of them re-arranged the given fractions in ascending order of the numerators and denominators instead of using the concept of LCM, decimals or percentages to obtain the correct order. As a result, they wrote $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ and $\frac{5}{8}$. This reveals that, the students lacked knowledge of comparing fractions. Others managed to convert the fractions into percentages or decimals or whole numbers by

using the concept of LCM but failed to use the obtained results to re-arrange the given fractions in order of magnitude. Some did not understand the requirement of the question as they ended up adding the given fractions. In part (a) (ii), some students were not able to multiply fractions correctly. Some students subtracted and others just added all the values in the numerator and denominator of the given fractions contrary to the given instruction. Those students were not familiar with the term "product" which means multiplication in the context of mathematical operation. In part (b), some students were not able to find $0.0\dot{2}$ of 270,000 as well as 36% of 50,000 correctly. This was mainly caused by lack of knowledge in converting the repeating decimal $0.0\dot{2}$ into fractions. For instance, they wrote $0.0\dot{2} = \frac{2}{100} = \frac{1}{50}$ instead of $0.0\dot{2} = 0.0222... = \frac{1}{45}$ before multiplying by 270,000. Again, some students failed to interpret the role of the word "of" in the statements "$0.0\dot{2}$ of 270,000" and "36% of 50,000". For example, they incorrectly wrote $0.0\dot{2} + 270,000$ and $36\% + 50,000$ instead of $0.0\dot{2} \times 270,000$ and $36\% \times 50,000$ before subtracting as instructed. Extract 2.1 shows the responses from a student who failed to answer this question correctly.



Extract 2.1: A sample of incorrect responses in question 2.

Extract 2.1 shows that, the student converted incorrectly the fractions $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{8}$ into decimals in part (a) (i). He/she just listed the given fractions instead of multiplying them to get the required product in part (a)

(ii). In part (b), the student also could not find the correct fraction for $0.0\dot{2}$ as well as 36% of 50,000 something which led him/her into incorrect answer.

Despite the weak performance, there were the students who answered this question correctly and scored full marks. The students were able to write the given fractions in order of magnitude starting from the smallest by different approaches in part (a) (i). Some determined the LCM of the denominators and then multiplied by each of the given fractions. Others multiplied each fraction by 100% or changed the fractions given into decimals, which was a necessary step that could help them to arrange the fractions in ascending order. In part (a) (ii), the students managed to multiply the given fractions, that is $\frac{2}{3}\times\frac{3}{4}\times\frac{5}{8}\times\frac{1}{2}=\frac{5}{32}$. This shows that, those students had adequate knowledge of multiplying the fractions. In part (b), they managed to convert the recurring decimal and percentage into fraction and later performed the required operations correctly. This indicates that students had knowledge and skills of applying the fractions, decimals and percentages in solving problems. Extract 2.2 shows a sample response from one of the students who answered this question correctly.

2. (a) (i) Write the fractions: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{2}$ in order of magnitude starting with the smallest fraction.

Solution

Find LCM of 3, 4, 8, 2

2	3	4	8	2
2	3	2	4	1
2	3	1	2	1
3	3	1	1	1
	1	1	1	1

LCM = 2 × 2 × 2 × 3

= 24

$\frac{2}{3} \times 24 = 16$

$\frac{3}{4} \times 24 = 18$

$\frac{5}{8} \times 24 = 15$

$\frac{1}{2} \times 24 = 12$

∴ In ascending order is $\frac{1}{2}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}$

(ii) Find the product of the fractions given in part (a) (i).

Solution

$\frac{2}{3} \times \frac{3}{4} \times \frac{5}{8} \times \frac{1}{2} = \frac{1}{4} \times \frac{5}{8}$

$= \frac{5}{32}$

The product of the fractions is $\frac{5}{32}$

2 (b) Subtract $0.0\dot{2}$ of Tsh. 270,000 from 36% of Tsh. 50,000.

First step: change $0.0\dot{2}$ into fraction

$x = 0.0\dot{2}$ - - - (i)

Multiply by 10 both sides

$10 \times x = 0.0\dot{2} \times 10$

$10x = 0.2\dot{2}$ - - - (ii)

Subtract eqn (i) from eqn (ii)

$10x - x = 0.2\dot{2} - 0.0\dot{2}$

$\frac{9x}{9} = \frac{0.2}{9}$

$= \frac{2}{90}$

Second step: take $0.0\dot{2}$ of 270,000 =

$= \frac{2}{90} \times 270{,}000$

= 6,000 /=

Then take 36% of 50,000

$= \frac{36}{100} \times 50000$

= 18000 /=

Lastly subtract the results.

18000 /= - 6000 /=

= 12,000 /=

Extract 2.2: A sample of the correct responses in question 2.

In Extract 2.2, the student used the LCM correctly to find the order of magnitude of each fraction and then arranged the fractions according to their order of magnitude in part (a) (i). In part (a) (ii), he/she calculated the product of the given fractions correctly. In part (b), the student used the required mathematical operations to get the answer.

2.3 Question 3: Units and Ratio, Profit and Loss

The question consisted of parts (a) and (b). In part (a), students were instructed to find the value of $500\,\text{cm} + 3150\,\text{mm} + 3.5\,\text{m}$ and to give the answer in metres. In part (b), students were required to find the number of years (time taken) in which Tshs. 20,000 will earn an interest of Tshs. 4,800 if the interest rate per annum is 4%.

The analysis shows that, the question was answered by 590,042 (97.9%) students out of whom 175,438 (29.7%) students scored from 3 to 10 marks indicating that the performance in this question was weak. Further analysis reveals that, only 69,900 (11.8%) students scored full marks while a total of 340,004 (57.6%) scored zero. Figure 3 shows the summary of the students' performance in question 3.

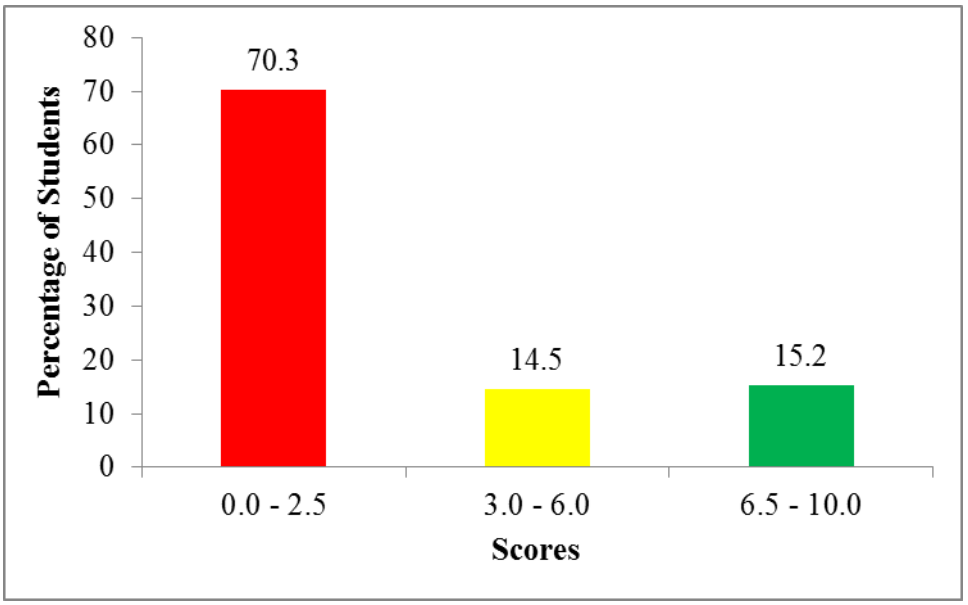
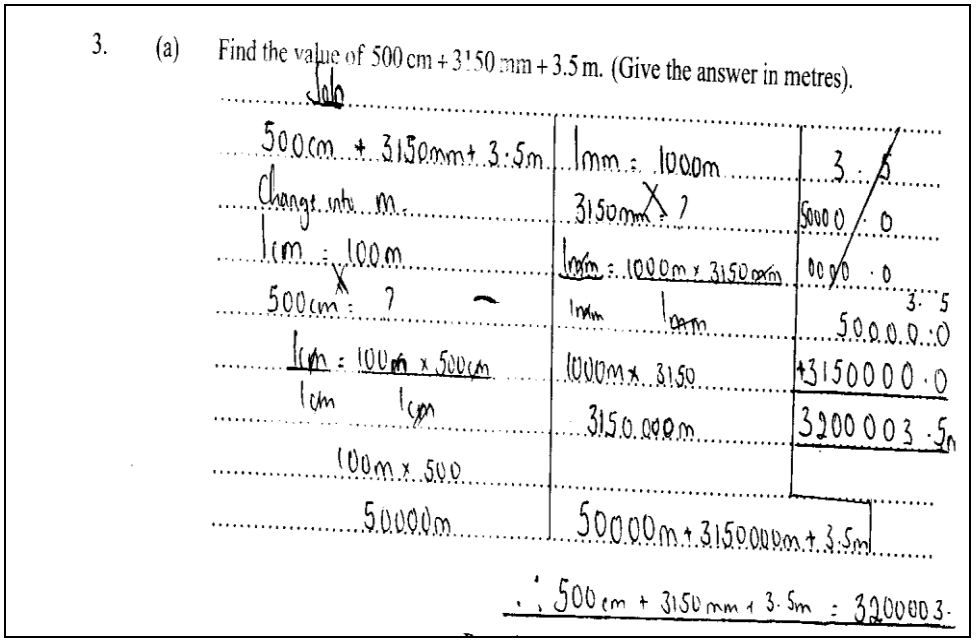


Figure 3: *Students' performance in question 3*

The weak performance was mainly contributed by lack of knowledge of units conversion. In part (a), the majority of students were unable to convert one unit of length to another. For example, some of them wrote that $1\,cm = 100\,m$ and $1\,mm = 1000\,m$ instead of using the conversion $1\,m = 100\,cm = 1000\,mm$ to get the required answer. Also, there were the students who just added the given numbers with different units, that is $500 + 3150 + 3.5 = 3,653.5\,m.$ Others were able to write $\left(\frac{500}{100} + \frac{3150}{1000} + 3.5\right)\,m$ but failed to perform correct calculations due to lack of arithmetic skills. In part (b), most of them failed to either recall or correctly write the formula for simple interest. Some students had inadequate understanding on the value of the rate (R) in relation to the simple interest formula. They applied incorrect formulae like $I = PRT$ and $I = \frac{TR}{100}$ instead of $T = \frac{100I}{PR}$ which was an important step to calculate the required time (T). Others managed to recall the correct formula but faced difficulties during computation. Extract 3.1 shows an incorrect response in this question.



(b) Find the number of years in which Tshs. 20,000 will earn an interest of Tshs. 4,800 if the interest rate is 4% per annum.

Soln

$I = \frac{PTR}{100}$

$I = \frac{20000 \times 4800 \times 4}{1000}$

$I = \frac{960,00 \cdot \times 4}{100}$

$I = \frac{96000 \times 4}{100}$

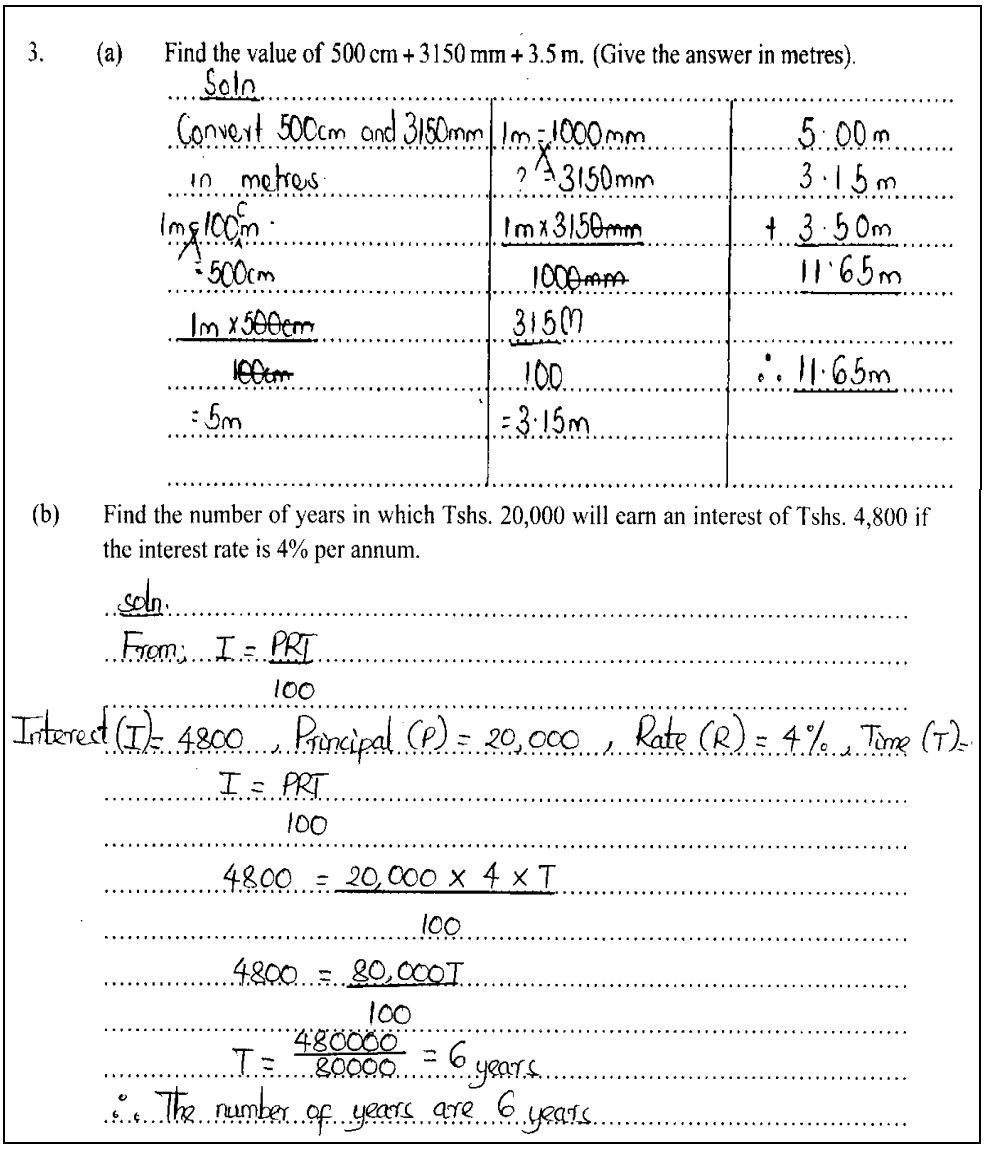
$I = 960,0 \times 4$

$I = 3840$

Extract 3.1: A sample of incorrect responses in question 3.

