THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEMS RESPONSE ANALYSIS REPORT FOR THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2018

041 BASIC MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEMS RESPONSE ANALYSIS REPORT FOR THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2018

041 BASIC MATHEMATICS

Published by
The National Examinations Council of Tanzania,
P.O. Box 2624,
Dar es salaam, Tanzania.

Table of Contents

FOREWORD

The National Examinations Council of Tanzania is pleased to issue the report on Students' Item Response Analysis (SIRA) in 041 Basic Mathematics items for the Form Two National Assessment (FTNA) 2018. This report is written to provide feedback to students, teachers and other education stakeholders, on how the students responded to the questions.

The analysis of the students’ responses was done in order to identify the areas in which the students faced problems, did well or averagely. Basically, the report highlights the factors for good or poor performance in order to understand what the education system managed or was unable to offer to the learners in their two years of Secondary Education.

The analysis shows that, question 1 and 3 were averagely performed and the other eight questions were poorly performed. The weak performance was due to the students’ inability to identify the requirements of the questions, perform mathematical operations and failure of the students to formulate equations from the given information. Also, failure of the students to represent the given information diagrammatically or graphically, adhere to the instructions of the questions and incorrect use of laws, formulae, theorems and other mathematical facts in answering the questions contributed to the poor performance.

Finally, the Council would like to thank everyone who participated in the preparation of this report.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report has analyzed the items responses for the students who sat for the 041 Basic Mathematics assessment in FTNA 2018. The analysis mainly focuses on the areas on which the students faced challenges and those which they performed well.

The 041 Basic Mathematics assessment paper consisted of ten (10) questions carrying 10 marks each. The students were required to answer all the questions.

In 2018, a total of 503,761 students sat for 041 Basic Mathematics assessment out of which 115,597 (22.95%) students passed. In 2017, a total of 485,494 students sat for 041 Basic Mathematics assessment out of which 155,092 (32.00%) students passed. This indicates that the performance in 2018 has dropped by 9.05 percent. The students who passed these assessments got different grades ranging from grade F to A as shown in Figure 1.

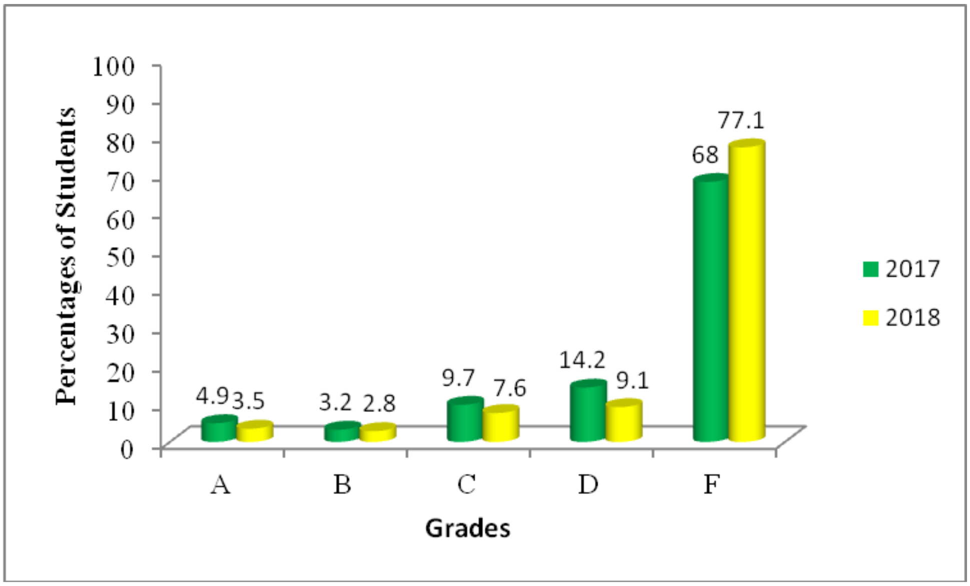


Figure 1: *Distribution of Grades for the 2017 and 2018 Basic Mathematics Assessments.*

The analysis of the students' performance in each question is presented in section 2 of this report. The analysis briefly includes descriptions of the requirements of the items, summary on how the students answered the questions, extracts showing the samples of the students' best and worst solutions and the possible reasons for good, average or weak performance in each question.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE IN EACH QUESTION

This section provides the analysis of the students’ performance for each question. The students' performance was categorized using the percentage of students who scored at least 30 percent of the marks that were allocated to a particular question. Thus, the performance was classified in the following groups: 65 – 100 for good performance; 30 – 64 for average performance and 0 – 29 for weak performance. Such categories of students’ performance are represented by green, yellow and red colours respectively in the figures and tables used in this report.

2.1 Question 1: Numbers and Approximations

This question required the students to (a) use prime factorization method to find the smallest possible mass of the block which was cut into equal units of 10g, 20g and 35g and (b) evaluate $0.864 \div 0.0246$ correct to 2 significant figures.

The question was answered by 485,486 (96.0%) students out of whom 64.38 percent scored below 3 marks. Further analysis indicates that 23.86 percent of the students scored from 3 to 6 marks and 11.76 percent scored above 6 marks. The analysis also shows that 53 percent of the students scored zero while 2.8 percent scored all the 10 marks. The performance of the students in this question was average as shown in Figure 1.

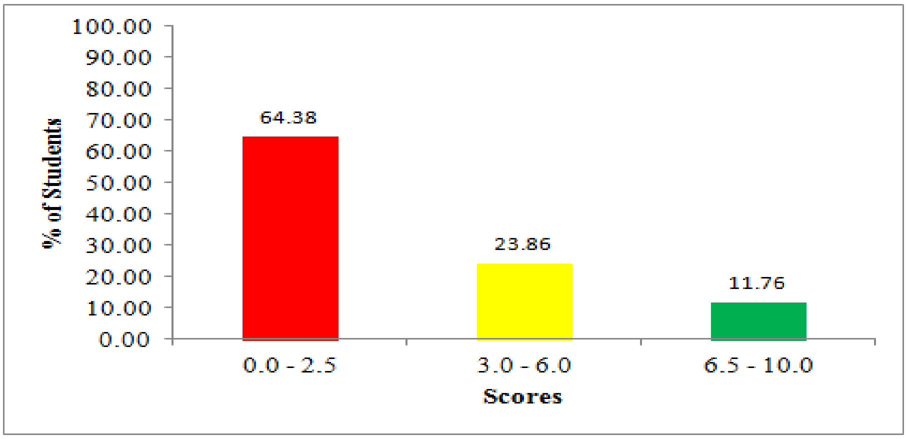


Figure 1: *Students performance in question 1*

The analysis of responses for this question shows that in part (a), the students who scored all the marks obtained prime factors of 10, 20 and 35; took the highest power of each prime factorization and multiplied them as $2^2 \times 5 \times 7$ to obtain the correct answer of 140 g.

The students who attempted part (b) correctly were able to evaluate $0.864 \div 0.0246$ to 2 significant figures by multiplying the divisor (0.864) and dividend (0.0246) by 10,000 so as to have whole numbers as follow: $\frac{8640}{246} = 35.1219 \approx 35$. Extract 1.1 is a sample solution of a student who answered this question correctly.

Extract 1.1

1. (a) A block is cut into equal units of 10 g, 20 g and 35 g. Use prime factorization method to find the smallest possible mass of the block from which the pieces can be cut.

Solution:

Equal units ⇒ 10g, 20g, 35g

By Prime factorization method:

Take,

2	10	20	35
2	5	10	35
5	5	5	35
7	1	1	7
	1	1	1

Then, Find LCM

LCM = 2×2×5×7

L.C.M = 140g

Smallest possible mass = 140g

∴ The smallest possible mass of the block is 140 grams

(b) Evaluate $0.864 \div 0.0246$ giving your answer correct to 2 significant figures.

Solution:

$0.864 \div 0.0246$

From,

BODMAS

Division Method:

$= 0.864 \div 0.0246$

$= \frac{0.864}{0.0246}$ ⟨Into a/b form⟩

Multiply by 10000,

$= \frac{0.864 \times 10000}{0.0246 \times 10000}$

$= \frac{8640}{246}$

Divide;

$$\begin{array}{r} 35.121 \\ 246 \overline{)8640} \\ -738 \\ \hline 1260 \\ -1230 \\ \hline 300 \\ -246 \\ \hline 540 \\ -492 \\ \hline 480 \end{array}$$

Into 2 significant figures

35.121

35.000

∴ $0.864 \div 0.0246 = 35$

Extract 1.1 represents the solution of a student who demonstrated good understanding of the concepts assessed in question 1.

On the other hand, majority of the students had weak performance in this question. The students who did part (a) poorly did not understand the concept of LCM and GCF. This situation led them to get the smallest possible mass of the block as 5 g instead of 140 g. Further analysis shows that some students were able to use the prime factorization method to find the factors of 10, 20 and 35 as 2×5, $2\times2\times5$ and 5×7 but lacked the understanding that the Least Common Multiple is the product of the highest powers of prime factors that is $2\times2\times5\times7=140$. Moreover, other students did not understand the requirements of the question. In this case, some of them were unable to interpret the key words in the question which stand for the Least Common Multiple while others used the listing method contrary to the given instructions. For instance, several students added the given quantities to get 65 g. In part (b), the students who scored low marks were unable to divide decimal numbers using long division method. Such students gave wrong answers such as 351.21, 35.20322, 3.411, 43.77 etc. A significant proportion of students, however, divided the numbers correctly but did not round 35.1219 to the required number of significant figures. Extract 1.2 is a sample answer from the script of a student who answered this question incorrectly.

Extract 1.2

1. (a) A block is cut into equal units of 10 g, 20 g and 35 g. Use prime factorization method to find the smallest possible mass of the block from which the pieces can be cut.

Soln

10 = 1, 2, 5 and 10

20 = 1, 2, 4, 5, 10 and 20

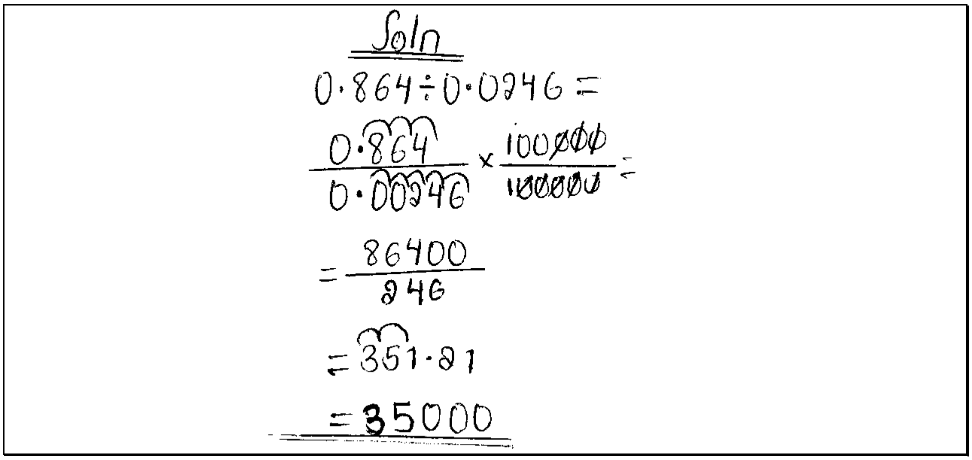
35 = 1, 5, and 35

10: 2, 5; 5: 5, 1
20: 2, 10; 10: 2, 5; 5: 5, 1
35: 5, 7; 7: 7, 1

= 5

The smallest possible number is 5

(b) Evaluate $0.864 \div 0.0246$ giving your answer correct to 2 significant figures.



