

**THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**



**CANDIDATES' ITEM RESPONSE ANALYSIS  
REPORT FOR THE CERTIFICATE OF SECONDARY  
EDUCATION EXAMINATION (CSEE) 2019**

$$e^{i\pi} + 1 = 0$$

$\pi$

**041 BASIC MATHEMATICS**

**THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**



**CANDIDATES' ITEM RESPONSE ANALYSIS REPORT  
FOR THE CERTIFICATE OF SECONDARY EDUCATION  
EXAMINATION (CSEE) 2019**

**041 BASIC MATHEMATICS**

## Table of Contents

FOREWORD.....	iii
1.0 INTRODUCTION .....	1
2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION .....	2
2.1 Question 1: Numbers, Approximations, Decimals and Percentages .....	2
2.2 Question 2: Exponents, Radicals and Logarithms .....	6
2.3 Question 3: Sets and Probability .....	11
2.4 Question 4: Coordinate Geometry and Vectors .....	16
2.5 Question 5: Congruence, Similarity and Geometry .....	20
2.6 Question 6: Rates and Variations.....	25
2.7 Question 7: Ratio, Profit and Loss and Accounts .....	30
2.8 Question 8: Sequences and Series.....	35
2.9 Question 9: Trigonometry and Pythagoras Theorem.....	40
2.10 Question 10: Algebra and Quadratic Equations.....	46
2.11 Question 11: Statistics and Circles.....	50
2.12 Question 12: The Earth as a Sphere and Three Dimensional Figures .....	56
2.13 Question 13: Matrices and Transformations .....	62
2.14 Question 14: Relations, Functions and Linear Programming .....	70
3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC	76
4.0 CONCLUSION AND RECOMMENDATIONS .....	76
4.1 Conclusion .....	76
4.2 Recommendations.....	77
Appendix .....	79

## **FOREWORD**

The National Examinations Council of Tanzania is very grateful to issue this Candidates' Item Response Analysis (CIRA) report for Basic Mathematics examination for the Certificate of Secondary Education Examination (CSEE) 2019. The report aims at giving feedback to students, teachers, policy makers and other educational stakeholders on how candidates performed in each question and topic that were examined in the Basic Mathematics examination.

The Basic Mathematics paper had 14 compulsory questions. According to data analysis, only 2 questions (question 11 that was set from the topics of Statistics and Circles and question 6 that was set from the topic of Rates and Variation) had average performance. The remaining questions had weak performance. The reasons for candidates' weak performance are summarized in section 4.1 of this report. The candidates' strengths and weaknesses highlighted in this report will give reflection to students and teachers in their self-evaluation and assessment.

Finally, the Council wishes to thank the examiners, examination officers and all others who participated in the preparation of this report.



Dr. Charles E. Msonde

**EXECUTIVE SECRETARY**

## 1.0 INTRODUCTION

This report is based on the analysis of the candidates' performance on Basic Mathematics Examination for CSEE 2019. The examination paper comprised of fourteen (14) compulsory questions from two sections A and B. Section A consisted of ten (10) questions each carrying six (6) marks and section B consisted of four (4) questions each carrying ten (10) marks. The analysis cited mainly on areas in which the candidates did not perform well when answering examination questions.

The number of candidates who sat for the examination in CSEE 2019 was 424,652 out of which 84,578 (20.03%) candidates passed. A total of 360,225 candidates sat for the CSEE 2018, out of which 71,703 (20.02%) candidates passed. Therefore, the performance has increased by 0.02 percent.

Section 2.0 of this report gives a brief description of the requirement of the question, the candidates' performance, the main reasons that contributed to average and weak performance, including the extracts for candidates' good and weak performance for each question. Section 3.0 presents the analysis of candidates' performance in each topic. However, section 4.0 gives the conclusion and recommendations that will help students, teachers and government to improve the performance in future Basic mathematics examinations.

In presenting the data, three colours - red, yellow and green were used to represent the categories of performance intervals in both Figures and Appendices.

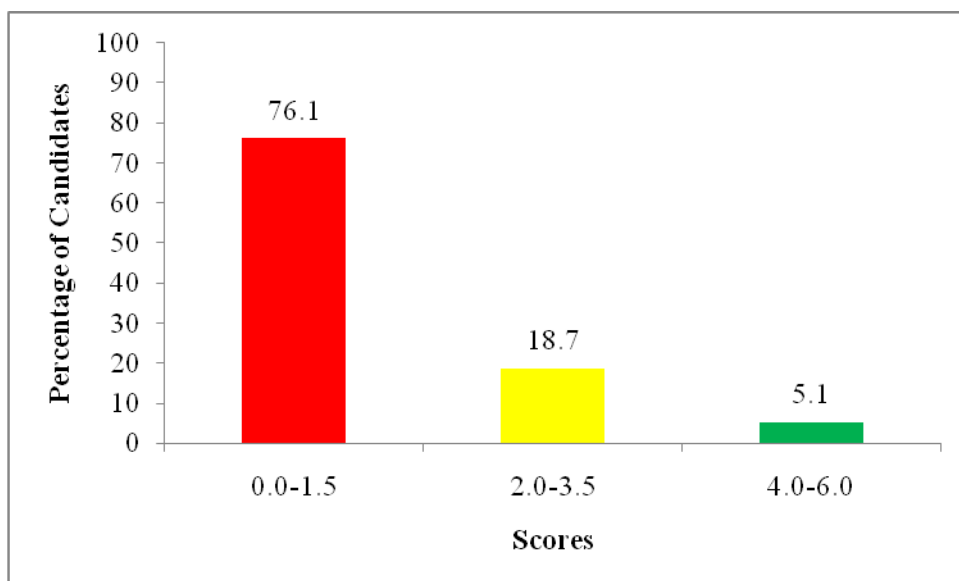
Colour	Percentage intervals for the whole examination	Score intervals per question		Remarks
		Section A	Section B	
Green	60 - 100	4.0 - 6.0	6.0 - 10.0	Good
Yellow	35 - 59	2.0 - 3.5	3.0 - 6.0	Average
Red	0.0 - 34	0.0 - 1.5	0.0 - 2.5	Weak

## 2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

### 2.1 Question 1: Numbers, Approximations, Decimals and Percentages

The question comprised parts (a) and (b). In part (a), the candidates were required to find the minimum number of mangoes which could be exactly divided into groups of 20, 30 or 36. In part (b), the question was as follows; "Mary was given 60,000 shillings by her mother. She spent 35 percent of the money to buy shoes and 10 percent of the remaining money to buy books. How much money remained?"

This question was attempted by 372,912 (87.8%) candidates, out of which only 89,068 (23.9%) candidates scored from 2 to 6 marks showing that the performance was weak as summarised in Figure 1.



**Figure 1:** *The summary of candidates' performance in question 1*

Further analysis shows that 283,844 (76.1%) candidates scored below 2 marks, out of which 231,093 (62.0%) candidates scored 0 mark.

In part (a), most candidates were not aware that the minimum number of mangoes which could be exactly divided into groups of 20, 30 or 36 is the Lowest Common Multiple (LCM) of the given numbers. Others were able to write the product of the correct prime factors as  $20 = 2 \times 2 \times 5$ ,  $30 = 2 \times 3 \times 5$ ,  $36 = 2 \times 2 \times 3 \times 3$  but they could not evaluate

the LCM. It was also noted that, some of the candidates computed the Greatest Common Factor (GCF) contrary to the given instructions.

There were some candidates who calculated the mean of the given numbers and got 28.67, hence concluded that the minimum number of mangoes is approximately equal to 29. This indicates that these candidates lacked knowledge and skills on how to use factors or multiples when finding LCM.

In part (b), some candidates were able to obtain the correct amount of money for buying shoes and subtracted it from the total amount of money to get the remaining amount but ended up with wrong solution. Others added the percentages spent to buy shoes and books and used it to calculate the amount of money remained, that is;  $\frac{45\%}{100} \times 60,000 = \text{Sh } 27,000$  and the amount of money remained =  $60,000 - 27,000 = \text{Sh } 33,000$  which was wrong. In addition, few candidates subtracted 35% from 100% when finding the percentage remaining which is 65% and used it to find the amount of money remaining, which is an incorrect step. Again, there were candidates who calculated the amount of money to buy shoes correctly, but wrongly computed the amount of money for buying books as ten percent of  $60,000 / =$  instead of ten percent of  $39,000 / =$ . Extract 1.1 illustrates a sample work of a candidate who failed to answer this question correctly.

Q1	8) <u>Soln</u>																								
	Mangoes are divided into groups																								
	20, 30 or 36.																								
	minimum number of mangoes required.																								
	= GCF.																								
	<table><tr><td>✓ 2</td><td>20</td><td>30</td><td>36.</td></tr><tr><td>2</td><td>10</td><td>15</td><td>18</td></tr><tr><td>5</td><td>5</td><td>15</td><td>9</td></tr><tr><td>3</td><td>1</td><td>3</td><td>9</td></tr><tr><td>3</td><td>1</td><td>1</td><td>3</td></tr><tr><td></td><td>1</td><td>1</td><td>1</td></tr></table>	✓ 2	20	30	36.	2	10	15	18	5	5	15	9	3	1	3	9	3	1	1	3		1	1	1
✓ 2	20	30	36.																						
2	10	15	18																						
5	5	15	9																						
3	1	3	9																						
3	1	1	3																						
	1	1	1																						
	hence minimum number of Mangoes required = 2.																								
	(b) Given 35% of money for shoes and 10% of money for books when reduced from 60,000/=-																								
	$\frac{35}{100} \times 60,000/- = 21,000/-$																								
	$\frac{10}{100} \times 60,000/- = 6,000/-$																								
	$21,000/- + 6,000/- = 27,000/-$																								
	$60,000/- - 27,000/- = 33,000/-$																								
	$\therefore$ She remained with 33,000/=-																								

**Extract 1.1:** A sample of the candidate's incorrect responses in question 1

Extract 1.2 clearly shows that the candidate found the GCF instead of LCM of the given numbers. The candidate also failed to calculate the amount of money for buying books, as a result he/she got the incorrect answer.

On the other hand, 19,195 (5.1%) candidates scored from 4 to 6 marks, out of which only 11,538 (3.1%) candidates scored full marks.

The candidates were able to find the LCM of the given numbers correctly in part (a) by using either listing or prime factorization method and got the required minimum number of mangoes which can be divided exactly into groups of 20, 30 or 36.

In part (b), the candidates were able to use the concept of fraction and percentages correctly when determining the amount of money remained. Extract 1.2 shows a sample response of a candidate who answered this question correctly.

1 (a)	SOLUTION:			
	✓ 2	20	30	26
	2	10	15	18
	3	5	15	9
	3	5	5	3
	5	5	5	1
		1	1	1
	L.C.M = $2 \times 2 \times 3 \times 3 \times 5$			
	$= 4 \times 45$			
	$= 180$			
	$\therefore$ The minimum number of mangoes required is 180.			
2. (b)	SOLUTION:			
	The Total Money = 60,000/-			
	• Money for shoes = 25% $\times$ 60,000/-			
	$= \frac{25}{100} \times 60,000/-$			
	$= 21,000/-$			
	Remaining money after buying shoes = 60,000/- - 21,000/-			
	$= 39,000/-$			
	• Money for books = 10% $\times$ 39,000/-			
	$= \frac{10}{100} \times 39,000/-$			
	$= 3,900/-$			
	- Money remained = Total Money - (Money for shoes + Money for books)			
	$= 60,000/- - (21,000/- + 3,900/-)$			
	$= 60,000/- - 24,900/-$			
	$= 35,100/-$			
	$\therefore$ The remained money is 35,100/-			

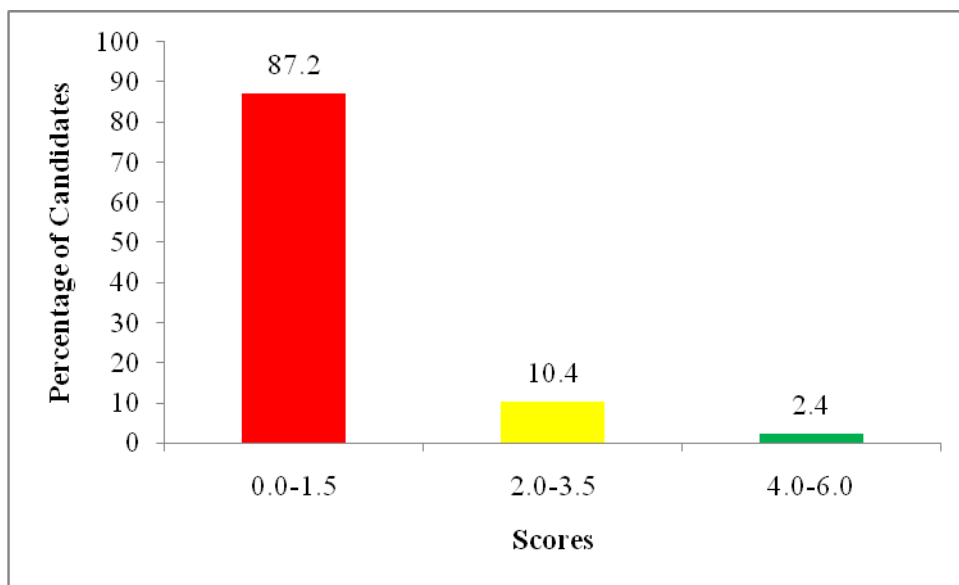
**Extract 1.2:** A sample of the candidate's correct responses in question 1

Extract 1.1 shows that the candidate was able to show the correct steps when finding the minimum number of mangoes in part (a) and the amount of money remained in part (b).