In Extract 3.1, the student performed incorrect conversions of the given units in part (a). He/she also applied incorrect formula for finding the simple interest in part (b).

Though the performance was weak, there were students who managed to answer this question correctly. Those students were able to: (a) convert the given units correctly, that is $500\ cm = 5\ m$ and $3150\ mm = 3.15\ m$, leading to $5\ m + 3.15\ m + 3.5\ m = 11.65\ m$ as required. (b) apply the formula for simple interest, that is $T = \frac{100I}{PR}$ and managed to make correct substitution of the given values, that is $P = 20{,}000$, $R = 4$ and $I = 4800$ to get the value of T, which is 6 years. Extract 3.2 represents a sample of the correct answer from one of the students.



3. (a) Find the value of 500 cm + 3150 mm + 3.5 m. (Give the answer in metres).

Soln

Convert 500cm and 3150mm to metres

1m = 100cm

∴ 500cm = 1m × 500cm / 100cm = 5m

1m = 1000mm

∴ 3150mm = 1m × 3150mm / 1000mm = 3150 / 100 = 3·15m

5·00 m
3·15 m
+ 3·50 m
11·65 m

∴ 11·65m

(b) Find the number of years in which Tshs. 20,000 will earn an interest of Tshs. 4,800 if the interest rate is 4% per annum.

Soln.

From: $I = \frac{PRT}{100}$

Interest (I) = 4800, Principal (P) = 20,000, Rate (R) = 4%, Time (T) =

$I = \frac{PRT}{100}$

$4800 = \frac{20,000 \times 4 \times T}{100}$

$4800 = \frac{80,000T}{100}$

$T = \frac{480000}{80000} = 6$ years

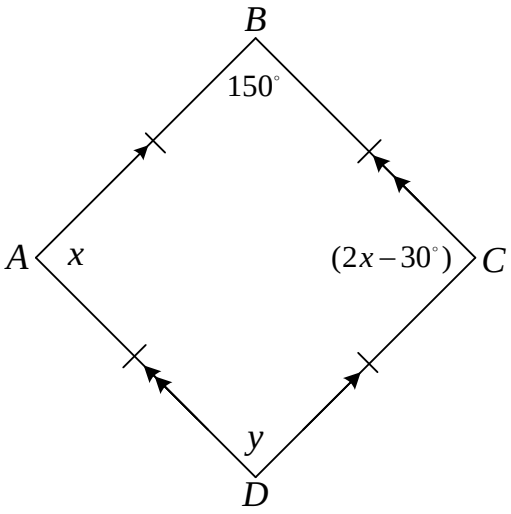
∴ The number of years are 6 years

Extract 3.2: A sample of the correct responses in question 3.

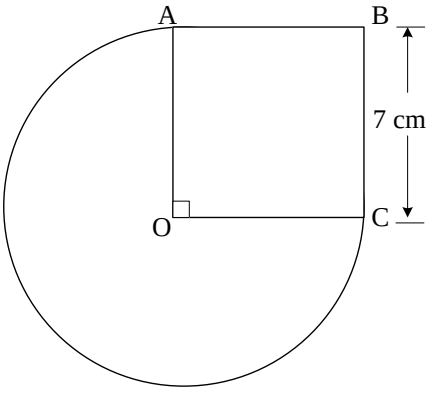
Extract 3.2 shows that, the student was able to convert the given units, millimetres and centimetres into metres and lastly performed correct addition in part (a). He/she also applied the correct formula for simple interest and performed correct computations to obtain the required number of years in part (b).

2.4 Question 4: Geometry, Perimeters and Areas

The question had part (a) and (b). In part (a) (i), students were required to write the name of the polygon ABCD represented in the following figure.



The students also were required to find the values of x and y from the figure given in part (a) (i). In part (b), the students were required to calculate the area of the following figure, if O is the centre of the circle and OABC is a square.



The question was answered by 558,562 (92.7%) students and among them, 489,423 (87.6%) students scored below 3 marks. This shows that the students' performance in this question was weak. Only 2,079 (0.4%) students scored full marks while a total of 347,649 (62.2%) scored zero in this question. Figure 4 portrays a summary of students' performance in this question.

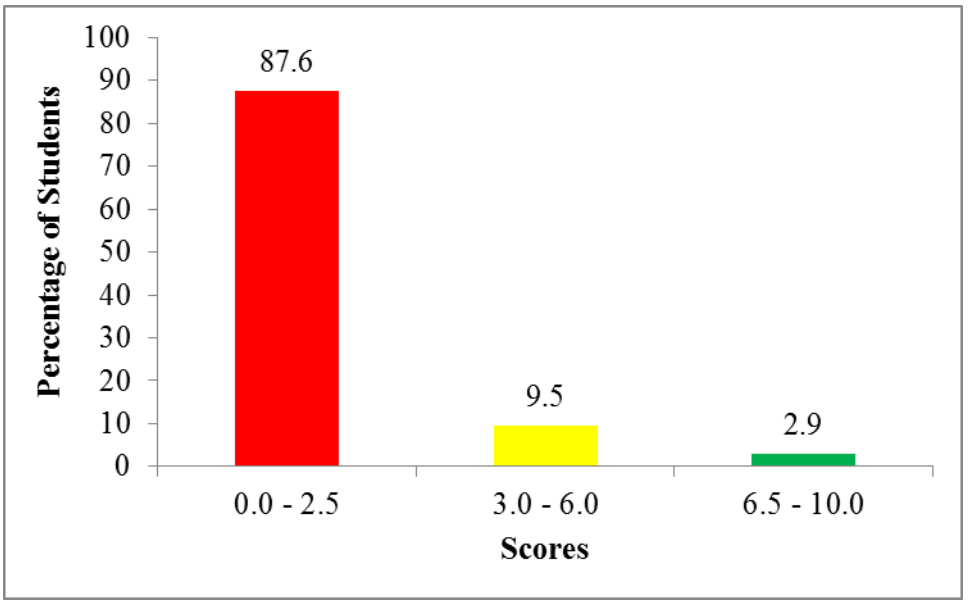
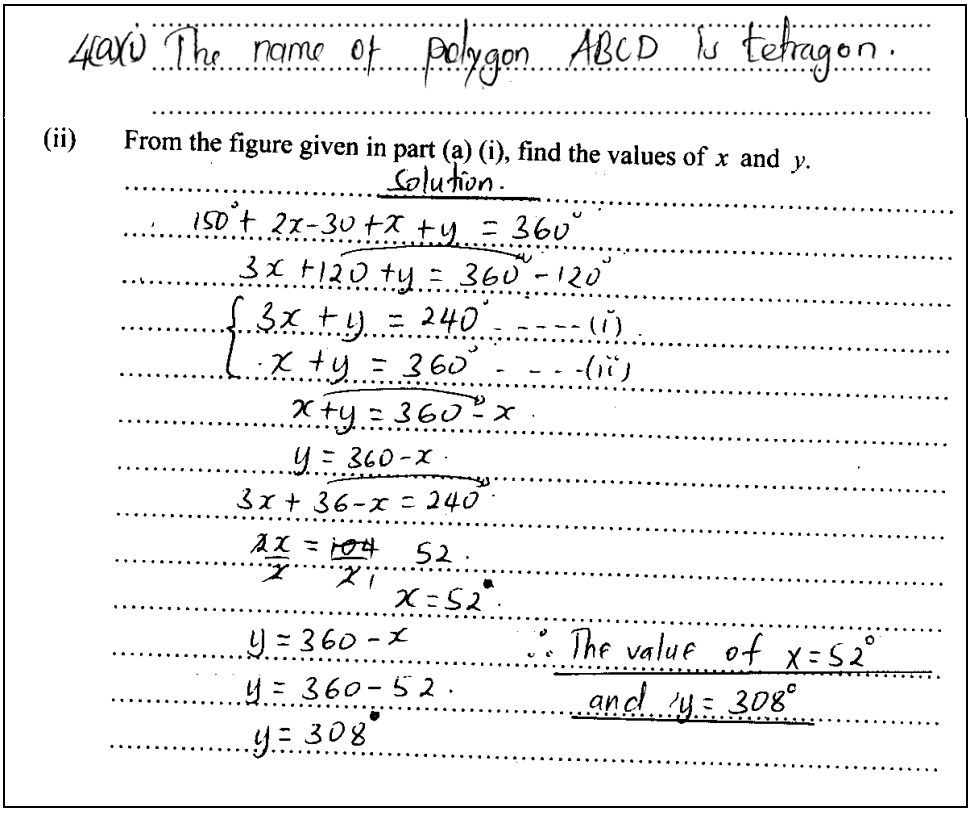
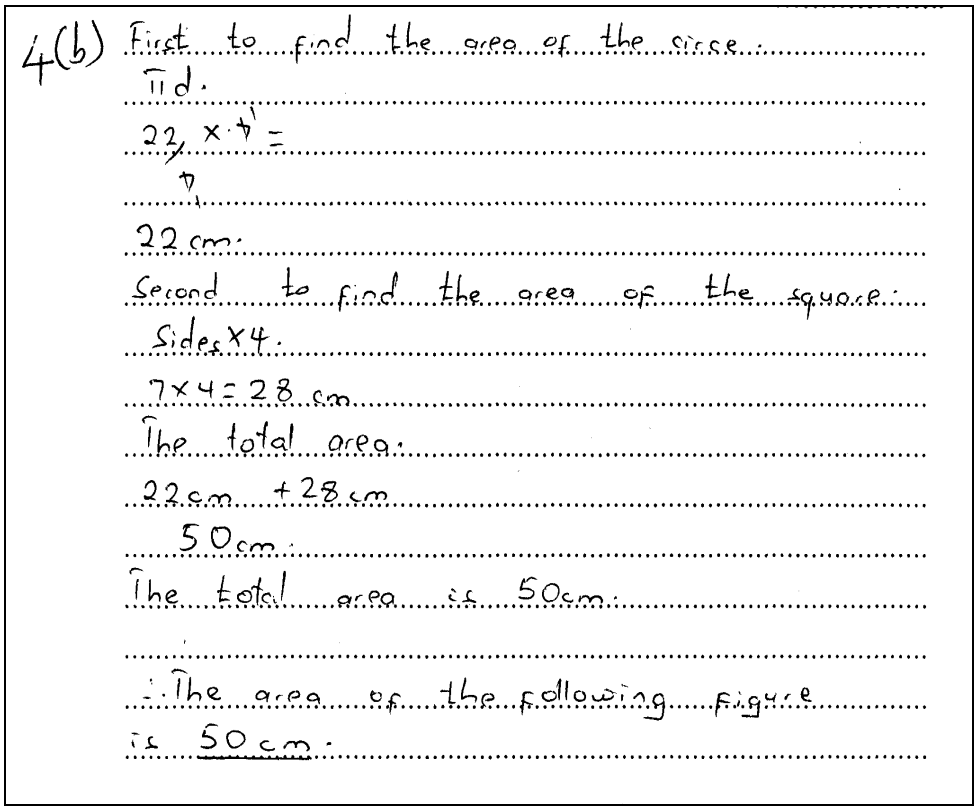


Figure 4: *Students' performance in question 4*

The students' weak performance in this question was contributed by the following factors: in part (a) (i), they were unable to name the given polygon ABCD. For example, some students named the figure as a square, rectangle and parallelogram. Those students failed to differentiate rhombus from other quadrilaterals due to lack of knowledge of geometrical figures. This attribute hindered them from getting the correct values of x and y in part (a) (ii). Others managed to correctly name the given polygon ABCD as rhombus but failed to determine the angles of the given figure due to insufficient knowledge of properties of rhombi. Those students were not aware that, the opposite interior angles of a quadrilateral are equal so that $x = 2x - 30^{\circ}$ and $y = 150^{\circ}$. In part (b), the majority failed to calculate the area of the given figure as they used incorrect formulae like $\text{total area} = (\text{area of the circle}) + (\text{area OABC})$. Instead, they were supposed to calculate the area of the three quarters of the given circle and then add the area of the square by using the formula $\text{total area} = \left(\frac{3}{4} \times \text{area of the circle}\right) + (\text{area OABC})$. In some cases, there were students who calculated correctly the area of the square but failed to

get the area of $\frac{3}{4}$ of a circle as they were unable to identify the radius of a circle using the length of a square. Those students were supposed to know that the radius of the circle is equal to the length of a square which is 7 cm. Others subtracted the area of a square from the area of a circle, that is, $\left(\frac{22}{7}\times 7\times 7\right)-(7\times 7)$ which would give the area of the three quarters of the circle contrary to the given instructions. Others used the required formula and substituted the required information correctly to get $\left(\frac{3}{4}\times\frac{22}{7}\times 7\times 7\right)+(7\times 7)$ but failed to perform the correct computations. All these results were an indication that the students lacked knowledge and skills of finding the areas of a circle and a square. Extract 4.1 shows a sample of student who answered this question incorrectly.





Extract 4.1: A sample of incorrect responses in question 4.

Extract 4.1 shows that, the student named the given figure as a tetragon instead of rhombus. He/she also equated the sum of all interior angles to 360° instead of 180° in part (a). In part (b), the student applied the formula for circumference of a circle and perimeter of a square instead of calculating the total area of the given figure.

Apart from weak performance, there were students who managed to answer the given question correctly. In part (a) (i), they were able to name the given polygon ABCD correctly as rhombus indicating that they had enough knowledge of the naming regular polygons. In part (a) (ii), they were aware of the properties of rhombi that helped them to obtain the angles of the given geometrical figure. In part (b), the students calculated the total area of the given figure as follows: $\frac{3}{4}\times\frac{22}{7}\times 7^2+7\times 7=115.5\text{ cm}^2+49\text{ cm}^2$ and obtained 164.5 cm^2. Extract 4.2 shows a sample of a correct solution given by a certain student.