Soln

$0.864 \div 0.0246 =$

$\frac{0.864}{0.00246} \times \frac{100000}{100000} =$

$= \frac{86400}{246}$

$= 351.21$

$= 35000$

Extract 1.2 shows how the student failed to answer the question correctly

2.2 Question 2: Fractions and Percentages

This question comprised parts (a) and (b). In part (a), the students were instructed to find out which fraction between $\frac{5}{7}$ and $\frac{6}{9}$ is greater. In part (b), the students were given that "0.125 of all students in a mixed class are girls" and were required to find the percentage of boys.

The statistical data shows that 28.94 percent of the students scored 3 marks or more, amongst only 4.6 percent scored all 10 marks. Also, the analysis indicates that 55.3 percent of the students scored zero. The performance of the students in this question is summarized in Figure 3.

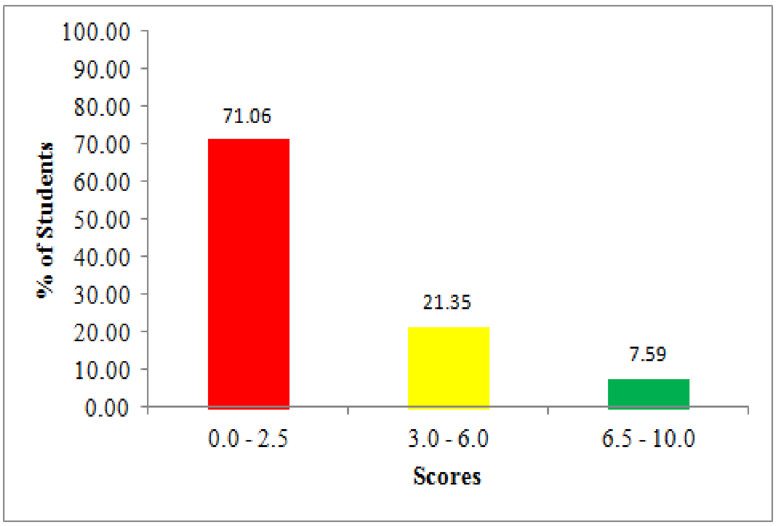


Figure 3: *Students' performance in question 2*

As Figure 3 indicates, 71.06 percent of the students scored below 3 marks. Therefore, the performance of the students in this question was generally weak. The students who scored low marks made mistakes that led to incorrect solution. In part (a), most students misconceived the question, whereby some students multiplied or added the given fractions while few students divided $\frac{5}{7}$ by $\frac{6}{9}$ as illustrated in Extract 2.1. Also, a considerable number of the students had a good start as they multiplied the fractions $\frac{5}{7}$ and $\frac{6}{9}$ by 63, which is the LCM of 7 and 9, but did mistakes when performing operations or they wrongly used the signs of inequalities to make conclusion. For example, some of them wrote $\frac{5}{7} \leq \frac{6}{9}$ or $\frac{5}{7} \geq \frac{6}{9}$ or $\frac{5}{7} < \frac{6}{9}$. This shows that these students had inadequate knowledge on the concept of inequalities. Further analysis shows that other students wrote that $\frac{5}{7} > \frac{6}{9}$ without showing any procedure to argue for. The students did not change the given fractions into whole numbers or percentages for easier comparison; instead they concluded using the numbers which are in the same form of fractions as given in the question.

In part (b), the majority of the students had low marks because they treated 0.125 as a representative decimal for boys while it was for girls. Such students converted 0.125 into percentages by taking $0.125 \times 100\%$ to get 12.5% and indicated it as percentage of boys. Besides this misconception, several students were unable to convert 0.125 into percentage whereby most of them failed to compute $0.125 \times 100\%$ ending up with incorrect answers including 0.125%, 1.25% and 125%. Other students had wrong approach in changing 0.125 into percentage as they divided 0.125 by 100 to get 0.00125%. With these errors, even the students who clearly understood the question ended up with incorrect answers as they got the values of percentage for boys different from 87.5% after subtracting the percentages of girls from 100%. Extract 2.2 is a sample response from one of the students who answered part (b) incorrectly.

Extract 2.1

2. (a) Find out which of the two fractions, $\frac{5}{7}$ or $\frac{6}{9}$ is greater.

Solution

$\frac{5}{7}$ or $\frac{6}{9}$

$\frac{5}{7} \times \frac{6}{9} = \frac{30}{63}$

$= \frac{30}{63}$ or $\frac{63}{30}$.

Extract 2.1 shows responses of a student who lacked the skills to compare fractions.

Extract 2.2

(b) If 0.125 of all students in a mixed class are girls, what percentage of the students are boys?

Soln

$0{\cdot}125 \times 100\%$

125%

$\therefore 0{\cdot}0{\cdot}125 = 12{\cdot}5\%$

Extract 2.2 shows responses of a student who misinterpreted the question.

On the other hand, the analysis shows that a small number of students were able to answer the question correctly. In part (a), some students correctly converted the given fractions, $\frac{5}{7}$ and $\frac{6}{9}$, into whole numbers through multiplication of each fraction by the LCM of the denominators of the given fractions which is 63 as follows: $\frac{5}{7} \times 63 = 45$ and $\frac{6}{9} \times 63 = 42$. With these whole numbers, the students realized that 45 is greater than 42 and consequently concluded that $\frac{5}{7}$ is greater than $\frac{6}{9}$. Other students attempted the question by expressing the given fractions as percentages as follows: $\frac{5}{7}\times100\% = 71.43\%$ and $\frac{6}{9}\times100\% = 66.67\%$. Thereafter, they were able to recognize that $\frac{5}{7}$ is greater than $\frac{6}{9}$.

The students who managed to answer part (b) correctly were able to convert 0.125 into percentage by taking $0.125\times100\% = 12.5\%$ and realized that it was percentage for girls. With the fact that the total percentage for both boys and girls is 100%, the students subtracted 12.5% from 100% to get 87.5% which was the required percentage for boys. A sample solution of a student who attempted the question correctly is shown in Extract 2.3.

Extract 2.3

2. (a) Find out which of the two fractions, $\frac{5}{7}$ or $\frac{6}{9}$ is greater.

Solution
- LCM of denominator
7 and 9 = 63

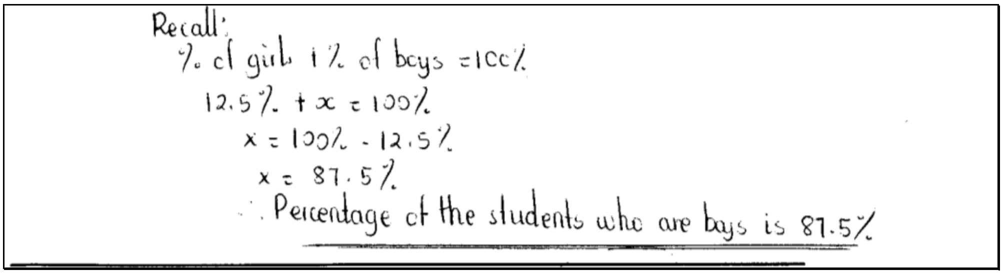
$\frac{5}{7}\times 63 = 5\times 9$
$= 45$

$\frac{6}{9}\times 63 = 6\times 7$
$= 42$

$\therefore$ 5/7 is greater than 6/9

(b) If 0.125 of all students in a mixed class are girls, what percentage of the students are boys?

Solution.
The number of students = 100%
% of girls + % of boys = 100%
0.125 of students = girls.
0.125 to percentage
$0.125\times100\% = 12.5\%$


Recall;
$\%$ of girls + $\%$ of boys $= 100\%$
$12.5\% + x = 100\%$
$x = 100\% - 12.5\%$
$x = 87.5\%$
$\therefore$ Percentage of the students who are boys is 87.5%

Extract 2.3: A sample response of a student who answered the question correctly.

2.3 Question 3: Units and Simple Interest

The question had parts (a) and (b). In part (a), the students were asked to subtract 8m, 9dm, 38cm and 9mm from 10m, 9dm, 31cm and 2mm. In part (b), the students were required to calculate the simple interest on shs.10,000,000/= invested for 5 years at the rate of 6% per annum.

The data shows that question 3 was best performed in this assessment. A total of 314,213 students, equivalent to 62.86 percent had average and good performance making the question to have average performance. The percentages of the students who scored low, average and high marks are presented in Figure 4.

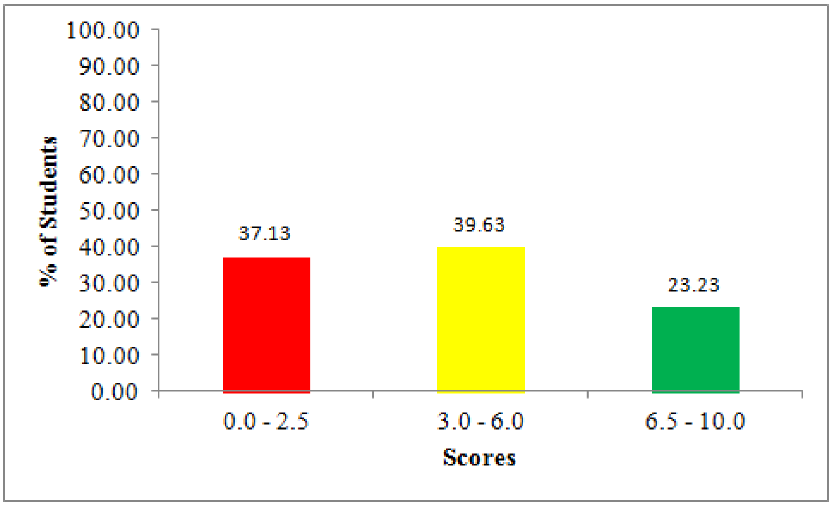


Figure 4: *Students' performance in question 3*

Furthermore, the analysis depicts that 8.1 percent of the students attempted the question correctly and scored full marks. In part (a), the students realized that $1\,\mathrm{m} = 10\,\mathrm{dm}$, $1\,\mathrm{dm} = 10\,\mathrm{cm}$ and $1\,\mathrm{cm} = 10\,\mathrm{mm}$, which was the key and logical idea for performing subtraction for the given units. Employing these conversions, the students were able to reconstruct the question such that they were required to subtract 8 m, 9 dm, 38 cm and 9 mm from 9 m, 18 dm, 40 cm and 12 mm; hence they got 1 m, 9 dm, 2 cm and 3mm as required.

In part (b), the students who performed well were able to substitute the values of the given variables $P = 10{,}000{,}000$, $R = 6$ and $T = 5$ into the formula $I = \frac{PRT}{100}$. Then, they calculated the simple interest (I) correctly to obtain $I = \text{shs } 3{,}000{,}000$. A sample solution of the student who did the question correctly is shown in Extract 3.1.

Extract 3.1

3. (a) Subtract:

	m	dm	cm	mm
	~~10~~ 9 10+8	~~9~~	~~31~~ 30 +10	2 + 10
-	8	9	38	9

Solution

	m	dm	cm	mm
	9	18	40	12
−	8	9	38	9
	1	9	2	3

∴ = 1m 9dm 2cm 3mm.

(b) Find the simple interest on sh. 10,000,000 invested for 5 years at the rate of 6% per annum.

Solution

Given.

P= 10,000,000.

t = 5years

R= 6% p.a

From $\boxed{I = \frac{P \times R \times T}{100}}$

$I = \frac{10{,}000{,}\cancel{000} \times 6 \times 5}{1\cancel{00}}$

$I = 100{,}000 \times 30$

$I = 3000{,}000$

∴ Interest was sh 3000,000

Extract 3.1 shows a student who demonstrated adequate knowledge and skills on the assessed concepts.