## 2.2 Question 2: Exponents, Radicals and Logarithms

This question had parts (a) and (b). In part (a), the candidates were required to find the value of  $x$  if  $\sqrt{5^{2x-3}} - 9 = 116$ . In part (b), they were required to find the value of the expression  $\frac{3.143 \times (0.81)^2}{\sqrt{35}}$  by using mathematical tables.

This question was attempted by 310,124 (73.0%) candidates, out of which only 39,639 (12.8%) candidates scored 2 to 6 marks, showing that the performance was weak. Figure 2 represents the candidates' performance in this question.



**Figure 2:** The summary of candidates' performance in question 2

Further analysis shows that 270,485 (87.2%) candidates scored from 0 to 1.5 marks, out of which 236,004 (76.1%) candidates scored 0 mark.

In Part (a), some candidates failed to equate powers of the same base and exponents since most of them ignored the radical sign as follows;

$\sqrt{5^{2x-3}} - 9 = 5^3 \Rightarrow 2x - 3 = 3$  which is incorrect step. Others squared both sides of the equation before taking 9 to the other side of the equation such as,  $\sqrt{5^{2x-3}} - 9 = 116$  thus  $(\sqrt{5^{2x-3}})^2 - 9^2 = 116^2 \Rightarrow 5^{2x-3} - 81 = 13,456$  and eventually ended up with  $5^{2x} \times 5^{-3} = 13,537$  and failed to get the value of  $x$  as required. It was also noted that few candidates were able to do the correct operations by squaring both sides of the equation but failed to perform correct computations as they commit errors like:  $(\sqrt{5^{2x-3}})^2 = (125)^2 \Rightarrow 5^{2x-3} = 15,625 = 5^5$ .

In part (b), some candidates failed to apply the laws of logarithms when finding the logarithms of the numbers with radical sign and square, for example some wrote  $\log(0.81)^2 = 2.9085$ , others wrote incorrectly  $\log(0.81)^2$  as  $2(\bar{1}.9085) = \bar{3}.8170$  and others wrote  $\log(0.81)^2 = 2(\bar{1}.9085) = \bar{1}.8170$  and  $\log(\sqrt{35}) = 0.7721$ , instead of writing correctly as  $\log(\sqrt{35}) = \frac{1}{2}(1.5441) = 0.7721$ . As a result, they failed to get the correct solution of the given expression, indicating that they lacked the skills and knowledge on using logarithmic tables to find the roots and powers of numbers. Extract 2.1 is a sample solution from one of the candidates who failed to answer this question correctly.  $\log(0.81)^2 = 2(\bar{1}.9085) = \bar{3}.8170$ .

$$2 \text{ as } \sqrt{5^{2x-3}} - 9 = 216$$

Soln

$$(\sqrt[3]{5^{2x-3}})^2 - (9)^2 = (116)^2$$

$$5^{2x-3} - 81 = (416)^2$$

$$= 5^{2x-3} = 13486 - 81$$

$$5^{2x-3} = 13375$$

$$5^{2x-3} = 5^3 \cdot 407$$

$$21-3=3$$

$$2x = 3 + 3$$

$$\frac{2x}{2} = \frac{6}{2}$$

$x = 3$

(b)

$$\frac{3.143 \times (0.81)^2}{\sqrt{35}}$$

Number		Log:
3.143	$3.143 \times 10^1$	1.4973
0.81	$(8.1 \times 10^{-1})_2$	$2(\bar{7}.9085) =$
		0.8170 +
Numerator	$\longrightarrow$	2.3143
$\sqrt{35}$	$3.5 \times 10^1$	$3(1.5441)$
Denominator	$\longrightarrow$	4.6323
	$2.070 \times 10^{-2}$	$-2.3180$

$$3.143 \times (0.81)^2$$

$\sqrt{35}$

$$= 2.070 \times 10^{-2}$$

**Extract 2.1:** *A sample of the candidate's incorrect responses in question 2*

Extract 2.1 shows the answer of a candidate who failed to remove the square root when solving the given equation. The candidate lacked adequate knowledge and skills on exponents to express the given numbers in standard notation.

Contrarily, 7,353 (2.4%) candidates scored from 4 to 6 marks, out of which 1,882 (0.6%) candidates scored full marks.

The candidates managed to apply the laws of exponents and logarithms correctly and got the correct answers. Extract 2.2 represents the work of a candidate who demonstrated the skills and knowledge in answering this question.

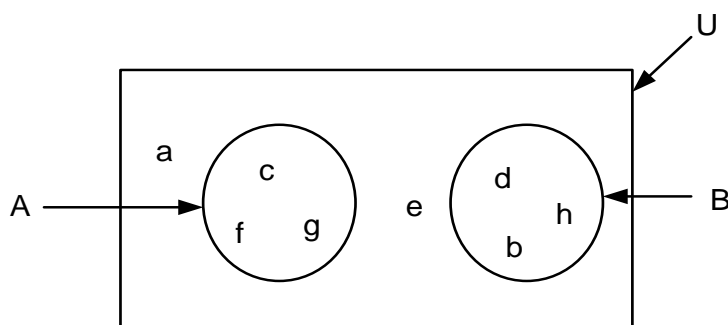
2	Soln!		
	(a) Given	$\sqrt{5^{2x-3}} - 9 = 116$	
		$x = ?$	
		$\sqrt{5^{2x-3}} = 116 + 9$	
		$(\sqrt{5^{2x-3}})^2 = 125^2$	
		$5^{2x-3} = (5^3)^2$	
		$5^{2x-3} = 5^6$	
4		$2x - 3 = 6$	
		$\frac{2x}{2} = \frac{9}{2}$	
		$\therefore x = \frac{9}{2}$	
	(b) Given	$\frac{3.143 \times (0.81)^2}{\sqrt{35}}$	
	No	Std	Log
	3.143	$3.143 \times 10^0$	0.4973
11	$(0.81)^2$	$(8.1 \times 10^{-1})^2$	$\bar{1}.9085 \times 2$
			$\Rightarrow \bar{1}.8170$
	Numerator		0.31430
	$\sqrt{35}$	$\sqrt{3.5 \times 10^1}$	$\frac{1.5441}{2}$
	Denominator		0.77205
	0.3485	$3.485 \times 10^{-1}$	$\bar{1}.54225$
	$\therefore \frac{3.143 \times (0.81)^2}{\sqrt{35}} = 0.3485$		

**Extract 2.2:** A sample of the candidate's correct responses in question 2

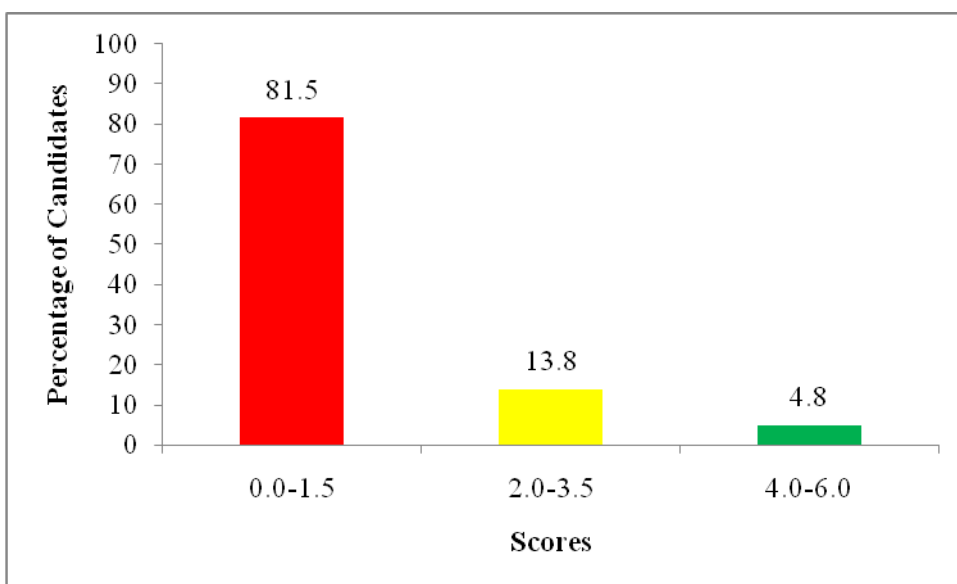
Extract 2.2 shows a response of a candidate who correctly applied the laws of exponents, powers and logarithms when solving the given problem.

### 2.3 Question 3: Sets and Probability

The question had parts (a) and (b). In part (a), the candidates were required to use the following Venn diagram to find; (i) the number of subsets of set  $B'$ , (ii) the elements of set  $A' \cap B$  and (iii) the probability of an element picked at random from the universal set ( $U$ ), which is not the element of set  $B$ . In part (b), the question was as follows; The Ministry of Business and Industries has planned to employ 54 people who will work in the business sector, 36 people who will work in industries sector only, and 12 people who will work in both sectors and 21 people who will neither work in business sector nor in industries sector. The candidates were required to find the number of people who will be employed by the Ministry using a Venn diagram.



This question was attempted by 334,721 (78.8%) candidates, out of which only 62,080 (18.5%) candidates scored from 2 to 6 marks showing that the performance was weak. Figure 3 summarises the candidates' performance in this question.



**Figure 3:** *The summary of candidates' performance in question 3*

Further analysis shows that 272,641 (81.5%) candidates scored from 0 to 1.5 marks, out of which 153,088 (45.7%) candidates scored 0 mark.

In part (a) (i), the majority of candidates were not able to find the number of subsets of set  $B'$ . Instead, they considered the number of elements of set  $B'$  as the number of subsets. For example, some candidates wrote  $B' = \{a, c, e, f, g\}$  and concluded that the subsets of  $B' = 5$  instead of taking  $2^n = 2^5 = 32$ , while others failed to identify set  $B'$ ; they wrote  $B' = \{c, f, g\}$  and got subsets  $2^n = 2^3 = 8$ . In part (a) (ii), most of the candidates failed to identify set  $A'$  from the given Venn diagram and as a result they wrote wrong sets like  $A' = \{c, f, g\}$  and  $B = \{b, d, h\}$ , hence they obtained  $A' \cap B = \{\}$ . In part (a) (iii), some candidates who failed to correctly identify  $n(B')$  as they wrote  $B' = \{c, f, g\}$  instead of  $B' = \{a, c, e, f, g\}$ . The candidates lacked knowledge and skills on joint and disjoint sets, since they did not consider the elements  $a$  and  $e$  as part of universal set.

In part (b), there were candidates who applied the formula  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$  contrary to the given instructions. Instead, they were supposed to use a Venn diagram to find the number of people to be employed by the Ministry. However, some of them drew the

Venn diagram as instructed but failed to represent the given information appropriately. For instance, regarding 36 as the number of people who work in the industry instead of treating it as the number of people who work in the industry only while others regarded 54 as the number of people who work in the business sector only and as a result, they perform wrong calculations to get the number of people to be employed by the ministry like;  $54 + 12 + 36 + 21 = 123$  which is wrong.