(a) (i) The polygon ABCD is a Rhombus

(ii) From the figure given in part (a) (i), find the values of x and y.

In the rhombus ABCD

$A\hat{B}C = A\hat{D}C\ (y)$

$D\hat{A}B = D\hat{C}B$

$A\hat{B}C = 150° = A\hat{D}C(y)$

$\therefore y = 150°$

$D\hat{A}B = x$

$D\hat{C}B = 2x - 30°$

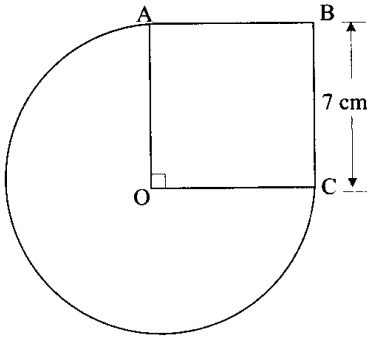
$2x - 30° = x$

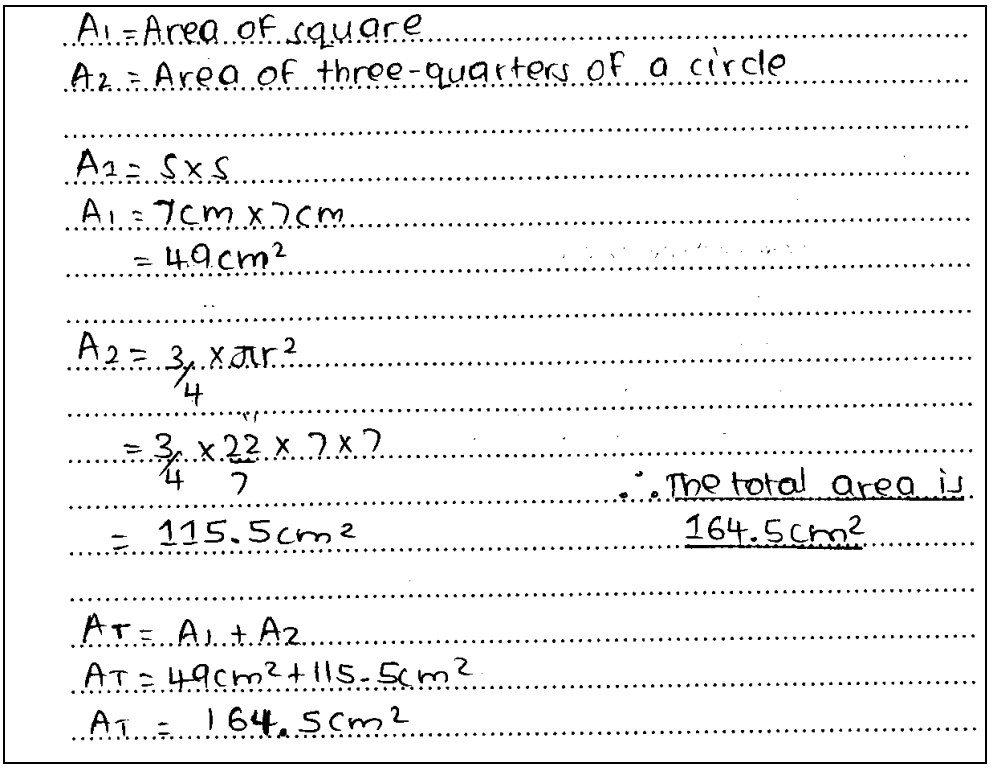
$x = 30°$

$\therefore x = 30°$

$y = 150°$

(b) Calculate the area of the following figure, if O is the centre of the circle and $OABC$ is a square.





Extract 4.2: A sample of the correct responses in question 4.

In Extract 4.2, the student identified the correct name of the polygon which is Rhombus and calculated the values of x and y correctly using the properties of rhombi in part (a). He/she also calculated correctly the area of each part of the given figure and then added up to get the required total area in part (b).

2.5 Question 5: Algebra and Quadratic Equations

The question was composed parts (a) and (b). In part (a), the students were given that the age of the father is three times the age of his son. If the sum of their ages is 64 years, then students were demanded to find their ages. In part (b), students were asked to solve the quadratic equation $x^2 + 7x + 12 = 0$ by using the factorization method.

The question was answered by 577,516 (95.8%) students out of whom 431,063 (74.6%) students scored below 3 marks. This indicates that the students' performance in this question was weak. It was further noted that, only 37,222 (6.4%) students scored full marks while 397,859 (68.9%)

scored zero in this question. Figure 5 shows the summary of the students' performance in question 5.

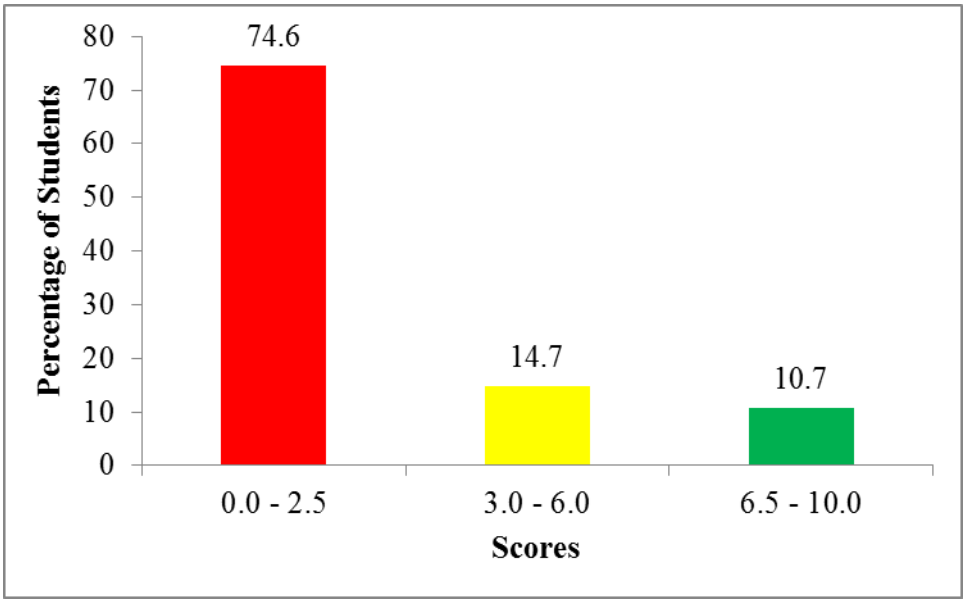
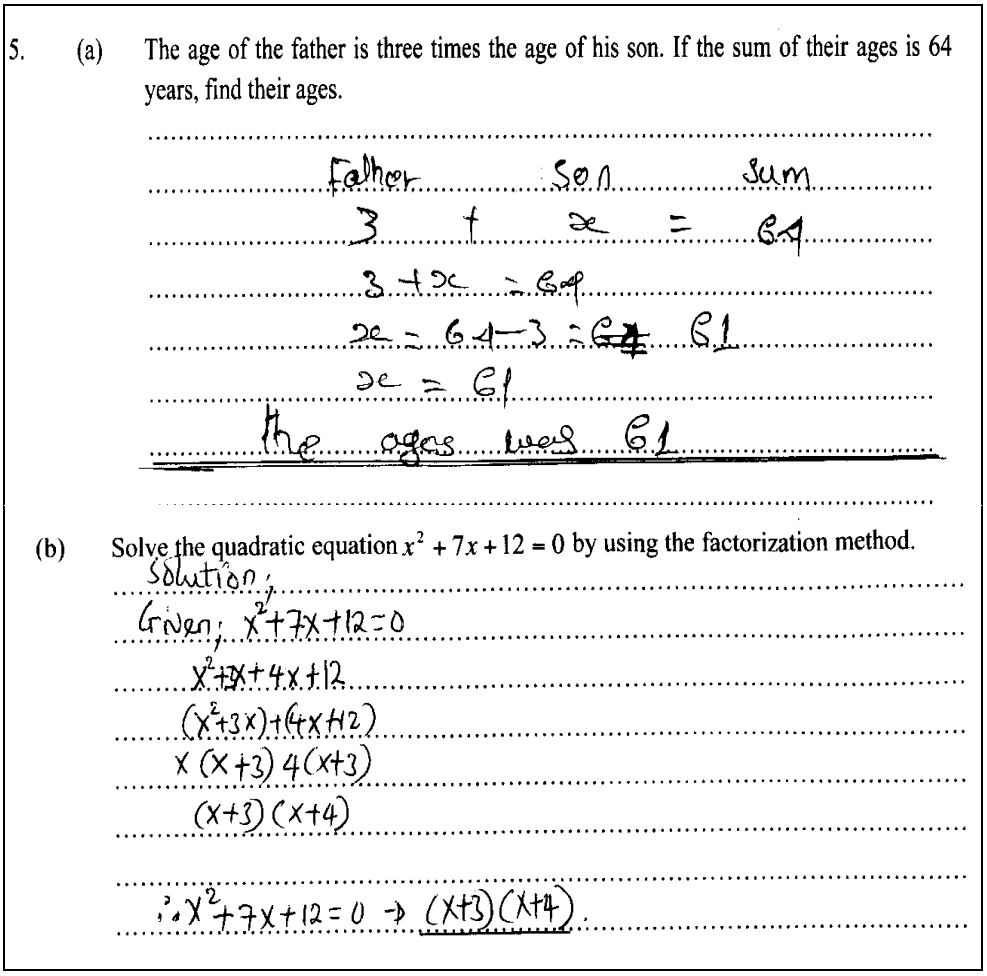


Figure 5: *Students' performance in question 5*

The analysis shows that, the majority of students scored low marks because they were unable to give the correct responses. In part (a), they failed to formulate a correct equation from the given word problem. For example, they wrote incorrect equations like: $2x+3=64$, $3x-x=64$ and $y+3x=64$ instead of $3x+x=64$, where x stands for the age of the son. Other students just multiplied or divided the given numbers, that is $64\div 3$ due to wrong interpretation of the given word problem. In part (b), most students used the method of completing the square and quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve the given equation instead of using factorization method as it was instructed. Other students termed x^2 as $2x$ which led them to change the equation from quadratic equation $x^2+7x+12=0$ to linear equation $2x+7x+12=0$. Further analysis shows that, some students formulated the expression $\left(x^2+3x\right)+\left(4x+12\right)$ instead of equation $\left(x^2+3x\right)+\left(4x+12\right)=0$ because they mixed up the concepts of *expression* and *equation*. However, some students adhered to the given instruction but failed to determine the factors $3x$ and $4x$ that

would replace the middle term $7x$ during factorization. Extract 5.1 shows a sample of a students' incorrect solution in this question.

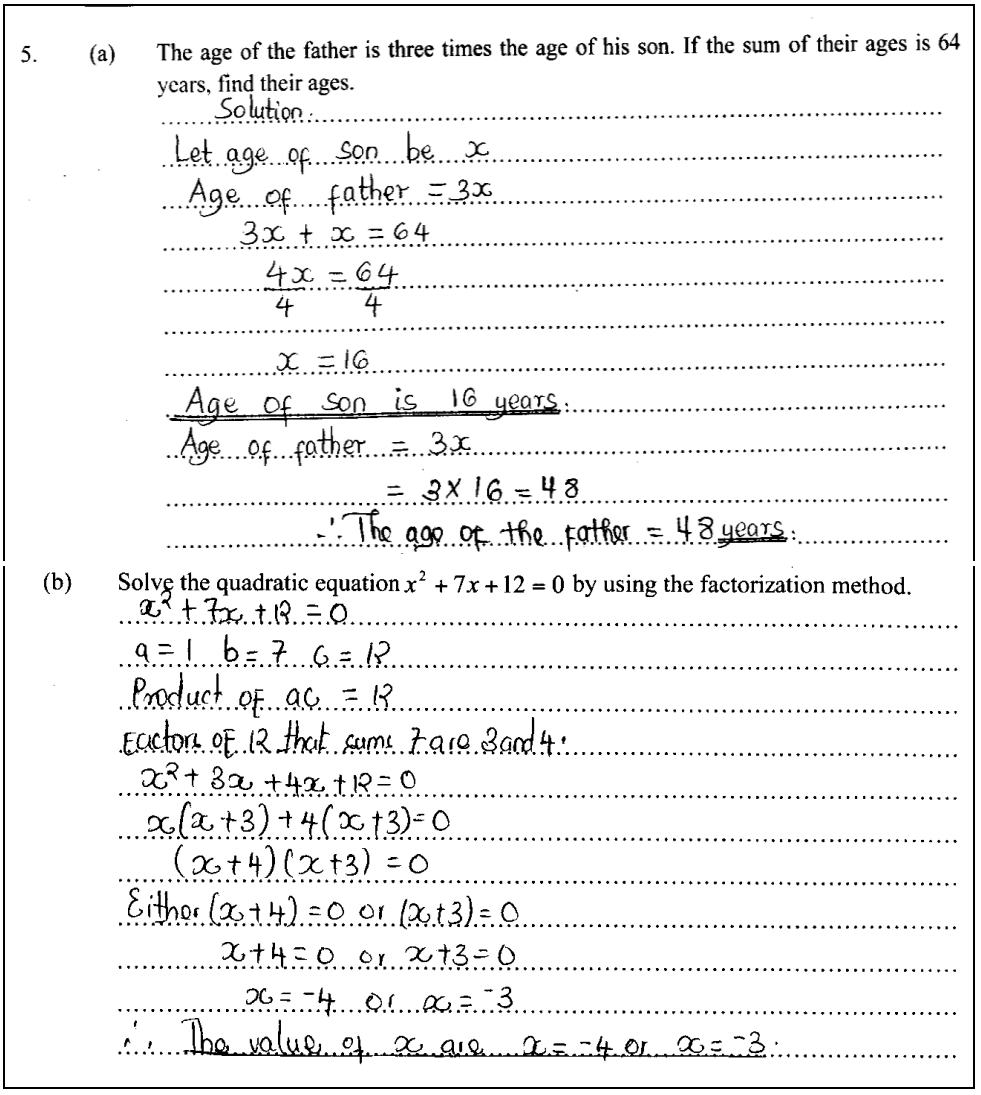


Extract 5.1: A sample of incorrect responses in question 5.