In spite of the good performance, a total of 65,783 students, equivalent to 13.2 percent, got the question wrong. In part (a), majority of the students used the relationship between the given metric units incorrectly. Most of them reduced 1 from 10 m, 9 dm, 31 cm and placed it behind the next unit number starting with 2 mm to get 9 m, 18 dm, 130 cm and 12 mm. This procedure led them to get a wrong reconstructed question which required them to subtract 8 m, 9 dm, 38 cm and 9 mm from 9 m, 18 dm, 130 cm and 12 mm. Such students ended up with wrong answers of 1 m, 9 dm, 92 cm and 3 mm. Other students subtracted the numbers from the left to the right; these students got 02 m, 00 dm, 92 cm, 03 mm which is wrong. Also few students applied operations which are not subtraction as per requirement of the question such as addition and multiplication. In very rare cases, some students managed to subtract the metric units correctly but they multiplied the obtained answer by 100, 1000 or 10,000. For example few who multiplied by 1000 obtained 1,000 m, 9,000 dm, 2,000cm and 3000 mm.

The weak performance in part (b) was greatly contributed by failure of the students to recognize the value of the rate (R) in relation to the stated formula. In this case, some students applied the formula $\mathrm{I} = \mathrm{PRT}$ with $R = 6$ instead of $R = \frac{6}{100}$ while others used the formula $\mathrm{I} = \frac{\mathrm{PRT}}{100}$ with $R = \frac{6}{100}$ instead of $R = 6$. For example the students who applied the formula $\mathrm{I} = \frac{\mathrm{PRT}}{100}$ with $R = \frac{6}{100}$ had the incorrect solution $\mathrm{I} = \frac{10000000 \times \frac{6}{100} \times 5}{100} = 30{,}000$. Also, a considerable number of students applied wrong formulae such as $\mathrm{I} = \frac{\mathrm{PR}}{100}$ and $\mathrm{A} = P\left(1 + \frac{R}{100}\right)^{n}$. Together with computational errors, the students lost all or some marks in part (b). Extract 3.2 is a sample response illustrating how a student failed to answer part (b) of the question.

Extract 3.2

(b) Find the simple interest on sh. 10,000,000 invested for 5 years at the rate of 6% per annum.

Soln

Principle = 10,000,000

time = 5 years

Rate = 6%

Interest = ?

Interest = PRT

I = PRT

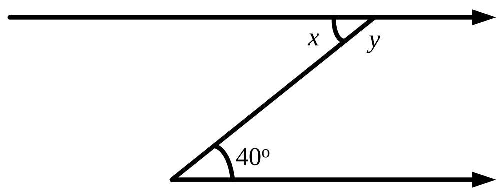
I = 10,000,000 x 6 x 5

Simple Interest = 300,000,000

Extract 3.2 shows the response from one of the students who performed the question poorly.

2.4 Question 4: Geometry and Perimeters

This question had parts (a) and (b). In part (a), the students were required to calculate the size of angle *x* and *y* in the following figure:



In part (b), they were required to find the perimeter of a right angled triangle whose base is $(4-\sqrt{2})$ cm and height is $(4+\sqrt{2})$ cm.

A total of 485,359 (96%) students answered this question out of which 77.16 percent scored below 3 marks, 20 percent scored from 3 to 6 marks and 2.84 percent scored above 6 marks. Based on these data, the question was poorly performed as shown in Figure 5.

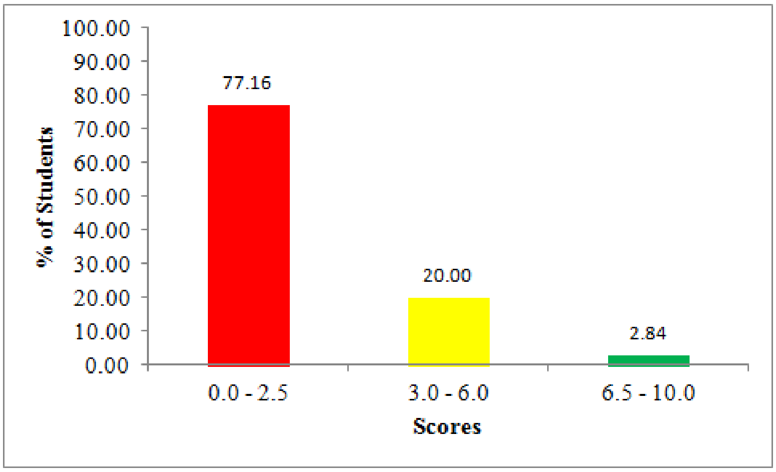


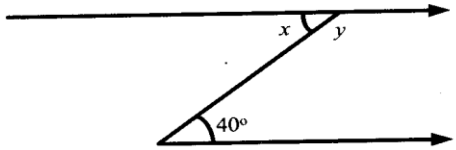
Figure 5: *Students' performance in question 4*

The analysis of responses indicates that the students who performed part (a) incorrectly encountered the following challenges. Firstly, they did not understand that x and $40°$ are alternate interior angles and thus have the same size. Secondly, they could not realize that x is also a supplementary angle of y implying that $x+y=180°$. For example, several students regarded x and 40^{o} as supplementary angles and thus getting $x=140°$ and $y=40°$ instead of $x=40°$ and $y=140°$, see Extract 4.1.

In part (b), the most common error was using the formula for calculating the area of a right angled triangle. Such students obtained 7 cm instead of getting 14 cm as the required perimeter. Another common error was formulation of the fraction $\frac{4-\sqrt{2}}{4+\sqrt{2}}$ by using the given measurements of the right angled triangle, rationalizing the numerator of the resulting fraction to get $\frac{9-4\sqrt{2}}{7}$ which was not part of the question. Furthermore, a considerable number of students had difficulty in computing the length of the hypotenuse side as they failed to expand expressions involving radicals hence were unable to get the correct answer. For example, they wrongly expanded $\left(4-\sqrt{2}\right)^{2}+\left(4+\sqrt{2}\right)^{2}$ as 32 instead of 36. Moreover, some students did not care to find the length of the hypotenuse side instead they computed the perimeter by adding the length of base and height ignoring the hypotenuse side obtaining 8 cm instead of 14 cm. Extract 4.2 is a sample answer from the script of a student who approached the question incorrectly.

Extract 4.1

(a) Calculate the size of angle x and y in the following figure:



Soln

$x + 40^\circ = 180$

$x = 180^\circ - 40^\circ$

$x = 140^\circ$

$x = 140^\circ$

$y + 140 = 180^\circ$

$y = 180 - 140$

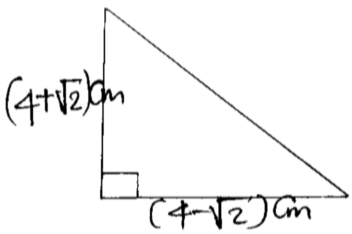
$y = 40$

$x = 140,\ y = 40$

Extract 4.1 shows how the student lacked knowledge on the properties of angles formed by a transversal.

Extract 4.2

(b) Find the perimeter of a right angled triangle whose base is $(4 - \sqrt{2})$ cm and height is $(4 + \sqrt{2})$ cm.



$P = \frac{1}{2} bh$

$P = \frac{1}{2}(4+\sqrt{2})(4-\sqrt{2})$

$P = \left(\frac{1}{2}\right) 4(4-\sqrt{2}) + \sqrt{2}(4-\sqrt{2})$

$P = \left(\frac{1}{2}\right)(16 - 4\sqrt{2} + 4\sqrt{2} - 2)$

$P = \frac{1}{2}(16-2)$

$P = \frac{1}{2} \times 14$

$P = 7$ cm

$\therefore$ Perimeter = 7 cm.

Extract 4.2 shows the responses of the student who had inadequate knowledge and skills on the tested concepts.

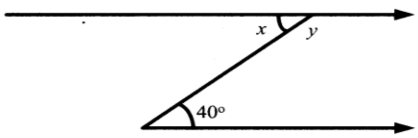
On the other hand, 6216 students equivalent to 1.3 percent scored full marks. This indicates that they had sufficient knowledge on Geometry and

Perimeter. These students demonstrated the following strengths. In part(a), they were able to apply the concept of alternate interior angles and the sum of degree measures in a straight angle on the given figure to calculate the values of x and y as 40^0 and 140^0 respectively.

In part (b), the students used the Pythagoras theorem correctly to calculate the length of the hypotenuse side as 6 cm, added the length of base, height together with the length of the hypotenuse as follow $((4-\sqrt{2})+\left(4+\sqrt{2}\right)+6$ and obtained the perimeter of 14 cm. Extract 4.3 shows the work of one of the students who answered parts (a) and (b) correctly.

Extract 4.3

4. (a) Calculate the size of angle x and y in the following figure:



Solution:

$x = 40°$ < Alternate Interior Angles > Z

$x + y = 180°$ < Sum of angles in a straight line > $\angle$

But, $x = 40°$

So,

$40° + y = 180°$

$y = 180° - 40°$

$y = 140°$

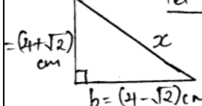
∴ The size of angle x and y are $40°$ and $140°$ respectively

(b) Find the perimeter of a right angled triangle whose base is $(4-\sqrt{2})$ cm and height is $(4+\sqrt{2})$ cm.

Solution:

From the right angled triangle:

let the hypotenuse be x



Perimeter:

Perimeter of triangle = L+L+L

$P = b + h + x$

Apply Pythagoras theorem:

$c^2 = a^2 + b^2$

$x^2 = (4+\sqrt{2})^2 + (4-\sqrt{2})^2$

$x^2 = (4+\sqrt{2})(4+\sqrt{2}) + (4-\sqrt{2})(4-\sqrt{2})$

$x^2 = 16 + 4\sqrt{2} + 4\sqrt{2} + 2 + 16 - 4\sqrt{2} - 4\sqrt{2} + 2$

$x^2 = 16 + 16 + 2 + 2 + 8\sqrt{2} - 8\sqrt{2}$

$x^2 = 36 + (8-8)\sqrt{2}$

$\sqrt{x^2} = \sqrt{36}$

$x = \pm 6$ cm

From, $P = b + h + x$

$= (4-\sqrt{2}) + (4+\sqrt{2}) + 6$

$= 4 - \sqrt{2} + 4 + \sqrt{2} + 6$

$= 4 + 4 + 6 + (1-1)\sqrt{2}$

$P = 14$ cm

∴ Perimeter of a right angled triangle is 14 cm

Extract 4.3 shows that a student had adequate knowledge on the concepts tested.

2.5 Question 5: Algebra and Quadratic Equations

The question consisted of parts (a) and (b). In part (a), the students were instructed to use elimination method to solve a pair of simultaneous equation $\begin{cases} 2x + y = 20 \\ x = 35 - 3y \end{cases}$. In part (b), the students were required to solve the equation $4(p+1)(1-p) = 3$.

A total of 487,111 students which is equivalent to 96.3 percent attempted the question whereby 19.69 percent of them got 3 marks or more. Figure 6 shows that 80.31 percent of the students scored below 3 marks, therefore the general performance of the question was weak.

