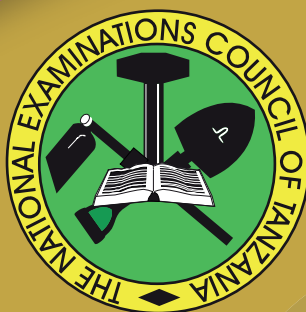


**THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**



**CANDIDATES' ITEM RESPONSE ANALYSIS  
REPORT FOR THE CERTIFICATE OF SECONDARY  
EDUCATION EXAMINATION (CSEE) 2019**

**042 ADDITIONAL MATHEMATICS**

**THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**



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## **FOREWORD**

The National Examinations Council of Tanzania has prepared this report on the Candidates' Items Responses Analysis (CIRA) for the Additional Mathematics paper of the Certificate of Secondary Education Examinations (CSEE) of 2019 in order to provide feedback to education stakeholders on how the candidates responded to the items of this paper.

The report identifies the strengths and weaknesses observed in the candidates' responses. Therefore, it indicates the competencies which were achieved and those which were not achieved by the candidates in their four years of ordinary secondary education.

The candidates' good performance was due to their ability to create patterns of numbers, solve algebraic equations, solve mathematical problems on variations, construct truth tables, perform operations on union, intersection and complement as well as apply De Morgan's law of sets. Furthermore, they were able to perform basic operations on rational functions and determine horizontal and vertical asymptotes graphically. Likewise, they were able to perform dot product and cross product on vectors and apply matrices in performing linear transformation. Conversely, the weak performance was attributed to failure of candidates to describe locus of a point moving in specified conditions and the inability to apply coordinate geometry in solving problems related to points, distances and angles.

The Council would like to thank everyone who participated in the process of writing this report.



Dr. Charles Msonde

**EXECUTIVE SECRETARY**

## **1.0 INTRODUCTION**

This report is a result of the analysis of candidates' responses to the items examined in 042 Additional Mathematics for the Certificate of Secondary Education Examination (CSEE) 2019. The paper was set according to the 2018 Examination format and the 2010 Additional Mathematics Syllabus for Secondary Schools. The report focuses on areas in which the candidates faced challenges as well as the areas in which candidates performed well.

The paper consisted of two sections, A and B with a total of fourteen (14) questions. Section A comprised 10 questions carrying 6 marks each while Section B had 4 questions carrying 10 marks each. The candidates were required to answer all questions in both Sections.

In 2019 a total of 336 candidates sat for the examination, of which 277 (84.45%) candidates passed. In 2018 a total of 393 candidates sat for the examination, of which 277 (70.48%) candidates passed. This indicates 13.97 percent increase in performance.

The analysis of the candidate's performance in all questions is presented in section 2.0. It consists of descriptions of the requirements of the questions and how the candidates responded. It also includes extracts showing the strengths and weaknesses demonstrated by the candidates in answering each question.

The candidates' performance in each question is categorized by using percentage of candidates who scored 30 percent or more of the total marks allotted to a particular question. The performance was categorized into three groups: 65 to 100 percent for good; 30 to 64 percent for average; and 0 to 29 percent for weak performance. Furthermore, green, yellow and red colors were used to denote good, average and weak performance respectively.

In section 3.0 the factors which contributed to weak performance in some topics are highlighted and the recommendations for improvement of the performance in future examinations have been provided. Also, the analysis of candidates' responses per topic is presented.

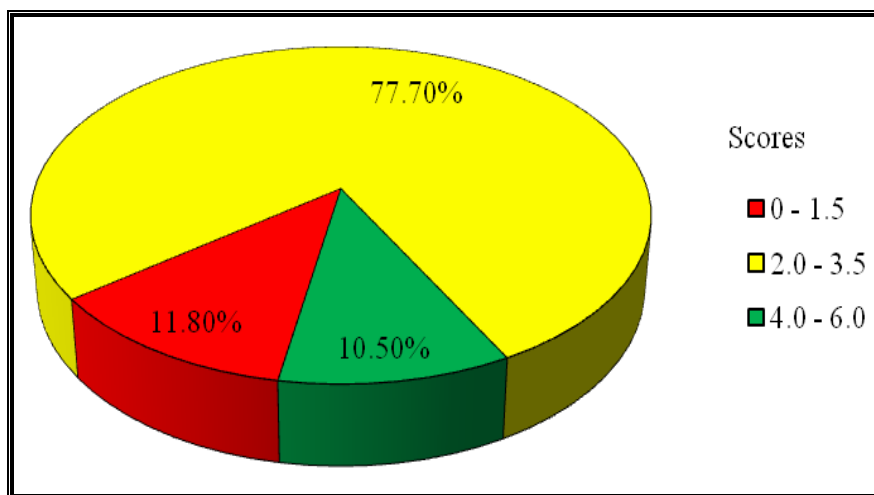
## 2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

### 2.1 Question 1: Variations

The question had parts (a) and (b). The candidates were required to:

- (a) show that  $x \propto y^{\frac{5}{6}}$  if  $x \propto yz^{\frac{1}{3}}$  and  $y \propto z^{-2}$ .
- (b) express  $y$  in terms of  $x$ ,  $k$  and  $m$  given that a quantity  $(y - m)$  is directly proportional to the square of  $x$ .

The question was attempted by 323 (98.2%) candidates. Among them, 285 (88.2%) candidates scored from 2.0 to 6.0 marks implying that candidates' performance was good. According to the analysis, this was the best performance compared to performance in other questions. Figure 1 shows percentage of candidates who got low, average and high marks in this question.



**Figure 1:** The candidates' performance on question 1

A total of 20 (6.2%) candidates scored full marks. In part (a), the candidates formulated the equations  $y = \frac{k_1}{z^2}$  and  $x = k_2 y z^{\frac{1}{3}}$  from  $x \propto y z^{\frac{1}{3}}$  and  $y \propto z^{-2}$  respectively. Then, they used these equations to eliminate  $z$  and simplified the resulting equation to get  $x = k_2 k_1^{\frac{1}{6}} y^{\frac{5}{6}}$ . Also, the candidates realized that  $k_2 k_1^{\frac{1}{6}}$  is a constant as  $k_1$  and  $k_2$  are constants, thus

they wrote  $x \propto y^{\frac{5}{6}}$ . In part (b), the candidates correctly interpreted the statement "A quantity  $(y-m)$  is directly proportional to the square of  $x$ " as  $(y-m) \propto x^2$  and consequently  $y = kx^2 + m$ .

1.	cb) $(y-m) \propto x^2$
	$(y-m) = kx^2.$
	$y = m + kx^2.$

**Extract 1.1:** A sample of correct responses for part (b) of question 1.

Extract 1.1 is a solution of one of the candidates who correctly wrote the given statement using proportionality sign and rearranged the resulting equation as per requirement.

In spite of candidates' good performance, 38 (11.8%) candidates obtained low scores. In part (a), majority of these candidates applied the laws of exponents incorrectly. For instance, some candidates simplified

$$x = y \left( \left( \frac{k}{y} \right)^{\frac{1}{2}} \right)^{\frac{1}{3}} \text{ into } x = ky \left( \frac{k}{y} \right)^{\frac{1}{2} + \frac{1}{3}} \text{ and consequently } x = ky \left( \frac{k}{y} \right)^{\frac{5}{6}} \text{ instead}$$

of  $x = ky \left( \frac{k}{y} \right)^{\frac{1}{6}}$ . This indicates that the candidates wrongly interpreted

$(a^n)^m$  as  $(a^n)^m = a^{n+m}$  instead of  $(a^n)^m = a^{n \times m}$ . Also, there were candidates who confused negative exponents with fractional exponent. They wrongly interpreted  $y \propto z^{-2}$  as  $y = k\sqrt{z}$  instead of  $y = \frac{k}{z^2}$ . Other candidates did not introduce different constants,  $k_1$  and  $k_2$ . They used the same letter ( $k$ ) in formulating the equations as they wrote  $y = \frac{k}{z^2}$  and  $x = kyz^{\frac{1}{3}}$ .

In part (b), majority interpreted the statement "quantity  $(y-m)$  is directly proportional to the square of  $x$ " wrongly as seen in Extract 1.2. Moreover, some candidates wrote  $(y-m) \propto \sqrt{x}$  instead of  $(y-m) \propto x^2$ . In addition, other candidates failed to make  $y$  the subject of  $y-m = kx^2$  whereby  $y = kx^2 - m$  was frequently observed.



Handwritten solution for part (a) of question 1:

$$y - m \propto x$$

$$y - m = x k$$

$$y = x k + m$$

**Extract 1.2:** A sample of incorrect solution for part (a) of question 1.

As shown in Extract 1.2, the candidate failed to interpret the statement "square of  $x$ " correctly, hence he/she got the wrong equation for the given statement.

## 2.2 Question 2: Statistics

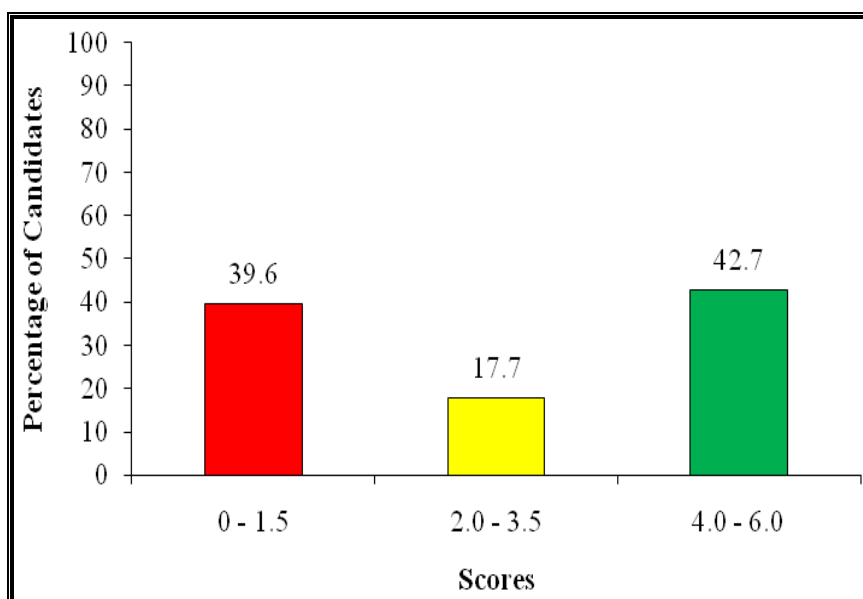
In this question, the candidates were given the following masses (in grams) of 50 apples:

86	108	118	92	101	113	97	107	111	100
100	114	109	96	116	104	99	101	105	117
103	92	107	100	102	99	106	98	96	108
101	118	87	93	110	102	93	101	113	88
106	101	95	103	105	92	116	105	86	92

They were required to:

- prepare a frequency distribution table if the lower limit of the first class interval is 85 and class width is 5.
- calculate the lower and upper quartiles in two decimal places.

The analysis of data indicates that 300 (91.2%) candidates attempted this question. The data also reveal that 181 (60.4%) candidates scored from 2.0 to 6.0 marks. Therefore, the overall candidates' performance in this question was average. Figure 2 illustrates candidates' performance in this question.



**Figure 2:** The candidates' performance on question 2

The analysis also reveals that 38 (12.7%) candidates responded to the question correctly. In part (a), the candidates prepared the table with classes 85 - 89, 90 - 94, 95 - 99, 100 - 104, 105 - 109, 110 - 114 and 115 - 119 whose frequency were 4, 6, 7, 13, 10, 5 and 5 respectively. In part (b), the candidates realized that 95 - 99 and 105 - 109 are the reference classes for calculating particular lower and upper quartiles respectively. Therefore, they determined their lower boundary and frequency correctly and substituted to a correct formula, as shown in Extract 2.1.

(b) Lower and upper quartiles in two decimal places.

$$Q = L_{Q_1} + \left( \frac{\frac{1}{4}N - N_b}{N_w} \right) i$$

where  $L_{Q_1} = 94.5$   $\frac{1}{4}N = 12.5$   $N_w = 7$   $N_b = 10$   $i = 5$

$$= 94.5 + \left( \frac{12.5 - 10}{7} \right) 5$$

$$= 94.5 + \left( \frac{2.5}{7} \right) 5$$

$$\Rightarrow 94.5 + 1.79 = 96.29$$

lower quartile is, 96.29

2. (b) Upper quartile

$$Q_3 = L_{Q_3} + \left( \frac{\frac{3}{4}N - Nb}{N_w} \right) i$$

where  $L_{Q_3} = 104.5$   $\frac{3}{4}N = 37.5$   $Nb = 30$   $N_w = 10$   $i = 5$

$$= 104.5 + \left( \frac{37.5 - 30}{10} \right) 5$$

$$= 104.5 + \left( \frac{7.5}{10} \right) 5$$

$$= 104.5 + \frac{37.5}{10}$$

$$= 104.5 + 3.75$$

$$= 108.25$$

$\therefore$  The upper quartile is 108.25

**Extract 2.1:** A sample of correct solution for part (b) of question 2.

In Extract 2.1, the candidate correctly calculated the lower quartile and upper quartile in two decimal places.

On the other hand, 119 (39.6%) candidates scored low marks whereby 43 (14.3%) scored zero. In part (a), majority of these candidates constructed incorrect frequency distribution table. For example, some candidates presented the following incorrect classes: 85 - 90, 91 - 96, 97 - 102, 103 - 108, 109 - 114 and 15 - 120. In addition, some candidates wrote correct classes but got incorrect frequency. In part (b), some candidates

used incorrect formula particularly  $Q_n = L \left( \frac{\frac{nN}{4} - \sum f_b}{f_w} \right) \times i$ .

Others candidates failed to determine the correct values of the components of the formula for calculating lower and upper quartiles. Extract 2.2 illustrates this case.