In Extract 5.1, the student failed to determine the relationship between the age of the son and that of the father. He/she also failed to factorize correctly the given equation in solving for x.

On the contrary, some of the students were able to answer the question correctly. In part (a), some students solved the given word problem correctly by letting the son's age be x and father's age be $3x$. They also formulated and solved the equation $x+3x=64$ and obtained $x=16$ years being the age of the son and hence the age of the son's father given by $3\times16=48$ years. This shows that, the students had sufficient knowledge and skills of the application of algebra in real life situation. In part (b), the

students were able to factorize the equation $x^2 + 7x + 12 = 0$ to obtain $x = -3$ or $x = -4$ indicating that, they were competent in solving quadratic equations by using factorization method as shown Extract 5.2.


5. (a) The age of the father is three times the age of his son. If the sum of their ages is 64 years, find their ages.

Solution

Let age of son be x

Age of father $= 3x$

$3x + x = 64$

$\frac{4x}{4} = \frac{64}{4}$

$x = 16$

Age of son is 16 years

Age of father $= 3x$

$= 3 \times 16 = 48$

∴ The age of the father = 48 years

(b) Solve the quadratic equation $x^2 + 7x + 12 = 0$ by using the factorization method.

$x^2 + 7x + 12 = 0$

$a = 1 \quad b = 7 \quad c = 12$

Product of $ac = 12$

Factors of 12 that sums 7 are 3 and 4.

$x^2 + 3x + 4x + 12 = 0$

$x(x+3) + 4(x+3) = 0$

$(x+4)(x+3) = 0$

Either $(x+4) = 0$ or $(x+3) = 0$

$x + 4 = 0$ or $x + 3 = 0$

$x = -4$ or $x = -3$

∴ The value of x are $x = -4$ or $x = -3$

Extract 5.2: A sample of the correct responses in question 5.

In Extract 5.2, the student calculated correctly the ages of the son and the father respectively in part (a). He/she also solved correctly the given equation by factorization method in part (b).

2.6 Question 6: Coordinate Geometry and Geometrical Transformations

The question had two parts. In part (a), the students were required to find the slope and the equation of a line joining the points A (6, 4) and B (12, 6).

In part (b) (i), it was given that, a translation takes the origin to $(-3,-4)$. The students were required to find where it takes $Q(1,-2)$ without drawing. In part (b) (ii), they were required to find the image of the points A (-5, 2) and B (4, -7) after reflection in the y – axis.

This question was attempted by 566,323 (94.0%) students out of whom 464,970 (82.1%) scored below 3 marks. It was also noted that, only 13,922 (2.5%) students scored full marks whereas a total of 372,806 (65.8%) scored zero. This indicates that the students' performance in this question was weak. Figure 6 shows the summary of the students' performance in in question 6.

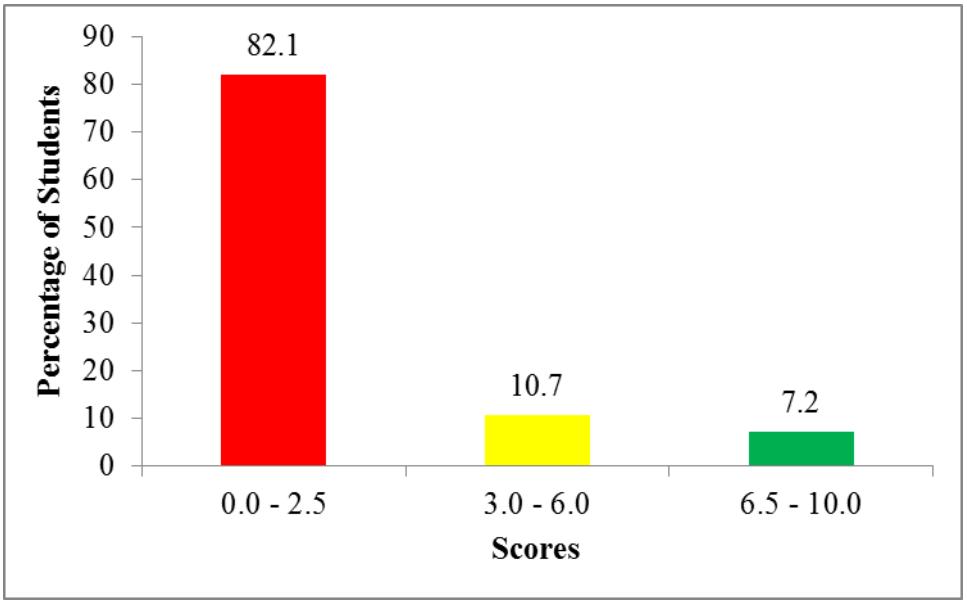
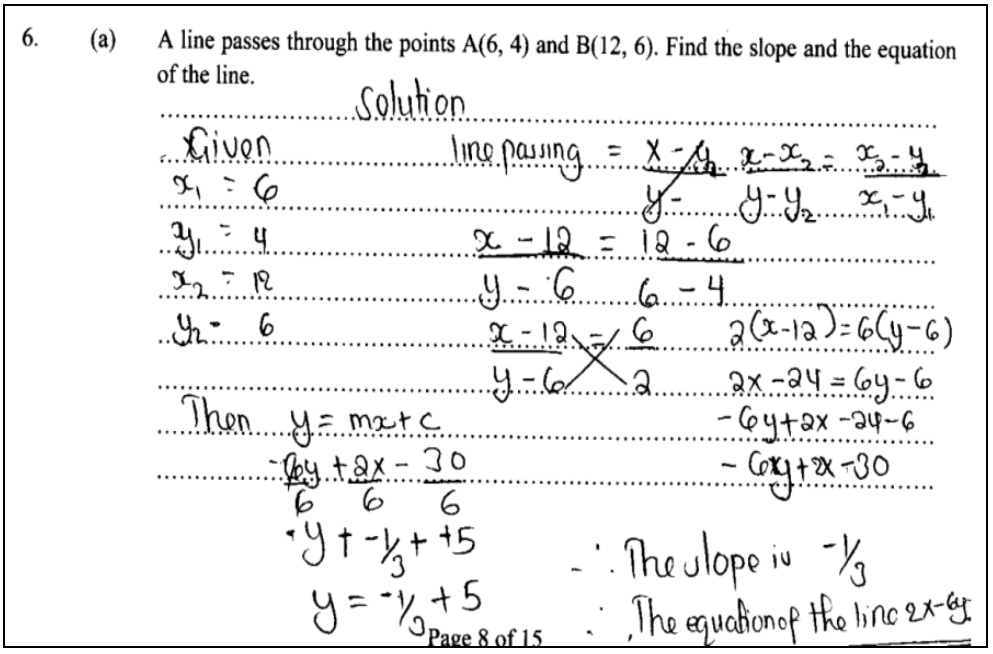


Figure 6: *Students' performance in question 6*

The analysis reveals that, the weak performance was contributed by different factors. In part (a), the majority of students failed to recall/apply the correct formula for the slope of a line which is $\text{slope} = \dfrac{\text{change in y}}{\text{change in x}}$ or $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$. For example, they applied incorrect formulae like $\text{slope} = \dfrac{\text{change in x}}{\text{change in y}}$ and $\text{slope} = \dfrac{y_2 - x_2}{y_1 - x_1}$ leading to incorrect slopes and equations. In part (b) (i), the majority failed to recall the formula required

for determining the image of the point Q as they applied incorrect formulae like $\begin{pmatrix} y' \\ x' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$ instead of $\begin{pmatrix} y' \\ x' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$. In part (b)(ii), some of the students were not able to apply the concept of the properties of reflection in a plane when finding the image of a given point. They applied incorrect formula like $M_y(x, y) = (x, -y)$ instead of $M_y(x, y) = (-x, y)$. As a result, they ended up with incorrect images such as $A'(-5, -2)$ instead of $(5, 2)$ and $B'(4, 7)$ instead of $B'(-4, -7)$. Extract 6.1 is a sample of the student's incorrect response to this question.



6. (a) A line passes through the points A(6, 4) and B(12, 6). Find the slope and the equation of the line.

Solution

Given

$x_1 = 6$

$y_1 = 4$

$x_2 = 12$

$y_2 = 6$

line passing $= \frac{x - x_1}{y - y_1} = \frac{x_2 - x_1}{y_2 - y_1}$

$\frac{x - 12}{y - 6} = \frac{12 - 6}{6 - 4}$

$\frac{x - 12}{y - 6} = \frac{6}{2}$

$2(x - 12) = 6(y - 6)$

$2x - 24 = 6y - 6$

$-6y + 2x - 24 - 6$

$-6y + 2x - 30$

Then $y = mx + c$

$\frac{-6y}{6} + \frac{2x}{6} - \frac{30}{6}$

$y + -\frac{1}{3} + 5$

$y = -\frac{1}{3} + 5$

∴ The slope is $-\frac{1}{3}$

∴ The equation of the line $2x - 6y$

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(b) (i) A translation takes the origin to (-3, - 4). Without drawing, find where it takes Q(1,-2).

$(x', y') = (a, b) - (x, y)$

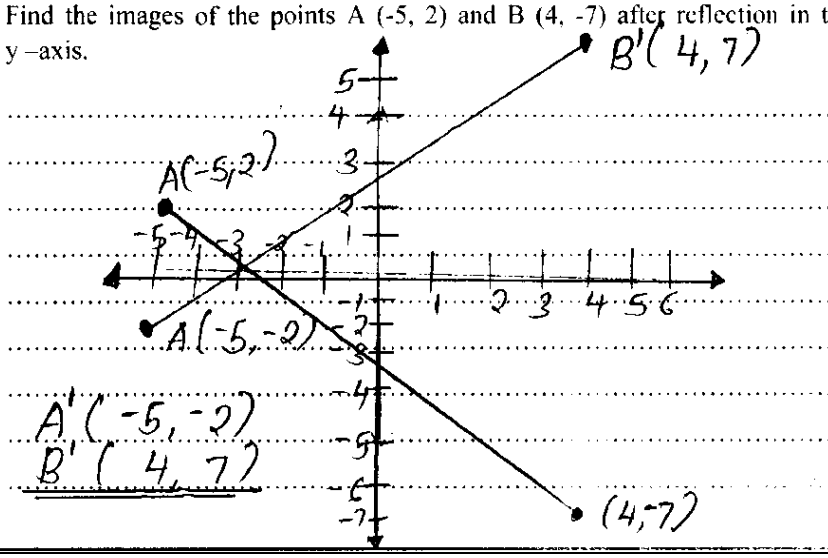
$= (-3, -4) - (1, -2)$

$= (-3-1)(-4+2)$

$\Rightarrow -4, -2$

$(x', y') = (-4, -2)$

(ii) Find the images of the points A (-5, 2) and B (4, -7) after reflection in the y –axis.



Extract 6.1: A sample of incorrect responses in question 6.

In Extract 6.1, the student applied incorrect formula for finding the slope of a line by using two points in part (a). In part (b) (i), the student wrote $(x', y') = (a, b) - (x, y)$ instead of $(x', y') = (a, b) + (x, y)$. In part (b) (ii), He/she performed the reflection of the given points in the x-axis instead of y-axis as it was required in the question.

Conversely, some students attempted this question correctly. In part (a), they managed to find the slope of the line joining the two points by using the formula $\text{slope (m)} = \dfrac{\text{change in y}}{\text{change in x}}$ to get $m = \dfrac{1}{3}$ and hence applied the slope obtained to find the equation of the line using the formula $m = \dfrac{y - y_\circ}{x - x_\circ}$ where $(x_\circ, y_\circ)$ is any point between the two given points or

$y - y_1 = m(x - x_1)$, where m is the slope of the line. Consequently, they were able to obtain the required equation $y = \frac{1}{3}x + 2$. In part (b) (i), the students applied correctly the properties of translation to find the new position of Q. Firstly, they determined the translation factor from origin to (-3, - 4), given as $T(a,b) = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ and applied it to find the new position of Q, that is $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ leading to $(-2,-6)$. In part (b) (ii), they applied correctly the properties of reflection in a plane by using the formula $M_y(x,y) = (-x,y)$ to get the image of A (-5, 2) as $A'(5,\ 2)$ and the image of B (4, -7) as $B'(-4,-7)$. Extract 6.2 represents the correct solution from one of the students in question 6.

6. (a) A line passes through the points A(6, 4) and B(12, 6). Find the slope and the equation of the line.

Soln:

$(x_1, y_1) = (6, 4)$

$(x_2, y_2) = (12, 6)$

Slope $(m) = ?$

From

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-4}{12-6} = \frac{2}{6} = \frac{1}{3}$

$m = \frac{1}{3}$

$\therefore$ The slope is $\frac{1}{3}$

The equation.

From

$y = m(x - x_1) + y_1$

$3\left(y = \frac{1}{3}(x-6) + 4\right)$

$3y = 1(x-6) + 12$

$3y = 1x - 6 + 12$

$\frac{3y}{3} = \frac{1x}{3} + \frac{6}{3}$

$y = \frac{1}{3}x + 2$

$\therefore$ The equation of the line is $y = \frac{1}{3}x + 2$

(b) (i) A translation takes the origin to (-3, - 4). Without drawing, find where it takes Q(1,-2).