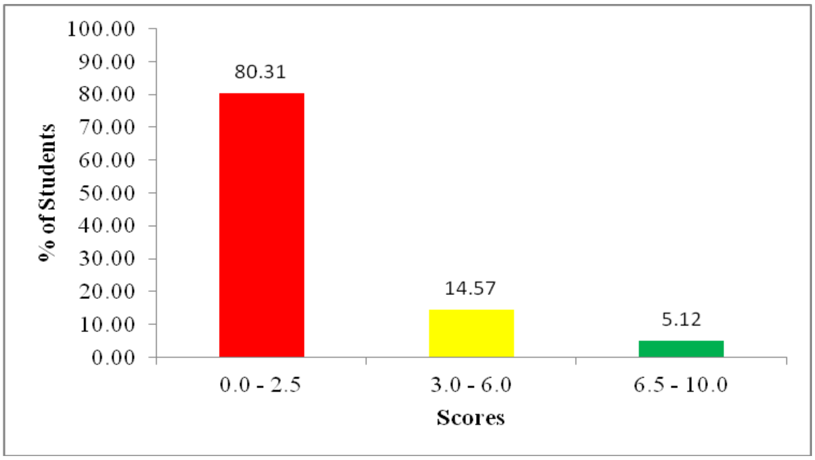


Figure 6: *Students' performance in question 5*

The analysis of data shows that more than two-thirds (69.6%) of the students scored zero. The analysis of students' responses in this category revealed various misconceptions and errors in answering the question. In part (a), many students realized that they were required to rearrange the equation $x = 35 - 3y$ in the form $ax + by = c$ but rearranged it incorrectly without considering the changing of signs and hence ending up with incorrect equation as $x - 3y = 35$ as shown in Extract 5.1. Some students correctly rearranged $x = 35 - 3y$ into $x + 3y = 35$ but were unable to eliminate x or y. Also, a significant number of students lost some marks in part (a) as they used elimination method to find one of the variables, mostly *y*, and then used substitution method to find the value of x. Moreover, there were few students who completely used substitution method or other methods rather than elimination method. Such students attempted the question contrary to its requirements; hence scored 1 mark or less.

Compared to part (a), the performance of students in part (b) was worse. Most students had weaknesses in multiplying two linear factors whereby failure to open brackets and misconception of $(p+1)(1-p)=(p-1)^2$ were commonly observed. For instance, several students were expanding $4\ p+1\ 1-p$ as $4p+4\ 4-4p$ instead of $4-4p\ 1+p$ as illustrated in Extract 5.2. Also, some students wrongly treated $4p+4$ and $1-p$ as factors of $(4p+4)(1-p)=3$ as they wrote $4p+4=3$ or $1-p=3$, which resulted to incorrect values of p. Such students were not aware of the fact that, to have correct factors is necessary for equation to be rearranged in such a way that the expression of one side is equated to 0. Further analysis of students' responses shows that there were some students who managed to manoeuvre the given equation into $4p^2-1=0$ but they did not appreciate the use of quadratic methods, of factorization (using the difference of two squares) or general quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve it. Most of them ended up with only one value of $p=\dfrac{1}{2}$ lacking the second value of $p=-\dfrac{1}{2}$.

Extract 5.1

5. (a) Solve $\begin{cases} 2x+y=20 \\ x=35-3y \end{cases}$ by the elimination method.

Soln.

$\begin{cases} 2x+y=20 \\ x=35-3y \end{cases}$

$\begin{matrix} 1 \\ 2 \end{matrix}\begin{cases} 2x+y=20 \\ x-3y=35 \end{cases}$

$- \left| \begin{matrix} 2x+y=20 \\ 2x-6y=35 \end{matrix} \right.$

$\dfrac{-5y}{-5}=\dfrac{-15}{-5}$

$y=3$

Recall eqn ... 1

$2x+y=20$

but $y=3$

$2x+3=20$

$2x=20-3$

$\dfrac{2x}{2}=\dfrac{17}{2}$

$x=8.5$ or $8\frac{1}{2}$.

$\therefore x=8\frac{1}{2}$ or 8.5 and $y=3$

Extract 5.1 illustrates the response of a student who attempted part (a) incorrectly.

Extract 5.2

(b) Solve the equation $4(p+1)(1-p)=3$.

Soln

5 b) $4(p+1)(1-p)=3$

$(4p+4)(4-4p)=3$

$16p \pm 16p^2 \mp 16 - 16p = 3$

$16p - 16p \mp 16p^2 \mp 16 - 3 = 0$

$16p^2 + p + 13 = 0$

$a=16,\ b=1,\ c=13$

From

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1-592}}{32}$

$x = \frac{-1+\sqrt{1-591}}{32}$ or $\frac{-1-\sqrt{1-591}}{32}$

$x\ \frac{-1+\sqrt{1-591}}{32}$ or $\frac{-1-\sqrt{1-591}}{32}$

Extract 5.2 illustrates the response of a student who attempted part (b) incorrectly.

On the other hand, a total of 3125 students, equivalent to 0.6 percent, did the question correctly scoring all 10 marks. In part (a), those students clearly understood the requirements of the question as they used elimination method to find the values of both x and y. The students rearranged the equation $x = 35 - 3y$ into $x + 3y = 35$ to have $\begin{cases} 2x + y = 20 \\ x + 3y = 35 \end{cases}$ and solved it to get $x = 5$ and $y = 10$.

In part (b), the students multiplied the linear factors $(1+p)$ and $(1-p)$ using expansion of the difference of two squares or otherwise, then simplified the resulted equation into $4p^2 - 1 = 0$. To complete the solution, such students used quadratic methods, commonly factorization by the difference of two squares, to get $p = \frac{1}{2}$ and $p = -\frac{1}{2}$. Extract 5.3 is a sample answer from the script of a student who answered this question correctly.

Extract 5.3

5. (a) Solve $\begin{cases} 2x + y = 20 \\ x = 35 - 3y \end{cases}$ by the elimination method.

$\begin{cases} 2x + y = 20 \\ x + 3y = 35 \end{cases}$ $\left(\begin{array}{l} x = 35 - 3y \\ = x + 3y = 35 \end{array} \right)$

$\begin{cases} (2x + y = 20)1 \\ (x + 3y = 35)2 \end{cases}$

$- \begin{cases} 2x + y = 20 \\ 2x + 6y = 70 \end{cases}$

$2x - 2x + y - 6y = 20 - 70$

$-5y = -50$

$y = 10.$

$\begin{cases} (2x + y = 20)3 \\ (x + 3y = 35)1 \end{cases}$

$\begin{cases} 6x + 3y = 60 \\ x + 3y = 35 \end{cases}$

$6x - x + 3y - 3y = 60 - 35$

$5x = 25$

$x = 5.$

$\therefore x = \underline{5}$

$y = \underline{10.}$

(b) Solve the equation $4(p+1)(1-p) = 3$.

$4(p+1)(1-p) = 3$

$4(1+p)(1-p) = 3$

$4(1^2 - p^2) = 3$

$4(1 - p^2) = 3$

$4 - 4p^2 = 3$

$4p^2 - 4 + 3 = 0$

$4p^2 - 1 = 0$

$(2p)^2 - (1)^2 = 0.$

$(2p-1)(2p+1) = 0$

$2p - 1 = 0$

$2p + 1 = 0.$

$2p - 1 = 2p = 1$

$p = 1/2$

$2p + 1 = 0$

$2p = -1$

$p = -1/2$

$\therefore \underline{p = 1/2 \text{ or } -1/2}$

Extract 5.3 shows the responses of a student who had adequate knowledge and skills on the concepts tested in question 5.

2.6 Question 6: Coordinate Geometry and Geometrical Transformations

This question comprised parts (a) and (b). In part (a), the students were given that the slope of a straight line through points $(7, 4)$ and $(-2, k)$ is 1 and were required to find the value of k. In part (b), the students were required to find the image of the point A (5, 2) under reflection in the line $y = 0$ followed by another reflection in the line $y = x$ using a sketch.

A total of 453,475 (89.7%) students attempted the question and 52,155 students which is equivalent to 10.3 percent did not do it showing lack of knowledge on Coordinate Geometry and Geometrical Transformations. Further analysis of data shows that 81.14 percent scored from 0 to 2.5 marks, 12.59 percent from 3 to 6 marks and 6.26 percent from 6.5 to 10 marks. Thus, the students' performance is rated as weak as portrayed in Figure 7.

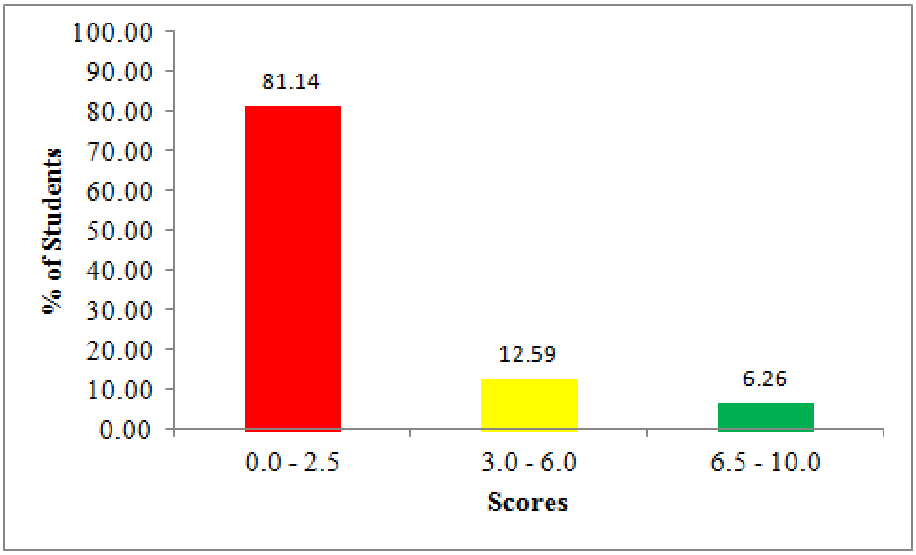


Figure 7: *Students' performance in question 6*

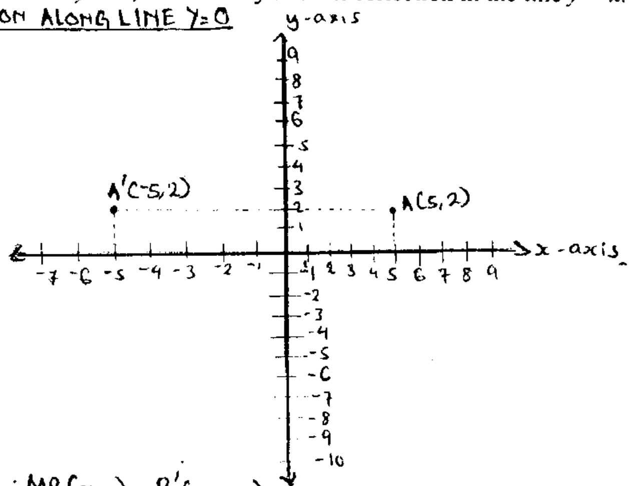
The weak performance of the students was attributed by various factors. In part (a), several students demonstrated limited understanding of gradient using the ratio $\frac{\text{change in } x}{\text{change in } y}$ instead of $\frac{\text{change in } y}{\text{change in } x}$. Such students were supposed to understand that the value of $\frac{\text{change in } y}{\text{change in } x}$ is called the slope of the straight line. However, it was pleasing to note that a number of students recalled the formula, $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$ correctly but could not insert x and y coordinates into this formula appropriately in order to find the value of *k*. Other students were unable to identify x and y - coordinates. These students ended up getting wrong answers such as $k = -2$ or $k = -1$ instead of $k = -5$. In addition, most of the students computed correctly the value of *k* but wrote the final answer as $k = 5$ ignoring the negative sign. Such students lost one mark which was allocated for the final answer.