Furthermore, the majority of candidates regarded 36 as the number of people who will work in the industry sector which is an incorrect step. For example, they wrote incorrect steps like  $(54 - 12) + 12 + (36 - 12) + 21 = 42 + 12 + 24 + 21 = 99$  instead of writing correctly as  $(54 - 12) + 12 + 36 + 21 = 42 + 12 + 36 + 21 = 111$ . This reveals that the candidates lacked knowledge and skills in solving problems on sets by using Venn diagrams as illustrated in Extract 3.1

3. a) Solution
$U = [a, b, c, d, e, f, g, h]$
i) $B' = (a, c, e, f, g)$
ii) $A' = (a, b, d, e, h)$
$B = [b, d, h]$
$\therefore A \cap B = [ ]$
iii) $n(S) = 8$
$E = (b, d, h)$
$n(E) = 3$
$P(E) = \frac{n(E)}{n(S)}$
$P(E) = \frac{3}{8}$
$\therefore$ The probability that it is not the element of set B is $\frac{3}{8}$

3 b) solution

Let  $B$  be number of people for Business sector  
 $I$  be number of people for industries sector

$n(B) = 54$   
 $n(I) = 36$   
 $n(B \cap I) = 21$

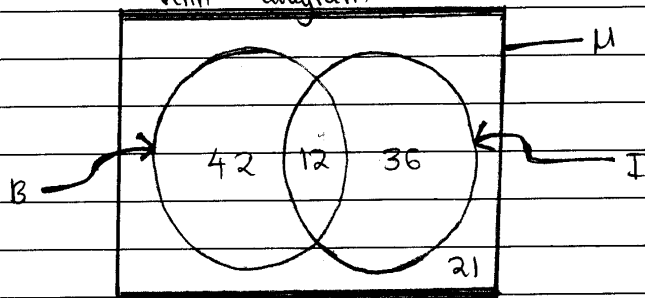
$\therefore n(A \cup B) = 33 + 15$   
 $= 48 + 21$   
 $\therefore \underline{\underline{69 \text{ people will be employed.}}}$

**Extract 3.1:** A sample of the candidate's incorrect responses in question 3

Extract 3.1 shows the candidate who lacked the knowledge of probability and failed to translate the set problem correctly by interpreting 'neither business nor industrial' as  $n(B \cap I)$  instead of  $n(B' \cap I')$ .

Despite the weak performance, the analysis shows that 15,999 (4.8%) candidates scored from 4 to 6 marks, out of which only 1,238 (0.4%) candidates scored full marks.

The candidates were able to apply the required knowledge and skills and got the correct answers as indicated in Extract 3.2

3	Soln:
(a)	Given the Venn diagram
(i)	Number of elements of set $B$ is 5
	Number of subsets = $2^n$
	$= 2^5$
	$= 32$
	$\therefore 32$ subsets
(ii)	$A' = \{a, e, d, h, b\}$
	$B = \{d, h, b\}$
	$\therefore A' \cap B = \{d, h, b\}$
	$B = \{d, h, b\} \quad n(E) = 3$
	$U = \{a, c, f, g, e, d, b, h\} \quad n(S) = 8$
	$P(E) = \frac{n(E)}{n(S)}$
	$= \frac{3}{8}$
	$P(E') = 1 - P(E)$
	$= 1 - \frac{3}{8}$
	$= \frac{5}{8}$
	$\therefore$ Probability it is not element of $B = \frac{5}{8}$
3	(b)
	Venn diagram
	
	Total, $(U) = 42 + 12 + 36 + 21$
	$= 111$
	$\therefore 111$ people

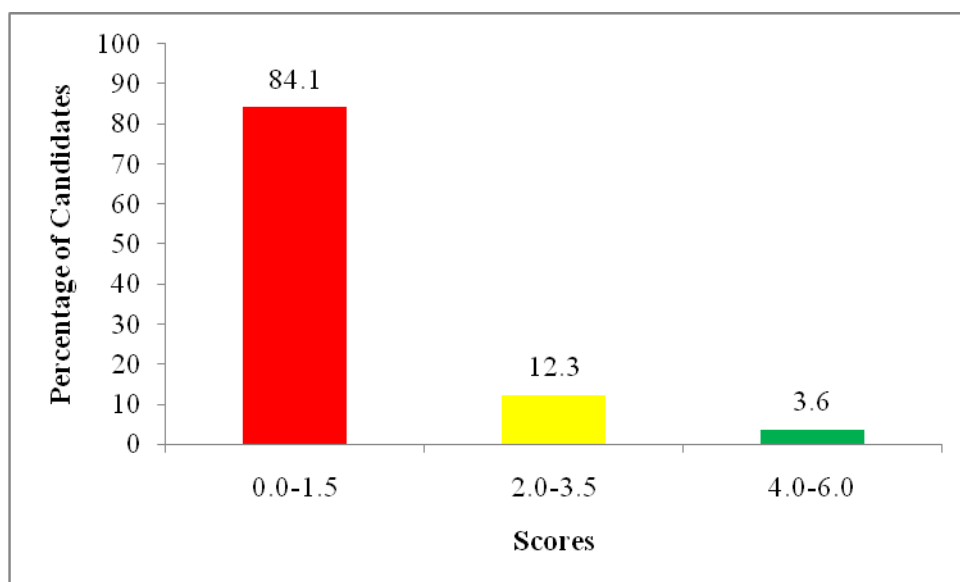
**Extract 3.2:** A sample of the candidate's correct responses in question 3

Extract 3.2 shows a response of a candidate who correctly answered the question involving probability and sets.

## 2.4 Question 4: Coordinate Geometry and Vectors

This question had two parts (a) and (b). In part (a), the candidates were required to find the equation of a line which passes through the point  $A(-3, 4)$  and which is parallel to the line  $3x + 4y - 15 = 0$ . In part (b), they were required to show that the points  $P(5, -3)$ ,  $Q(-6, 1)$  and  $R(1, 8)$  form an isosceles triangle.

This question was attempted by 282,199 (66.4%) candidates, out of which only 44,983 (15.9%) candidates scored from 2 to 6 marks showing that the performance was weak as summarised in Figure 4.



**Figure 4:** *The summary of candidates' performance in question 4*

Further analysis shows that 237,216 (84.1%) candidates scored from 0 to 1.5 marks, out of which 200,622 (71.1%) candidates scored 0 mark.

In part (a), most of the candidates who attempted this question were unable to identify correctly the slope of the given line  $3x + 4y - 15 = 0$ . For example, there were some candidates who incorrectly took the coefficient of the term containing  $x$  as the slope  $m$ , that is  $m_1 = 3$  and applied the

condition for two lines to be parallel, that is  $m_1 = m_2$  by taking  $m_2 = 3$  instead of rearranging the equation of the given line in the form of  $y = mx + c$  that is,  $y = -\frac{3}{4}x + \frac{15}{4}$  and taking the slope  $m_1 = -\frac{3}{4}$ . Others managed to rearrange the equation in the form  $y = mx + c$  and got the correct slope but they failed to apply the condition for parallel lines so as to proceed. However, others confused between the condition for perpendicular lines and parallel lines as they applied the formula  $m_1 m_2 = -1$ , contrary to the demands of the question. These candidates lacked knowledge and skills on the conditions for parallel lines and perpendicular lines.

In part (b), most of the candidates failed to understand the requirement of the question. Some candidates used the formula for finding the gradient to prove that the points  $P(5, -3)$ ,  $Q(-6, 1)$  and  $R(1, 8)$  form an isosceles triangle which is a wrong approach. For example, they did as follow:  $\overline{PQ} = \frac{1+3}{-6-5} = -\frac{4}{11}$ ,  $\overline{QR} = \frac{8-1}{1+6} = \frac{7}{7} = 1$  and  $\overline{PR} = \frac{8+3}{1-5} = -\frac{11}{4}$  instead of using the distance formula. Others wrote incorrect distance formulae like  $d = \sqrt{(x_1 + x_2)^2 - (y_1 + y_2)^2}$  instead of  $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ . Also, there were candidates who applied the formula for finding the mid-point to prove that the given points form an isosceles triangle. For example, they wrote:  $PQ = \left( \frac{5 + -6}{2}, \frac{-3 - 1}{2} \right) = \left( \frac{-1}{2}, -2 \right)$ ,  $PR = \left( \frac{5 + 1}{2}, \frac{-3 + 8}{2} \right) = \left( 3, \frac{5}{2} \right)$  and  $QR = \left( \frac{-6 + 1}{2}, \frac{1 + 8}{2} \right) = \left( \frac{-5}{2}, \frac{9}{2} \right)$ , hence concluded that triangle PQR is an isosceles triangle, which is not true and not the correct approach to this question. This indicates that the candidates lacked knowledge and skills in finding the distance between two points in a plane as illustrated in Extract 4.1.

4	@ points = A(3, 4)
	equation $3x + 4y - 15 = 0$
	from
	$y = mx + c$
	slope = 3
4	@ from
	slope = $\frac{y_2 - y_1}{x_2 - x_1}$
	$\frac{y - 4}{x + 3} = 3$
	$y - 4 = (x + 3)3$
	$y - 4 = 3x + 9$
	$y = 3x + 13$
	$y = 3x + 1$
	$y = 3x - 1$
	∴ The equation is $y = 3x - 1$
	<b> for an isosceles triangle
	mid point PQ = PR = QR
	PQ = (5, 3) and (-6, 1)
	$PQ = \left( \frac{5 + (-6)}{2}, \frac{3 + 1}{2} \right) =$
	$PR = \left( \frac{5 + 1}{2}, \frac{3 + 8}{2} \right) =$
	$QR = \left( \frac{-6 + 1}{2}, \frac{1 + 8}{2} \right) =$
	∴ ΔPQR is an isosceles triangle

**Extract 4.1:** A sample of the candidate's incorrect responses in question 4

Extract 4.1 represents the work of a candidate who failed to determine the correct slope from the given equation of a line and applied the wrong formula (that is, the formula for finding the mid-point of a line segment) to find the lengths of sides of the given triangle.

Apart from weak performance, the analysis shows that 10,265 (3.6%) candidates scored from 4 to 6 marks, out of which 3,498 (1.2%) candidates scored full marks.

The candidates were able to correctly find the equation of line as well as showing that the given points form an isosceles triangle using the correct conditions, formulae and approach. Extract 4.2 represents a sample solution from one of the candidates.

4	a	$3x + 4y = 15$
		$4y = \frac{-3x}{4} + \frac{15}{4}$
		$y = \frac{-3x}{4} + \frac{15}{4}$
		Gradient of the line, $M = -3/4$ .
		From;
		Parallel lines have the same gradient
		$A(-3, 4), P(x, y), M = -3/4$
		From; $M = \frac{\Delta y}{\Delta x}$
		$\frac{-3}{4} = \frac{y - 4}{x - -3}$
		$\frac{-3}{4} = \frac{y - 4}{x + 3}$
		$-3(x + 3) = 4(y - 4)$
		$-3x - 9 = 4y - 16$
		$4y = -3x - 9 + 16$
		$4y = \frac{-3x}{4} + \frac{7}{4}$
		$y = \frac{-3x}{4} + \frac{7}{4}$
		$\therefore$ Equation of a line passing through $A(-3, 4)$ is
		$y = \frac{-3x}{4} + \frac{7}{4}$

4	<del>b</del> $P(5, -3), Q(-6, 1)$
	$PQ = \sqrt{(5 - (-6))^2 + (-3 - 1)^2}$
	$PQ = \sqrt{(11)^2 + (-4)^2}$
	$PQ = \sqrt{121 + 16}$
	$PQ = \sqrt{137}$ units
	$P(5, -3), R(1, 8)$
	$PR = \sqrt{(5 - 1)^2 + (-3 - 8)^2}$
	$PR = \sqrt{(4)^2 + (-11)^2}$
	$PR = \sqrt{16 + 121}$
	$PR = \sqrt{137}$ units
	$Q(-6, 1), R(1, 8)$
	$QR = \sqrt{(-6 - 1)^2 + (1 - 8)^2}$
	$QR = \sqrt{(-7)^2 + (-7)^2}$
	$QR = \sqrt{49 + 49}$
	$QR = \sqrt{98} \approx 7\sqrt{2}$ units
	Since one of the properties of an isosceles triangle is the length of two sides are equal and from above, $PR = PQ$
	$\therefore$ The points P, Q and R form an isosceles triangle

**Extract 4.2:** A sample of the candidate's correct responses in question 4

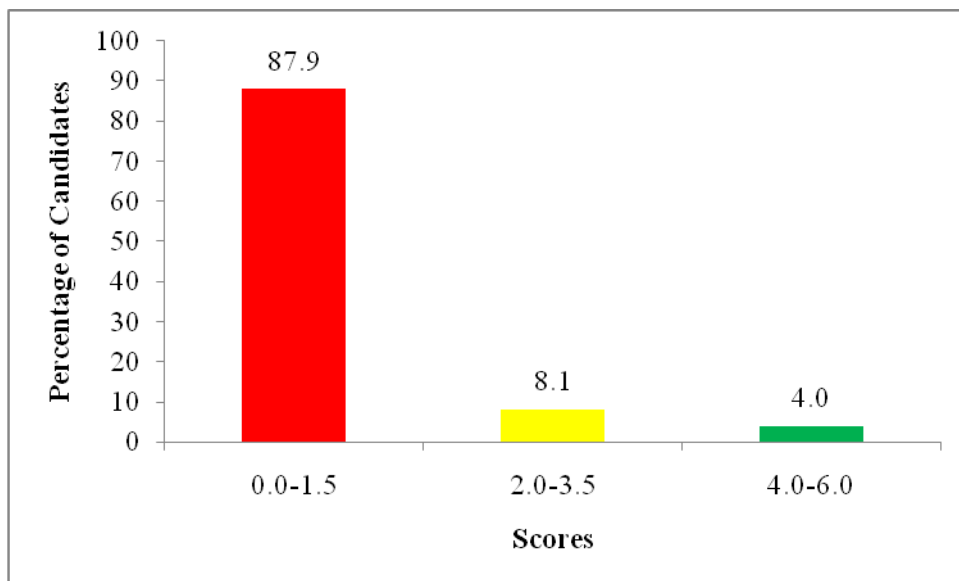
Extract 4.2 shows a response of a candidate who correctly determined the equation of a line and proved that the given three points form an isosceles triangle by using the correct approach.