$P'(x',y') = (x,y) + (a,b)$

$(-3,-4) = (0,0) + (a,b)$

$(-3,-4) - (0,0) = (a,b)$

$(-3,-4) = (a,b)$

∴ Translation vector $= (-3,-4)$

$Q'(x',y') = (x,y) + (a,b)$

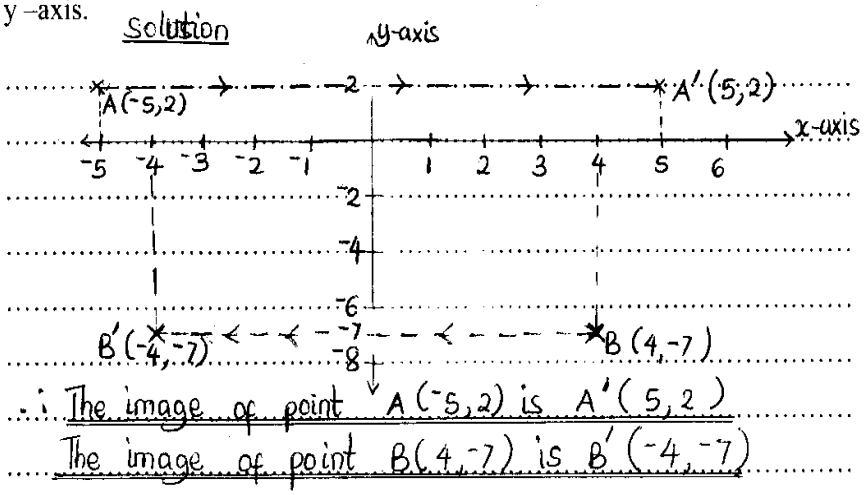
$Q'(x',y') = (1,-2) + (-3,-4)$

$Q'(x',y') = (-2,-6)$

∴ It takes $Q(1,-2)$ to $Q'(-2,-6)$

(ii) Find the images of the points A (-5, 2) and B (4, -7) after reflection in the y –axis.

Solution



∴ The image of point A (-5,2) is A' (5, 2)

The image of point B(4,-7) is B' (-4,-7)

Extract 6.2: A sample of the correct responses in question 6.

In extract 6.2, the student used the appropriate formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to obtain the gradient of the line and correctly substituted the gradient and one of the points in the equation $y = m(x - x_1) + y_1$ to get required equation $y = \frac{1}{3}x + 2$ in part (a). In part (b), he/she translated the point *Q* (1, -2) correctly using the given condition; and reflected the points *A* (-5, 2) and *B* (4, -7) in the y-axis by using the correct procedure.

2.7 Question 7: Exponents and Radicals, and Logarithms

The question consisted of parts (a) and (b). In part (a), the students were required to find the value of x in the equation $\left(\frac{1}{3}\right)^{\sqrt{x}} = 81^{-x}$. In part (b), they were required to solve the equation $\log_{10}\left(\frac{x}{5}\right) = \log_{10}\left(\frac{2}{x}\right) + 1.$

The question was attempted by a total of 548,784 (91.1%) students out of whom 25,264 (4.6%) students scored at least 3 marks. Only 1,251 (0.2%) students scored full marks while 471,294 (85.9%) scored zero, indicating a weak performance. Figure 7 shows the summary of the students' performance.

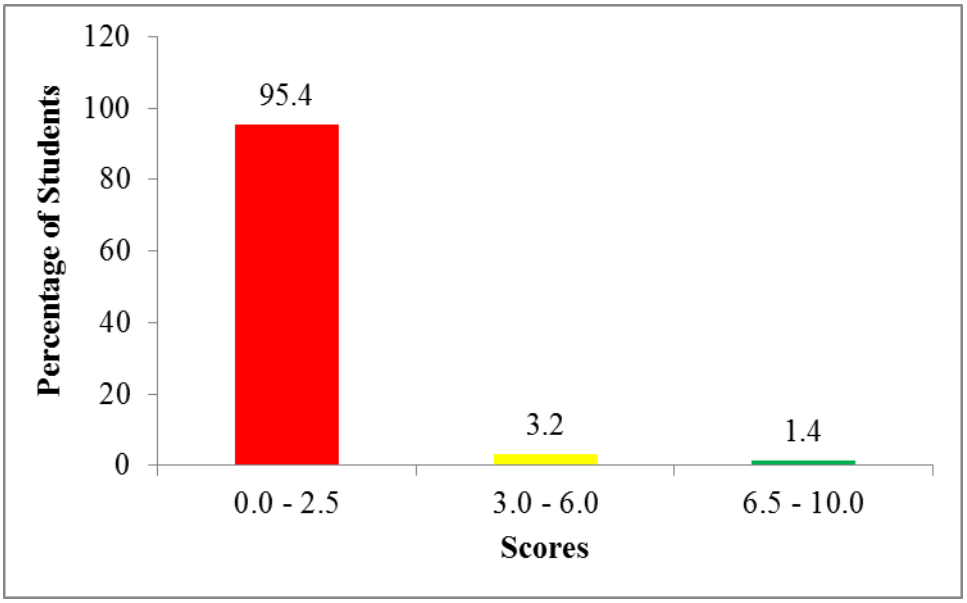
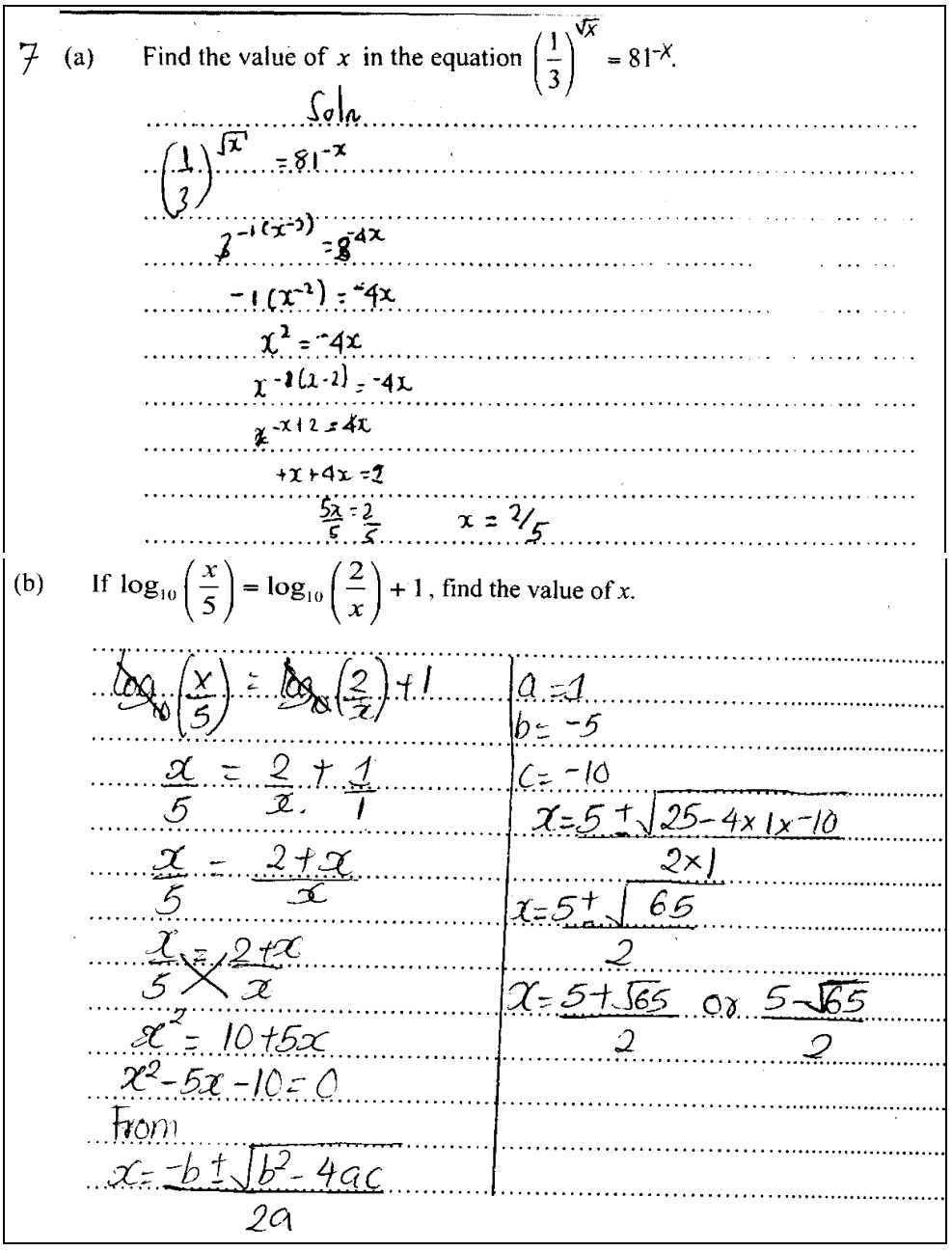


Figure 7: *Students' performance in question 7*

The data analysis reveals that, the students' performance in this question was the weakest. The students failed to get the required answers due to various reasons. In part (a), the students seemed to lack adequate knowledge of the application of the laws of exponents. For example, some students expressed $\sqrt{x}$ as $\frac{1}{2}x$ instead of $x^{\frac{1}{2}}$ while others expressed 81^{-x} as $\sqrt{9^{2-x}}$ instead of 3^{-4x}. However, there were some students who

simplified the given equation correctly as $\left(3^{-1}\right)^{\sqrt{x}} = \left(3^{4}\right)^{-x}$ but failed to compare the exponents. For example, they wrote $\sqrt{x} = -x,\ -\sqrt{x} = 4x$ and $\sqrt{x} = 4$ instead of $\sqrt{x} = 4x$. Others were able to get the equation $x = 16x^2$ but they wrote only one value of *x*, that is $x = \frac{1}{16}$. Those students did not recognize that $x = 0$ was also part of the required solution. This shows that, the students lacked knowledge and skills of solving quadratic equations. In part (b), the students failed to apply the laws of logarithms in solving the given logarithmic equation. For example, some wrote $\log_{10}\left(\frac{x}{5}\right) = \log(x-5)$ instead of $\log_{10}\left(\frac{x}{5}\right) = \log_{10}(x) - \log_{10}(5)$ and $\log_{10}\left(\frac{2}{x}\right) = \log(2-x)$ instead of $\log_{10}\left(\frac{2}{x}\right) = \log_{10}(2) - \log_{10}(x)$ by using the law of quotient of logarithm. Other Students cancelled the term "$\log_{10}$" from both sides of the equation and got the equation $\frac{x}{5} = \frac{2}{x} + 1$. Those students were supposed to collect like terms when simplifying, that is $\log_{10}\left(\frac{x}{5}\right) - \log_{10}\left(\frac{2}{x}\right) = 1$ and then apply the quotient rule to get $\log_{10}\left(\frac{x^2}{10}\right) = 1$ or $10^1 = \frac{x^2}{10}$ which was an important step to get the required answer, $x = 10$. Extract 7.1 is a sample of an incorrect solution in question 7.



Extract 7.1: A sample of incorrect responses in question 7.

In Extract 7.1, the student compared the exponents incorrectly by writing $x^2 = -4x$ instead of $x^{\frac{1}{2}} = 4x$ in part (a). In part (b), he/she just cancelled the term "$\log_{10}$" from both sides of the given equation instead of applying the law of quotient of logarithm to solve the equation.

Notwithstanding, a small number of students answered this question correctly. In part (a), the students correctly applied the appropriate laws to compare the exponent under the same base. They correctly re-write the equation $\left(\frac{1}{3}\right)^{\sqrt{x}} = 81^{-x}$ in the form $\left(3^{-1}\right)^{\sqrt{x}} = \left(3^{4}\right)^{-x}$. Later, they applied the product rule of exponent to get $3^{-\sqrt{x}} = 3^{-4x}$. Then, they equated the exponents and applied some concepts of radicals which resulted into a quadratic equation $16x^2 - x = 0$ and thereafter solved it to get the required values of x, that is 0 and $\frac{1}{16}$. In part (b), they were able to apply the laws of logarithm to solve the given equation. Firstly, they applied the quotient law to re-arrange $\log_{10}\left(\frac{x}{5}\right) = \log_{10}\left(\frac{2}{x}\right) + 1$ into $\log_{10}\left(\frac{x}{5}\right) - \log_{10}\left(\frac{2}{x}\right) = 1$ to get $\log_{10}\left(\frac{x}{5} \div \frac{2}{x}\right) = 1$ or $\log_{10}\left(\frac{x^2}{10}\right) = 1$ and lastly solved it to obtain $x = 10$. Generally, the students had adequate knowledge and skills of solving logarithmic and exponential equations by employing the appropriate laws. Extract 7.2 shows a sample solution from one of the students who answered question 7 correctly.

7. (a) Find the value of x in the equation $\left(\frac{1}{3}\right)^{\sqrt{x}} = 81^{-x}$.

soln

$\left(\frac{1}{3}\right)^{\sqrt{x}} = 81^{-x}$

$\left(\frac{1}{3}\right)^{\sqrt{x}} = (3^4)^{-x}$

$\left(\frac{1}{3}\right)^{\sqrt{x}} = 3^{-4x}$

$\left(\frac{1}{3}\right)^{\sqrt{x}} = \left(\frac{1}{3}^{-1}\right)^{-4x}$

$\left(\frac{1}{3}\right)^{\sqrt{x}} = \left(\frac{1}{3}\right)^{4x}$

$(\sqrt{x})^2 = (4x)^2$

$x = 16x^2$

$x - 16x^2 = 0$

$x(1-16x) = 0$

$x = 0$ OR $1-16x = 0$

$\frac{-16x}{-16} = \frac{-1}{-16}$

$x = \frac{1}{16}$

$\therefore x = 0$ OR $x = \frac{1}{16}$

(b) If $\log_{10}\left(\frac{x}{5}\right) = \log_{10}\left(\frac{2}{x}\right) + 1$, find the value of x.