Q2 b) Lower quartile =  $L + \frac{(N/4 - F_b)}{f_w} c$   
 $= 95 + \frac{(59.4 - 11)}{9} 5$   
 Lower quartile =  $95 + \frac{(10.5 - 11)}{9} 5$   
 $= 95 + \frac{(1.5)}{9} 5$   
 $= 95 + \frac{(7.5)}{9}$   
 $= 95 + 0.8$   
 $= 95.8$   
 $\therefore$  Lower quartile = 95.8

Q2 b) Upper quartile -  
 from Upper quartile =  $L + \frac{(3N/4 - F_b)}{f_w} c$   
 Upper quartile =  $105 + \frac{(87.5 - 33)}{8} 5$   
 Upper quartile =  $105 + \frac{(4.5)}{8} 5$   
 $= 105 + \frac{(22.5)}{8}$   
 $= 105 + 2.8$   
 $= 107.8$   
 $\therefore$  Upper quartile = 107.8

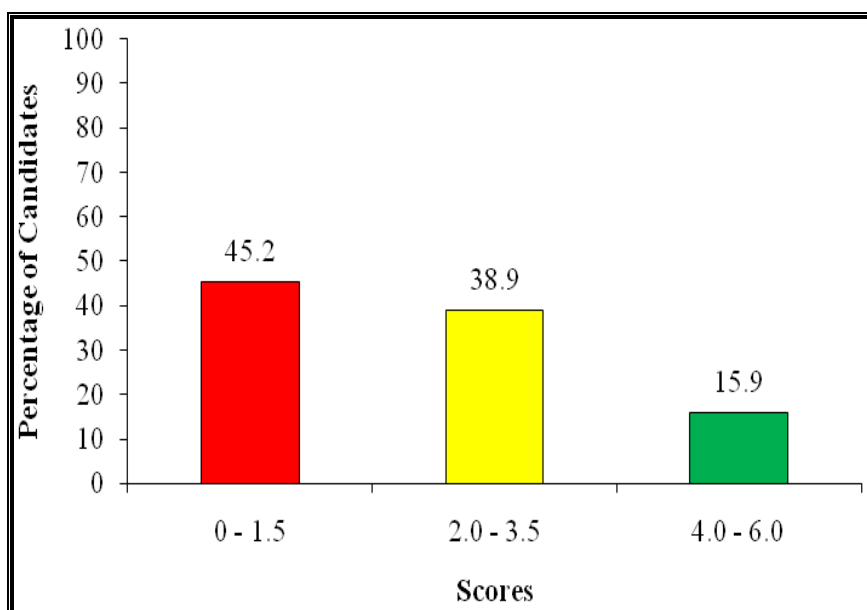
**Extract 2.2:** A sample of incorrect solution for part (b) of question 2.

Extract 2.2 is a response of one of the candidates who failed to determine the number of total frequency ( $N$ ), frequency within the respective quartile class ( $f_w$ ), sum of frequency of classes with lesser values than quartile class as well as lower boundary ( $f_b$ ).

### 2.3 Question 3: Coordinate Geometry

The question consisted of parts (a) and (b). In part (a), the candidates were informed that "The straight line  $y = x - 6$  cuts the curve  $y^2 = 8x$  at the points P and Q". Then, they were instructed to use graphical method to determine the coordinates of P and Q and calculate the length of PQ in the form  $a\sqrt{b}$ . In part (b), the candidates were required to find the acute angle between lines  $y = x + 2$  and  $3x - 4y + 4 = 0$ .

The question was attempted by 303 (92.1%) candidates. Their performance is summarized in Figure 3.



**Figure 3:** *The candidates' performance on question 3*

About 166 (54.8%) candidates scored from 2.0 to 6.0 marks. Therefore, the overall candidates' performance in this question was average.

The candidates who responded to part (a) correctly were able to express  $y^2 = 8x$  into  $y = \pm\sqrt{8x}$  and constructed tables of values for  $y = x - 6$  and  $y = \pm\sqrt{8x}$ . Then, they drew the graphs of  $y = x - 6$  and  $y^2 = 8x$  on the same  $xy$  plane (see Extract 3.1) and read the coordinates of points

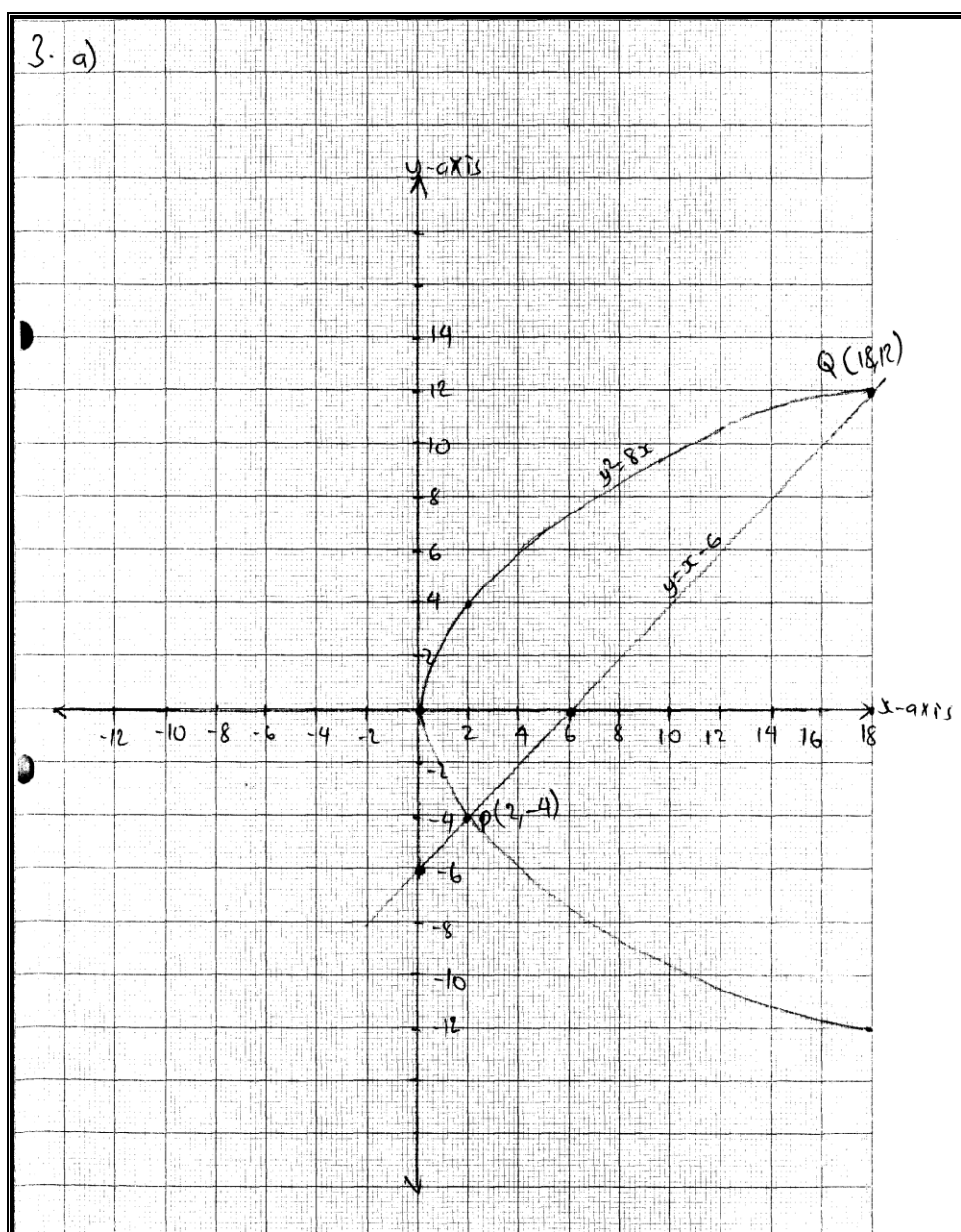
where two graphs met,  $P(2, -4)$  and  $Q(18, 12)$ . Finally, they used the formula for finding distance between two points

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  and the coordinates of P and Q to obtain  $\overline{PQ} = 16\sqrt{2}$  units. In part (b), the competent candidates correctly

determined the slope of  $y = x + 2$  and  $3x - 4y + 4 = 0$  as  $m_1 = 1$  and  $m_2 = \frac{3}{4}$  respectively. Then, they substituted the values into the formula for

finding an acute angle between two lines  $\theta = \tan^{-1}\left(\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|\right)$  and

computed to get  $\theta = 8.13^\circ$ .



**Extract 3.1:** A sample of correct graph for part (a) of question 3.

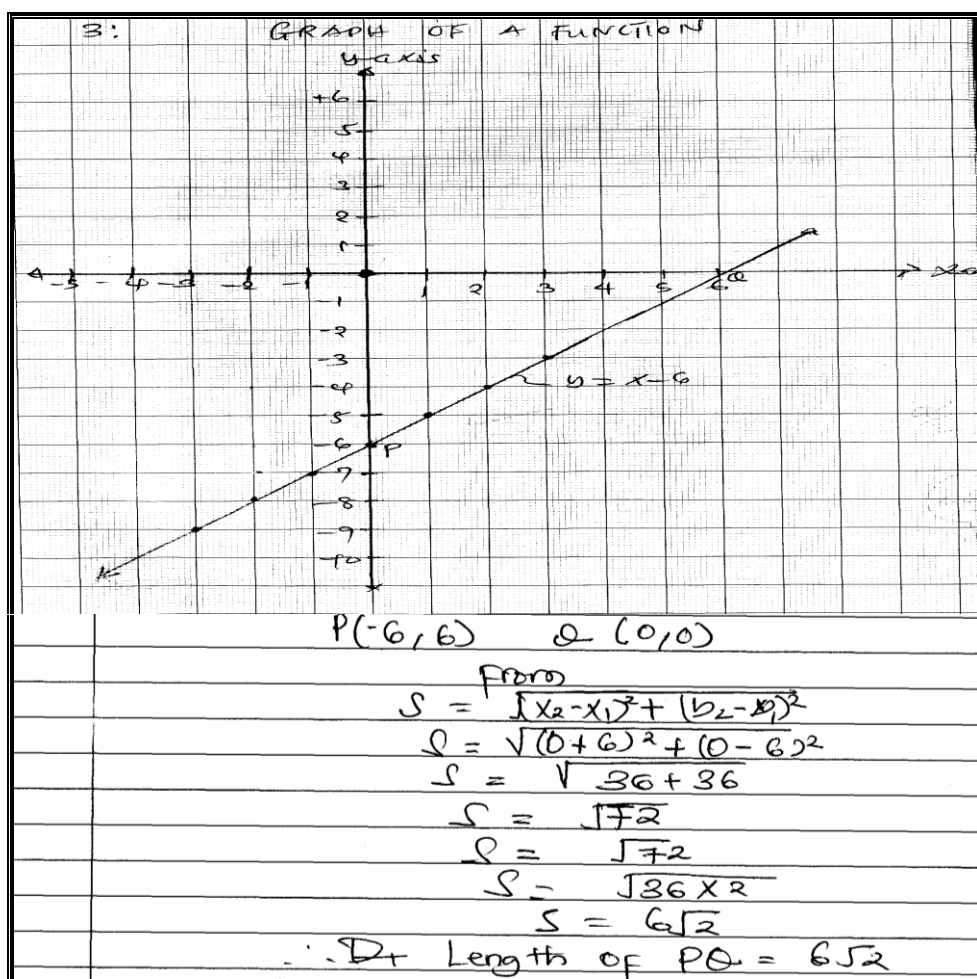
Extract 3.1 illustrates one of the correct graphs for  $y^2 = 8x$  and  $y = x - 6$  and the coordinates of P and Q presented by one of the candidates.

However, 137 (45.2%) candidates scored 1.5 marks or less. In part (a), most of these candidates computed the values of  $y^2$  instead of  $y$ . Such candidates could not express  $y^2 = 8x$  into  $y = \pm\sqrt{8x}$ ; therefore, they wrote  $y = 8x$ . As a result, they drew incorrect graphs, coordinates P and

Q as well as length  $\overline{PQ}$ . Also, some candidates only manages to obtain positive values (omitted negative values) of  $y$  after formulating inappropriate relation  $y = \sqrt{8x}$ . Other candidates solved algebraically  $y = x - 6$  and  $y^2 = 8x$ , which was contrary to the instructions. Moreover, there were candidates who incorrectly evaluated first derivative of  $y = x - 6$  and  $y^2 = 8x$  while others substituted the incorrect coordinates of P and Q into the formula for finding perpendicular distance from a line to a point,  $d = \frac{ax+by+c}{\sqrt{a^2+b^2}}$ . This indicates the failure to understand the requirements of the question.

In part (b), many candidates failed to recall the formula for finding the angle between two lines. The majority wrote  $\theta = \tan^{-1}\left(\frac{m_1 + m_2}{1 - m_1 m_2}\right)$  and got incorrect answer  $\theta = 81.8^\circ$ . Also, there were candidates who got  $\theta = 171.87^\circ$  (which is an obtuse angle) and could not change it into an acute angle. They were supposed to subtract  $171.87^\circ$  from  $180^\circ$  so as to obtain the acute angle  $\theta = 8.13^\circ$  as per requirements.

3	QD	soln						
		$y = x - 6$						
		$y^2 = 8x$						
		<table border="1"> <tr> <td>x</td><td>0</td><td>-6</td></tr> <tr> <td>y</td><td>0</td><td>6</td></tr> </table>	x	0	-6	y	0	6
x	0	-6						
y	0	6						
		$y^2 = 8x$						
		<table border="1"> <tr> <td>x</td><td>0</td><td>0</td></tr> <tr> <td>y</td><td>0</td><td>0</td></tr> </table>	x	0	0	y	0	0
x	0	0						
y	0	0						



**Extract 3.2:** A sample of incorrect solution for part (a) of question 3.

Extract 3.2 is a response from one of the candidates who failed to draw the graph of  $y^2 = 8x$  and interpreted the intercepts of the line  $y = x - 6$  wrongly as the coordinates P and Q; thus obtaining incorrect value of length  $\overline{PQ}$ .