## 2.5 Question 5: Congruence, Similarity and Geometry

This question consisted of parts (a) and (b). In part(a), the candidates were given that  $\overline{AB}$  is parallel to  $\overline{CD}$  and  $\overline{PQ}$  is a transversal and were required to sketch the line segments and label on the same diagram the following pairs of angles: (i) corresponding angles  $a$  and  $g$ , (ii) alternate interior angles  $f$  and  $g$  and (iii) vertically opposite angles  $c$  and  $d$ . In part (b), they were required to find the area of the triangle RQR which is similar to

triangle ABC, such that  $\overline{AB} = 4\text{ cm}$ ,  $\overline{BC} = 5\text{ cm}$ ,  $\overline{PQ} = 18\text{ cm}$  and angle  $PQR$  is  $30^\circ$ .

This question was attempted by 177,117 (41.7%) candidates, out of which only 21,446 (12.1%) candidates scored from 2 to 6 marks showing that the performance was weak. The summary of the candidates' performance in this question is shown in Figure 5.



**Figure 5:** The summary of candidates' performance in question 5

Further analysis shows that 155,671 (87.9%) candidates scored from 0 to 1.5 marks, out of which 134,022 (75.7%) candidates scored 0 mark.

In part (a), most of the candidates were unable to sketch and label the parallel lines  $\overline{AB}$ ,  $\overline{CD}$ , transversal line  $\overline{PQ}$  and indicate the angles as instructed. This shows that they lacked adequate knowledge and skills to relate the angles formed by the transversal and parallel lines. There were some candidates who were able to sketch the parallel lines and transversal lines correctly but failed to correctly locate the corresponding angles, alternate interior angles and vertically opposite angles.

In part (b), the majority of candidates were not able to apply the concept of ratio for similar triangles, as a result they failed to calculate the area of the triangle PQR. Most of these candidates were not aware of the concept that,

when two triangles are similar the ratios of the lengths of the corresponding sides are the same. This indicates that, the candidates lacked knowledge and skills on the concept of similarity. However, few candidates managed to recall correctly the formula for finding the area of similar triangles but they substituted the wrong values leading them to an incorrect answer. Further analysis shows that, few of them used inappropriate formula for finding the area as;  $A = \frac{1}{2}abc \cos \hat{C}$  or  $a^2 = b^2 + c^2 - 2b \times c \times \cos \hat{A}$  instead of  $A = \frac{1}{2}ab \sin \hat{C}$ , which led them to incorrect answers. Extract 5.1 represents a sample solution of a candidate who failed to answer this question correctly.

5.	$\overline{AB} \parallel \overline{CD}$ and $\overline{PQ}$
	i) Corresponding angles a and g shown.
	ii) alternate interior angles f and g shown
	iii) Vertically opposite angles c and d show. Ans

5	(b)	Area = $\frac{1}{2}(PQ)(QR) \sin 30^\circ$
		$AB = PQ$ and $BC = QR$ (similar)
		$\frac{1}{2} \times 4 \times 5 \times \sin 30$
		$2 \times 5 \times \frac{1}{2}$
		$10 \times \frac{1}{2}$
		$5 \text{ cm}^2$
		$\therefore \text{Area of PQR is } 5 \text{ cm}^2$

**Extract 5.1:** A sample of the candidate's incorrect responses in question 5

Extract 5.1 represents the answer of a candidate who showed misunderstanding of pairs of corresponding, alternate and vertically opposite angles and confused between similarity and congruence of the given triangles as well as using the dimensions of triangle ABC to find the area of triangle PQR.

Although the general performance was weak, the analysis shows that 7,023 (4.0%) candidates scored from 4 to 6 marks, out of which 1,557 (0.9%) candidates scored full marks.

The candidates were unable to sketch and label the parallel lines  $\overline{AB}$ ,  $\overline{CD}$ , transversal line  $\overline{PQ}$  and identify the required angles. This shows that the candidates had adequate knowledge and skills to relate the angles formed by the transversal and parallel lines. In part (b), the candidates were able to apply the concept of ratio for similar triangles and the correct formula calculate the area of the triangle PQR. Extract 5.2 represents the work of one of these candidates.

5 a)

5 b)

$$\frac{\overline{PQ}}{\overline{AB}} = \frac{\overline{QR}}{\overline{BC}}$$

Therefore

$$\frac{18 \text{ cm}}{4 \text{ cm}} = \frac{\overline{QR}}{5 \text{ cm}}$$

$$\overline{QR} = \frac{18 \text{ cm} \times 5 \text{ cm}}{4 \text{ cm}}$$

$$\overline{QR} = \frac{45 \text{ cm}}{2}$$

$$\overline{QR} = 22.5 \text{ cm}$$

therefore

$$\text{Area} = \frac{1}{2} ab \sin \theta$$

$$\text{Area} = \frac{1}{2} \times 22.5 \text{ cm} \times 18 \text{ cm} \times \sin 30^\circ$$

$$\text{Area} = 22.5 \text{ cm} \times 9 \text{ cm} \times \frac{1}{2}$$

$$\text{Area} = 202.5 \times \frac{1}{2} \text{ cm}^2$$

$$\text{Area} = 101.25 \text{ cm}^2$$

$\therefore$  The area of triangle PQR is  $101.25 \text{ cm}^2$ .

**Extract 5.2:** A sample of the candidate's correct responses in question 5

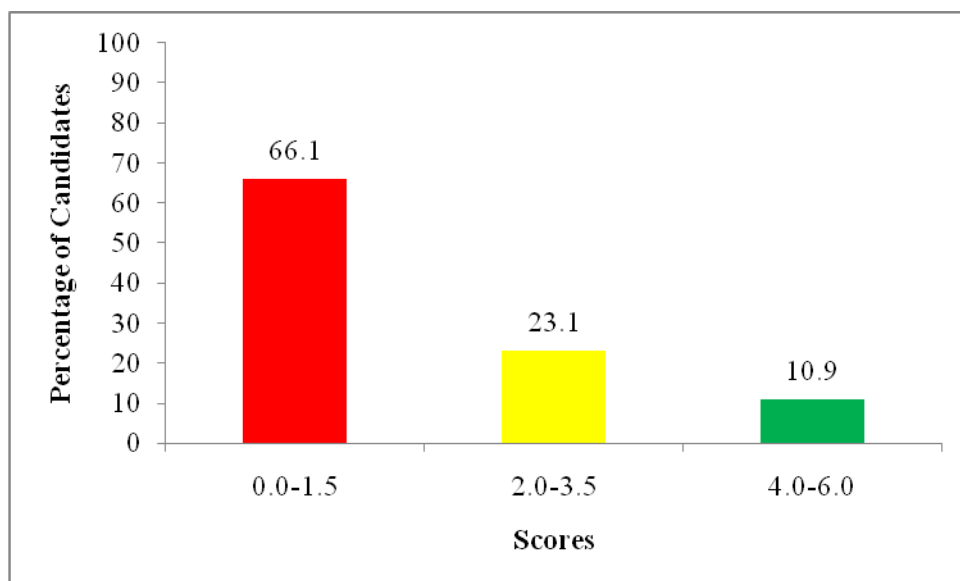
Extract 5.2 shows a response of a candidate who correctly sketched the line segments and correctly labelled the corresponding, alternate and vertically

opposite angles and also used the concept of similarity correctly to find the area of the triangle PQR.

## 2.6 Question 6: Rates and Variations

In part (a) of this question, it was given that: Mr. Ogango from Kenya visited Tanzania. He had 5,000 Kenya Shillings (Kshs) and wanted to change the money into US dollars. If US dollar 1 was equivalent to 2500 Tanzania Shillings (Tshs) and Kshs 1 was equivalent to Tshs 20. The candidates were required to determine how much US dollars he got. In part (b), it was given that, a gardener has found that the time  $t$  to cut the grass on a square field varies directly as the square of its length  $L$  and inversely as the number of men  $m$  doing that job. If 5 men cut grass on a field of side 50 m in 3 hours, how many more men are required to cut grass on a field of side 100 m in 5 hours? Assume that the men are working on the same pace.

This question was attempted by 255,484 (60.2%) candidates, out of which only 86,649 (33.9%) candidates scored from 2 to 6 marks showing that the performance was average as represented in Figure 6.



**Figure 6:** *The summary of candidates' performance in question 6*

Further analysis shows that 27,755 (10.9%) candidates scored from 4 to 6 marks, out of which 3,195 (1.3%) candidates scored full marks.

The candidates were able to correctly convert 5,000 Kenya Shillings (Kshs) into US dollars as instructed. They were also able to formulate and apply the correct joint variation equation and eventually ended up with the correct answer. A sample answer from one of those candidates is illustrated in Extract 6.1.

(a)	Data:
	1 US dollar = 2500 Tshs
	1 Ksh = 20 Tshs
	5000 Ksh = ? US dollars
	1 Ksh = 20 Tshs
	5000 Ksh = x
	$x = \frac{5000 \text{ Ksh} \times 20 \text{ Tshs}}{1 \text{ Ksh}}$
	$x = 100000 \text{ Tshs}$
	1 US dollar = 2500 Tshs
	$x = 100000 \text{ Tshs}$
	$x = \frac{100000 \times 1}{2500} \text{ US dollar}$
	$x = 40 \text{ US dollars}$
	∴ 40 US dollars

6	(b)	Data:
		$t \propto L^2$
		$t \propto \frac{1}{m}$
		$t \propto \frac{L^2}{m}$
		$t = \frac{KL^2}{m}$
		$\frac{KL^2}{L^2} = \frac{tm}{L^2}$
		$k = \frac{tm}{L^2}$
		$t_1 = 3 \quad m_1 = 5 \quad L_1 = 50$
		$t_2 = 5 \quad m_2 = \quad L_2 = 100$
		$\frac{t_1 m_1}{L_1^2} = \frac{t_2 m_2}{L_2^2}$
		$\frac{3 \times 5}{50^2} = \frac{5 \times m}{100^2}$
		$\frac{15}{2500} = \frac{5m}{10000}$
		$\frac{5m \times 2500}{5 \times 2500} = \frac{15 \times 10000}{5 \times 2500}$
		$m = 12$
		$12 - 5 = 7$
		$\therefore 7$ more men are required

**Extract 6.1:** A sample of the candidate's correct responses in question 6

Extract 6.1 represents a sample response of a candidate who correctly converted the Kenya Shillings into US dollars and correctly formulated joint variation equation and applied it solve the given problem.

In spite of the average performance, 168,835 (66.1%) candidates scored from 0 to 1.5 marks, out of which 144,673 (56.6%) candidates scored 0 mark.