Soln

$\log_{10}\left(\frac{x}{5}\right) = \log_{10}\left(\frac{2}{x}\right) + 1$

too but $\log_{10} 10^1 = 1$

$\log_{10}\left(\frac{x}{5}\right) = \log_{10}\left(\frac{2}{x}\right) + \log_{10} 10$

$\log_{10}\frac{x}{5} = \log_{10}\left(\frac{2}{x} \times 10\right)$

$\log_{10}\left(\frac{x}{5}\right) = \log_{10}\left(\frac{20}{x}\right)$

$\frac{x}{5} \times \frac{20}{x}$

$\sqrt{x^2} = \sqrt{100}$

$x = \pm 10$

$\therefore x = 10$ or $x = -10$

but there is no log - so

$x = 10$

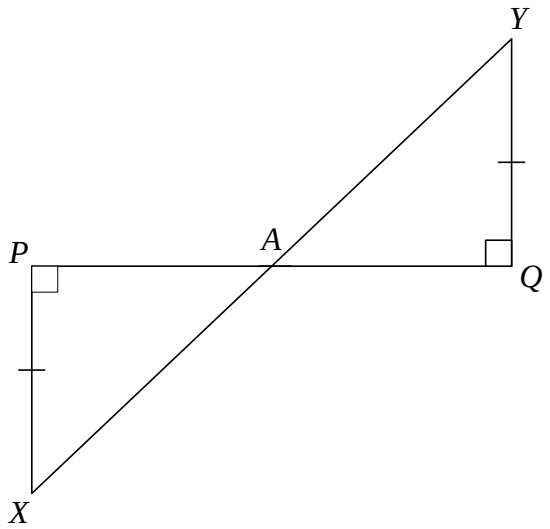
Extract 7.2: A sample of the correct responses in question 7.

In Extract 7.2, the student changed the equation into powers of the same base and compared the exponents to get the equation $x - 16x^2 = 0$. Lastly,

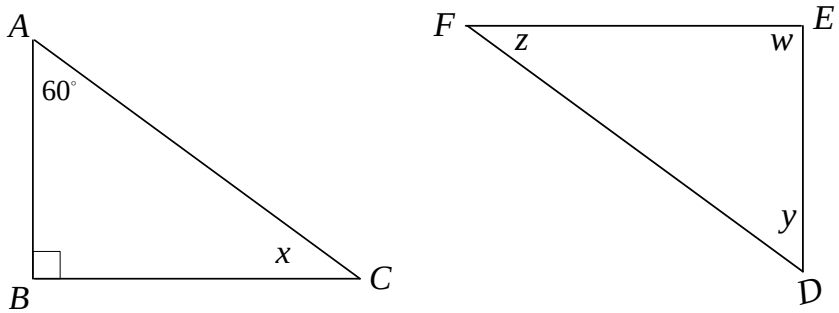
he/she solved the equation to get $x = 0$ and $x = \frac{1}{16}$ in part (a). He/she also applied law of product to obtain $\log_{10}\left(\frac{x}{5}\right) = \log_{10}\left(\frac{2\times 10}{x}\right)$ which resulted into $\frac{x}{5} = \frac{20}{x}$ and lastly solved for *x* in part (b).

2.8 Question 8: Congruence and Similarity

The question consisted of parts (a) and (b). In part (a), the question stated that, *"In the following figure,* $\overline{PX}$ *and* $\overline{QY}$ *are perpendicular to* $\overline{PQ}$ *and* $\overline{PX} = \overline{QY}$*."* The students were required to show that the two triangles *XPA* and *YQA* are congruent. The figure provided was a follows:



In part (b), the students were asked to find the size of the angles labeled *x*, *y*, *z* and *w* from the similar triangles *ABC* and *DEF*.



The question was answered by 533,597 (88.6%) students out of whom 381,300 (71.5%) students scored below 3 marks. Only 2,243 (0.4%) students scored full marks while a total of 248,185 (46.5%) scored 0 mark. This indicates that the students' performance in this question was below the average performance. Figure 8 shows the summary of the students' performance in question 8.

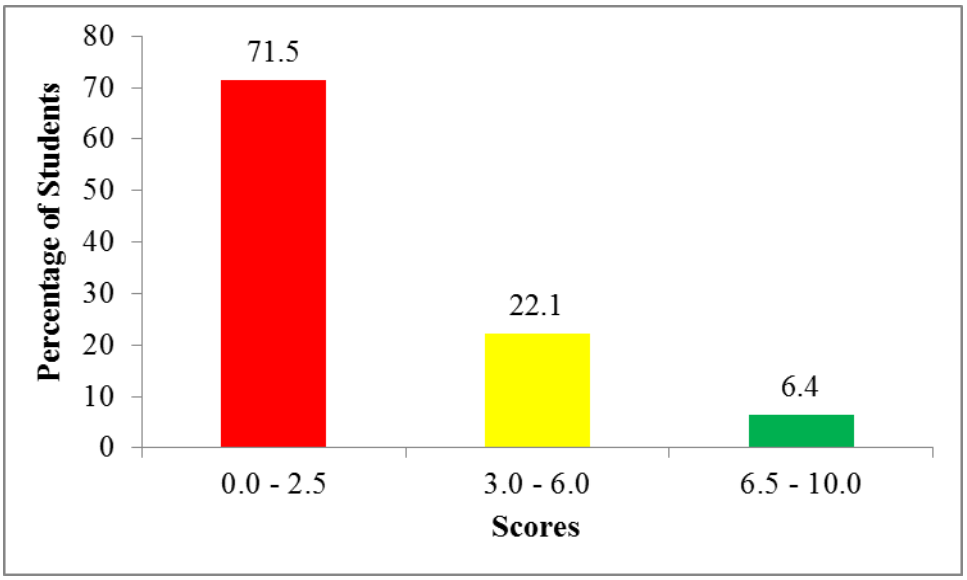
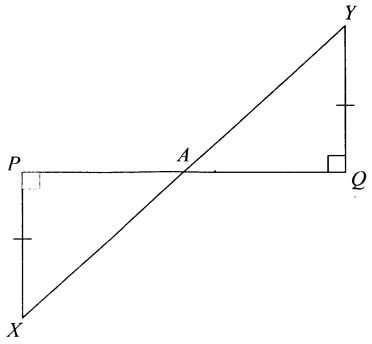


Figure 8: *Students' performance in question 8*

Majority of the students failed to answer this question correctly due to various reasons. In part (a), most of the students proved the congruence of the two triangles XPA and YQA by using incorrect theorems or conditions. For example, they used wrong conditions like *Right Angle Hypotenuse Side (RHS) theorem* and *Side Side Angle (SSA) theorem* instead of *Angle Angle Side (AAS) theorem* which was appropriate to show that the two triangles are congruent. In most cases, the students failed to identify the corresponding sides and angles of the two triangles. For instance, they incorrectly identified $\overline{XY}$ as the common side. In part (b), the majority failed to get the size of angles labeled by x, y, z and w due to lack of knowledge of the properties of similar triangles. Others showed the similarity of the given triangles instead of finding the size of the angles given. Additionally, others did not recognize the sum of the interior angles of a right angled triangle which led them to the incorrect answers. For

example, some students wrote $60^\circ + 90^\circ + x = 360^\circ$ instead of $60^\circ + 90^\circ + x = 180^\circ$. It was also noted that, the students lacked knowledge of identifying the corresponding angles that could help them to get the required angles. Extract 8.1 shows a sample of the student's incorrect response in this question.

8. (a) In the following figure, $\overline{PX}$ and $\overline{QY}$ are perpendicular to $\overline{PQ}$ and $\overline{PX} = \overline{QY}$. Show that the two triangles XPA and YQA are congruent.



8(a) Solution

$\bar{P}X = \bar{Q}Y$ --- given

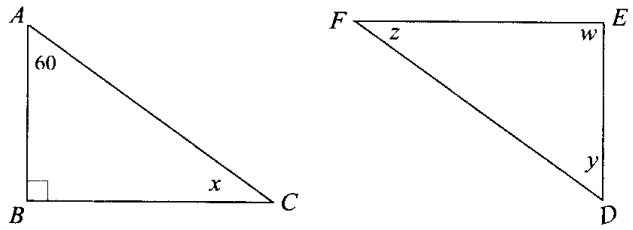
$\bar{A}X = \bar{O}A$ -- Given

$\bar{P}O = \bar{P}X$ -- Given

$XPA = YQA$ -- Common

$\triangle XPA = \triangle YQA$ BY SAS

(b) Triangles ABC and DEF are similar. Find the size of the angles labeled x, y, z and w.



solution

$60^\circ + x = 180^\circ$

$x = 180^\circ - 60^\circ$

$x = 120^\circ$

$x = 120^\circ$

$y = 60^\circ + x = 180^\circ$

$y = 60^\circ + 120 = 180^\circ$

$y = 180^\circ - 180^\circ$

$y = 10.$

$z = 60^\circ + 10 + x = 180^\circ$

$70 + x = 180^\circ$

$x = 180^\circ - 70^\circ$

$x = 10$

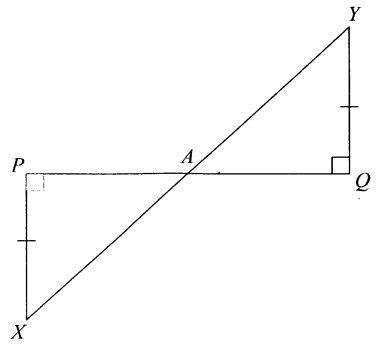
$z = 10^\circ$

Extract 8.1: A sample of incorrect responses in question 8.

Extract 8.1 shows that, the student used the *Side Side Side (SSS) theorem* instead of *Angle Angle Side (AAS) theorem* in part (a). In part (b), he/she was not able to identify the corresponding angles when finding the size of the angles given in two similar triangles due to lack of knowledge of the conditions of similar triangles.

On the other hand, there were students who managed to demonstrate their competence in applying the rules/conditions of congruence and similarity of figures to respond to the items of given question. In part (a), they were able to apply the AAS theorem to show the congruence for triangles, that is $\Delta PAX \equiv \Delta QAY$ by using the given figure. In part (b), they correctly apply the properties of similar triangles to supply the required responses. For example, their responses based on the following arguments: since $\Delta ABC \sim \Delta DEF$, then, $B\hat{A}C = E\hat{D}F = 60^o$ (corresponding angles of similar triangles), therefore $y = 60^o$, $A\hat{C}B = D\hat{F}E = 30^o$ (corresponding angles of similar triangles), therefore $z = 30^o$, $A\hat{B}C = D\hat{E}F = 90^o$ (corresponding angles of similar triangles), hence $w = 90^o$. Furthermore, they recognized that the angles $90^\circ, 60^\circ,$ and x add up to 180°, that was a sufficient condition to get the correct value of x. Extract 8.2 shows a sample of the student's correct responses in this question.

8. (a) In the following figure, $\overline{PX}$ and $\overline{QY}$ are perpendicular to $\overline{PQ}$ and $\overline{PX} = \overline{QY}$. Show that the two triangles XPA and YQA are congruent.



8(a) Required to prove that $\Delta XPA \cong YQA$

PROOFS

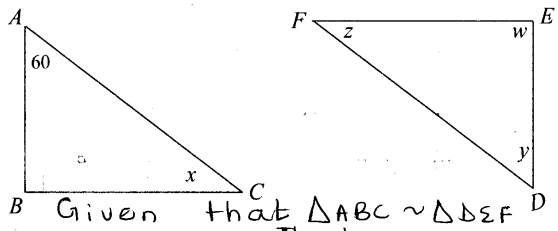
~~PQ~~ $\overline{PX} = \overline{QY}$ (Given)

$X\hat{P}A = Y\hat{Q}A = 90°$ (Given)

$P\hat{A}X = Y\hat{A}Q$ (Vertically Opposite Angles)

$\therefore \Delta XPA \cong \Delta YQA$ (AAS) hence shown

8 (b) Triangles ABC and DEF are similar. Find the size of the angles labeled x, y, z and w.



Given that $\Delta ABC \sim \Delta DEF$

Then!

$y = 60°$ (Corresponding Angles)

$w = 90°$ (Corresponding angles)

$60° + 90° + x = 180°$

$x = 180° - 150°$

$x = 30°$

$z = 30°$ (Corresponding Angles)

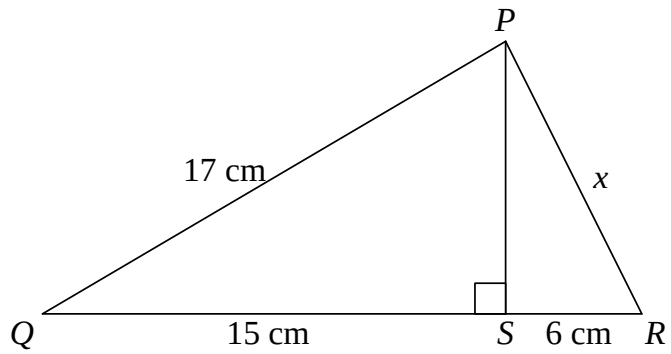
$\therefore y = 60°, x = 30°, z = 30°, w = 90°$

Extract 8.2: A sample of the correct responses in question 8.

In Extract 8.2, the student managed to show that triangles XPA and YQA are congruent by using the *AAS theorem*. The student also was able to find the size of the intended angles by using the conditions of similar triangles.

2.9 Question 9: Pythagoras Theorem and Trigonometry

The question had parts (a) and (b). In part (a), the students were required to find the value of x from figure PQR.



In part (b), they were given the information that "the angle of elevation of the top of a vertical building from a point on the ground is 25^{o}. The point on the ground is 80 m away from the base of the building." They were required to sketch the diagram representing this information and hence calculate the height of the building, giving the answer correct to one decimal place.

The data analysis shows that, this question was attempted by a total of 561,309 (93.2%) students out of whom 100,754 (17.9%) students scored at least 3 marks. Further analysis indicates that, only 13,241 (2.4%) students scored all 10 marks whereas 393,335 (70.1%) students scored zero. This indicates that the students' performance in this question was weak. Figure 9 summarizes the students' performance in question 9.