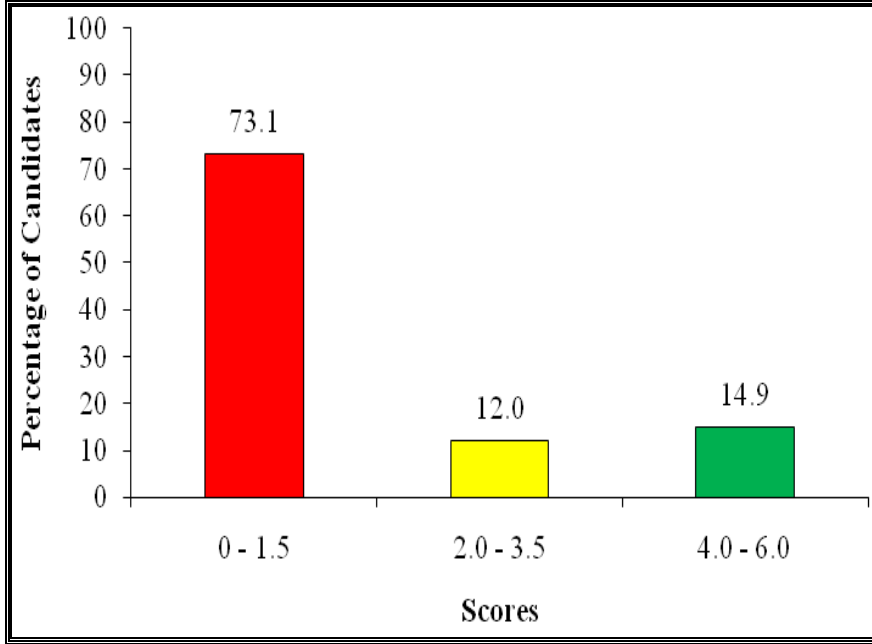
## 2.4 Question 4: Locus

The question stated that; the coordinates of points A and B are  $(-5, n)$  and  $(2, 4)$  respectively. If  $P(x, y)$  moves in such a way that  $PA : PB = 3 : 2$ , the locus traced out by P is given by the equation  $5x^2 + 5y^2 - 76x - 48y + 44 = 0$ . Find the value of n.

The data reveal that 249 (75.7%) candidates attempted this question whereby only 67 (26.9%) candidates got average or good scores. For this



case, the candidates' performance was weak. Figure 4 is a summary of candidates' performance on this question.



**Figure 4:** The candidates' performance on question 4

The analysis also reveals that among 182 (73.1%) candidates who scored low marks, 160 (64.3%) got zero. In part (a), several candidates interpreted " $PA:PB=3:2$ " wrongly (see Extract 4.1). Also, some candidates wrote incorrect relation  $PA=PB$  and used it to obtain incorrect answers  $n=4$ . Furthermore, there were candidates who failed to open the brackets correctly. Such candidates were only able to reach at  $4(PA)^2=9(PB)^2$  but thereafter they wrote  $4(x+5)^2+(y-n)^2=9(x-2)^2+(y-4)^2$  instead of  $4((x+5)^2+(y-n)^2)=9((x-2)^2+(y-4)^2)$ . Therefore, they got an incorrect equation  $5x^2-76x-(8-2n)y-48=0$  and consequently incorrect answer

$n=-20$ . Moreover, a number of candidates wrote  $\frac{\overline{PA}}{\overline{PB}}=\frac{3}{2}$  and substituted

$m=\frac{3}{2}$ ,  $(x_1, y_1)=(-5, n)$  and  $(x_2, y_2)=(2, 4)$  into the formula for finding

slope of a straight line  $m=\frac{y_1-y_2}{x_1-x_2}$  leading to an incorrect answer  $n=-13$ .

In addition, few candidates approached the question inappropriately by equating the centre of  $5x^2+5y^2-76x-48y+44=0$  and the midpoint of  $(-5, n)$  and  $(2, 4)$ . These candidates also did computational errors and

equated unequal points,  $\left(\frac{38}{5}, \frac{-24}{5}\right) = \left(\frac{6-10}{5}, \frac{12+2n}{5}\right)$  that led to incorrect answer  $n = -18$ .

4.	$3PA : 2PB$ $3\sqrt{(x-x_1)^2 + (y-y_1)^2} = 2\sqrt{(x-x_2)^2 + (y-y_2)^2}$ $6((x-x_1)^2 + (y-y_1)^2) = 4((x-x_2)^2 + (y-y_2)^2)$ $6((x-5)^2 + (y-n)^2) = 4((x-2)^2 + (y-4)^2)$ $6(x^2 + 10x + 25 + y^2 - 2yn + n^2) = 4(x^2 - 4x + 4 + y^2 - 8y + 16)$ $6x^2 + 60x + 150 + 6y^2 - 12yn + 6n^2 = 4x^2 - 16x + 16 + 4y^2 - 32y + 64$ $2x^2 + 108x + 30 + 2y^2 - 12yn + 6n^2 = 0$ $\begin{cases} 5x^2 + 5y^2 - 76x - 48y + 44 = 0 \\ 2x^2 + 2y^2 + 108x - 12yn + 6n^2 + 30 = 0 \end{cases}$ <p style="text-align: right;">By Comparison</p> $\frac{-48y}{-12y} = \frac{-12yn}{-12y}$ $n = 4$
----	--

**Extract 4.1:** A sample of incorrect solution for question 4.

Extract 4.1 shows a solution of the candidate who wrongly interpreted  $PA:PB = 3:2$  as  $3\overline{PA} = 2\overline{PB}$  instead of  $3\overline{PB} = 2\overline{PA}$ .

Despite candidates' weak performance, 14 (5.6%) candidates responded to the question correctly. These candidates expressed " $PA:PB = 3:2$ " into  $3\overline{PB} = 2\overline{PA}$  and used the formula for finding the distance between two points  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  and the points  $P(x, y)$ ,  $A(-5, n)$  and  $B(2, 4)$  to get  $5x^2 + 5y^2 - 76x + (8n - 72)y + 180 - 4n^2 = 0$ . Then, they equated it to  $5x^2 + 5y^2 - 76x - 48y + 44 = 0$  and got  $72 - 8n = 48$  and  $80 - 4n^2 = 44$  which were solved to obtain  $n = 3$ . Extract 4.2 is a sample response of one of the candidates who attempted the question correctly.

4	$A = (-5, n) \quad B = (2, 4)$
	$\frac{PA}{PB} = \frac{3}{2}$
	$3PB = 2PA$
	$3\sqrt{(x-2)^2 + (y-4)^2} = 2\sqrt{(x+5)^2 + (y-n)^2}$
	$3\sqrt{x^2 - 4x + 4 + y^2 - 8y + 16} = 2\sqrt{x^2 + 10x + 25 + y^2 - 2ny + n^2}$
	$(3\sqrt{x^2 + y^2 - 4x - 8y + 20})^2 = (2\sqrt{x^2 + y^2 + 10x - 2ny + 25 + n^2})^2$
	$9(x^2 + y^2 - 4x - 8y + 20) = 4(x^2 + y^2 + 10x - 2ny + 25 + n^2)$
	$9x^2 + 9y^2 - 36x - 72y + 180 = 4x^2 + 4y^2 + 40x - 8ny + 100 + 4n^2$
	$5x^2 + 5y^2 - 76x - 72y + 8ny + (180 - 100 - 4n^2) = 0$
	$5x^2 + 5y^2 - 76x - 72y + 8ny + 80 - 4n^2 = 0$
	On comparison with the obtained locus
	$80 - 4n^2 = 44$
	$4n^2 = 80 - 44$
	$4n^2 = 36$
	$n^2 = \frac{36}{4}$
	$n^2 = 9$
	$\sqrt{n^2} = \sqrt{9}$
	$n = 3$
	$\therefore$ The value of $n$ is 3

**Extract 4.2:** A sample of correct solution for question 4.

Extract 4.2 is a response of the candidate who were able to determine the locus of a point equidistant from two fixed points.

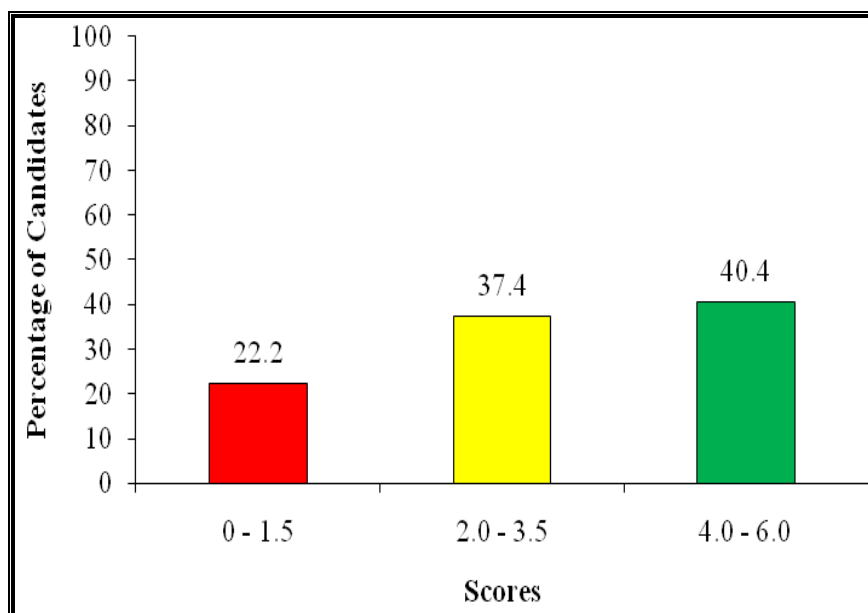
## 2.5 Question 5: Algebra

The question had parts (a) and (b). The candidates were instructed to:

- (a) solve the pair of simultaneous equations  $\frac{5}{x} - \frac{3}{y} = \frac{7}{2}$  and  $\frac{2}{x} + \frac{1}{y} = \frac{5}{2}$  by using elimination method.

- (b) (i) find the value of  $h$  if the algebraic expression  $5x^2 + hx + 5$  is a perfect square.
- (ii) use factorization method to solve the equation  $5x^2 + hx + 5 = 0$  using the results obtained in part (i).

This question was attempted by 324 (98.5%) candidates. The performance of candidates on this question was good because 252 (77.8%) candidates scored from 2.0 to 6.0 marks. Figure 5 shows percentage of candidates who obtained low, average and high scores.



**Figure 5:** The candidates' performance on question 5

Candidates who responded correctly to part (a) were able to perform appropriate operations to eliminate  $x$  and  $y$ . Majority of these candidates

let  $\frac{1}{x} = a$  and  $\frac{1}{y} = b$ , hence changed the given equations into  $5a - 3b = \frac{7}{2}$  and  $2a + b = \frac{5}{2}$ . Then, they solved the equations simultaneously by

elimination method to get  $a = 1$  and  $b = \frac{1}{2}$  and consequently  $x = 1$  and

$y = 2$ . Also, some candidates multiplied the given equations by  $xy$  to get

$5y - 3x = \frac{7}{2}xy$  and  $2y + x = \frac{5}{2}xy$ . Thereafter, they eliminated either  $x$  or

$y$  by performing both subtraction (or addition) and division to get  $x = 1$  and  $y = 2$ . Other candidates answered the question by eliminating  $x$  and

y directly from the given equations, as shown in Extract 5.1. In part (b) (i), the competent candidates substituted  $a=5$ ,  $b=h$  and  $c=5$  into the condition for perfect square  $b^2=4ac$  to get the equation  $h^2=100$  and solved it to get  $h=\pm 10$ . In part (b) (ii), the candidates formulated the equations  $5x^2+10x+5=0$  for  $h=10$  and  $5x^2-10x+5=0$  for  $h=-10$  and factorized them to get  $(5x+5)(x+1)=0$  and  $(5x-5)(x-1)=0$  respectively. Therefore, they got  $x=-1$  and  $x=1$ .

5	a)	$\begin{cases} \frac{5}{x} - \frac{3}{y} = \frac{7}{2} \\ \frac{2}{x} + \frac{1}{y} = \frac{5}{2} \end{cases} \quad \begin{array}{l} \times 1 \\ \times 3 \end{array}$
	(f)	$\begin{cases} \frac{5}{x} - \frac{3}{y} = \frac{7}{2} \\ \frac{6}{x} + \frac{2}{y} = \frac{15}{2} \end{cases}$ $\frac{11}{x} = \frac{11}{1}$ $\frac{11x}{11} = \frac{11}{11}$ $x = 1$
		$\begin{array}{l} 2/1 \left\{ \begin{array}{l} 5/x - 3/y = 7/2 \\ 2/x + 1/y = 5/2 \end{array} \right. \end{array}$
		$\begin{array}{l} (c) \left\{ \begin{array}{l} 10/x - 6/y = 14/2 \\ 10/x + 6/y = 25/2 \end{array} \right. \\ - \end{array}$ $\begin{array}{r} -6/y - 5/y = 7 - 25/2 \\ \frac{-6-5}{y} = \frac{14-25}{2} \\ \frac{-11}{y} = \frac{-11}{2} \\ \frac{-22}{-11} = \frac{-11 \cdot 2}{-11} \\ y = 2 \end{array}$

**Extract 5.1:** A sample of correct solution for part (a) of question 5.

Extract 5.1 shows a solution of the candidate who correctly eliminated the variables forming denominators of the terms.

On contrary, 72 (22.2%) candidates scored 1.5 marks or less. As illustrated in Extract 5.2, majority of these candidates used both elimination and substitution methods to solve the equations given in part (a). This was contrary to the instructions on the item. Also, some candidates confused fractions with negative powers. For instance, there were few candidates who

expressed the equations  $\frac{5}{x} - \frac{3}{y} = \frac{7}{2}$  and  $\frac{2}{x} + \frac{1}{y} = \frac{5}{2}$  wrongly as

$5^{-\frac{1}{x}} - 3^{-\frac{1}{y}} = 7^{-\frac{1}{2}}$  and  $2^{-\frac{1}{x}} + 1^{-\frac{1}{y}} = 5^{-\frac{1}{2}}$  respectively.

In part (b) (i), many candidates wrote  $h = \sqrt{100} = 10$  (ignoring  $h = -10$ ) resulting to one equation,  $5x^2 + 10x + 5 = 0$ . In addition, there were

candidates who used the condition for perfect square  $c = \left(\frac{b}{2a}\right)^2$

inappropriately. They did not rewrite the equation into  $x^2 + \frac{h}{5}x + 1 = 0$  so as

to have  $a = 1$  and consequently  $b = \frac{h}{5}$  and  $c = 1$ . Instead, they substituted

$a = 5$ ,  $b = h$  and  $c = 5$  into the formula and got incorrect answer  $h = 10\sqrt{5}$ .

In part (b) (ii), many candidates did not present the equation  $5x^2 - 10x + 5 = 0$  and its solution. Moreover, some candidates did not adhere to the instructions. They solved the obtained quadratic equation(s) using

general quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  instead of factorization

method (see Extract 5.2).