In part (a), most candidates failed to convert the currencies leading to an incorrect solution. For example, they performed incorrect calculations like;

$$\text{Kshs } 1 = \text{Tshs } 20$$

$$\text{Kshs } 500 = ?$$

$$\frac{500}{1} \times 20 \equiv \text{Tshs } 10,000$$

$$\text{Thus; US dollar } 1 \equiv \text{Tshs } 2500$$

$$? \equiv \text{Tshs } 10,000$$

$$\Rightarrow \frac{10,000 \times 1}{2,500} = \text{US dollar } 4 \text{ US\$ which is an incorrect answer.}$$

There were also few candidates who were not able to relate different currencies from one to another leading to an incorrect solution. For example, there were incorrect calculations like;

$$\text{USD } 1 \equiv \text{TShs } 2500$$

$$x \equiv 20 \text{ Tshs} \Rightarrow \frac{20}{2500} \times 1 \text{ US dollar} = 0.004 \text{ US dollar}$$

Thus,  $500 \text{ Kshs} \times 0.004 \text{ US dollar} = 2 \text{ US dollar}$  and concluded that  $500 \text{ Kshs} \equiv 2 \text{ US dollars}$ .

In part (b), the majority of the candidates failed to correctly formulate the joint variation equation. They wrote incorrect equations like:  $t = \frac{kl}{m}$  or

$$k = \frac{tm}{l}, \text{ hence they ended up with incorrect answer. Other candidates}$$

confused the term “*square*” and “*square root*” when writing the variation equation as they formulated incorrect equations like  $t = \frac{k\sqrt{l}}{m}$  instead of

$$t = \frac{kl^2}{m}. \text{ Others were able to formulate the required joint variation equation}$$

and substituted correctly but they failed to make correct computations. Extract 6.2 shows a sample solution of one the candidates who failed to answer this question correctly.

6	(a)	<u>Solution</u> Onyango had 5000 Kshs $1 \text{ US} = 2500 \text{ Tsh}$ $1 \text{ Ksh} = 20 \text{ Tsh}$
		$\frac{4}{1 \text{ Ksh} = 20 \text{ Tsh}}$ $5000 \text{ Ksh} = ?$ $\frac{5000 \text{ Ksh} \times 20 \text{ Tsh}}{1 \text{ Ksh}}$ $200000 \text{ Tsh}$
		$1 \text{ US} = 2500 \text{ Tsh}$ $? = 200,000 \text{ Tsh}$ $\frac{1 \text{ US} \times 200,000 \text{ Tsh}}{2500 \text{ Tsh}}$ $\frac{1 \text{ US} \times 2000}{25}$ $80 \text{ US}$
		$\therefore$ He got 80 US dollars
6	(b)	<u>Soln</u> $t \propto \frac{\sqrt{L}}{M}$ $t = \frac{k\sqrt{L}}{M}$ $\frac{t_m}{\sqrt{L}} = k$
		Given: $t_1 = 3 \text{ hrs}$ , $L_1 = 50 \text{ m}$ , $M_1 = 5$ . $t_2 = 5 \text{ hrs}$ , $L_2 = 100 \text{ m}$ , $M_2 = ?$ .
		$M = \frac{k\sqrt{L}}{t}$ $k = \frac{3 \times 5}{\sqrt{50}} = \frac{15}{\sqrt{50}}$ $\frac{3 \times 5}{\sqrt{50}} = \frac{5 \times M}{\sqrt{100}}$ $\frac{3 \times 5}{\sqrt{50}} \times \frac{5 \text{ m}}{10}$ $5M\sqrt{50} = 150$ $5M \times \sqrt{2} = 150$ $25M\sqrt{2} = 150$ $25M \times 1.41 = 150$ $35M = 150$ $M = \frac{150}{35}$ $M = 4 \text{ men.}$
		$\therefore$ 4 men will work on the field.

**Extract 6.2:** A sample of the candidate's incorrect responses in question 6

In Extract 6.2 the candidate failed to perform the correct currency conversion and formulate the required joint variation equation from the given word problem.

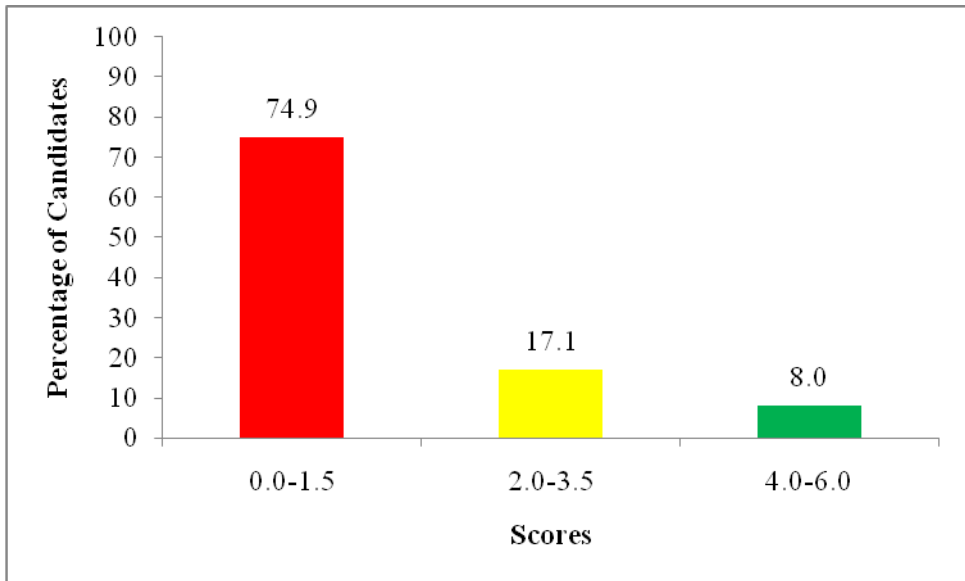
## 2.7 Question 7: Ratio, Profit and Loss and Accounts

This question had two parts, (a) and (b). In part (a), the candidates were given that; Misumbwi, Shuma and Kiyando contributed 770,000, 560,000 and 1,050,000 shillings respectively to start a business. They were required to find the ratio of their contribution in its simplest form. In part (b), the candidates were supposed to use the following trial balance to prepare trading, profit and loss account of Mr. Rwaichi as at 31<sup>st</sup> December, 2015.

### TRIAL BALANCE AS AT 31<sup>ST</sup> DECEMBER, 2015

Account name	Dr	Cr
Cash	1,750,000	
Capital		2,500,000
Purchases	2,300,000	
Rent	200,000	
Furniture	550,000	
Shelves	350,000	
Sales		3,000,000
Salary	250,000	
Wages	100,000	
	<b>5,500,000</b>	<b>5,500,000</b>

This question was attempted by 288,598 (68.0%) candidates, out of which only 72,576 (25.1%) candidates scored from 2 to 6 marks showing that the performance was weak. Figure 7 shows the summary of the candidates' performance in this question.



**Figure 7:** *The summary of candidates' performance in question 7*

Further analysis shows that 216,022 (74.9%) candidates scored from 0 to 1.5 marks, out of which 129,612 (44.9%) candidates scored 0 mark.

In part (a), the candidates failed to divide the given contributions into proportional parts and subsequently were unable to express the ratio in its simplest form. Others divided throughout by the smallest share instead of dividing each share by their common factor. For example, they wrote  $\frac{770,000}{560,000}$ ,  $\frac{560,000}{560,000}$  and  $\frac{1,050,000}{560,000}$  which is incorrect approach.

Others candidates considered the ratio of the three people as 1:2:3 and then computed the total of the ratio as  $1+2+3=6$  and converted 1:2:3 into shillings as follow;  $\frac{1}{6} \times 2,380,000 = 396,666.6667$ ,

$\frac{2}{6} \times 2,380,000 = 793,333.3333$ ,  $\frac{3}{6} \times 2,380,000 = 1,190,000$  then they considered :396,666.6667, 793,333.3333 and 1,190,000 as the ratio. Also there were some candidates who incorrectly divided the total amount by each individual contribution as follows;

Misumbwi:  $\frac{2,380,000}{770,000} = 3.09 \approx 3.1$

Shuma:  $\frac{2,380,000}{560,000} = 4.25 \approx 4.3$

Kiyando:  $\frac{2,380,000}{1,050,000} = 2.26 \approx 2.3$  which is incorrect.

This shows that the candidates lacked knowledge on the concept and application of ratios.

In part (b), the majority of candidates were unable to ascertain the gross profit and loss account accordingly by using the given trial balance. Some candidates prepared cash accounts while others committed errors when posting the entries in the debit side and credit side of the trading, profit and loss account. For example, the expenses were posted in the credit side instead of debit side. Extract 7.1 shows a sample solution of one of the candidates who performed poorly in this question.

7. (a)	Total Contributions = 2,380,000.
	Hera,
	2,380,000 for Msimbani
	770,000 = 3.
	2,380,000 for Shana
	560,000 = 4
	2,380,000 for Kiyando.
	1,050,000 = 1
	= 3:4:1.
	Therefore ratio for Hera Contributions = 3:4:1.

b TRADING, PROFIT AND LOSS ACCOUNT OF AS AT 31 <sup>st</sup> December 2015				
	DR	Amount	CR	Amount
	Purchases Capital	2,300,000	Sales	3,000,000
	Add: Purchases	2,300,000	Net Sales (Turnover)	3,000,000
	Cost of Goods for Sale	4,800,000		
	Cost of Goods Sold	4,800,000		
			Gross loss c/d	1,200,000
		4,800,000		4,800,000
	Gross loss b/d	1,200,000		
	Cash	1,750,000		
	Rent	200,000		
	Furniture	550,000		
	Shelves	350,000		
	Salary	250,000		
	Wages	100,000	Net loss c/d	4,400,000
		4,400,000		4,400,000

**Extract 7.1:** A sample of the candidate's incorrect responses in question 7

Extract 7.1 shows a candidate who failed to divide the amount of contribution in the simplest form and lacked the knowledge on how to prepare the trading, profit and loss account.

Though the performance the performance was weak, 23,213 (8.0%) candidates scored from 4 to 6 marks, out of which 2,600 (0.9%) candidates scored full marks.

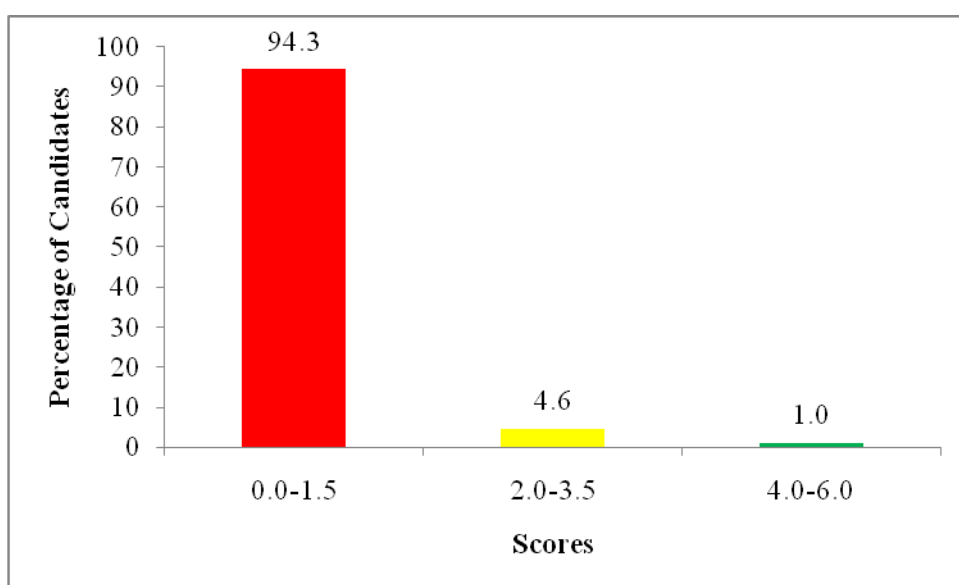
The candidates were able to correctly find the ratio of the given shares in its simplest form and prepared the trading, profit and loss account from the trial balance provided. Extract 7.2 shows the work of one of a candidate who answered the question correctly.



## 2.8 Question 8: Sequences and Series

This question had parts (a) and (b). In part (a), the candidates were instructed to find: (i) the value of  $x$  and (ii) the geometric mean, given that 49,  $x$  and 81 are consecutive terms of a geometric progression. Part (b) stated that: A wall is in the shape of a trapezium. The first level of the wall is made up of 50 bricks whereas the top level has 14 bricks. The candidates were required to determine the number of: (i) the levels of the bricks and (ii) the bricks used to make the wall, if the levels differ from each other by 4 bricks.

This question was attempted by 157,422 (37.1%) candidates, out of which only 8,904 (5.7%) candidates scored from 2 to 6 marks showing that the performance was weak. Figure 8 summarises the candidates' performance in this question.



**Figure 8:** *The summary of candidates' performance in question 8*

According to the analysis the candidates were unable to answer the question correctly due to various reasons.