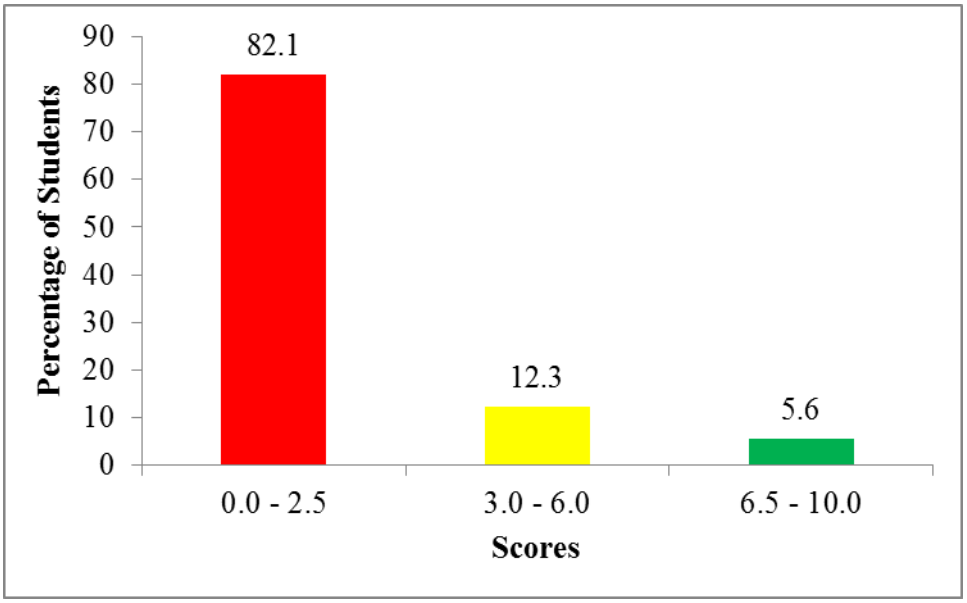
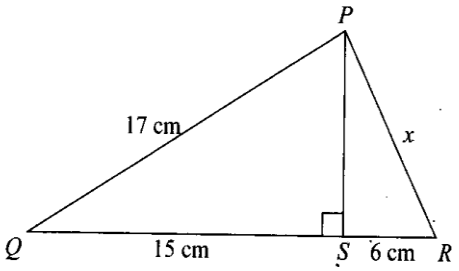


Figure 9: *Students' performance in question 9*

The majority of students had weak performance in this question due to the following factors: In part (a) (i), the students failed to demonstrate their competence in applying the Pythagoras theorem to get the length of the intended side from the figure. For example, the incorrect formula $\overline{QS}^2 - \overline{PS}^2 = \overline{QP}^2$ was frequently noted from their responses. Instead, they were supposed to use $\overline{QS}^2 + \overline{PS}^2 = \overline{QP}^2$ in order to obtain $\overline{PS} = 8cm$ that was an important step to get the required length, that is, $x = 10cm$. Some applied the concept of similarities to find the value of x by taking the ratios $\frac{\overline{PQ}}{\overline{PR}} = \frac{\overline{QR}}{\overline{SR}}$ implying that $\frac{17}{x} = \frac{(15+6)}{6}$ which is also a wrong approach. Such students lacked knowledge of the conditions for similar triangles. In part (b), they failed to indicate the right location of the angle of elevation (25°) in the sketched diagrams. Others were able to sketch the required diagram but failed to apply the appropriate trigonometric ratio for $\tan 25^\circ$, instead they used *sine* and *cosine* that could not bring the intended height of the building. Extract 9.1 is a sample of an incorrect solution from one of the students who attempted this question.

9. (a) In the following figure, $\overline{PQ} = 17$ cm, $\overline{QS} = 15$ cm, $\overline{RS} = 6$ cm and $\overline{PR} = x$. Find the value of x.



solution

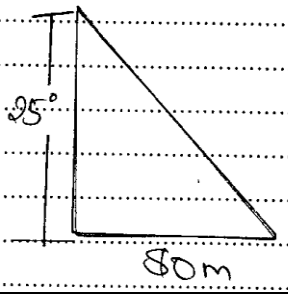
17cm + 15cm + 6cm + x

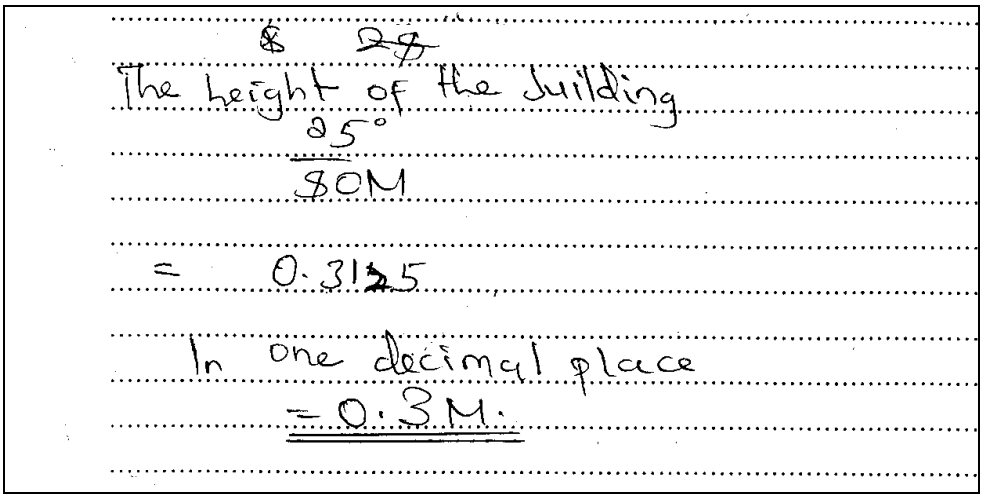
32cm + 6x + x

x = 38 cm

The $\overline{PR}$ = 38cm

(b) The angle of elevation of the top of a vertical building from a point on the ground is 25°. The point on the ground is 80 m away from the base of the building. By sketching a diagram representing this information, calculate the height of the building. Write the answer correct to one decimal place.





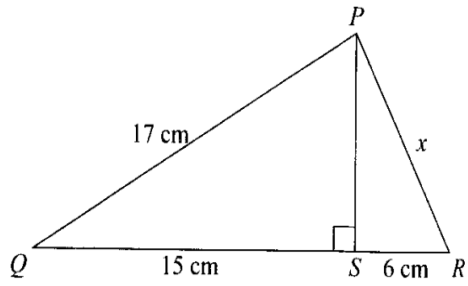
Extract 9.1: A sample of the incorrect responses in question 9.

In Extract 9.1, the student added the given dimensions when finding the value of x indicating lack of knowledge of application of Pythagoras theorem in part (a). He/she also divided the given angle of elevation by horizontal length of 80 m. This confirms that, the student lacked knowledge of solving problems involving angle of elevation and angle of depression in part (b).

In contrary, there were students who managed to answer this question correctly. In part (a), they initially applied Pythagoras theorem $\overline{PQ}^2 = \overline{PS}^2 + \overline{SP}^2$ to obtain the length $\overline{PS}$ in the triangle PQS, where $\overline{QS} = 15cm$, $\overline{PQ} = 17cm$. Thereafter, they made correct substitution to get 8cm. They also proceeded with Pythagoras theorem by using the right angled triangle PSR by applying $\overline{PS}^2 + \overline{SR}^2 = \overline{PR}^2$ to get the required value, $x = 10$ cm. In part (b), they correctly sketched the diagram representing height of the building by using the given information and located the angle of elevation from the point on the ground which is 80 m away from the base of the building. This step helped them to determine the tangent ratio, that is $\tan 25^\circ = \dfrac{\overline{PQ}}{\overline{QR}}$ as a formula for obtaining the height of the building, that is $h = 37.304$ m. Finally, the students wrote the answer in one decimal place which is 37.3 m as it was instructed. The students had knowledge and skills of applying Pythagoras theorem in solving problems related to right angled triangles and the concepts related to angles of

elevation or depression in solving word problems. Extract 9.2 is a sample of the correct solution in this question.

9. (a) In the following figure, $\overline{PQ} = 17$ cm, $\overline{QS} = 15$ cm, $\overline{RS} = 6$ cm and $\overline{PR} = x$. Find the value of x.



Solution

From pythagoras theorem

$\overline{QS}^2 + \overline{PS}^2 = \overline{QP}^2$

$15^2 + \overline{PS}^2 = 17^2$

$225 + \overline{PS}^2 = 289$

$\overline{PS}^2 = 289 - 225$

$\sqrt{\overline{PS}^2} = \sqrt{64}$

$\overline{PS} = 8$ cm

Also

$\overline{PS}^2 + \overline{SR}^2 = \overline{PR}^2$

$8^2 + 6^2 = Px\ x^2$

$64 + 36 = x^2$

$\sqrt{100} = \sqrt{x^2}$

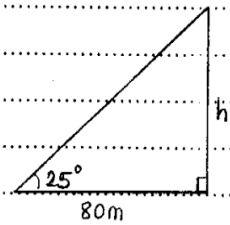
$10 = x$

$\therefore x = 10$ cm

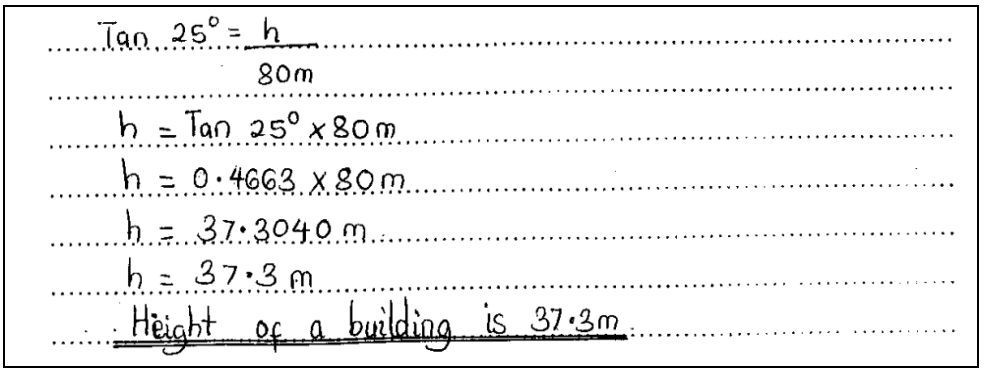
$\therefore$ The value of x is 10 cm.

(b) The angle of elevation of the top of a vertical building from a point on the ground is 25°. The point on the ground is 80 m away from the base of the building. By sketching a diagram representing this information, calculate the height of the building. Write the answer correct to one decimal place.

Solution



$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$



Extract 9.2: A sample of the correct responses in question 9.

In Exact 9.2, the student applied Pythagoras theorem correctly to get the length of side $\overline{PS}$ and hence the required value of x in part (a). He/she also interpreted correctly the given information and drew the right angled triangle showing the angle of elevation, that is 25° and then, applied the *tangent* of 25° to get the required height in part (b).

2.10 Question 10: Sets and Statistics

The question consisted of parts (a) and (b). Part (a) stated that, "In a class of 30 students, 17 participate in English debate and 12 participate in both English debate and Mathematics club". In part (a), the students were required to find the number of students who participate in (i) English debate only and (ii) Mathematics club only if every student is required to participate in at least one of these two events. Part (b) stated that, the ages of students selected to participate in a debate competition were recorded as follows:

13	15	17	16	15	14	16	18	17	16
15	14	13	16	14	17	15	16	15	16

The students were required to (i) prepare a frequency table showing the ages of students and their corresponding frequencies and (ii) draw a frequency polygon representing the information given in part (b) (i).

The question was attempted by 568,170 (94.3%) students out of whom 403,821 (71.1%) students scored below 3 marks. Only 7,810 (1.4%) students scored full marks while 298,095 (52.5%) scored 0 mark. Generally, the students' performance in this question was weak. Figure 10 shows the summary of the students' performance in question 10.

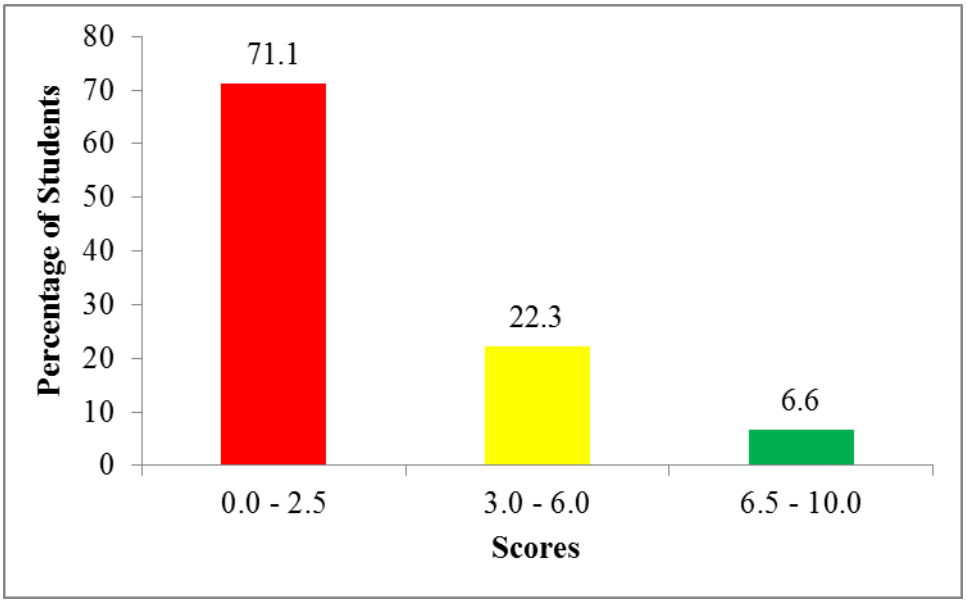


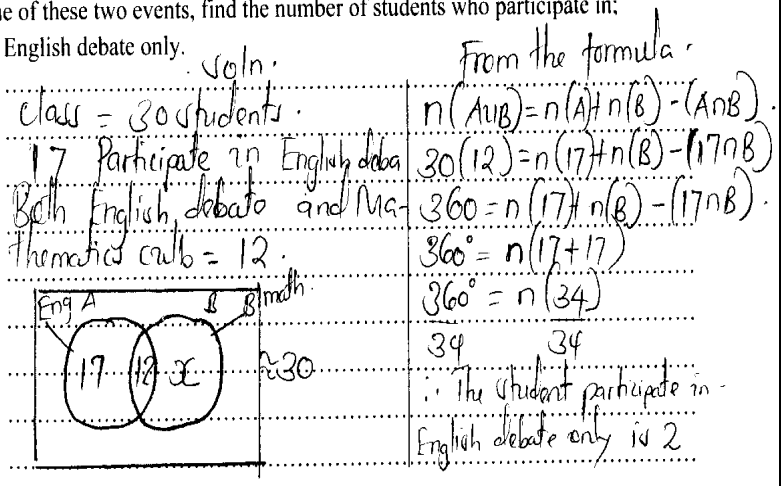
Figure 10: *Students' performance in question 10*