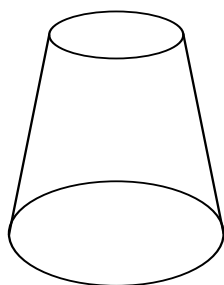
5.	a) soln:
	let $a = \frac{1}{x}$ and $b = \frac{1}{y}$ .
	$\frac{5}{x} - \frac{3}{y} = \frac{7}{2}$
	$5(\frac{1}{x}) - 3(\frac{1}{y}) = \frac{7}{2}$
	$5a - 3b = \frac{7}{2}$
	$10a - 6b = 7$ ----- i)
	$\frac{2}{x} + \frac{1}{y} = \frac{5}{2}$
	$2(\frac{1}{x}) + (\frac{1}{y}) = \frac{5}{2}$
	$2a + b = \frac{5}{2}$
	$4a + 2b = 5$ ----- ii)
	Solving eqn i) and ii) simultaneously
	$\begin{array}{r} 10a - 6b = 7 \\ 4a + 2b = 5 \end{array}$
	Eliminating b.
	$\begin{array}{r} 1 \mid 10a - 6b = 7 \\ 3 \mid 4a + 2b = 5 \end{array}$
	$\begin{array}{r} + \mid 10a - 6b = 7 \\ \quad 12a + 6b = 15 \\ \hline 22a + 0 = 22 \end{array}$
	$22a = 22$
5	a) $a = \frac{22}{22}$
	$a = 1$
	Substituting the value of "a" into eqn ii,
	$4a + 2b = 5$
	$4(1) + 2b = 5$
	$4 + 2b = 5$
	$2b = 5 - 4$
	$2b = 1$
	$b = \frac{1}{2}$
	But $a = \frac{1}{x}$
	$x = \frac{1}{a}$
	$x = \frac{1}{1}$
	$x = 1$
	also $b = \frac{1}{y}$
	$y = \frac{1}{b}$
	$y = \frac{1}{\frac{1}{2}}$
	$y = 2$
	$\therefore$ The value of $x = 1$ and $y = 2$ .

**Extract 5.2:** A sample of incorrect response for part (a) of question 5.

As Extract 5.2 shows, the candidate used elimination method when finding the value of  $a$  and substitution method when finding the value of  $b$ .

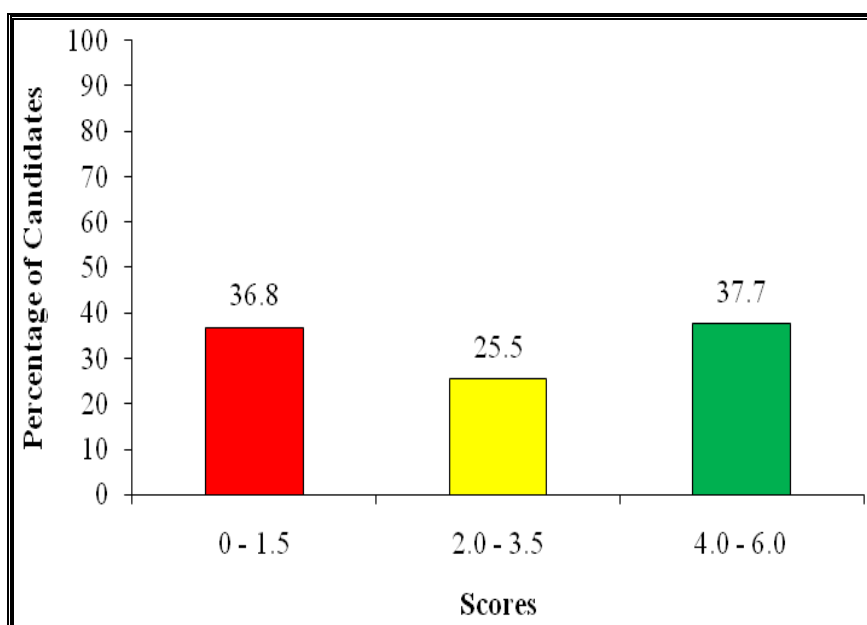
## 2.6 Question 6: Plan and Elevations; and Geometrical Constructions

The question had parts (a) and (b). In part (a), the candidates were required to draw the plan, front and side elevations of the following cone.



In part (b), the candidates were informed that "one interior angle of an octagon is  $100^\circ$  and the remaining angles are of the same size". They were required to find the value of each of the remaining interior angles.

Out of 302 (91.8%) candidates who attempted this question, the marks of 191 (63.2%) candidates ranged from 2.0 to 6.0. Therefore, the general performance of the candidates was average. The performance of the candidates in this question is shown in Figure 6.

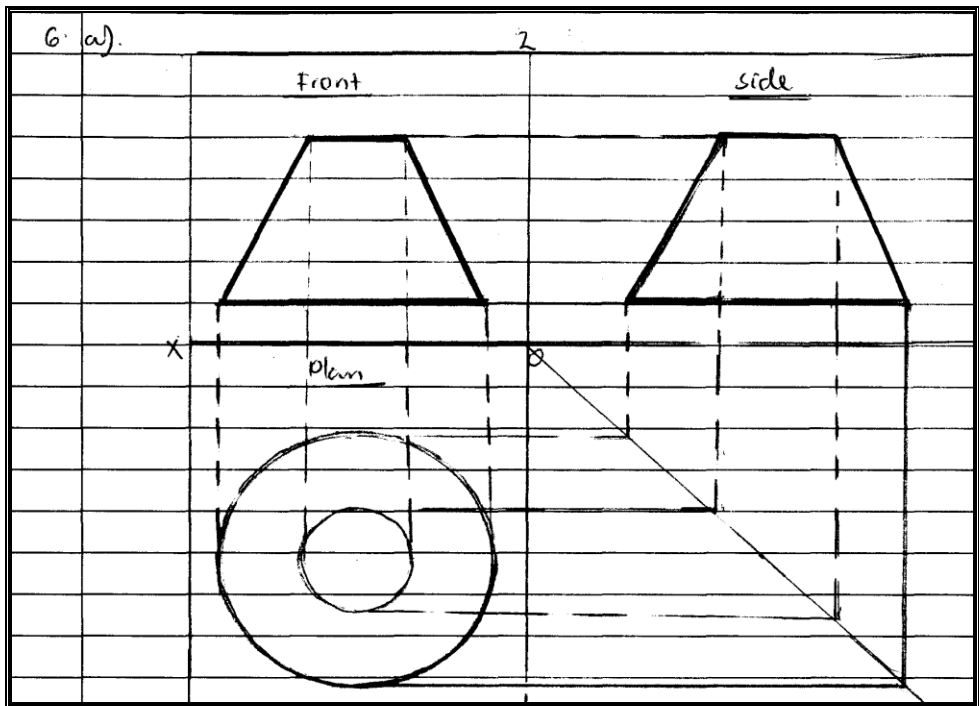


**Figure 6:** *The candidates' performance on question 6.*

The analysis also reveals that 59 (19.5%) candidates responded to the question correctly scoring all 6 marks. In part (a), the candidates correctly drew the plan, front and side elevations, as shown in Extract 6.1. In part (b), the candidates realized that the octagon has eight sides, therefore, they substituted  $n = 8$  into the formula for finding the sum of interior angles of



the polygon  $S = (n - 2) \times 180^\circ$  and computed to get  $S = 1080^\circ$ . Then, they subtracted  $100^\circ$  from  $1080^\circ$  and divided the difference ( $980^\circ$ ) by 7 to get  $140^\circ$ , which is the degree measure of each remaining interior angle.



**Extract 6.1:** A sample of correct response for part (a) of question 6

Extract 6.1 shows a response of the candidate who was competent in identifying plan, front and side elevations of the cone as well as in drawing skills.

Nevertheless, 111 (36.8%) candidates scored 1.5 or less whereas 63 (20.9%) got zero. In part (a), many candidates drew rectangle and two ovals implying that they failed to identify the plan, front and side elevations correctly. In part (b), some candidates interpreted the word "octagon" wrongly. They mistook it for the polygon with ten sides (decagon). Such candidates substituted  $n = 10$  into the formula  $S = (n - 2) \times 180^\circ$  and got incorrect answer for the sum of interior angles of octagon,  $1440^\circ$  instead of  $1080^\circ$ . As a result, they also obtained incorrect answer for interior angle particularly  $144^\circ$ . It was also noted that some candidates substituted  $n = 8$  into the formula  $Interior\ angle = \left( \frac{n - 2}{n} \right) \times 180^\circ$  and got  $225^\circ$ . Then, they subtracted  $100^\circ$  from  $225^\circ$  ending up with incorrect answer

*Interior angle* =  $125^\circ$ . These candidates failed to formulate an equation that interprets the given information correctly. In addition, there were candidates who applied inappropriate formula for calculating the degree measure of an interior angle (see Extract 6.2).

6.	b) $(\text{Exterior}) + (\text{Interior}) = 180^\circ$
	let $x$ be Exterior angle
	$x + 100^\circ = 180^\circ$
	$x = 180^\circ - 100^\circ$
	$x = 80^\circ$
	$\therefore$ The value of the remaining interior angles is $80^\circ$

**Extract 6.2:** A sample of incorrect solution for part (b) of question 6.

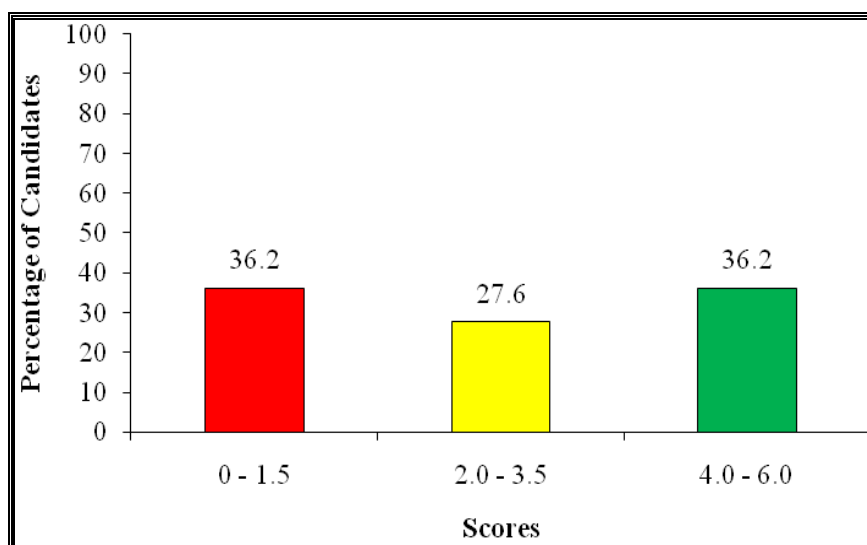
In Extract 6.2, the candidate applied incorrect formula *Interior angle* + *Exterior angle* =  $180^\circ$ .

## 2.7 Question 7: Trigonometry

In this question, the candidates were required to:

- find  $\tan x$  in terms of  $\alpha$  and  $\beta$  if  $\sin(x - \alpha) = \cos(x + \beta)$ .
- solve the equation  $3\cos 2\theta - \sin \theta + 2 = 0$  for values of  $\theta$  from  $0^\circ$  to  $360^\circ$  inclusive.

Figure 7 illustrates the performance of 276 (83.9%) candidates who attempted this question.



**Figure 7:** The candidates' performance on question 7

The analysis reveals that 176 (63.8%) candidates obtained from 2.0 to 6.0 marks. This implies that the candidates performed averagely in this question.

The analysis further showed that 33 (12.0%) candidates responded to the question correctly scoring all 6 marks. In part (a), the candidates expanded  $\sin(x-\alpha)$  and  $\cos(x+\beta)$  in  $\sin(x-\alpha)=\cos(x+\beta)$  using the compound angle formulae and got  $\sin x \cos \alpha - \cos x \sin \alpha = \cos x \cos \beta - \sin x \sin \beta$ .

Thereafter, they manoeuvred it to get  $\frac{\sin x}{\cos x} = \frac{\sin \alpha + \cos \beta}{\cos \alpha + \sin \beta}$  and

consequently  $\tan x = \frac{\sin \alpha + \cos \beta}{\cos \alpha + \sin \beta}$ . In part (b), the candidates replaced

$\cos 2\theta$  with  $1-2\sin^2 \theta$  in  $3\cos 2\theta - \sin \theta + 2 = 0$  resulting to  $6\sin^2 \theta + \sin \theta - 5 = 0$ . Then, they solved it correctly and obtained  $\theta = 56^\circ 26'$ ,  $123^\circ 34'$ ,  $270^\circ$ , as shown in Extract 7.1.

Handwritten solution for part (b):

$$\begin{aligned} \text{b) } & 3\cos 2\theta - \sin \theta + 2 = 0 \\ & \text{but } \cos 2\theta = 2\cos^2 \theta - \sin^2 \theta \\ & \quad = 1 - 2\sin^2 \theta \\ & \text{Then } 3(1 - 2\sin^2 \theta) - \sin \theta + 2 = 0 \\ & 3 - 6\sin^2 \theta - \sin \theta + 2 = 0 \\ & 6\sin^2 \theta + \sin \theta - 5 = 0 \\ & 6\sin^2 \theta + 6\sin \theta - 5\sin \theta - 5 = 0 \\ & 6\sin \theta (\sin \theta + 1) - 5(\sin \theta + 1) = 0 \\ & (6\sin \theta - 5)(\sin \theta + 1) = 0 \end{aligned}$$

$$\begin{aligned}
 7.1(b) \quad 6 \sin \theta - 5 &= 0 & \text{or} & \quad \sin \theta + 1 = 0 \\
 \sin \theta &= \frac{5}{6} & \text{or} & \quad \sin \theta = -1 \\
 \theta &= \sin^{-1}(0.8333) & \text{or} & \quad \theta = \sin^{-1}(-1) \\
 &= 56^{\circ} 26' & \text{or} & \quad \theta = 270^{\circ} \\
 \text{In the 2nd quadrant.} \\
 \theta &= 180^{\circ} - 56^{\circ} 26', \\
 &= 123^{\circ} 34' \\
 \text{Hence the required values of } \theta &\text{ are } 56^{\circ} 26', 123^{\circ} 34' \text{ and } 270^{\circ}.
 \end{aligned}$$

**Extract 7.1:** A sample of correct solution for part (b) of question 7.

Extract 7.1 shows a solution of the candidate who correctly solved the given equation by applying appropriate trigonometric identity and reading the trigonometric inverses of sine correctly.