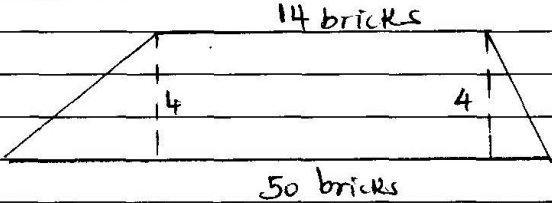
Further analysis shows that 148,518 (94.3%) candidates scored from 0 to 1.5 marks, out of which 120,787 (76.7%) candidates scored 0 mark.

In part (a) (i), the candidates were unable to calculate the value of  $x$  by using the formula for finding the geometric mean. Some candidates used the formula for finding the arithmetic mean as they started with steps like:  $x - 49 = d$  or  $81 - x = d$  leading to  $x - 49 = 81 - x$ . Instead, they were supposed to write  $\frac{x}{49} = \frac{81}{x}$ , implying that  $x = \sqrt{49 \times 81} = 7 \times 9 = 63$ . In part (a) (ii), some candidates incorrectly calculated the geometric mean as;  $M = \frac{G_1 + G_2 + G_3}{3} = \frac{49 + 63 + 81}{3} = \frac{193}{3} = 64.3$ , instead of using the correct formula, that is  $M = \sqrt{ab}$  such that  $M = \sqrt{49 \times 81} = 63$ . Those candidates lacked knowledge on the application of the formula for finding the geometric mean to solve problems.

In part (b), the candidates were unable to understand the requirement of the question which required them to apply knowledge of Arithmetic Progression (AP) but the majority applied incorrect formulae like the formula for the  $n$ th term of geometric progression. Some candidates applied the formula for finding the area of trapezium, that is  $\text{area} = \frac{1}{2}h[a + b]$  but they failed to recognize the correct value of height, since they used 4 cm as the height and wrote  $\text{area} = \frac{1}{2} \times 4 \times (14 + 50)$  instead of writing  $\text{area} = \frac{1}{2} \times 10 \times (14 + 50)$ , a step that could enable them to arrive at the correct answer. However, the majority of candidates did not know that they were supposed to apply the formulae for the  $n$ th term of AP, that is  $A_n = A_1 + (n - 1)d$  and substitute the values as  $A_1 = 50$ ,  $A_n = 14$  and  $d = -4$  to get  $n = h = 10$  as the number of levels of the bricks, then compute the number of bricks required by using the formula,  $\text{number of bricks} = \frac{1}{2}h[a + b] = \frac{1}{2} \times 10 \times (14 + 50) = 320$ .

However, others considered the common difference as  $d = 4$  instead of  $d = -4$  which led to an incorrect answer. Extract 8.1 represents the work of one the candidates who failed to answer this question correctly.

8. a/ii)	$G_1 \rightarrow A_1$
	$G_2 \rightarrow A_2$
	$G_3 \rightarrow A_3$
	$G_1 = G_2$
	$G_2 \quad G_3$
	$A_1 = A_1 + d$
	$A_1 + d \quad A_1 + 2d$
	$A_1(A_1 + 2d) = (A_1 + d)(A_1 + d)$
	$A_1^2 + A_1 \cdot 2d = A_1^2 + A_1 \cdot d + A_1 \cdot d + d^2$
	$A_1^2 + A_1 \cdot 2d = A_1^2 + A_1 \cdot d + A_1 \cdot d + d^2$
	$= A_1 \cdot d + A_1 \cdot d + d^2$
	$A_1 = d^2$
	$\sqrt{49} = \sqrt{d^2}$
	$7 = d$
	$G_2 \rightarrow A_2 \rightarrow A_1 + d$
	$= 49 + 7$
	$= 56$
	$\therefore$ The Value of $x$ is 56
ii/	$G_1 + G_2 + G_3 = G_{\text{mean}}$
	3
	$49 + 56 + 81 = 186$
	186
	3
	$= 62$
	$\therefore$ The Geometric mean is 62

8b)	<u>Solution.</u>
(i)	
	<p>i) The level of the bricks. = Area of trapezium</p> $A = \frac{1}{2} h (a+b)$ $A = \frac{1}{2} \times 4 (14+50)$ $A = 2 \times 64$ $A = 128 \text{ levels. } \therefore \text{There are 128 levels}$
	<p>ii) The bricks used to make the wall</p> <p><u>Solution.</u></p> $= (\text{Number of levels}) (\text{Sum of bricks of lower and upper levels})$ $= 128 \text{ levels} \times 14+50$ $= 128 \times 64$ $= 128$ $\times 64$ $\hline 512$ $768$ $\hline 8192$ <p><math>\therefore</math> There are 8192 bricks.</p>

**Extract 8.1:** A sample of the candidate's incorrect responses in question 8

Extract 8.1 reveals that the candidate failed to differentiate between arithmetic progression and geometric progression. He/she also failed to identify the correct value of  $h$  when calculating the number of levels of bricks.

Although the performance was generally weak, 1,633 (1.0%) candidates scored from 4 to 6 marks, out of which 301 (0.2%) candidates scored full marks.

The candidates applied the correct formulae and methods when finding the geometric mean and solving the given word problem as shown in Extract 8.2.

8. a. i. 49,  $x$ , 81

$$r = \frac{x}{49} = \frac{81}{x}$$

$$\frac{x}{49} = \frac{81}{x}$$

$$x^2 = 49 \times 81$$

8. a.  $x = \sqrt{49 \times 81}$

$$x = 7 \times 9$$

$$x = 63$$

$\therefore$  The value of  $x = 63$

ii. Geometric mean

$$= \pm \sqrt{ac}$$

$$= \pm \sqrt{49 \times 81}$$

$$= \pm 63$$

$\therefore$  Geometric mean is 63 or -63

b. > Data.

Level<sub>1</sub> = 50 bricks

Level<sub>2</sub> =  $L_1 - 4$  bricks

Level<sub>3</sub> =  $L_2 - 4$  bricks

Last level = 14 bricks

It is an Arithmetic Progression

$$A_1 = 50, A_n = 14, d = -4$$

i. Find  $n$

$$\text{From } A_n = A_1 + (n-1)d$$

$$14 = 50 + (n-1)(-4)$$

$$14 = 50 - 4n + 4$$

$$14 = 54 - 4n$$

$$4n = 54 - 14$$

$$\frac{4n}{4} = \frac{40}{4}$$

$$n = 10 \text{ levels}$$

$\therefore$  There are 10 levels to make the wall.

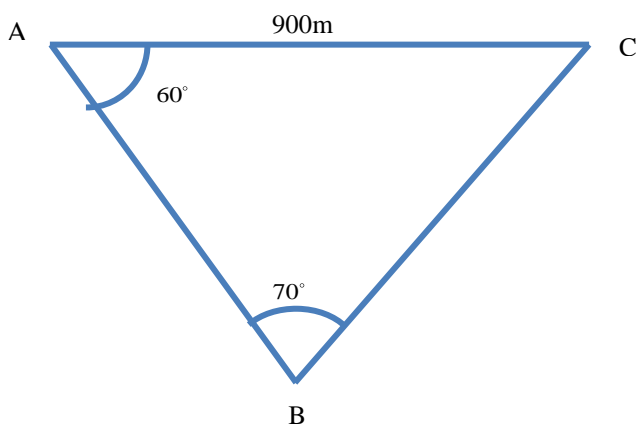
8	b>ii. Sum
	$S_n = n/2 (A_1 + A_n)$
	$= 10/2 \times (50 + 14)$
	$= 25 \times 64$
	$= 320 \text{ bricks}$
	$\therefore 320 \text{ bricks were used to make the wall.}$

**Extract 8.2:** A sample of the candidate's correct responses in question 8

In Extract 8.2 the candidate was able to apply the correct formulae and methods to answer all parts of the question. This indicates that the candidate had adequate knowledge and skills on application of geometric mean formula and the formulae for the  $n$ th terms of AP and the sum of the first  $n$  terms in solving the given problems.

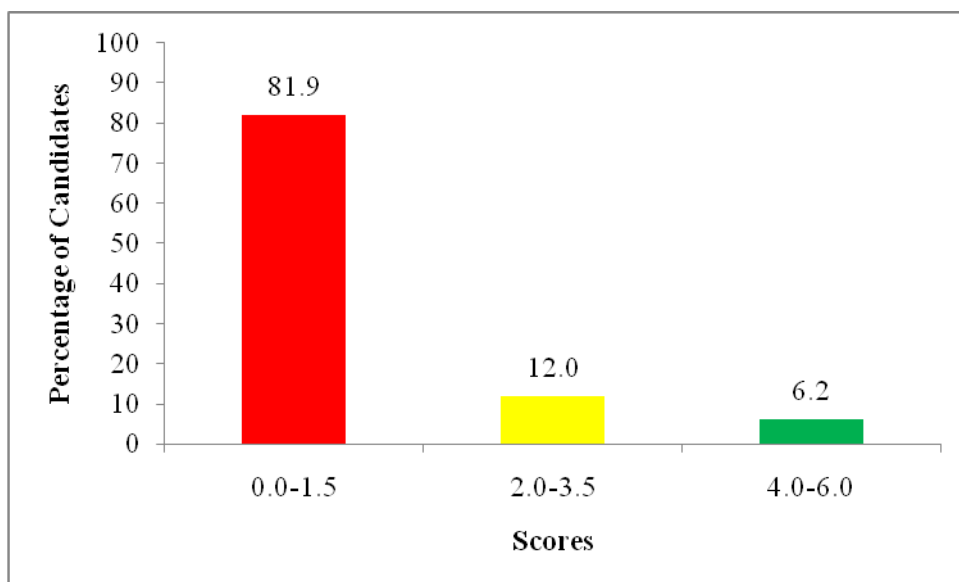
## 2.9 Question 9: Trigonometry and Pythagoras Theorem

This question had parts (a) and (b). In part (a), the candidates were given the following diagram showing the location of the houses A, B and C. They were required to determine how far is house A from house B, giving the answers to the nearest metre.



In part (b), they were required to find the length of the rectangular frame made of wooden bars such that the diagonal of the frame is 25 cm long and its width is 15 cm.

This question was attempted by 191,753 (45.2%) candidates, out of which only 34,797 (18.1%) candidates scored from 2 to 6 marks showing that the performance was weak. Figure 9 shows the candidates' performance in this question.



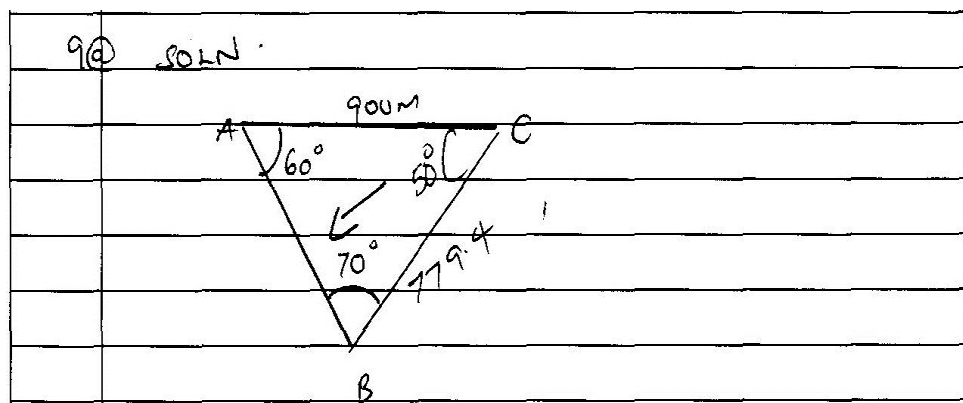
**Figure 9:** The summary of candidates' performance in question 9

Further analysis shows that 156,956 (81.9%) candidates scored from 0 to 1.5 marks, out of which 129,617 (67.6%) candidates scored 0 mark.

In part (a), some candidates were able to correctly apply sine rule  $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c}$  and substituted correctly  $\frac{\sin 50^\circ}{a} = \frac{\sin 70^\circ}{b}$  but failed to evaluate  $\sin 50^\circ$  and  $\sin 70^\circ$  by using mathematical tables, as a result they either got incorrect answers or failed to proceed. Further analysis indicates that, other candidates failed to know the proper method to use and hence, as a result they just measured the length  $\overline{AB}$  by using a ruler instead of applying the sine rule or cosine rule. Others considered the triangle  $ABC$  as a right angled triangle and applied the trigonometric ratio of

$\sin 60^\circ = \frac{\overline{AB}}{900}$  or  $\sin 50^\circ = \frac{\overline{AB}}{900}$  when finding the length  $\overline{AB}$  and ended with incorrect answers.