In part (b), some students reflected the point $A(5, 2)$ incorrectly in the line $x = 0$ and could not proceed with the next reflection while others skipped the first transformation and reflected the given point directly in the line $y = x$. Similarly, there were students who reflected the given point in the x axis correctly but did not reflect the resulting point in the line $y = x$. These students could not comply with the requirements of this question and hence they ended up getting incorrect images such as $(-5, 2)$, $(5, -2)$ and $(2, 5)$. Moreover, several students did not sketch the xy-plane correctly as they indicated the *x* and y coordinates to inappropriate axes. For instance, some of them showed x-axis as vertical axis and y-axis as horizontal axis. Extract 6.1 is a sample solution from the script of a student who performed poorly in part (b) of question 6.

Extract 6.1

(b) By using a sketch, find the image of the point $A(5, 2)$ under a reflection in the line $y = 0$, followed by another reflection in the line $y = x$.

REFLECTION ALONG LINE Y=0



$\therefore$ MR(x,y) = R'(-x, y)
(5,2) = R'(-5, 2)

Extract 6.1 shows the solution of a student who had insufficient knowledge in Geometrical transformations.

However there were 1293 (0.3%) students who scored full marks. In part (a), these students were able to identify the slope as 1 and the values of x and y coordinates appropriately as $x_1 = 7$, $x_2 = -2$, $y_1 = 7$ and $y_2 = k$. Then, they were able to insert these values of the variables into the formula, slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ to obtain $k = -5$. In part (b), the students who scored all marks were able to reflect the point A (5, 2) in the line $y = 0$ to get $A'(5, -2)$ which was then reflected in the line $y = x$ to get $A''(-2, 5)$. Extract 6.2 is a sample solution from the script of a student who answered question 6 correctly.

Extract 6.2

(a) If the slope of the straight line through the points (7, 4) and (-2, k) is 1, find the value of k.

Solution

Slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ $\quad \underset{x_1\ \ y_1}{(7,\ 4)} \quad \underset{x_2\ \ y_2}{(-2,\ k)}$

$1 = \dfrac{k-4}{^-2-7}$

$\dfrac{1}{1} = \dfrac{k-4}{-9}$

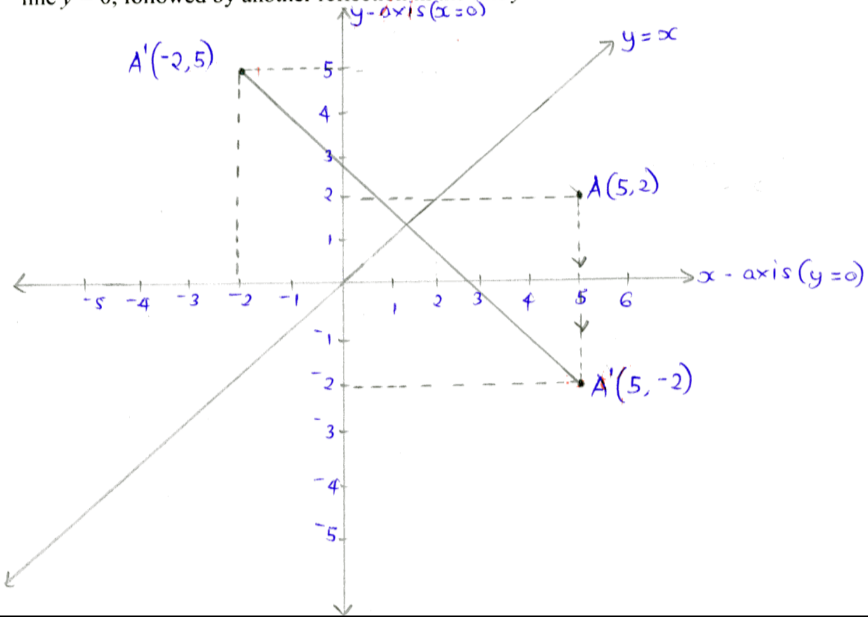
$k - 4 = {}^-9$

$k = {}^-9 + 4$

$k = {}^-5$

$\therefore k = {}^-5$

(b) By using a sketch, find the image of the point $A(5, 2)$ under a reflection in the line $y = 0$, followed by another reflection in the line $y = x$.



Extract 6.2 shows how a student worked out correctly the answers for question 6

2.7 Question 7: Exponents and Logarithms

The question comprised parts (a) and (b). In part (a), the students were required to simplify $\frac{(2r^3)^2}{(2r)^3}$ using the laws of exponents. In part (b), the students were given $\log 2 = 0.3010$, $\log 3 = 0.4771$ together with $\log 7 = 0.8451$ and were required to find $\log 42$.

A total of 470,783 students which is equivalent to 93.1 percent attempted the question out of which 367,697 students scored below 3 marks. Further analysis of data shows that 70.6 percent of the students scored zero while 1.4 percent scored all 10 marks. Figure 8 summarizes the performance of the students in this question.

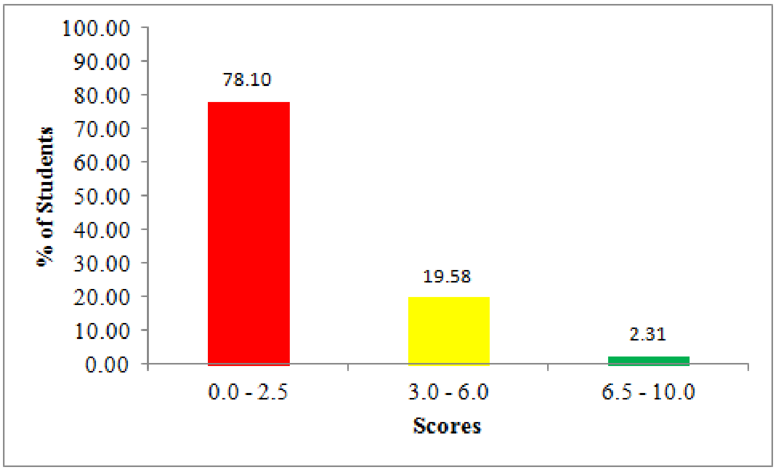


Figure 8: *Students' performance in question 7.*

As seen in Figure 8, 78.10 percent of students performed poorly, hence the general performance of the students in this question was weak. This weak performance was due to students' inability to apply the laws of exponents and logarithms. In part (a), the majority of students wrote $\frac{(2r^3)^2}{(2r)^3} = \frac{2r^6}{2r^3} = 2r^{6-3} = 2r^3$. This indicates that the students used a wrong concept by evaluating $(ab)^n$ as $a \times b^n$ instead of using the correct law of exponent $(ab)^n = a^n \times b^n$, see Extract 7.1. Also, the observation shows that many students were aware of the correct law of exponents $\frac{a^n}{a^m} = a^{n-m}$ but could not apply it to constant numbers. For example they wrote $2^0 = 2$ instead of $2^0 = 1$. Additionally, the analysis of responses shows that a significant number of students wrote $\frac{(2r^3)^2}{(2r)^3} = \frac{2r^3 + 2r^3}{2r + 2r + 2r}$ exposing the misconception of $a^2 = a + a$. They were supposed to expand $\left(2r^3\right)^2$ as $2r^3 \times 2r^3$ and $\left(2r\right)^3 = 2r \times 2r \times 2r$. Furthermore, the analysis depicts that few students who worked out the exponents incorrectly ignored the bases. The students came up with a solution which gives the final answer as a constant number. For example $\frac{(2r^3)^2}{(2r)^3} = \frac{2r^6}{2r^3} = \frac{6}{3} = 2$.

In part (b), the analysis of responses shows that majority of the students expressed 42 as the product of prime factors of 2, 3 and 7, that is, $42 = 2 \times 3 \times 7$ correctly. The common challenge in this part was inability to apply some laws of logarithms. Most of the students wrote $\log 42$ as $\log 2 \times \log 3 \times \log 7$ instead of writing it as $\log 2 + \log 3 + \log 7$. Such students were unable to use the product rule of logarithm, $\log(abc) = \log a + \log b + \log c$. Furthermore, there were few students who directly read $\log 42$ from mathematical tables instead of using the given values of $\log 2$, $\log 3$ and $\log 7$. However, some students were unable to express 42 as the product of prime factors 2, 3 and 7; hence they obviously obtained incorrect answers. Extract 7.2 illustrates some mistakes done by one of the students who attempted part (b) of this question.

Extract 7.1

7. (a) Use laws of exponents to simplify $\frac{(2r^3)^2}{(2r)^3}$.

Soln

$$\frac{(2r^3)^2}{(2r)^3} = \frac{2r^{3\times 2}}{2r^3}$$

$$= \frac{2r^6}{2r^3}$$

$$= a^n \div a^m = a^{n-m}$$

$$= \frac{2r^6}{2r^3} = 2r^{6-3}$$

$$= 2r^3$$

$$\therefore \frac{(2r^3)^2}{(2r)^3} = 2r^3$$

Extract 7.1 shows a sample solution of a student who had inadequate knowledge on the laws of exponents

Extract 7.2

(b) If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, find $\log 42$.

Soln

$\log 42 =$

2	42
3	21
7	7
	1

$\log 42 = (2^1)(3^1)(7^1)$

$\log 42 = 0.3010 \times 0.4771 \times 0.8451$

$\log 42 = 7.30866210 ..$

$\log 42 = 7.30866210.$

$\therefore \text{Log } 42 = 7.30866210.$

Extract 7.2 shows an incorrect response of a student who failed to apply the laws of logarithms.

On the other hand, 1.4 percent of the students had adequate knowledge and skills on using the laws of exponents and logarithms. In part (a), they used the laws of exponents, $(ab)^n = a^n \times b^n$ and $(a^r)^n = a^{rn}$, to express $\frac{(2r^3)^2}{(2r)^3}$ into $\frac{4r^6}{8r^3}$. Thereafter, they applied the fact that $\frac{a^n}{a^m} = a^{n-m}$ on $\frac{4r^6}{8r^3}$ to get $\frac{r^3}{2}$ as the simplest form of $\frac{(2r^3)^2}{(2r)^3}$.

In part (b), these students expressed 42 as the product of prime factors of 2, 3 and 7, that is, $42 = 2 \times 3 \times 7$. Then, they used the product rule of logarithms to obtain $\log 42 = \log 2 + \log 3 + \log 7$. Finally, they replaced

$\log 2$, $\log 3$ and $\log 7$ with 0.3010, 0.4771 and 0.8451 respectively to obtain $\log 42 = 1.6232$. Extract 7.3 is a sample response of correct solutions to question 7.

Extract 7.3

7. (a) Use laws of exponents to simplify $\frac{(2r^3)^2}{(2r)^3}$.

Solution.

from: $(ab)^2 = a^2 \times b^2$

$$\frac{(2r^3)^2}{(2r)^3}$$

$$= \frac{2^2 \times r^{3\times 2}}{2^3 \times r^3}$$

$$= \frac{4^1 \times r^6}{8_2 \times r^3}$$

$$= \frac{r^{6-3}}{2} = \frac{r^3}{2}$$

$$\therefore \frac{(2r^3)^2}{(2r)^3} = \frac{r^3}{2}$$

(b) If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, find $\log 42$.

Solution.