Conversely, 100 (36.2%) candidates scored 1.5 marks or less. In part (a), most of these candidates could not recall the compound angle formulae for either  $\sin(x - \alpha)$  or  $\cos(x + \beta)$ . For instance, some candidates expanded  $\sin(x - \beta)$  and  $\cos(x + \alpha)$  as  $\sin x \cos \alpha + \cos x \sin \alpha$  and  $\cos x \cos \beta + \sin x \sin \beta$  respectively. Therefore, they got incorrect answer

$$\tan x = \frac{\cos \alpha - \sin \beta}{\cos \beta - \sin \alpha} \text{ instead of } \tan x = \frac{\sin \alpha + \cos \beta}{\cos \alpha + \sin \beta}.$$

In part (b), the majority could not recall the identity  $\cos 2\theta = 1 - 2\sin^2 \theta$ ; therefore, they failed to eliminate  $\cos 2\theta$  or  $\cos \theta$  from the given equation. Instead, they expressed  $\cos 2\theta$  in terms of  $\sin \theta$  ending up with irrelevant work such as  $\cos 2\theta = \frac{\sin \theta - 2}{3}$ . Moreover, a number of candidates replaced  $\cos 2\theta$  in  $3\cos 2\theta - \sin \theta + 2 = 0$  with incorrect expression  $\cos^2 \theta + \sin^2 \theta$ , as seen in Extract 7.2.

$$\begin{aligned}
 076. \quad & 3 \cos 2\theta - \sin \theta + 2 = 0. \\
 & \text{but } \cos 2\theta = \cos(\theta + \theta) \\
 & \quad = \cos^2 \theta + \sin^2 \theta. \\
 & 3 \cos^2 \theta + 3 \sin^2 \theta - \sin^2 \theta + 2 = 0. \\
 & \text{but } \cos^2 \theta = 1 - \sin^2 \theta. \\
 & 3(1 - \sin^2 \theta) + 3 \sin^2 \theta - \sin^2 \theta + 2 = 0. \\
 & 3 - 3 \sin^2 \theta + 3 \sin^2 \theta - \sin^2 \theta + 2 = 0. \\
 & 3 - \sin^2 \theta + 2 = 0. \\
 & 5 - \sin^2 \theta = 0. \\
 & 0.5 = \sin^2 \theta. \\
 & \theta = \sin^{-1}(\sqrt{0.5}) \\
 & \theta = 30^\circ. \\
 & \text{Considering the four quadrants;} \\
 & \text{The 1st quadrant: } \theta = 30^\circ. \\
 & \text{2nd quadrant: } 180^\circ - \theta = 30^\circ. \\
 & \theta = 180^\circ + 30^\circ.
 \end{aligned}$$

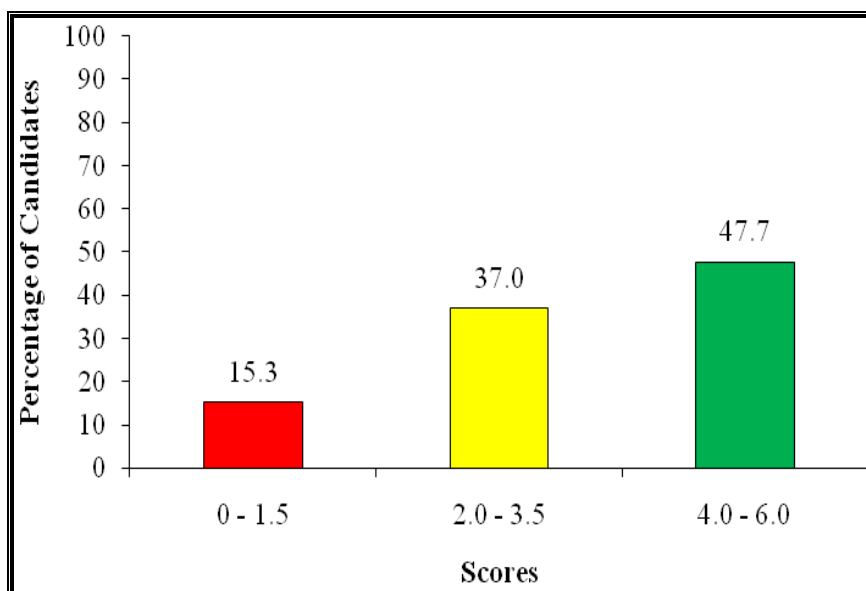
**Extract 7.2:** A sample of incorrect solution for part (b) of question 7.

As shown in Extract 7.2, the candidate failed to recall the double angle formula, hence ended up with incorrect answer.

## 2.8 Question 8: Numbers

The question comprised parts (a) and (b). In part (a), the candidates were required to use the divisibility rule to show that 35120 is divisible by 5. Part (b) read "The sum of the squares of the first  $n$  numbers is given by  $\frac{n(n+1)(2n+1)}{6}$ . Find the sum of the first three squares when  $n$  is a natural number.

The question was attempted by 308 (93.6%) candidates, whereby 262 (84.7%) candidates scored between 2.0 and 6.0 marks. This performance was good. Figure 8 shows the candidates' performance on this question.



**Figure 8:** The candidates' performance on question 8

The analysis showed that 5 (1.6%) candidates scored all allotted marks to this question. In part (a), the candidates stated correctly that the number 35120 is divisible by 5 since its last digit is zero. In part (b), the candidates substituted  $n = 3$  into  $\frac{n(n+1)(2n+1)}{6}$  and computed to get the sum of the first three squares of natural number, as shown in Extract 8.1.

8.	b) soln
	Given
	$S_n = \frac{n(n+1)(2n+1)}{6}$
	$n = 3$
	$S_3 = \frac{3(3+1)(2(3)+1)}{6}$
	$S_3 = \frac{3(4)(6+1)}{6}$
	$S_3 = \frac{3(4)(7)}{6}$
	$S_3 = 2(7)$
	$S_3 = 14$
	$\therefore$ The sum is 14

**Extract 8.1:** A sample of correct solution for part (b) of question 8.

In Extract 8.1, the candidate used correctly the given expression to find the sum of squares of the first 3 natural numbers.

On the other hand, the analysis showed that 47 (15.3%) candidates scored low marks. In part (a), some candidates could not state the divisibility rule correctly. For instance, there were candidates who stated that the whole number is divisible by 5 if and only if it gives remainder 0. Therefore, they divided 35120 by 5 using long division to get 7024 and commented that 35120 is divisible by 5. This indicates that they used the general meaning of divisibility of a number rather than divisibility rule. Also, there were few candidates who stated that a number is divisible by 5 if it is even and its last digit is zero. These candidates could not differentiate the divisibility rules of 2 and 5.

In part (b), many candidates interpreted the given expression wrongly.

Some candidates substituted  $n=1$ ,  $n=2$  and  $n=3$  into  $\frac{n(n+1)(2n+1)}{6}$

to get 1, 5 and 14 respectively. Then, they summed up 1, 5 and 14 and got 20. These candidates assumed that the given expression gives  $n$ th term while it was for sum of first  $n$  terms. It was also noted that there were few candidates who substituted  $n=4$ ,  $n=9$  and  $n=16$  in the formula  $\frac{n(n+1)(2n+1)}{6}$  and got incorrect answer 1711 (see Extract 8.2).

08 b) soln.

$$\frac{n(n+1)(2n+1)}{6}$$

$$n=1 = \frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6}$$

$$1^{st} = 6/6 = 1.$$

$$n=2 = \frac{2(2+1)(2(2)+1)}{6} = \frac{2(3)(5)}{6}$$

08 b)  $2^{nd} = 30$

$$\frac{6}{6} = 5.$$

$$n=3 = \frac{3(3+1)(2(3)+1)}{6}$$

$$3^{rd} = \frac{3(4)(7)}{6}$$

$$3^{rd} = \frac{84}{6}$$

$$3^{rd} = 14.$$

$$Sum = 1 + 5 + 14.$$

$$= 20.$$

$\therefore$  Sum of the first three squares = 20.

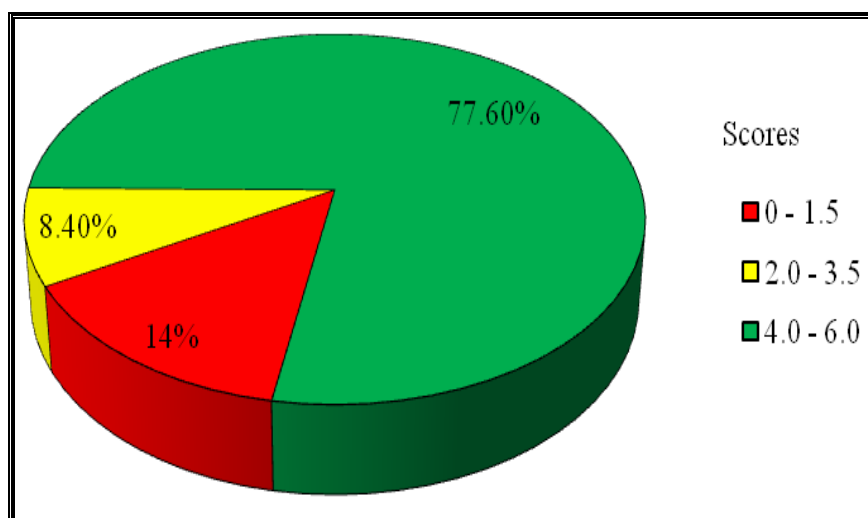
**Extract 8.2:** A sample of incorrect solution for part (b) of question 8.

Extract 8.2 shows, the response of the candidate who responded to the item wrongly by adding the first number, sum of first two numbers and the sum of first three numbers.

## 2.9 Question 9: Logic

This question comprised two parts from which the candidates were required to (a) use truth table to verify that  $(p \rightarrow q) \wedge (q \rightarrow p)$  is equivalent to  $p \leftrightarrow q$  and (b) simplify  $(p \vee q) \wedge \sim p$  by using the laws of algebra of propositions.

This question was attempted by 308 (93.6%) candidates, out of which, 265 (86.0%) scored marks ranging from 2.0 to 6.0. Therefore, the general performance by candidates in this question was good as Figure 9 shows.



**Figure 9:** The candidates' performance on question 9

As Figure 9 shows, the marks of 239 (77.6%) candidates ranged from 4.0 to 6.0. Amongst, 18 (5.8%) candidates responded correctly to the question scoring all 6 marks. In part (a), the candidates constructed a truth table with columns  $p$ ,  $q$ ,  $p \rightarrow q$ ,  $q \rightarrow p$ ,  $(p \rightarrow q) \wedge (q \rightarrow p)$  and  $p \leftrightarrow q$  correctly, as shown in Extract 9.1. Then, they observed the truth values in columns  $(p \rightarrow q) \wedge (q \rightarrow p)$  and  $p \leftrightarrow q$ ; and commented that they are equivalent. In part (b), the candidates used appropriately the laws to write  $(p \vee q) \wedge \sim p$  into its simplest form  $\sim p \wedge q$  (see Extract 9.2)



9	a)	$(p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$					
		truth table					
		P	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
		T	T	T	T	T	T
		T	F	F	T	F	F
		F	T	T	F	F	F
		F	F	T	T	T	T
		Since, the column of $(p \rightarrow q) \wedge (q \rightarrow p)$ and $p \leftrightarrow q$ are equal, therefore $(p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$					
		hence proved.					

**Extract 9.1:** A sample of correct response for part (a) of question 9.

Extract 9.1 shows a response of the candidate who was competent in constructing the truth table and using it to judge the equivalence of propositions.

9	b)	$(p \vee q) \wedge \sim p$	
		$\sim p \wedge (q \vee p)$	commutative law
		$(\sim p \wedge p) \vee (\sim p \wedge q)$	distributive law
		$F \vee (\sim p \wedge q)$	complement law
		$\sim p \wedge q$	identity law
		$\therefore (p \vee q) \wedge \sim p \equiv \sim p \wedge q$	

**Extract 9.2:** A sample of correct response for part (b) of question 9.

Extract 9.2 is a response of the candidate who was competent in applying laws to simplify the given proposition.

Meanwhile, 7 (2.3%) candidates scored the marks ranging from 0.5 to 1.5 and 36 (11.7%) candidates scored zero. In part (a), some candidates drew truth table with eight rows instead of four rows. Additionally, a number of candidates failed to perform logical operations of either implication, conjunction or double implication correctly. In part (b), many candidates could not state the appropriate names of the laws particularly complement and identity laws. Also, some candidates applied the laws incorrectly while other candidates constructed the truth table.

9.	$\Rightarrow (p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$					
	$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$A \wedge B$	$p \leftrightarrow q$
	T	T	T	T	T	T
	T	F	F	T	F	F
	F	T	T	F	F	F
	F	F	T	F	F	F
$\therefore (p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$						

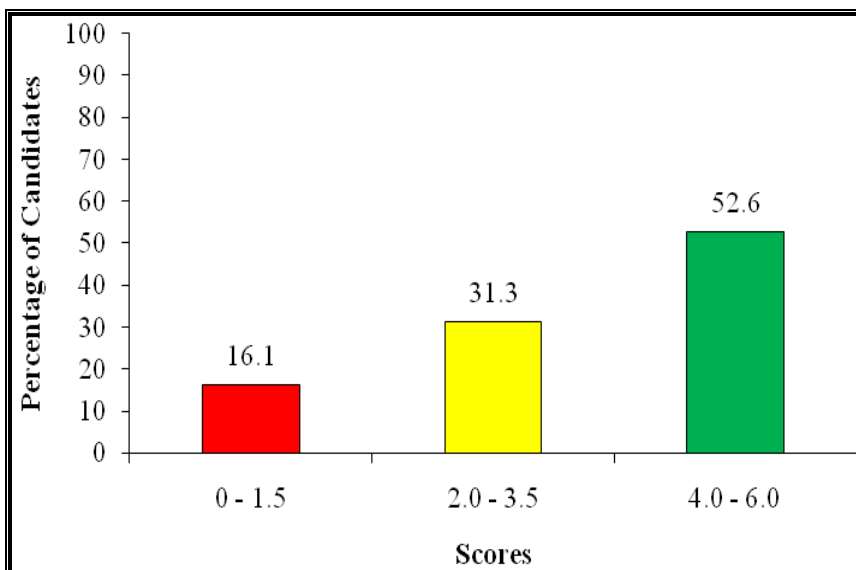
**Extract 9.3:** A sample of incorrect response for part (a) of question 9.

As Extract 9.3 shows, the candidate wrote incorrect truth value of  $F \rightarrow F$  as F in fourth row of column  $p \rightarrow q$  as well as incorrect truth value of  $F \leftrightarrow F$  as F in fourth row of column  $p \leftrightarrow q$ .

## 2.10 Question 10: Sets

The question had parts (a) and (b). In part (a), the candidates were required to simplify  $(A \cap B') \cup (A \cup B)'$  by using basic properties of set operations. In part (b), the candidates were required to find  $n(A \cap B)'$  by using a Venn diagram given that  $A$  and  $B$  are two sets such that  $n(A) = 42$ ,  $n(B) = 27$  and  $n(A \cup B) = 59$ .