The response analysis shows that, in part (a) (i), the student failed to recall or apply the required formula to find the number of students who participate in English debate only, that is $n(E)-n(E\cap S)$, where E represents the set of students who participate in English debate and S represents the set students who participate in Mathematics club. Instead, they used general formulae like $n(E\cup M)=n(E)+n(M)-n(E\cap M)$ that could not give the expected answer. Others confused *the number of students who participate in English debate* with *those who participate in English debate only* as they wrote 17 instead of $17-12=5$. Similarly, in part (a) (ii), most students incorrectly wrote 30 students as the participants of Mathematics club only. They did subtract 17 participants of both English debate and Mathematics club from 30 students so as to get 13 students who participate in Mathematics club only. Other students used a Venn diagram but failed to present the given information correctly. Others considered 12 as the number of students who participate in neither English debate nor Mathematics club which led them to incorrect answers. In part (b) (i) the majority of students failed to prepare a frequency table because they could not tally the given values to get their corresponding frequencies. Others prepared the tables for grouped data by using their own class intervals. They were not aware of the difference between the *grouped data* and *ungrouped data*; and the necessary steps of preparing a frequency

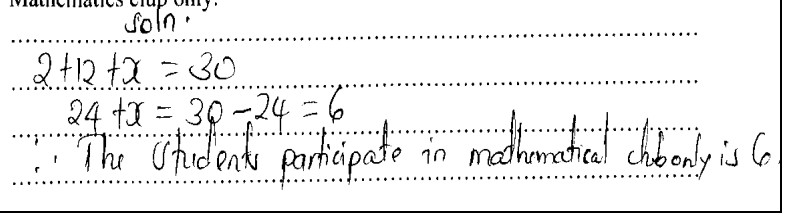
distribution table. Failure to prepare the correct frequency distribution table led to plotting of incorrect frequency polygon in part (b) (ii). Some students also drew the graph of class mark (age) against frequency instead of frequency against class mark (age). Others drew the graph using free hand instead of using a ruler while others drew histogram and cumulative frequency instead of frequency polygon. All these show that the students were not conversant with different methods of presenting statistical information. Extract 10.1 is one of the incorrect responses to this question.

10. (a) In a class of 30 students, 17 participate in English debate and 12 participate in both English debate and Mathematics club. If every student is required to participate in at least one of these two events, find the number of students who participate in;

(i) English debate only.



(ii) Mathematics club only.



(b) The ages of students selected to participate in a debate competition were recorded as follows:

13 15 17 16 15 14 16 18 17 16
15 14 13 16 14 17 15 16 15 16

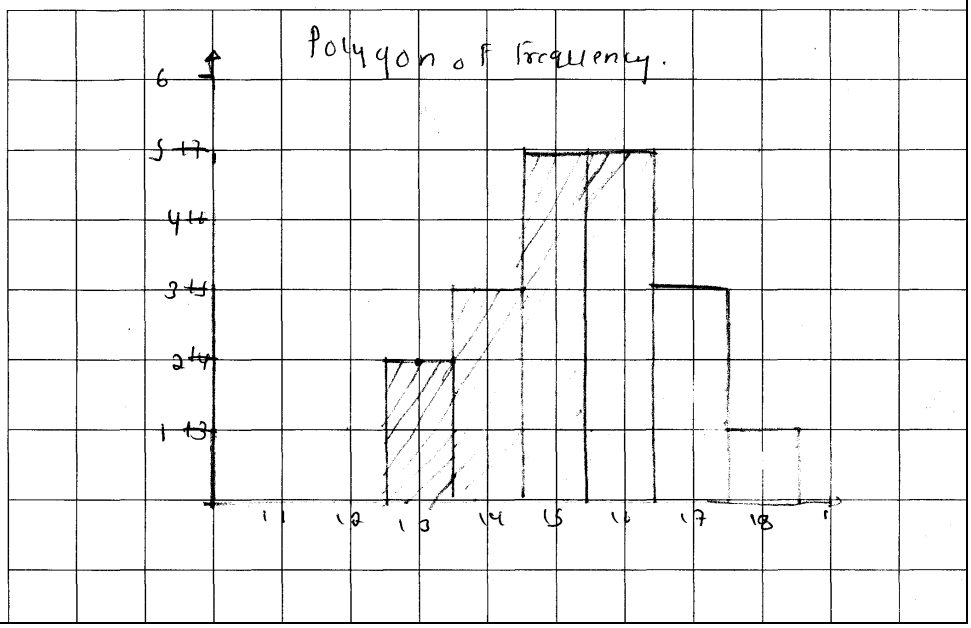
(i) Prepare a frequency table showing the ages of students and their corresponding frequencies.

Solution

Frequency table

Class interval	class mark	Tally	frequency
13 – 14	$\frac{13+14}{2} = 13.5$	𝍸	5
15 – 16	$\frac{15+16}{2} = 15.5$	𝍸𝍸 1	11
17 – 18	$\frac{17+18}{2} = 17.5$	1111	4

(06) (ii) Draw a frequency polygon representing the given information in part (b)(i).



Extract 10.1: A sample of the incorrect responses in question 10.

In Extract 10.1, the student performed incorrect calculations due to lack of understanding o union and intersection of sets. However, he/she used the

degree measure in the procedures when determining the union and intersection of sets. In part (b) (i), the student prepared an incorrect frequency table. In part (b) (ii), the student drew a histogram which was also incorrect, indicating lack of knowledge of drawing histogram and frequency polygon.

Despite the fact that the performance was weak, there were students who responded correctly to the question. In part (a), they were able to find the number of participants of English debate only as well as those who participated in Mathematics club only by using formula or Venn diagram. In part (b), the students who answered this question correctly were keen enough to prepare a frequency table showing students' ages and their corresponding frequencies by tallying. Moreover, they managed to draw the graph of the frequency against students' age. Such students were competent in sets and statistics related problems in different contexts. Extract 10.2 is a sample of the student's correct answer in this question.

10.(a) In a class of 30 students, 17 participate in English debate and 12 participate in both English debate and Mathematics club. If every student is required to participate in at least one of these two events, find the number of students who participate in;

(i) English debate only.

Soln:

let no. of student participate in English = E and math = M.

n(E) = 17 students, n(E∩M) = 12 and n(U) = 30 students = n(E∪M)

By using a formular

n(E∪M) = n(E) + n(M) − n(E∩M) also n(E) only...

= n(E) − n(E∩M)

= 17 − 12 = 5 students

∴ Number of student who study English only = 5 students

(ii) Mathematics club only.

from, n(E∪M) = n(E) + n(M) − n(E∩M)

30 = 17 + n(M) − 12

30 − 5 = n(M)

25 = n(M)

n(M)only = n(M) − n(E∩M)

= 25 − 12

= 13 students.

∴ Mathematics club only = 13 students.

10 (b) The ages of students selected to participate in a debate competition were recorded as follows:

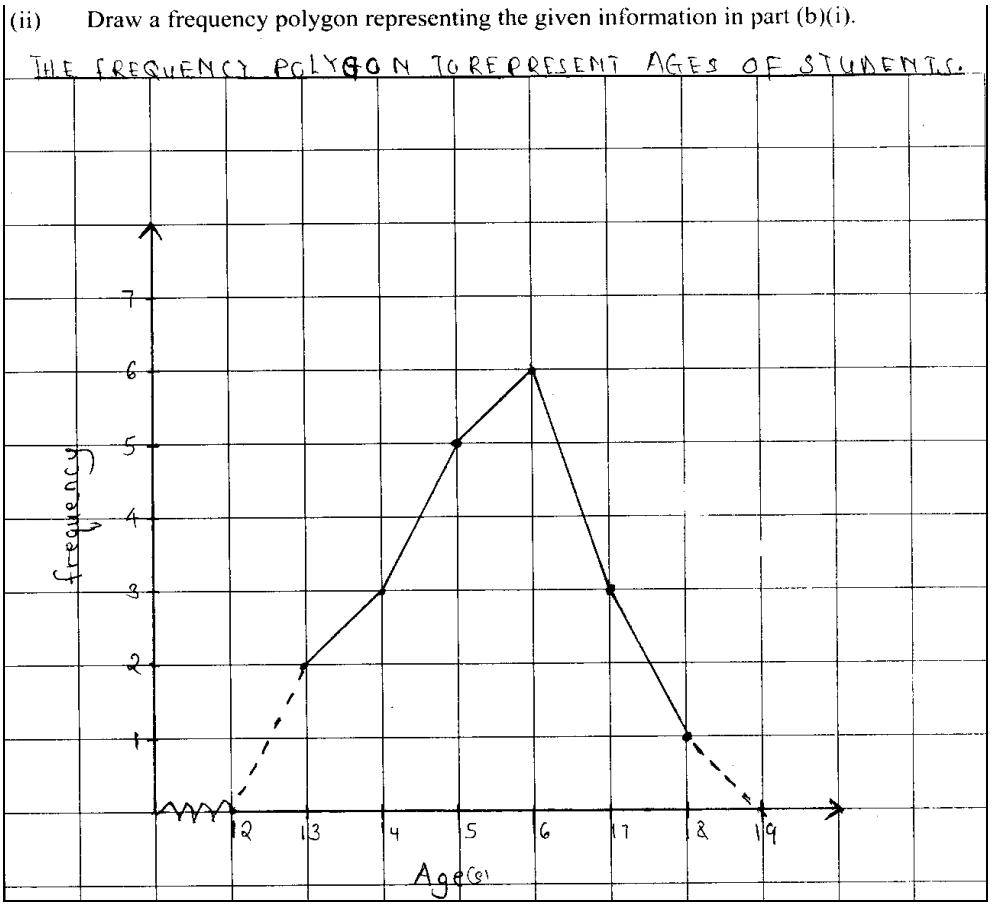
13	15	17	16	15	14	16	18	17	16
15	14	13	16	14	17	15	16	15	16

(i) Prepare a frequency table showing the ages of students and their corresponding frequencies.

THE FREQUENCY DISTRIBUTION TABLE OF STUDENTS AGES

Age	Tally	Frequency.
13	//	2
14	///	3
15	/////	5
16	//////	6
17	///	3
18	/	1

(ii) Draw a frequency polygon representing the given information in part (b)(i).



Extract 10.2: A sample of the correct responses in question 10.

In Extract 10.2, the student identified the number of students who participated in mathematics club only and English debate only by using the correct formulae in part (a). In part (b), he/she prepared a correct frequency distribution table and later used it to draw a correct frequency polygon.

3.0 ANALYSIS OF STUDENTS' PERFORMANCE IN EACH TOPIC

The Basic Mathematics assessment paper had ten (10) questions that were set from twenty (20) topics. The analysis shows that, among the topics from which the questions were set, none of them had either good or average performance. The students' performance in all the questions was below the average. Based on the percentage of students who scored at least 30 marks, the students' performance in each question as per topic(s) is as follows: question 3 from the topic of *Units* and *Ratio, Profit and Loss* (29.73%), question 10 that was set from *Sets* and *Statistics* (28.93%), question 8 from *Congruence* and *Similarity* (28.54%), question 2 from the topic of

Fractions, Decimals and Percentages (25.84%), question 1 from *Numbers* and *Approximations* (25.62%), question 5 from the topic of *Algebra* and *Quadratic Equations* (25.36%), question 6 that was set from *Coordinate Geometry* and *Geometrical Transformations* (17.90%), question 9 from *Pythagoras theorem* and *Trigonometry* (17.95%), question 4 from *Geometry, Perimeters and Areas* (12.38%) and question 7 from the topic of *Exponents, Radicals* and *Logarithms* (4.60%). The summary of the performance in each topic is shown in the Appendix.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

Generally, the Basic Mathematics SIRA report on FTNA 2021 shows that, 19.52 percent of the students passed the assessment compared to 15.94 percent of the students who passed in 2020. The analysis of students' performance shows that, out of twenty (20) topics that were tested in 2021, the topic of *Units* and *Ratio, Profit and Loss* had a bit higher level of performance of 29.73 percent than any other although it is below the average performance. On the other side, *Exponents, Radicals* and *Logarithms* had the lowest performance of 4.60 percent in that assessment. The rest of the topics had a weak performance ranging from 28.93 percent to 12.38 percent.

4.2 Recommendations

In order to improve the students' performance in Form Two Basic Mathematics assessment, the following recommendations are suggested to students and teachers.

Teachers are recommended to:

(a) teach by using improvised teaching aids or learning objects particularly when demonstrating or clarifying all concepts that need the use of real objects.

(b) apply suitable teaching strategies and techniques that actively involve learners in the learning process. This will pave way for learners to demonstrate their competences.

(c) make sure that they assess the learners immediately at the end of the period to identify the concepts that need more teaching improvements before teaching a new lesson.

(d) assess the students regularly and devise suitable mechanisms to assist them closely according to their learning ability.

Students are recommended to:

(a) seek assistance from their teachers and search for knowledge from different resources such as books and other online materials in order to widen their understanding of different concepts.

(b) develop a habit of making self-evaluation by frequently working on extra exercises and timely consult their teachers for assistance whenever they encounter difficulties on any concept.

(c) carefully adhere to assessment instructions. This will help them to respond correctly in relation to the requirements of given question.

APPENDIX

ANALYSIS OF STUDENTS' PERFORMANCE PER TOPIC IN BASIC MATHEMATICS ASSESSMENT – FTNA 2021

S/N	Topics	Question number	Percentage of students who scored 30 marks or more	Remarks
1	Units and Ratio, Profit and Loss	3	29.7	Weak
2	Sets and Statistics	10	28.9	Weak
3	Congruence and Similarity	8	28.5	Weak
4	Fractions, Decimals and Percentages	2	25.8	Weak
5	Numbers and Approximations	1	25.6	Weak
6	Algebra and Quadratic Equations	5	25.4	Weak
7	Coordinate Geometry and Geometrical Transformations	6	17.9	Weak
8	Pythagoras theorem and Trigonometry	9	17.9	Weak
9	Geometry, Perimeters and Areas	4	12.4	Weak
10	Exponents and Radicals, and Logarithms	7	4.6	Weak