$\log 42$

$= \log (7 \times 6)$

$= \log (7 \times 2 \times 3)$

Product rule

$\log (7 \times 2 \times 3)$

$= \log 7 + \log 2 + \log 3$

Recall:

$\log 2 = 0.3010$

$\log 3 = 0.4771$

$\log 7 = 0.8451$

Then

$= \log 7 + \log 2 + \log 3$

$= 0.8451 + 0.3010 + 0.4771$

$= 0.8451 + 0.7781$

$= 1.6232$

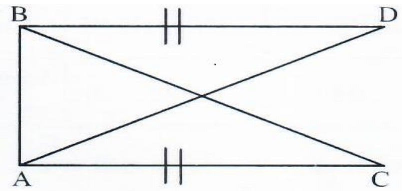
$\log 42 = 1.6232$

Extract 7.3 shows a response of a student who correctly applied the laws of exponents and logarithms.

2.8 Question 8: Similarity and Congruence

The question had parts (a) and (b). In part (a), the students were instructed that rectangle WXYZ is similar to rectangle ABCD, $\overline{BC} = 9cm$,

$\overline{AB} = 4cm$ and $\overline{WX} = 5cm$ and were required to calculate the length of $\overline{XY}$. In part (b), the students were given that $\overline{AC} = \overline{BD}$ and were required to prove that $A\hat{C}B = A\hat{D}B$ using the following figure:



This question was attempted by 91.4 percent of the students of whom 82.42 percent scored below 3 marks and among them 59.4 percent did not score any mark. Moreover, 43,594 students equivalent to 8.6 percent did not attempt this question. Thus, the question was poorly performed as shown in Figure 9.

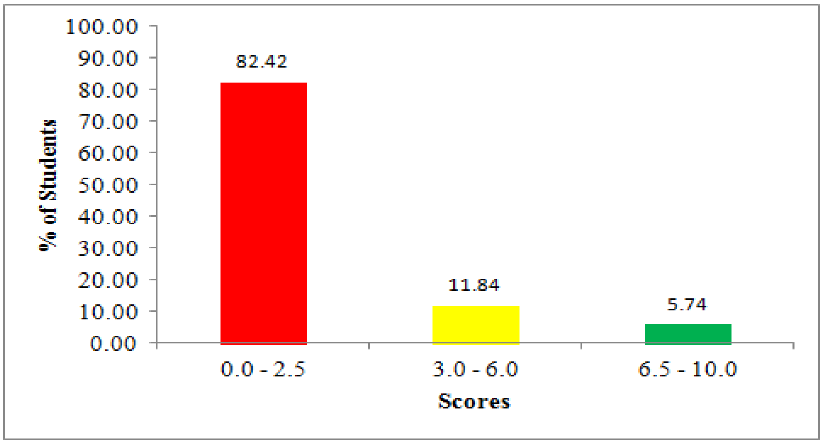


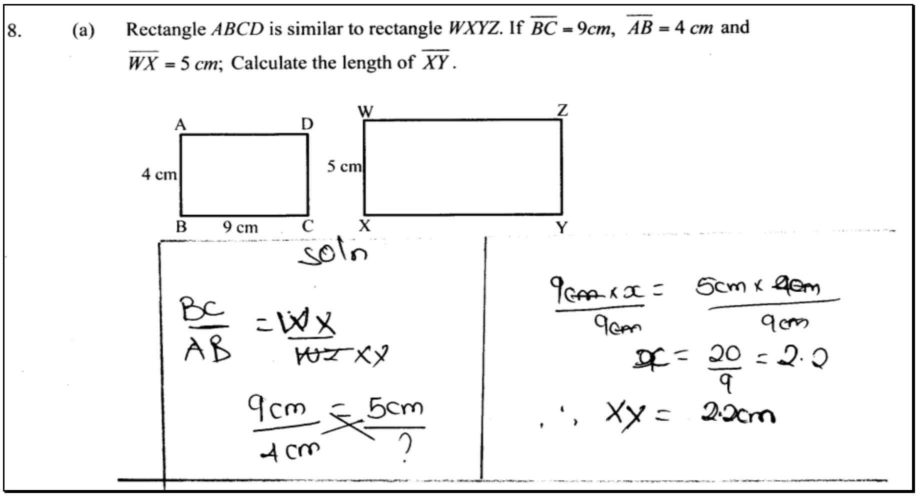
Figure 9: *Students' Performance in question 8*

The majority of the students who did part (a) incorrectly were unable to recognize that $\overline{AB}$ corresponds to $\overline{WX}$ and $\overline{BC}$ corresponds to $\overline{XY}$ and hence $\frac{4\text{ cm}}{5\text{ cm}} = \frac{9\text{ cm}}{\overline{XY}}$. For example, some students wrote incorrect ratios such as $\frac{AB}{BC} = \frac{WX}{XY}$, $\frac{AB}{WX} = \frac{XY}{BC}$ and $\frac{AB}{BC} = \frac{WX}{ZY}$. These incorrect relationships led them to obtain the wrong length of $\overline{XY}$. Further analysis of the responses revealed that most of the students did not understand the requirements of the question. For example, some students added the given measurements to get 18 cm while others calculated the area of rectangle

ABCD to get $36\ cm^2$. Additionally, a large number of students showed lack of understanding between the concepts of congruence and similarity. For example, they wrongly considered $\overline{BC}$ and $\overline{XY}$ to have the same length and therefore concluded that $\overline{XY} = 9$ cm. Such students were supposed to understand that when two figures are similar, the ratios of the lengths of their corresponding sides must be equal.

The students who scored low marks in part (b) had a poor understanding of the concept of congruence. They could not use the given figure to identify that triangle *ACB* is congruent to triangle ADB. Therefore, they were unable to prove that $A\hat{C}B = A\hat{D}B$. Furthermore, several students used wrong postulates such as SSS thus failing to prove the relationship between these angles. They were supposed to apply the SAS postulate on the identified triangles as follow: $\overline{AC} = \overline{BD}$ given side; $A\hat{B}D = B\hat{A}C$ (included angle between the two sides); $\overline{AB} = \overline{AB}$ common side; $\Delta\, ACB \equiv \Delta\, ADB$ (SAS) and hence $A\hat{C}B = A\hat{D}B$. Extract 8.1 is a sample solution from the script of a student who did not answer the question correctly.

Extract 8.1

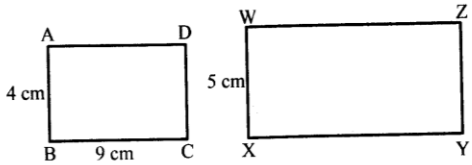

8. (a) Rectangle *ABCD* is similar to rectangle *WXYZ*. If $\overline{BC} = 9cm$, $\overline{AB} = 4\ cm$ and $\overline{WX} = 5\ cm$; Calculate the length of $\overline{XY}$.

Extract 8.1 shows that the student had inadequate knowledge on similarity.

Despite the weaknesses in the responses of the students, there were 2137 students which is equivalent to 0.5 percent who attempted the question correctly. In part (a), they used the properties of similar polygons to formulate the relationship $\frac{\overline{AB}}{\overline{WX}} = \frac{\overline{BC}}{\overline{XY}}$. Then, they substituted $\overline{BC} = 9cm$, $\overline{AB} = 4cm$, and $\overline{WX} = 5cm$ correctly into this relationship to get $\overline{XY} = 11.25\,\text{cm}$. Furthermore, they checked the correctness of the value of $\overline{XY}$ by showing that each of these corresponding sides bears the same ratio, i.e. $\frac{AB}{WX} = \frac{BC}{XY} = \frac{4}{5}$. In part (b), the students who scored all marks identified the triangles ACB and BDA from the given figure as two congruent triangles. Thereafter, they were able to argue correctly that $A\hat{C}B = A\hat{D}B$ using the SAS postulate. Extract 8.2 is a sample solution of a student who responded to this question correctly.

Extract 8.2

8. (a) Rectangle *ABCD* is similar to rectangle *WXYZ*. If $\overline{BC} = 9cm$, $\overline{AB} = 4\,cm$ and $\overline{WX} = 5\,cm$; Calculate the length of $\overline{XY}$.



Solution

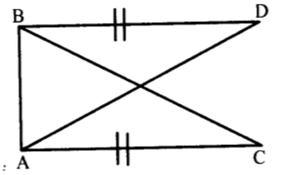
Since rectangle ABCD ~ rectangle WXYZ

$\frac{5}{4} = \frac{\overline{XY}}{9}$

$\frac{\overline{XY} \times 4}{4} = \frac{9 \times 5}{4} = \frac{45}{4}$

$\therefore \overline{XY} = 11.25\,cm$

(b) The figure below shows that $\overline{AC} = \overline{BD}$. Prove that $A\hat{C}B = A\hat{D}B$.



Solution

Consider Δ ABD and Δ BAC

$\overline{BD} = \overline{AC}$ (Given) ... S

$\overline{BA}$ is common ... S

$\hat{BAC} = \hat{ABD} = 90°$ --- A

$\therefore \Delta ABD \equiv \Delta BAC$ (by SAS)

Hence; All corresponding sides and angles are equal

$\hat{ACB} = \hat{ADB}$ (by definition of congruence)

Extract 8.2 shows that the student understood well the requirement of the question and had all the necessary skills to solve it correctly.

2.9 Question 9: Pythagoras Theorem and Trigonometry

The question consisted of parts (a) and (b). In part (a) (i), the students were asked to draw a diagram that represents the information; "A ladder on the ground leans against a vertical wall whose height is 5 metres. The ground distance between the ladder and the wall is 12m metres". In part (a) (ii), the students were required to use the diagram drawn in (i) to find the length of the ladder. In part (b), the students were given that $\sin A = \frac{3}{5}$ where A is an acute angle and were required to find without using mathematical tables the values of (i) $\cos A$, (ii) $\tan A$ and (iii) $\frac{1-\sin A}{1-\cos A}$.

The statistical data shows that 418,055 students which is equivalent to 82.7 percent attempted this question while a total of 87,575 (17.3%) students did not do it. Therefore it was the most skipped question in this assessment. Further analysis shows that 57.7 percent of the students scored zero. Figure 10 shows the performance of the students in question 9.

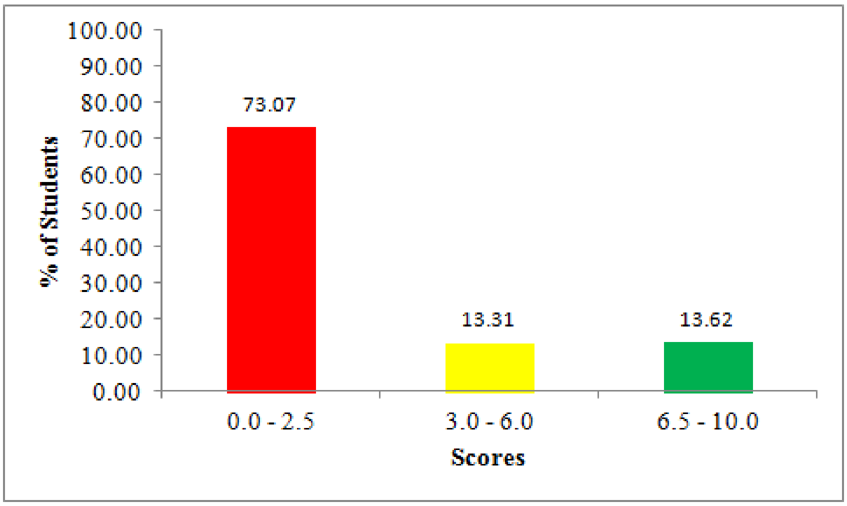


Figure 10: *Students' performance in question 9*

As observed in Figure 10 the performance of the students in this question was weak as 73.07 percent of students scored below 3 marks.