The question was attempted by 323 (98.2%) candidates whereby 271 (83.9%) candidates scored marks ranging from 2.0 to 6.0. The overall performance of candidates in this question was good. Figure 10 shows the candidates' performance.



**Figure 10:** The candidates' performance on question 10

The data shows that 58 (18.0%) candidates scored all six (6) marks. In part (a), the candidates applied De Morgan, Distributive, Compliment and Identity properties to simplify  $(A \cap B') \cup (A \cup B)'$  into  $B'$ . In part (b), the candidates drew Venn diagram and wrote the correct data in the respective regions and worked on the data to get a correct answer  $n(A \cap B)' = 49$ .

10	a)	$(A \cap B') \cup (A \cup B)'$	-----	Given
		$= (A \cap B') \cup (A' \cap B')$	-----	De Morgan property
		$= (A \cup A') \cap B'$	-----	Distributive property
		$= U \cap B'$	-----	Compliments property
		$= B'$	-----	Identity property

**Extract 10.1:** A sample of the correct response for part (a) of question 10.

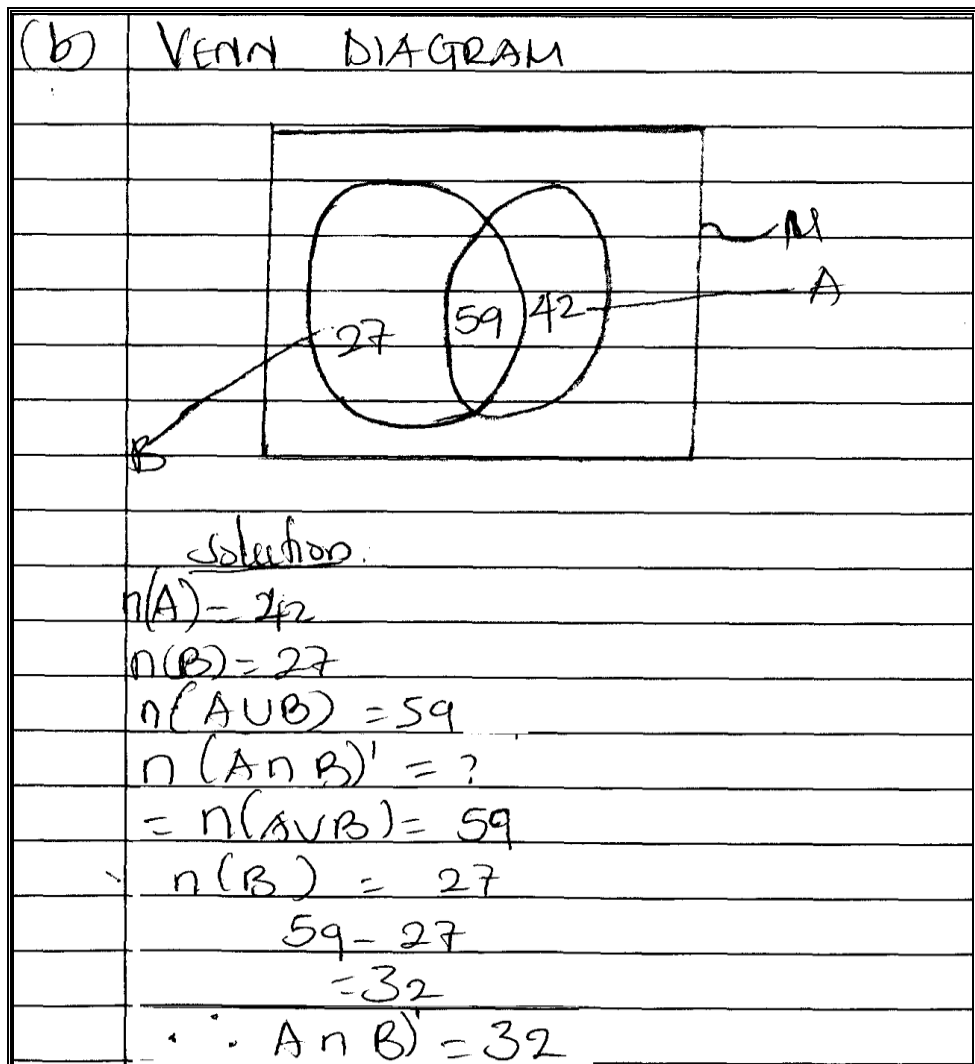
In Extract 10.1, the candidate applied the set properties and laws correctly to get the simplest expression of  $(A \cap B') \cup (A \cup B)'$ .

10	b)		
		$42 - x + x + 27 - x = 59,$ $69 - x = 59,$ $-x = 59 - 69,$ $-x = -10$ $x = 10,$ $n(A \cap B) = 10,$ $n(A \cap B)' = n(A' \cup B'),$ $(A') = n(B) = 17.$ $17 + 32 = 49$ $n(A \cap B)' = 49,$	

**Extract 10.2:** A sample of the correct response for part (b) of question 10.

Extract 10.2 shows a solution of the candidate who correctly performed union and intersection of two sets and their compliments.

On the other hand, 52 (16.1%) candidates scored 1.5 marks or less. In part (a), some candidates could not state the names of laws correctly. Also, there were candidates who applied the laws inappropriately. In part (b), some candidates used formula  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$  instead of Venn diagram. Also, there were candidates who failed to identify the regions representing  $A \cup B$ . Therefore, they used incorrect data for other regions of the Venn diagram and consequently obtained incorrect value of  $n(A \cap B)'$  (see Extract 10.3).



**Extract 10.3:** A sample of incorrect response for part (b) of question 10.

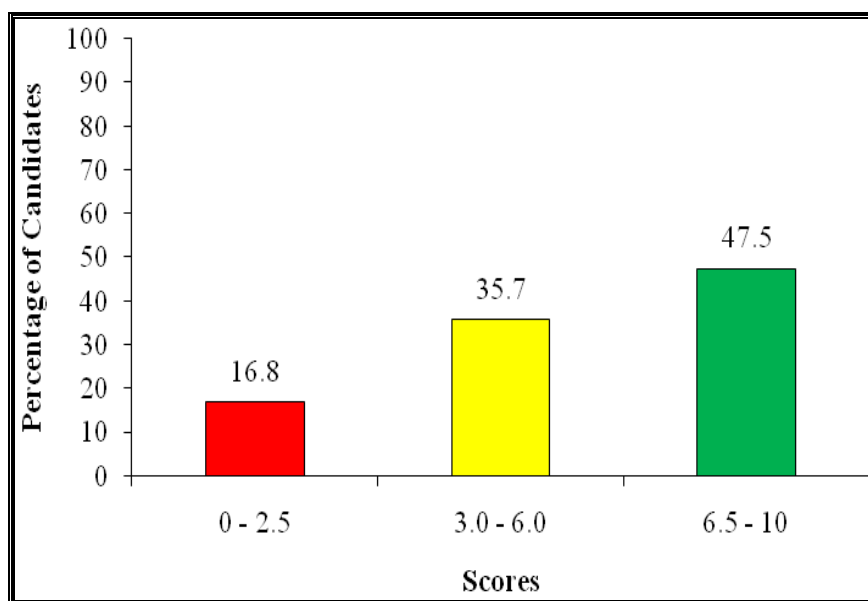
As Extract 10.3 shows, the candidates failed to identify the regions for set  $A \cup B$  as he/she indicated  $n(A \cup B)$  in the region representing  $A \cap B$ .

## 2.11 Question 11: Functions and Remainder Theorem

The question had parts (a), (b) and (c). The candidates were required to:

- (a) sketch the graph of  $g(x) = \frac{x+3}{2x-3}$ .
- (b) use the graph in part (a) to determine the domain and range of  $g(x)$ .
- (c) use the remainder theorem to compute the value of  $k$  given that when the function  $f(x) = 2x^4 + kx^3 - 11x^2 + 4x + 12$  is divided by  $x - 3$ , the remainder is 60.

This question was attempted by 322 (97.9%) candidates. The analysis reveals that the marks of 268 (83.2%) candidates ranged from 3.0 to 10.0. Therefore, the candidates' performance on this question was generally good. Figure 11 shows candidates' performance on this question.

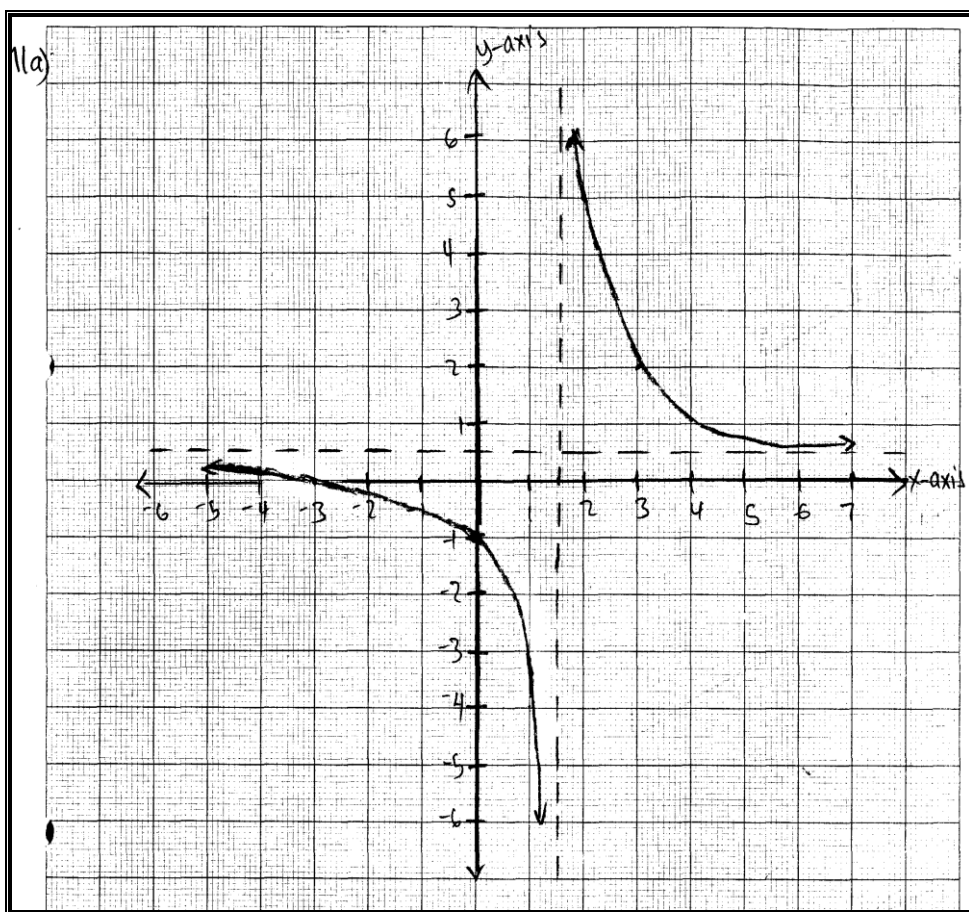


**Figure 11:** *The candidates' performance on question 11*

The data shows that 39 (12.1%) candidates responded to the question correctly scoring all 10 marks allotted to the question. In part (a), the candidates determined vertical asymptote, horizontal asymptote,  $x$  - intercept and  $y$  - intercept which enabled them to sketch a correct graph, as shown in Extract 11.1. Then, they carefully observed the graph and responded to part (b) correctly by writing  $Domain = \left\{ x \in \mathbb{R} : x \neq \frac{3}{2} \right\}$  and

$\text{Range} = \left\{ y \in \mathbb{R} : y \neq \frac{1}{2} \right\}$ . In part (c), the candidates wrote  $f(3) = 60$  and therefore, they replaced  $x$  with 3 in  $2x^4 + kx^3 - 11x^2 + 4x + 12 = 60$  to formulate the equation  $27k + 87 = 60$  and solved it to get  $k = -1$  (see Extract 11.1).

11.	a)	$g(x) = \frac{x+3}{2x-3}$
		V. A
		$2x - 3 = 0$
		$\frac{2x}{2} = \frac{3}{2}$
		V. A = $\frac{3}{2}$
		H. A
		$g(x) = \frac{\frac{x}{x} + \frac{3}{x}}{\frac{2x}{x} - \frac{3}{x}}$
		$= \frac{1}{2}$
		H. A = $\frac{1}{2}$
		Y-intercept
		$y = \frac{0+3}{0-3}$
		$y = -1$
		Y-intercept = $-1$
		X-intercept
		$0 = \frac{x+3}{2x-3}$
		$x+3 = 0$
		$x = -3$
		X-intercept = $-3$



**Extract 11.1:** A sample of correct response for part (a) of question 11.

Extract 11.1 shows the solution of the candidate who was knowledgeable on the features of the given rational function and correctly described them on graph.