In part (b), some candidates sketched properly the diagram of the rectangular frame and applied correctly Pythagoras theorem  $c^2 = a^2 + b^2$  but they failed to perform the arithmetic operation correctly to get the final answer. For example, they wrote incorrect steps like  $25^2 = a^2 + 15^2$  such that  $a^2 = 625 - 125 = 500$  instead of  $a^2 = 625 - 225 = 400$  and got  $a = \sqrt{500} = 20.236$  cm which is an incorrect answer instead of 20cm which is the correct answer. But, few of them applied improper method to find the length of a rectangular frame as;  $A = l \times w$  instead of using Pythagoras theorem  $c^2 = a^2 + b^2$  and hence they ended up with the wrong answer. Extract 9.1 shows a sample solution of a candidate who failed to answer this question correctly.



9@ SOLN

$$\sin 60^\circ = \frac{\text{Opp}}{X 900}$$

$$\text{Opp} = \sqrt{3} \times 900$$

$$= \sqrt{3} \times 450$$

$$= \sqrt{3} \times 15\sqrt{2}$$

$$\text{Opp} = 0.866 \times 900$$

$$= 86.6 \times 9.$$

$$= 779.4$$

$$60^\circ + 70^\circ + x = 180^\circ \text{ (interior angle of a triangle)}$$

$$130^\circ + x = 180^\circ$$

$$180^\circ - 130^\circ = x$$

$$x = 50^\circ$$

$$\sin 50^\circ = \frac{\text{Opp}}{X 900}$$

$$\text{Opp} = \sin 50^\circ \times 900$$

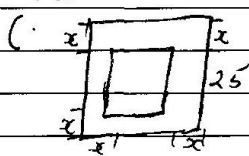
$$= 0.7660 \times 900$$

$$= 76.6 \times 9.$$

$$= 689.4$$

$$\therefore \overline{AB} = 689.4 \text{ m}$$

$$\overline{AB} = \underline{689.4 \text{ m.}}$$

9	b) l x w.
	
	$25 + 2x = \text{Length.}$
	$15 + 2x = \text{width.}$
	$(25 + 2x)(15 + 2x) = 0$
	$375 + 50x + 30x + 4x^2 = 0$
	$\frac{375}{4} + \frac{80x}{4} + \frac{4x^2}{4} = 0$
	$93.5 + 20x + x^2 = 0$
	$x^2 + 20x + 93.5 = 0$
	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
	$\frac{-20 \pm \sqrt{400 - 4 \times 1 \times 93.5}}{2 \times 1}$
	$\frac{-20 \pm \sqrt{400 - 374}}{2}$
	$\frac{-20 \pm \sqrt{126}}{2}$
	$\frac{-20 \pm 1.57}{2}$

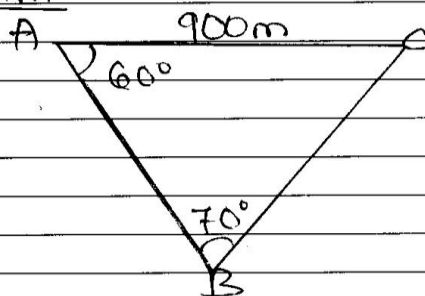
**Extract 9.1:** A sample of the candidate's incorrect responses in question 9

Extract 9.1 shows the work of a candidate who failed to apply sine rule to find the required length in the given triangle and used the given dimensions of the frame incorrectly when finding the length of the frame.

Regardless the weak performance, 11,818 (6.2%) candidates scored from 4 to 6 marks, out of which only 3,117 (1.6%) candidates scored full marks.

The candidates were able to correctly apply the sine rule and Pythagoras theorem to get the required length in part (a) and length of the frame in part (b) as shown in Extract 9.2

Q. (a.) Soln:



From,

$$60^\circ + 70^\circ + \angle C = 180^\circ$$

$$130^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 130^\circ$$

$$\angle C = 50^\circ$$

Thus,

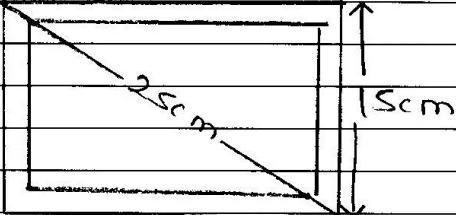
$$\frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 70^\circ}{900m} = \frac{\sin 50^\circ}{AB}$$

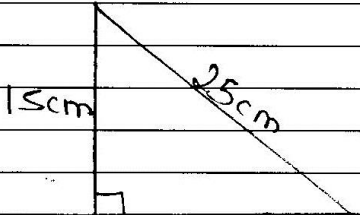
$$0.9397 = \frac{0.7660}{AB}$$

$$AB = \frac{76.6 \times 9m}{0.9397} = 733.6m$$

9. (b) soln:



Diagonal of frame = 25cm.  
width = 15cm.  
Length of frame = ?



From;                      Length.  
Pythagoras Theorem;  
 $a^2 + b^2 = c^2$   
 $(15)^2 + (L)^2 = (25)^2$   
 $225 + L^2 = 625$   
 $L^2 = 625 - 225$   
 $\sqrt{L^2} = \sqrt{400}$   
 $L = 20 \text{ cm}$

∴ The length of the frame is 20cm.

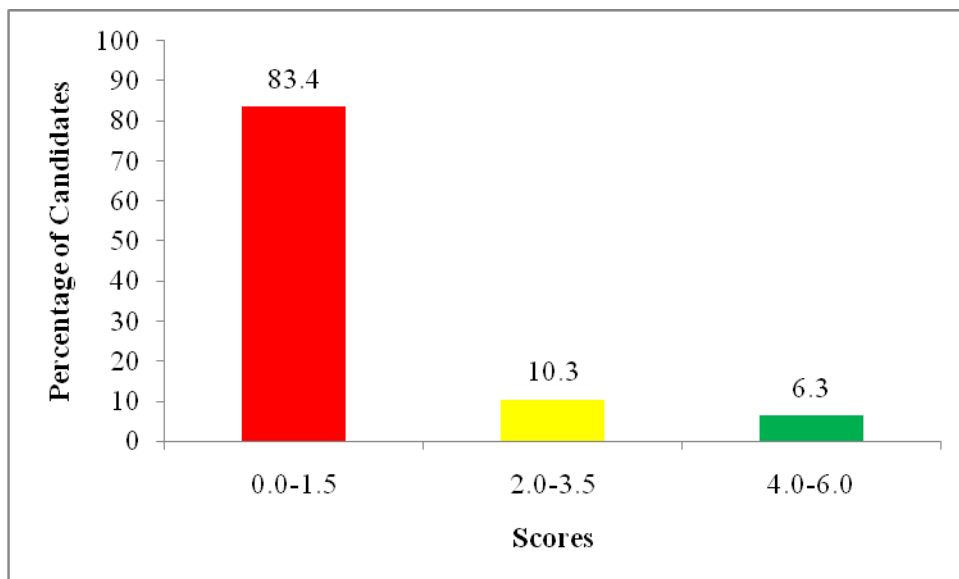
**Extract 9.2:** A sample of the candidate's correct responses in question 9

Extract 9.2 shows that the candidate was able to correctly apply the sine rule and Pythagoras theorem to solve the given problem.

## 2.10 Question 10: Algebra and Quadratic Equations

This question had parts (a) and (b). In part (a), the candidates were instructed to factorize the quadratic expression  $3x^2 - 11x - 20$  by splitting the middle term. In part (b), they were required to solve the equation  $2x^2 + 3x - 5 = 0$  by completing the square.

This question was attempted by 245,867 (57.9%) candidates, out of which 40,810 (16.6%) candidates scored from 2 to 6 marks indicating that the performance was weak. Figure 10 summarises the candidates' performance in this question.



**Figure 10:** *The summary of candidates' performance in question 10*

Further analysis shows that 205,057 (83.4%) candidates scored from 0 to 1.5 marks, out of which 177,947 (72.4%) candidates scored 0 mark.

In part (a), some candidates treated the expression  $3x^2 - 11x - 20$  as an equation as they wrote  $3x^2 - 11x - 20 = 0$  from which they solved for  $x$ , instead of factorizing  $3x^2 - 11x - 20$  to get  $(3x + 4)(x - 5)$  as the required solution. Other candidates failed to determine the correct factors for  $-11x$  as they wrote  $-11x$  as  $15x - 4x$  instead of  $4x - 15x$  thus giving the incorrect solution.

In part (b), most of the candidates lacked the knowledge and skills on how to solve the given quadratic equation by completing the square. Mostly, they ignored the signs " $\pm$ " when computing the square root. Some used the sign "+" only or sign "-" only as they wrote;  $x + 4 = \sqrt{\frac{49}{16}}$  and ended up

with only one solution, instead of  $x+4=\pm\sqrt{\frac{49}{16}}$ . There were few candidates who solved the given equation by using factorization method or quadratic formula contrary to the given instruction which required them to use the method of completing the square. Extract 10.1 shows the work of one of the candidates who failed to follow the given instructions when solving this question.

10a)	$(x - \frac{11}{6})^2 = \pm \sqrt{36}$
	$\sqrt{36}$
	$x - \frac{11}{6} = \pm \sqrt{36}$
	$\frac{6}{6}$
	$x = \frac{11 \pm \sqrt{36}}{6}$
	$\frac{6}{6}$
	$\therefore x = \frac{11 + \sqrt{36}}{6} \text{ or } \frac{11 - \sqrt{36}}{6}$
	$\frac{6}{6}$
10b)	$2x^2 + 3x - 5 = 0$
	Product = -10
	Sum = 3
	factors (5, -2)
	$2x^2 - 2x + 5x - 5 = 0$
	$2x(x-1) + 5(x-1) = 0$
	$(2x+5)(x-1) = 0$
	$2x+5 = 0$
	$2x = -5$
	$x = \frac{-5}{2}$
	$x = 1$
	$\therefore x = \frac{-5}{2} \text{ or } 1$
	$\frac{2}{2}$

**Extract 10.1:** A sample of the candidate's incorrect responses in question 10

As seen in Extract 10.1 the candidate confused between the methods of completing the square that was supposed to be applied in part (b) and factorization method which was to be used in part (a).

Although the performance was generally weak, 15,402 (6.3%) candidates scored from 4 to 6 marks, out of which 6,379 (2.6%) candidates scored full marks.

The candidates were able to factorize correctly the quadratic expression and managed to solve the given quadratic equation as illustrated in the Extract 10.2.

10	a) $3x^2 - 11x - 20$
	$9C = 3x - 20 = -60$
	Factors of 60 = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}
	Factors = {4, -15}
	$3x^2 - 15x + 4x - 20 = 0$
	$3x(x - 5) + 4(x - 5)$
	$\therefore (3x + 4)(x - 5)$
	b) $2x^2 + 3x - 5 = 0$
	$\frac{2x^2}{2} + \frac{3x}{2} = \frac{5}{2}$
	$x^2 + \frac{3}{2}x = \frac{5}{2}$
	$(x + \frac{3}{4})^2 = \frac{5}{2} + (\frac{3}{4})^2$
	$= \frac{5}{2} + \frac{9}{16}$
	$= \frac{40 + 9}{16}$
	$\sqrt{(x + \frac{3}{4})^2} = \sqrt{\frac{49}{16}}$
	$x + \frac{3}{4} = \sqrt{\frac{49}{16}}$
	$x = -\frac{3}{4} \pm \frac{7}{4}$
	$= \frac{4}{4} \text{ or } -\frac{10}{4}$
	$\therefore x = 1 \text{ or } -\frac{5}{2}$

**Extract 10.2:** A sample of the candidate's correct responses in question 10

In Extract 10.2 the candidate was able to correctly factorize the given quadratic expression and applied the method of completing the square when solving the given quadratic equation.