Based on the analysis of students' responses, various weaknesses were noted. In part (a) (i), many students presented the correct diagrams but did mistakes of interchanging the height of the wall with the ground distance. They assigned one of the given lengths (5m or 12m) to be the length of hypotenuse in the diagram which represents the length of the ladder as shown in Extract 9.1. Also, some students sketched diagrams which do not resemble the right angled triangle as it was required. In this case the students drew diagrams such as trapeziums, rectangles, isosceles triangles, squares, histograms, Venn diagrams, cylinders and pie charts. However, some students sketched the correct labelled diagrams but responded to part (a) (ii) wrongly. Many students were not aware of the terms in the Pythagoras' theorem $a^2+b^2=c^2$ together with their implication, that is c must be the hypotenuse of the right angled triangle. For example, some of them wrote $5^2+b^2=12^2$ instead of $5^2+12^2=c^2$. Furthermore, a considerable number of students lost the marks in this part as they subtracted 5m from 12m to get 7m while others applied the formula for calculating the area of right angled triangle and found the required length as $l=\frac{1}{2}\times 12m\times 5m=30m$. Though it was a rare case, some students changed the units whereby most of them used *cm* instead of *m*.

In part (b), many students realized the need of sketching a right angled triangle but did not indicate angle A and assigned the magnitude 5 to the side which is not hypotenuse. This shows that the students interpreted 5 from $\sin A=\frac{3}{5}$ incorrectly as a result of defining the trigonometric ratio of sine wrongly. Also, many students recognized the need of finding the length of the missing side of the right angled triangle, though the previous mistake led them to have incorrect equation $3^2+5^2=c^2$ instead of $3^2+b^2=5^2$ after applying the Pythagoras theorem. Other students interpreted angle A as special angle. It is clear that such students were not aware of the fact that an acute angle is any angle whose measure is less than 90^0 and not necessary to be special angle. Moreover, a significant number of students defined the trigonometric ratios of cosine and tangent wrongly by writing $\cos A = \dfrac{opposite}{hypotenuse}$ and $\tan A = \dfrac{hypotenuse}{adjacent}$ instead of $\cos A = \dfrac{adjacent}{hypotenus}$ and $\tan A = \dfrac{opposite}{adjacent}$ which gave out incorrect answers. In addition, some students lost marks in part (b) because they used mathematical tables to calculate the degree measure of angles A instead of finding $\cos A$, $\tan A$ and $\dfrac{1-\sin A}{1-\cos A}$. This situation shows that they did not understand the requirements of the question. It was further noted that, several students were able to compute $\cos A$ and $\tan A$ but did not evaluate $\dfrac{1-\sin A}{1-\cos A}$ correctly. Such students had insufficient skills in subtracting fractions from whole numbers as they incorrectly obtained $\dfrac{3}{2}$ when simplified the expression $\dfrac{1-\frac{3}{5}}{1-\frac{4}{5}}$. Extract 9.2 shows incorrect responses written by some students who attempted part (b).

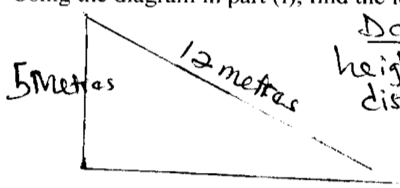
Extract 9.1

(a) A ladder on the ground leans against a vertical wall whose height is 5 metres. The ground distance between the ladder and the wall is 12 metres.

(i) Draw a diagram to represent this information.

(ii) Using the diagram in part (i), find the length of the ladder.

(i)



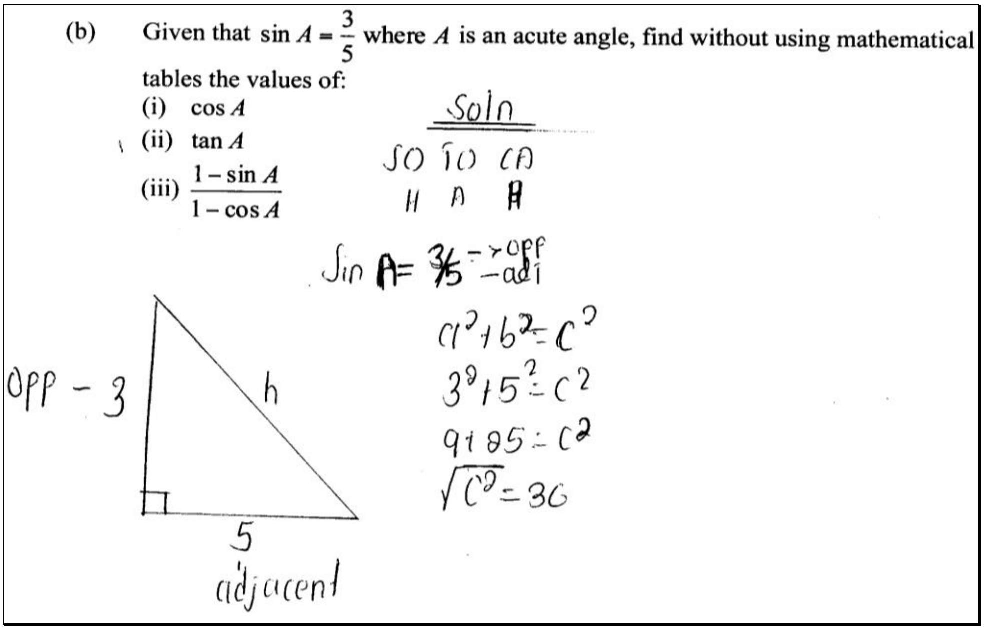
Data given
height of ~~the~~ ladder = 5 m
distance btn ladder and wall = 12 m

(ii) $12 - 5 = 7$

The length of the ladder is 7 metres

Extract 9.1 shows how the student lacked understanding of the Pythagoras theorem.

Extract 9.2



(b) Given that $\sin A = \frac{3}{5}$ where A is an acute angle, find without using mathematical tables the values of:

(i) $\cos A$

(ii) $\tan A$

(iii) $\frac{1 - \sin A}{1 - \cos A}$

Soln

SO TO CA

H A A

Sin A= 3/5 → opp → adj

$a^2 + b^2 = c^2$

$3^2 + 5^2 = c^2$

$9 + 25 = c^2$

$\sqrt{c^2} = 36$

Opp - 3

h

5

adjacent

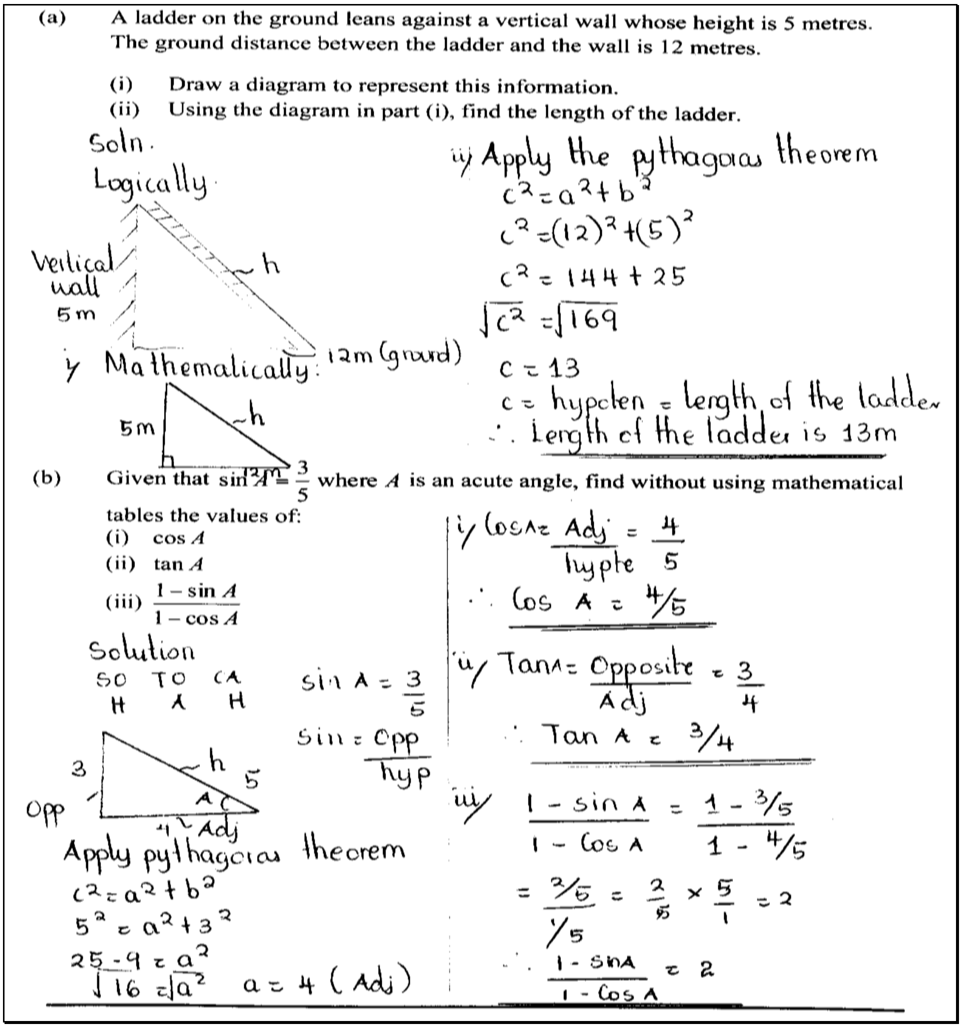
Extract 9.2 shows response of a student who interpreted the question wrongly.

Despite the weak performance in this question, there were 30,920 (7.4%) students who scored full marks. In part (a) (i), they correctly understood that the ground and wall form a right angle while a ladder completes a triangle as hypotenuse. Then, they sketched a right angled triangle and assigned 5 m and 12 m to opposite and adjacent sides of the triangle respectively. Furthermore, the students answered part (a) (ii) by applying Pythagoras theorem to the diagram sketched in part (a) (i) as follows; $5^2 + 12^2 = c^2$ where c is hypotenuse of the triangle, which gave $c = 13$ after solving it. Finally, they concluded that the length of the ladder is 13 m.

In part (b), the students correctly answered the question using a right angled triangle whose one of the acute angles was labelled A. Also, the students assigned 5 as the hypotenuse and 3 to the opposite side of an angle A. Then, they applied Pythagoras theorem, $3^2 + b^2 = 5^2$ to get 4 as the length of the adjacent side to angle A. Finally, they used trigonometric ratios $\cos A = \frac{adjacent}{hypotenuse}$ and $\tan A = \frac{opposite}{adjacent}$ to evaluate $\cos A$, $\tan A$ and

$\dfrac{1-\sin A}{1-\cos A}$ as $\dfrac{4}{5}$, $\dfrac{3}{4}$ and 2 respectively. Extract 9.3 is a sample response of a student who answered question 9 correctly.

Extract 9.3



(a) A ladder on the ground leans against a vertical wall whose height is 5 metres. The ground distance between the ladder and the wall is 12 metres.

(i) Draw a diagram to represent this information.

(ii) Using the diagram in part (i), find the length of the ladder.

Soln.

Logically.