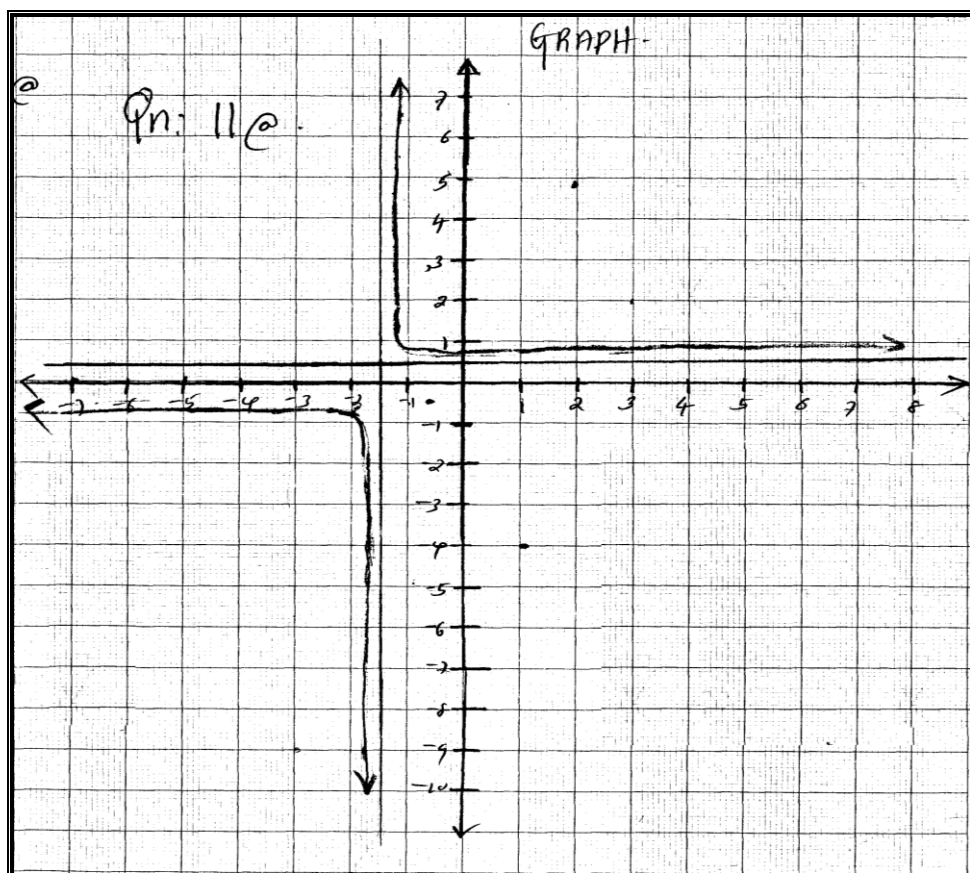
However, 54 (16.8%) candidates scored low marks and among them, 10 (3.1%) got zero. In part (a), some candidates could not find intercepts or asymptotes while other failed to get their correct values. For instance, there

were candidates who wrote  $y = \lim_{x \rightarrow 0} \left( \frac{x+3}{2x-3} \right)$  and obtained the horizontal

asymptote  $y = -1$ . This means that the candidates lacked understanding on the concept of limits especially for rational functions. In addition, some candidates formulated inequality to find vertical asymptotes. They wrote

$2x - 3 \geq 0$  and computed to get  $x \geq \frac{3}{2}$  which was incorrect answer.

In part (b), majority of the candidates stated incorrect domain and range whereby the statements  $\text{Domain} = \{\text{All real numbers}\}$  and  $\text{Range} = \{\text{All real numbers}\}$  were frequently observed in candidates' responses. Also, some candidates wrote  $\text{Domain} = \left\{x : x \geq \frac{3}{2}\right\}$  and  $\text{Range} = \left\{y : y \geq \frac{1}{2}\right\}$  instead of  $\text{Domain} = \left\{x \in \mathbb{R} : x \neq \frac{3}{2}\right\}$  and  $\text{Range} = \left\{y \in \mathbb{R} : y \neq \frac{1}{2}\right\}$ . In part (c), a number of candidates made errors when solving the equation  $87 - 9k = 60$  as most of them got  $k = 3$  instead of  $k = -1$ .



**Extract 11.2:** A sample of incorrect graph for part (a) of question 11.

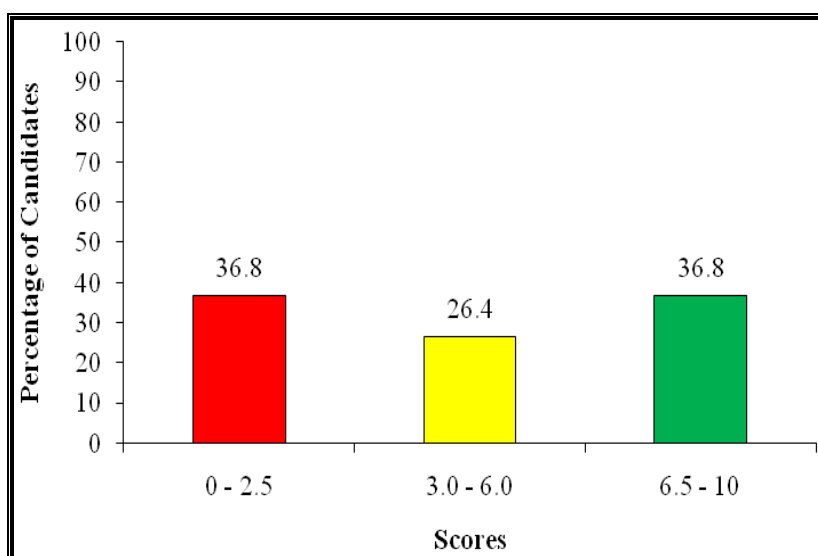
As Extract 11.2 shows, some candidates drew incorrect graph whose curves approached the line  $x = -\frac{3}{2}$  instead of  $x = \frac{3}{2}$ .



## 2.12 Question 12: Differentiation and Integration

This question consisted of parts (a), (b) and (c). In part (a), the candidates were required to differentiate  $f(x)=5$  from the first principles. Part (b) required the candidates to use the product rule to differentiate  $y=\cos^2 x$  with respect to  $x$ . In part (c), the candidates were required to find the area enclosed by the curve  $y=x^2-3x+2$  and the  $x$ -axis.

The question was attempted by 304 (92.4%) candidates whereby 192 (63.2%) candidates scored marks ranging from 3.0 to 10.0. Therefore, the candidates' performance in this question was average. Figure 12 summarizes the performance of the candidates on this question.



**Figure 12:** *The candidates' performance on question 12*

In this question, 11 (3.6%) candidates responded correctly scoring full marks. In part (a), the candidates rewrote  $f(x)=5$  as  $f(x)=5x^0$  and applied the correct formula for finding first derivative from the First Principles to get  $f'(x)=0$  (see Extract 12.1). In part (b), the candidates correctly expressed  $\cos^2 x$  into  $\cos x \cdot \cos x$  and applied the product rule  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$  which gave  $\frac{dy}{dx} = -2 \cos x \sin x$  or  $\frac{dy}{dx} = -\sin 2x$  correctly. In part (c), the candidates drew the graph or solved  $x^2-3x+2=0$  so as to determine the limits of the region,  $x=1$  and  $x=2$ .

Thereafter, they evaluated  $\int_1^2 (-x^2 + 3x - 2) dx$  and obtained  
 $Area = \frac{1}{6}$  square units.

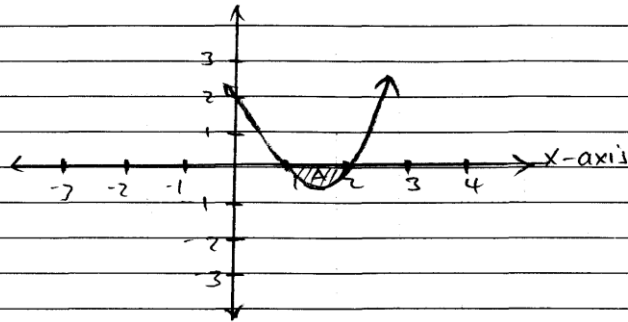
Handwritten mathematical derivation for the derivative of a constant function  $f(x) = 5$  using first principles. The steps are as follows:

$$\begin{aligned} 12. a) \quad & f(x) = 5 \\ & f(x) = 5x^0 \\ & f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ & f'(x) = \lim_{h \rightarrow 0} \frac{5(x+h)^0 - 5x^0}{h} \\ & f'(x) = \lim_{h \rightarrow 0} \frac{5(1) - 5(1)}{h} \\ & = \lim_{h \rightarrow 0} \frac{5 - 5}{h} \\ & = \lim_{h \rightarrow 0} \frac{0}{h} \\ & = 0 \\ & \therefore f'(x) = 0 \end{aligned}$$

**Extract 12.1:** A sample of correct response for part (a) of question 12.

In Extract 12.1, the candidate correctly determined  $f(x+h)$  from the given constant function and adhered to the procedures of finding derivative using first principles.

12 c) x-intercept = 2 and 1  
so, consider diagram below.



$$A = \int_1^2 y \, dx$$

$$= \int_1^2 (x^2 - 3x + 2) \, dx$$

$$= \int_1^2 x^2 \, dx - 3 \int_1^2 x \, dx + 2 \int_1^2 1 \, dx$$

$$= \left[ \frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_1^2$$

$$= - \left( \frac{2^3}{3} - \frac{3(2)^2}{2} + 2(2) \right) - \left( \frac{1^3}{3} - \frac{3}{2} + 2 \right)$$

$$= - \left( \left( \frac{8}{3} - 6 + 4 \right) - \left( \frac{2 - 9 + 12}{6} \right) \right)$$

$$= - \left( \left( \frac{8 - 6}{3} \right) - \left( \frac{5}{6} \right) \right)$$

$$12 \quad c) = - \left( \frac{2}{3} - \frac{5}{6} \right)$$

$$= - \left( \frac{4 - 5}{6} \right)$$

$$= - \left( -\frac{1}{6} \right)$$

$$= \frac{1}{6}$$

$\therefore$  The area =  $\frac{1}{6}$  square units.

**Extract 12.2:** A sample of correct response for part (c) of question 12.

In Extract 12.2, the candidate identified the limits of the required region by drawing graph and computed area of the region correctly.

The data further shows that 112 (36.8%) candidates scored marks ranging from 0 to 2.5. In part (a), some candidates failed to determine  $f(x+h)$  from  $f(x)=5$  (see Extract 12.1). Also, there were candidates who approached the item inappropriately as they applied the rule  $f'(x)=nx^{n-1}$  instead of the First Principles. They wrote  $f(x)=5x^0$  and obtained  $f'(x)=0 \times 5x^{-1}=0$ . In part (b), majority of the candidates applied chain rule  $\left(\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}\right)$  after letting  $u = \cos x$  and  $y = u^2$  instead of the product rule. Moreover, other candidates substituted  $u = \cos x$ ,  $v = \cos x$ ,  $\frac{du}{dx} = -\sin x$  and  $\frac{dv}{dx} = -\sin x$  into inappropriate formula  $\frac{dy}{dx} = v \frac{du}{dx} - u \frac{dv}{dx}$  and got incorrect answer  $\frac{dy}{dx} = 0$ .

In part (c), some candidates applied inappropriate formula particularly  $A = \int_b^a y^2 dx$ . This formula is used to find the volume of solid of revolution not area under the curve. Also, there were candidates who failed to determine the correct limits. It was further noted that few candidates applied the formula for finding maximum value  $y = \frac{b^2 - 4ac}{4a}$  taking  $a = 1$ ,  $b = -3$  and  $c = 2$  to obtain  $Area = \frac{1}{4}$  while other candidates calculated the second derivative and got  $\frac{d^2y}{d^2x} = 2$ . These candidates did not understand the requirements of the item.

12 (a)  $f(x) = 5$  from first principle

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= f'(x) = \lim_{h \rightarrow 0} \frac{f(5+h) - 5}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{5 - 5 + h}{h}$$

$$\lim_{h \rightarrow 0} = \frac{0 + h}{h}$$

$$f'(x) = \frac{h}{h}$$

$$f'(x) = 1$$

**Extract 12.3:** A sample of incorrect response for part (a) of question 12.

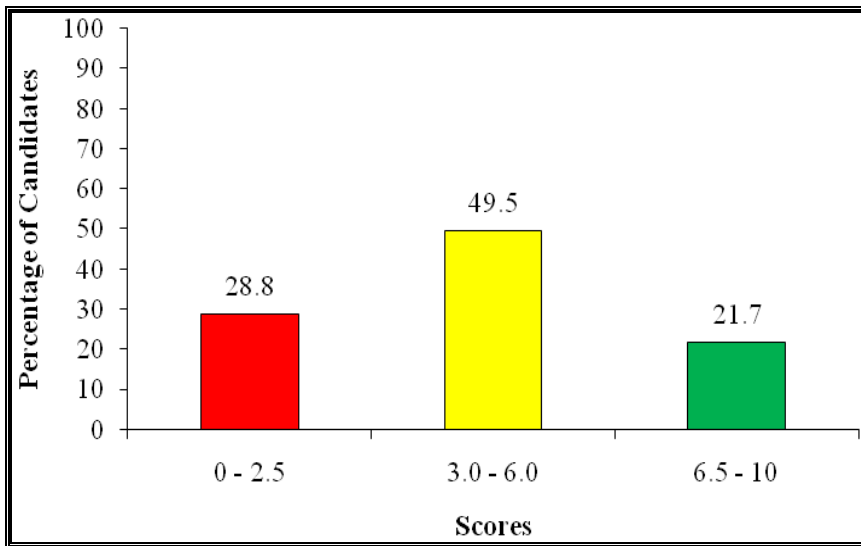
Extract 12.3 is a solution of the candidate who wrote  $f(x+h) = 5+h$  instead of  $f(x+h) = 5$  which led to incorrect answer.

## 2.13 Question 13: Probability

The question was as follows:

- A bag contains 3 white balls, 4 red balls and 2 yellow balls. How many white balls must be added in the bag so that the probability of drawing a white ball is  $\frac{1}{2}$ ?
- Find how many different numbers can be made by using four out of the six digits 0, 1, 2, 3, 4, 5.
- Two dice are thrown at the same time. Find the probability of obtaining a total which is less than 10.

This question was attempted by 309 (93.9%) candidates. Among them, 220 (71.3%) candidates scored marks ranging from 3.0 to 10.0. Therefore, the general performance in this question was good. The percentage of candidates who scored low, average and high marks is shown in Figure 13.



**Figure 13:** *The candidates' performance on question 13*

The candidates who responded to part (a) correctly realized that if  $x$  white balls is added, then the number of sample space will also increase by  $x$ .

Therefore, they formulated the equation  $\frac{3+x}{9+x} = \frac{1}{2}$  and solved it to obtain

$x=3$ . In part (b), a number of candidates applied the principles of permutation and multiplication ( ${}^5P_1 \times {}^5P_3$ ) and got 300 different numbers that could be formed. Also, there were candidates who responded to the item by firstly interpreting the given information in a table, as seen in Extract 13.1.

In part (c), the candidates used the principle of multiplication ( $6 \times 6$ ) to determine the number of sample space,  $n(S)$ , as 36. Then, they realised that 30 pairs of numbers in a table give a total which is less than 10.

Therefore, they applied the formula  $P(E) = \frac{n(E)}{n(S)}$  for finding probability of

an event and got  $P(E) = \frac{5}{6}$ . Additionally, there were candidates who summarized the outcomes in a table as shown in Extract 13.2.

b) soln:				
Digits:	0, 1, 2, 3, 4, 5			
Position	1	2	3	4
Possibles	5	5	4	3
Numbers formed are = $5 \times 5 \times 4 \times 3$				
= $25 \times 12$				
= 300 numbers				
$\therefore$ Different numbers that can be made are 300 numbers.				

**Extract 13.1:** A sample of correct response for part (b) of question 13.

As Extract 13.1 shows, the candidate correctly identified the number of digit(s) (out of the given digits) that qualify to occupy a particular place value of the number to be formed and computed their product.