Vertical wall 5m

h

12m (ground)

i) Mathematically:

5m

h

ii) Apply the pythagoras theorem

$c^2 = a^2 + b^2$

$c^2 = (12)^2 + (5)^2$

$c^2 = 144 + 25$

$\sqrt{c^2} = \sqrt{169}$

$c = 13$

c = hypoten = length of the ladder

∴ Length of the ladder is 13m

(b) Given that $\sin A = \frac{3}{5}$ where A is an acute angle, find without using mathematical tables the values of:

(i) $\cos A$

(ii) $\tan A$

(iii) $\dfrac{1-\sin A}{1-\cos A}$

Solution

SO TO CA
H A H

$\sin A = \dfrac{3}{5}$

$\sin = \dfrac{\text{Opp}}{\text{hyp}}$

3 Opp

h 5

A

4 Adj

Apply pythagoras theorem

$c^2 = a^2 + b^2$

$5^2 = a^2 + 3^2$

$25 - 9 = a^2$

$\sqrt{16} = \sqrt{a^2}$ $a = 4$ (Adj)

i) $\cos A = \dfrac{\text{Adj}}{\text{hypte}} = \dfrac{4}{5}$

∴ $\cos A = 4/5$

ii) $\tan A = \dfrac{\text{Opposite}}{\text{Adj}} = \dfrac{3}{4}$

∴ $\tan A = 3/4$

iii) $\dfrac{1-\sin A}{1-\cos A} = \dfrac{1-3/5}{1-4/5}$

$= \dfrac{2/5}{1/5} = \dfrac{2}{5} \times \dfrac{5}{1} = 2$

∴ $\dfrac{1-\sin A}{1-\cos A} = 2$

Extract 9.3 shows the student's correct response in question 9.

2.10 Question 10: Sets and Statistics

The question comprised parts (a) and (b). In part (a), the students were instructed that "In a class of 32 students, 18 play golf, 16 play piano and 7 play both golf and piano" and were required to find the number of students who play neither golf nor piano by using a formula. In part (b), the students were given the following frequency distribution table showing the results of a survey which was done to find out the most popular subject under the condition that each student voted once. They were required to show this information in a pie chart.

Subject	Maths.	English	Biology	History	Geography	Physics
Number of Pupils	50	80	120	40	80	30

The performance of the students in this question is shown in Figure 11.

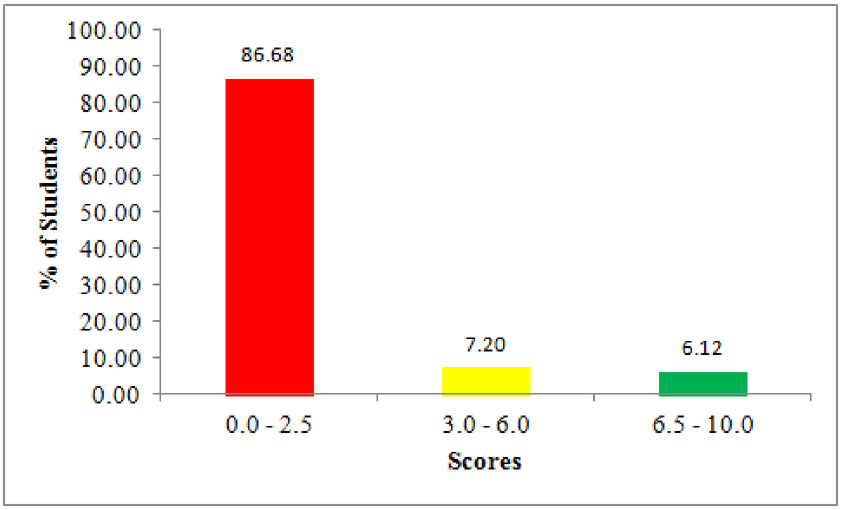


Figure 11: *Students' performance in question 11*

As indicated in Figure 11, only 13.32 percent of the students scored from 3 to 10 marks. Most students (86.68%) scored below 3 marks and among them 75.2 percent scored 0. The students with weak performance in part (a) were unable to follow the given instructions. Most of them used "Venn diagrams" instead of using the formula as it was instructed. This situation resulted into losing marks that were allocated for the formula. Also, many students computed only the number of students who play either golf or piano using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ but could not realize that they were required to substitute $n(A \cup B) = 27$ and $n(\mu) = 32$ into the formula $n(\mu) = n(A \cup B) + n(A \cup B)'$ to find the number of students who play neither golf nor piano. It was further noted that some students copied the question without writing answers while others used the number that were in the question to perform calculations that were not related to the requirements of the question. For instance, most of them summed up the given numbers to get 73 as shown in Extract 10.1.

In part (b), some students could not represent the given information in a pie chart as it was instructed instead they were representing it using histograms, bar charts, pictograms, line graphs etc. Other students represented the given information using number of pupils in a table without showing efforts to convert each number into degrees. Extract 10.2 is a sample answer of a student who answered part (b) of this question incorrectly.

Extract 10.1

10. (a) In a class of 32 students, 18 play golf, 16 play piano and 7 play both golf and piano. Use a formula to find the number of students who play neither golf nor piano.

Soln

In a class of 32 students, 18 play golf 16 play piano and 7 play both golf and piano. Use a formular to find the number of students who play neither golf nor piano.

$32 + 18 + 16 + 7 = 73$

Extract 10.1 shows the solution of a student who lacked understanding on the topic of Sets.

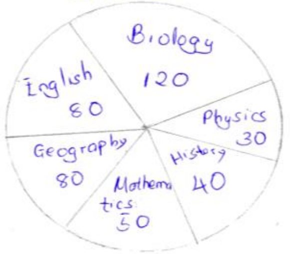
Extract 10.2

(b) A survey was done among students in a certain school in order to find the most popular subject. In this survey each student voted once and the results were as follows:

Subject	Mathematics	English	Biology	History	Geography	Physics
Number of Pupils	50	80	120	40	80	30

Show this information in a pie chart.

Solution



Extract 10.2 shows how poorly a student answered part (b) of question 10.

On the other hand, 1.7 percent of the students answered this question correctly. They were able to solve the set problem and represent the information using a pie chart as instructed. Extract 10.3 is a sample solution of a student who answered the question correctly.

Extract 10.3

10. (a) In a class of 32 students, 18 play golf, 16 play piano and 7 play both golf and piano. Use a formula to find the number of students who play neither golf nor piano.

Solution.

- let the number of those who play golf be set A(18)
- let the number of those who play piano be set B(16)

n(Universal set) = 32

from:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 18 + 16 - 7$

$n(A \cup B) = 34 - 7$

$n(A \cup B) = 27$

$n(\mathcal{U}) = n(A \cup B) + n(A \cup B)'$

$32 = 27 + n(A \cup B)'$

$n(A \cup B)' = 32 - 27$

$n(A \cup B) = 5$

∴ Those who play neither golf nor piano is 5 pupils /students

(b) A survey was done among students in a certain school in order to find the most popular subject. In this survey each student voted once and the results were as follows:

Subject	Mathematics	English	Biology	History	Geography	Physics
Number of Pupils	50	80	120	40	80	30

Show this information in a pie chart.

Soln.

$50 + 80 + 120 + 40 + 80 + 30 = 400$

400 pupils = 360°

$\frac{50}{400} \times 360 = 45°$

$\frac{40}{400} \times 360 = 36°$

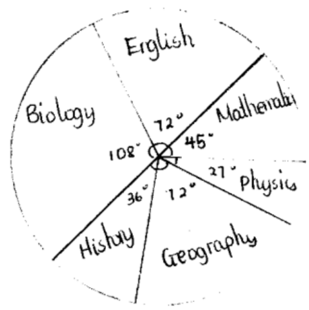
$\frac{80}{400} \times 360 = 72°$

$\frac{80}{400} \times 360 = 72°$

$\frac{120}{400} \times 360 = 108$

$\frac{30}{400} \times 360 = 27°$

$45° + 72° + 108° + 36° + 72° + 27° = 360°$



PIE CHART.

Extract 10.3 shows that the student had good understanding on the tested concepts.

3.0 CONCLUSION AND RECOMMENDATIONS

3.1 CONCLUSION

The analysis of the students' performance in each question has shown that out of ten questions that were assessed, question 1 on *Numbers and Approximations* and question 3 on *Units, Ratios, Profit and Loss* were averagely performed. The other eight questions had poor performance. The questions were set from *Fractions and Percentages; Pythagoras Theorem and Trigonometry; Geometry, Perimeter and Areas; Exponents and Logarithms; Algebra and Quadratic Equations; Coordinate Geometry and Geometrical Transformations; Similarity and Congruence* and *Sets and Statistics*. In addition, the questions on *Pythagoras' Theorem, Trigonometry, Coordinate Geometry, Geometrical Transformations, Similarity, Congruence, Exponents* and *Logarithms* were omitted by several students.

The students' weak performance on these topics was due to the students' inability to identify the requirements of the questions, perform mathematical operations and failure of the students to formulate equations from the given information. Also, failure of the students to represent the given information diagrammatically or graphically, adhere to the instruction of the questions and incorrect use of laws, formulae, theorems and other mathematical facts in answering the questions contributed to the poor performance.

3.2 RECOMMENDATIONS

In order to improve the standard of students' performance in future Basic Mathematics assessments; it is recommended that;

(a) The students should put more emphasis on the topics which had weak performance, namely *Fractions and Percentages; Pythagoras Theorem and Trigonometry; Geometry, Perimeter and Areas; Exponents and Logarithms; Algebra and Quadratic Equations; Coordinate Geometry and Geometrical Transformation; Similarity and Congruence* and *Sets and Statistics*. Such emphasis should include:

(i) studying all the topics to ensure that the underlined concepts and facts are well understood,
(ii) reading very carefully each question in order to identify the requirements of the question,
(iii) observing the given instructions when answering the assessment questions and
(iv) revising various concepts, theorems and mathematical properties by solving related problems.

(b) The teachers should;

(i) Deliberately address all the pointed out misconceptions during the teaching and learning process.
(ii) Strive to complete the syllabus much earlier so as to provide room for intensive revisions.
(iii) Set closed questions (not opened questions) in school continuous assessments and examinations as they are very specific to certain demands.
(iv) Emphasize the use of teaching resources instructed in the syllabus such as ruler, protractor, pair of compasses and coloured pencils.
(v) Use simple teaching and learning methods and daily life examples to enhance the students' understanding of various mathematical concepts and formulae.
(vi) Assess the students according to their abilities by providing the exercises frequently and planning a mechanism of assisting them to improve their performance.
(vii) Use participatory methods that allow interaction during the teaching-learning process.
(viii) Initiate and supervise the discussion groups and mathematics clubs so as to inculcate the spirit of students' cooperation in solving mathematical problems.

(c) The government should;

(i) Make follow up to find out why most students skipped questions on various topics as shown in Appendix II.

(ii) Facilitate in-house and in-service training for teachers in order to acquire additional knowledge and skills on topics which have weak performance.

Appendix I

ANALSYSIS OF THE STUDENTS' PERFORMANCE TOPIC-WISE FTNA 2018

S/n	Topics	Question Number	Percentage of Students who scored 30 marks or more	Remarks
1	Units, Ratio, Profit and Loss	3	62.9	Average
2	Numbers and Approximations	1	35.6	Average
3	Fractions, Decimals and Percentages	2	28.9	Weak
4	Pythagoras Theorem and Trigonometry	9	26.9	Weak
5	Geometry, Perimeters and Areas	4	22.8	Weak
6	Exponents and Logarithms	7	21.9	Weak
7	Algebra and Quadratic Equations	5	19.7	Weak
8	Coordinate geometry and Geometrical Transformations	6	18.9	Weak
9	Similarity and congruence	8	17.6	Weak
10	Sets and Statistics	10	13.3	Weak

Appendix II

THE PERCENTAGE OF STUDENTS WHO DID NOT ATTEMPT THE ASSESSED QUESTIONS IN PARTICULAR TOPICS