13. c) soln:							
		Die 1					
	*	1	2	3	4	5	6
	1	1,1	2,1	3,1	4,1	5,1	6,1
	2	1,2	2,2	3,2	4,2	5,2	6,2
	3	1,3	2,3	3,3	4,3	5,3	6,3
	4	1,4	2,4	3,4	4,4	5,4	6,4
	5	1,5	2,5	3,5	4,5	5,5	6,5
	6	1,6	2,6	3,6	4,6	5,6	6,6
	Total no of sample $n(S) = 36$						
	Total less than 10 $n(E) = 30$						
	$P(E) = \frac{n(E)}{n(S)}$						
	$P(E) = \frac{30}{36}$						
	$P(E) = \frac{5}{6}$						
	$\therefore$ The probability of getting a total less than 10 is $\frac{5}{6}$ .						

**Extract 13.2:** A sample of correct response for part (c) of question 13.

In Extract 13.2, the candidate correctly tabulated all possible outcomes and realized that out of all 36 pairs of numbers, there were 30 pairs whose sum is less than 10.

Further analysis of data shows that 89 (28.8%) candidates scored 0 to 2.5 marks in this question. In part (a), majority of these candidates wrote

$\frac{P(W)}{9} = \frac{1}{2}$  and solved to get 1.5 white balls. Also, some candidates wrote

$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$ . This indicates that they wrongly interpreted the

number of white balls in a box (3) as the number of asked event and the total number of red and yellow balls in a box (6) as the size of sample

space. In addition, there were few candidates who wrote  $P(E) = \frac{n(S)}{n(E)}$ ,

implying their failure to recall the correct formula for finding probability of

an event  $P(E) = \frac{n(E)}{n(S)}$ .

In part (b), a number of candidates substituted  $n=6$  and  $r=4$  into the

inappropriate formula  $\frac{n!}{r!}$  and got incorrect answer 30 numbers. Moreover,

some candidates wrongly by applied the principle of combination (see Extract 13.3).

In part (c), some candidates failed to interpret the statement “total which is less than 10” whereby they wrote  $n(E)=6$  instead of  $n(E)=30$ . There

were few candidates who constructed the incorrect table which signified 12 sides for a die instead of 6 sides. Therefore, they got incorrect answers

particularly  $n(E)=36$ ,  $n(S)=144$  and  $P(E) = \frac{1}{4}$ . Furthermore, Extract

13.4 shows the response of a candidate who performed poorly in this question.

(b)  ${}^6C_4$

$$= \frac{6!}{(6-4)! \cdot 4!} = \frac{6!}{2! \cdot 4!}$$

$$= \frac{6 \times 5 \times 4}{2 \times 4} = \frac{6 \times 5}{2 \times 1} = \frac{30}{2} = 15$$

15 numbers.

**Extract 13.3:** A sample of incorrect response for part (b) of question 13.

Extract 13.3 is the response of the candidate who computed the number of combinations instead of number of permutations.



(c). Let 1st die be A.  
2nd die be B.

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{1, 2, 3, 4, 5, 6\}$$

$n(S)$  of each is 6.

$$n(E) = 1.$$

$$P(E) = \frac{n(E)}{n(S)}.$$

$$P(E)_A = \frac{1}{6}.$$

$$P(E)_B = \frac{1}{6}.$$

$$P(E)_A + P(E)_B = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

The total probability =  $\frac{1}{6}$  therefore

**Extract 13.4:** A sample of incorrect response for part (c) of question 13.

In Extract 13.4, the candidate failed to apply the concept of combined events.

## 2.14 Question 14: Vectors and Matrices and Transformations

The question instructed candidates to:

(a) find  $\underline{a} \times \underline{b}$  and  $(\underline{a} \times \underline{b}) \cdot \underline{a}$  if  $\underline{a} = -2\underline{i} + 5\underline{j} - 3\underline{k}$  and  $\underline{b} = 3\underline{i} - \underline{j} + 2\underline{k}$ .

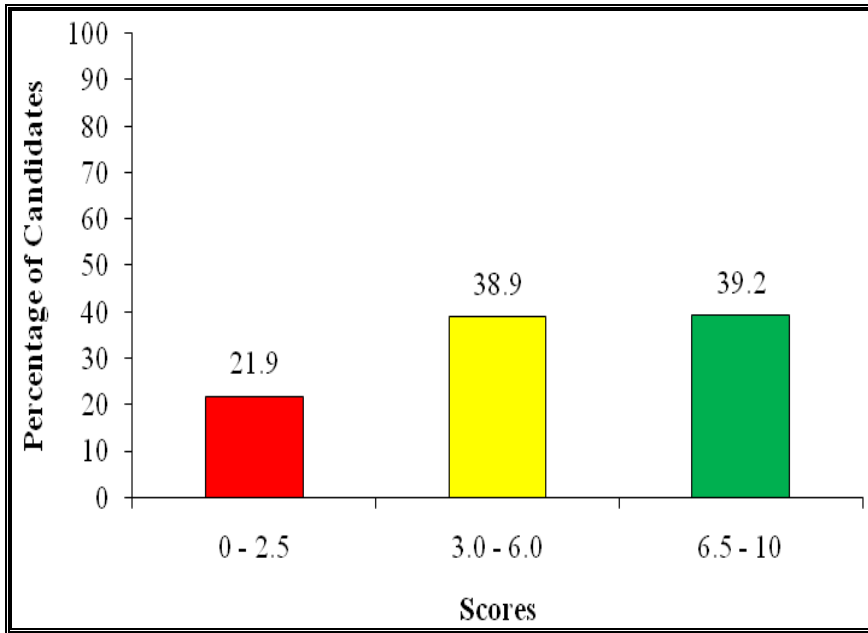
(b) show that  $\det(AB) = \det(A) \times \det(B)$  given that  $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$

and  $B = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -3 & 2 \\ 1 & 1 & 1 \end{pmatrix}.$

(c) determine the matrix corresponding to the linear reflection of the point  $P(x, y)$  on the line  $y - x = 0$  and use it to find the point whose image under the reflection is  $(3, -2)$ .

This question was attempted by 319 (97.0%) candidates. A total of 249 (78.1%) candidates scored marks ranging from 3.0 to 10.0. This implies

that the overall candidates' performance in this question was good. Figure 14 shows candidates' performance on this question.



**Figure 14:** *The candidates' performance on question 14*

In part (a), the candidates computed cross product  $(\underline{a} \times \underline{b})$  of  $\underline{a} = -2\underline{i} + 5\underline{j} - 3\underline{k}$  and  $\underline{b} = 3\underline{i} - \underline{j} + 2\underline{k}$  and got  $7\underline{i} - 5\underline{j} - 13\underline{k}$ . Then, they performed the dot product of  $7\underline{i} - 5\underline{j} - 13\underline{k}$  and  $-2\underline{i} + 5\underline{j} - 3\underline{k}$  and got 0

(see Extract 14.1). In part (b), the candidates got  $AB = \begin{pmatrix} 10 & -4 & 12 \\ 3 & 3 & 0 \\ 11 & -5 & 9 \end{pmatrix}$ ,

$\det(AB) = -198$ ,  $\det(A) = 33$  and  $\det(B) = -6$ . Therefore, they correctly showed that  $\det(AB) = \det(A) \times \det(B)$ . In part (c), the candidates

substituted  $\alpha = 45^\circ$  into the matrix  $M = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$  to get

$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ . This matrix enabled them to write the formula

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$  for finding the image  $\begin{pmatrix} x' \\ y' \end{pmatrix}$  of an object  $\begin{pmatrix} x \\ y \end{pmatrix}$ . Then,

they substituted  $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$  into the formula and formulated the matrix equation  $\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$  which was solved to get  $\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ .

1/4	a) i) $\underline{a} \times \underline{b}$ .
	$\underline{a} = -2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ .
	$\underline{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ .
	$\underline{a} \times \underline{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 5 & -3 \\ 3 & -1 & 2 \end{vmatrix}$
	$\underline{a} \times \underline{b} = \mathbf{i} \begin{vmatrix} 5 & -3 \\ -1 & 2 \end{vmatrix} - \mathbf{j} \begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} -2 & 5 \\ 3 & -1 \end{vmatrix}$
	$= \mathbf{i}(10 - 3) - \mathbf{j}(-4 + 9) + \mathbf{k}(2 - 15)$ .
	$= 7\mathbf{i} - 5\mathbf{j} - 13\mathbf{k}$ .
	$\underline{a} \times \underline{b} = 7\mathbf{i} - 5\mathbf{j} - 13\mathbf{k}$ .
	$\therefore (\underline{a} \times \underline{b}) = 7\mathbf{i} - 5\mathbf{j} - 13\mathbf{k}$ .
1/4	a) ii) $(\underline{a} \times \underline{b}) \cdot \underline{a}$ .
	$(\underline{a} \times \underline{b}) = 7\mathbf{i} - 5\mathbf{j} - 13\mathbf{k}$ .
	$\underline{a} = -2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ .
	$(\underline{a} \times \underline{b}) \cdot \underline{a} = \begin{pmatrix} 7 \\ -5 \\ -13 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix}$
	$= -14 - 25 + 39$
	$(\underline{a} \times \underline{b}) \cdot \underline{a} = -39 + 39$ .
	$(\underline{a} \times \underline{b}) \cdot \underline{a} = 0$ .
	$\therefore (\underline{a} \times \underline{b}) \cdot \underline{a} = 0$ .

**Extract 14.1:** A sample of correct response for part (a) of question 14.

In Extract 14.1, the candidate correctly performed cross product and dot product on vectors.

However, 70 (21.9%) candidates scored 0 to 2.5 marks. In part (a), some candidates confused the formulae for cross product and dot product. For example, there were candidates who wrote  $\underline{a} \times \underline{b} = (-2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}) \times (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = -17$ . Also, some candidates could not identify the correct sign connecting the determinants of reduced matrices. For instance, some candidates wrote

$$\underline{a} \times \underline{b} = \mathbf{i} \begin{vmatrix} 5 & -3 \\ -1 & 2 \end{vmatrix} + \mathbf{j} \begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} -2 & 5 \\ 3 & -1 \end{vmatrix} \quad \text{instead of}$$

$\underline{a} \times \underline{b} = i \begin{vmatrix} 5 & -3 \\ -1 & 2 \end{vmatrix} - j \begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix} + k \begin{vmatrix} -2 & 5 \\ 3 & -1 \end{vmatrix}$ . These candidates got

$\underline{a} \times \underline{b} = 7i + 13j - 13k$  instead of  $\underline{a} \times \underline{b} = 7i - 5j - 13k$ . Similarly, in part (b), some candidates computed the determinants of  $A$ ,  $B$  or  $AB$  incorrectly. Some candidates wrote  $\det(A) = 45$ , hence they concluded that  $\det(AB) \neq \det(A)\det(B)$ . Moreover, some candidates did computational errors when computing the product of  $A$  and  $B$  whereby the answer

$$AB = \begin{bmatrix} 10 & 14 & 12 \\ 5 & 3 & 0 \\ 11 & 7 & 9 \end{bmatrix} \text{ was frequently seen in the candidates' responses.}$$

In part (c), majority of the candidates used  $(3, -2)$  as an object while it was an image. They wrote  $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$  and concluded that

$(x, y) = (3, -2)$ . Moreover, some candidates failed to identify the angle of reflection. Therefore, they got incorrect matrix from the formula

$M = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$  and resulted to wrong answer, as seen in Extract 14.2.

Handwritten work showing incorrect calculations for a reflection transformation. The candidate uses an incorrect angle of 120 degrees and confuses the image with the object.

$$\begin{aligned} \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} \cos 90(12) & \sin 90(12) \\ \sin 90(12) & -\cos 90(12) \end{pmatrix} \\ \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} \cos 120 & \sin 120 \\ \sin 120 & -\cos 12 \end{pmatrix} \\ \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} \\ \text{or } \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} -3 \times 3 + 0 \times -2 \\ 0 \times 3 + 1 \times -2 \end{pmatrix} \\ \begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} -3 \\ -2 \end{pmatrix} \\ \therefore \text{Image } x' &= -3 \text{ and } y' = -2. \end{aligned}$$

**Extract 14.2:** A sample of incorrect response for part (c) of question 14.

In Extract 14.2, the candidate used an incorrect value of the angle under the given reflection and confused image with object for the reflection.

### **3.0 CONCLUSION AND RECOMMENDATIONS**

#### **3.1 Conclusion**

The analysis of the candidates' performance reveals that 277 (84.45%) candidates passed. The candidates had good performance in questions 1, 5, 8, 9, 10, 11, 13 and 14 which were set from the topics of *Variations, Algebra, Numbers, Logic, Sets, Functions and Remainder Theorem, Probability* and *Vectors and Matrices and Transformations*. The candidates' performance in questions 2, 3, 6, 7 and 12 was average. These questions were set from the following topics; *Statistics, Coordinate Geometry, Plan and Elevations, Geometrical Constructions, Differentiation* and *Integration*.

The candidates' performance in question 4 (which was set from *Locus*) was weak. The candidates' weak performance on this question was due to the inability of candidates to describe the locus of a point moving in specified distance from other two points.

#### **3.2 Recommendations**

In order to improve the candidate's performance in the future examinations, teachers should enable students to develop competency in:

- (a) describing the locus of a point moving from the two fixed points at given distances.
- (b) drawing graphs using ordered pairs of given equations.
- (c) calculating the quartiles of data using the formula.
- (d) finding the area of the region under the curve.
- (e) finding derivative of a constant function from first principle.
- (f) interpreting plan, front and side elevations of the cone.
- (g) applying trigonometric identities to simplify trigonometric expressions and solving trigonometric equations.

## Appendix

### Analysis of Candidates' Performance per Topic

S/N	Topic	Questions Number	Percentage of Candidates who Passed	Remarks
1	Variations	1	88.2	Good
2	Logic	9	86.0	Good
3	Numbers	8	84.7	Good
4	Sets	10	83.9	Good
5	Functions and Remainder Theorem	11	83.2	Good
6	Vectors; and Matrices and Transformations	14	78.1	Good
7	Algebra	5	77.8	Good
8	Probability	13	71.3	Good
9	Trigonometry	7	63.8	Average
10	Plan and Elevations; and Geometrical Constructions	6	63.2	Average
11	Differentiation and Integrations	12	63.2	Average
12	Statistics	2	60.4	Average
13	Coordinate Geometry	3	54.8	Average
14	Locus	4	26.9	Weak