## 2.11 Question 11: Statistics and Circles

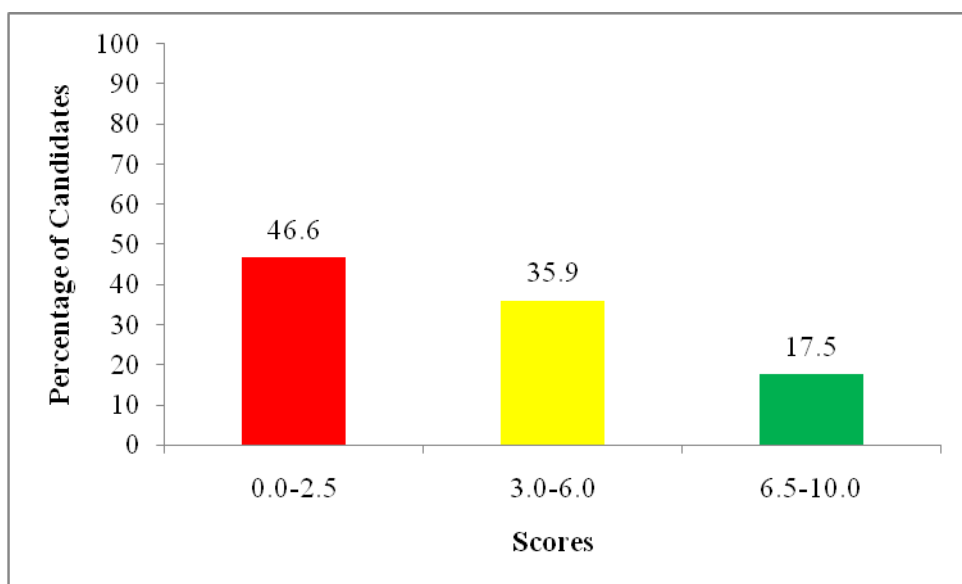
The candidates were given that; the number of patients who attended maternity clinic daily in the month of June 2017 in a certain village was recorded as follows:

52	61	42	27	38	44	56	36	73	22
41	48	77	30	46	43	72	63	43	76
47	53	38	55	60	51	47	58	33	37

In part (a), the candidates were required to make a frequency distribution table by grouping the given data in the class intervals: 20–29, 30–39, 40–49,...

By using the frequency distribution table obtained in part (a), they were required to (b) calculate the mean number of patients per day and (c) construct a pie chart.

This question was attempted by 347,453 (81.8%) candidates, out of which only 185,644 (53.4%) candidates scored from 3 to 10 marks showing that the performance was average. Figure 11 shows the candidates' performance in this question.

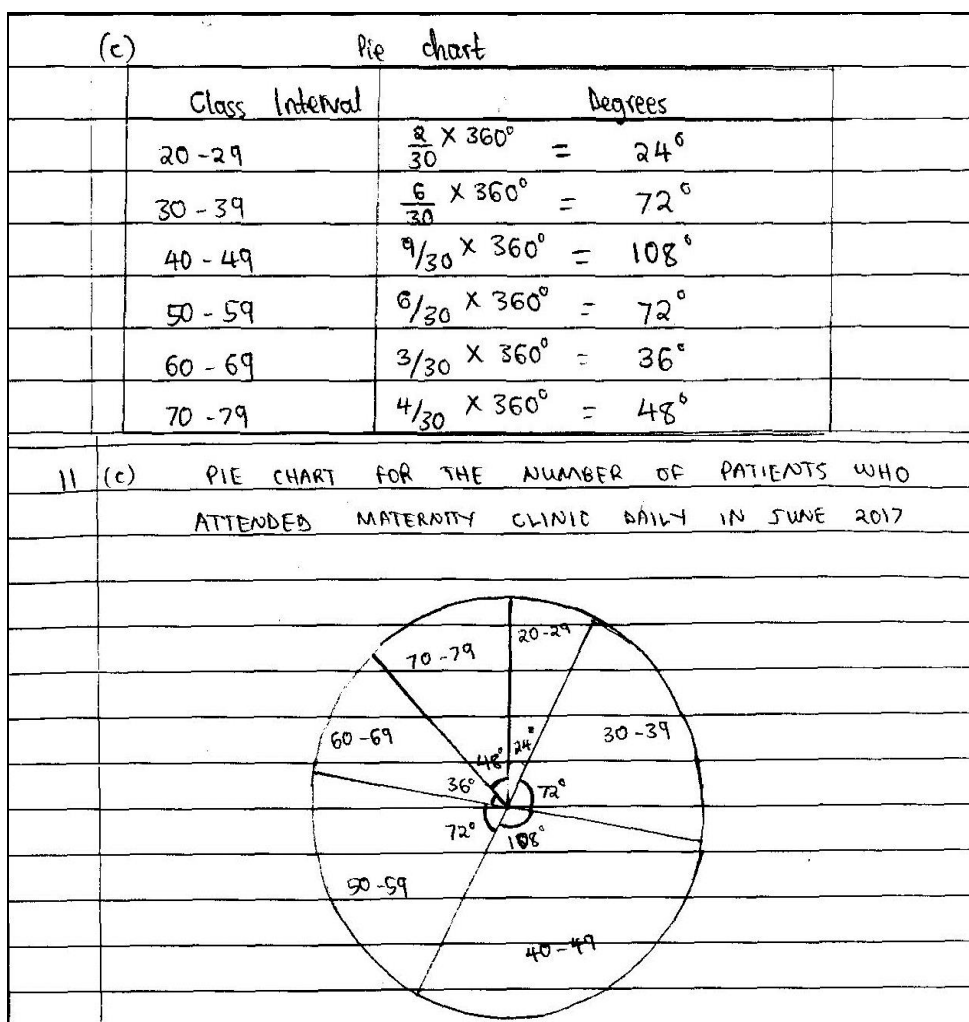


**Figure 11:** *The summary of candidates' performance in question 11*

Further analysis shows that 205,057 (83.4%) candidates scored from 6.5 to 10 marks, out of which 19,283 (5.5%) candidates scored full marks.

The candidates were able to correctly make the frequency distribution table from the given data and used it to: calculate the mean and construct the pie chart as illustrated on a sample solution of one of the candidate in Extract 11.1.

11	Soln:					
	(a) Frequency distribution table					
	Class interval	tally	f	x	fx	
	20-29		2	24.5	49.0	
	30-39	###	6	34.5	207.0	
	40-49	###	9	44.5	400.5	
	50-59	###	6	54.5	327.0	
	60-69		3	64.5	193.5	
	70-79		4	74.5	298.0	
			N=30		$\Sigma=1475.0$	
	(b) Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$					
	$= \frac{1475}{30}$					
	$= 49.17 \approx 49$					
	$\therefore$ 49 patients					



**Extract 11.1:** A sample of the candidate's correct responses in question 11

Extract 11.1 shows the work of a candidate who correctly constructed the frequency distribution table and managed to use it to determine the mean and construct a pie chart.

On the other hand, 161,809 (46.6%) candidates scored from 0 to 2.5 marks, out of which 690,037 (19.9%) candidates scored 0 mark.

In part (a), the candidates failed to make the frequency distribution table in the intervals; 20–29, 30–39, 40–49,... as instructed. Instead, they established wrong intervals while others skipped some intervals, like writing 50–59, 60–69, 70–79,... that does not include the given

intervals. However, other candidates failed to calculate the correct class mark,  $x$ , in each of the class intervals which resulted them to incorrect columns for  $fx$ ,  $fd$ ,  $\sum fx$  and  $\sum fd$ . For example, some of the incorrect class marks noted from the candidates' scripts include: 30.9, 40.9, 50.9, 60.9, 70.9 and 80.9 for the classes 20–29, 30–39, 40–49, 50–59, 60–69 and 70–79. Instead, they were supposed to take the average of the lower and upper class limits, that is,  $\frac{20-29}{2} = 24.5$ ,  $\frac{30-39}{2} = 34.5$ , and so on.

In part (b), the candidates failed to apply the correct the formulae for calculating the mean. For example, they applied incorrect formulae like

$$\bar{x} = \frac{\sum fd}{\sum f} \quad \text{instead of} \quad \bar{x} = A + \frac{\sum fd}{\sum f} \quad \text{as a result they ended up with}$$

incorrect answers. Others were able to recall the formula  $\bar{x} = \frac{\sum fx}{\sum f}$  or

$$\bar{x} = A + \frac{\sum fd}{\sum f} \quad \text{but they failed to get the correct solution as they got}$$

incorrect values of  $fx$ ,  $fd$ ,  $\sum fx$  or  $\sum fd$  due to lack of skills of adding the given frequencies, and inability to multiply a "class mark" or "deviation" by its corresponding frequency.

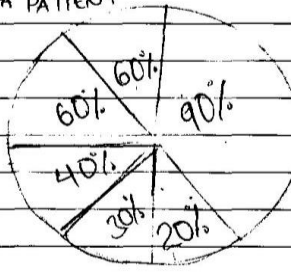
In part (c), the candidates were unable to express the given frequencies as angles in degree measure, which could enable them to draw the pie chart. Some of them changed each individual frequency of each class interval into percentages and used those percentages to draw the pie chart.

For example, they performed as follows;  $\left\{ \frac{2}{10} \times 100\% = 20\% , \right.$

$$\frac{6}{10} \times 100\% = 60\% , \quad \frac{9}{10} \times 100\% = 90\% , \quad \frac{6}{10} \times 100\% = 60\% ,$$

$$\left. \frac{3}{10} \times 100\% = 30\% , \frac{4}{10} \times 100\% = 40\% \right\} , \text{ instead of correctly multiplying the}$$

fraction of each frequency by  $360^\circ$  to get the angle corresponding to that particular frequency. This confirms that the candidates lacked the knowledge of representing frequencies by using pie chart. Extract 11.2 represents the solution of one of the candidates who failed to answer this question correctly.

11. Soln							
C-Interval	Class mark(x)	Tallies	F	fx	$d = x - A$	$Efd$	
20-29	30.9		2	2	-18		
30-39	40.9	1	6	8	-72	-90	
40-49	50.9		9	12	-108		
50-59	60.9	1	6	28	252		
60-69	70.9		3	31	279	801	
70-79	80.9		4	30	270		
N=50							
Mean							
$A + \frac{Efd}{N}$							
A = 9							
$Efd =$							
N =							
$9 + \frac{711}{50} = 14.4$							
$  \begin{array}{r}  9 + 711 = 720 \\  \underline{50} \\  720 \\  \underline{50} \\  14.4  \end{array}  $							
$  \begin{array}{r}  5 \overline{) 72} \\  \underline{-50} \\  22 \\  \underline{-20} \\  20 \\  \underline{-20} \\  0  \end{array}  $							
$\therefore \text{Mean} = 14.4$							
<div> <div> <div>PIE CHART TO SHOW</div> <div>AAA PATIENT</div> </div>  </div>							
<div> <div>From number of frequency</div> <div> <math>2 \times 100\% = 20\%</math>  <math>6 \times 100\% = 60\%</math>  <math>9 \times 100\% = 90\%</math>  <math>6 \times 100\% = 60\%</math>  <math>3 \times 100\% = 30\%</math>  <math>4 \times 100\% = 40\%</math> </div> </div>							

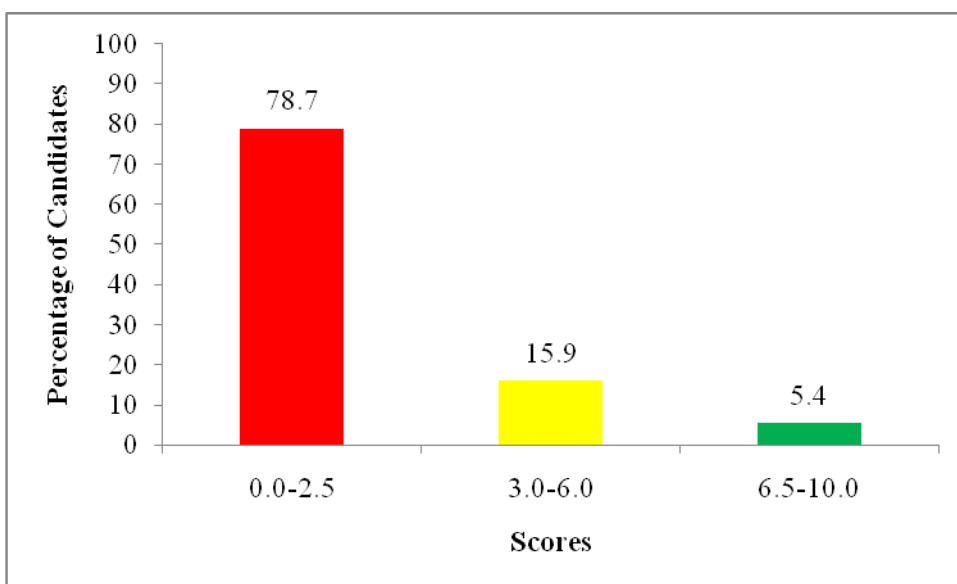
**Extract 11.2:** A sample of the candidate's incorrect responses in question 11

In Extract 11.2 the candidate took the highest frequency 9 to be the assumed mean and failed to find the central angles of each sector in the pie chart. Besides this, the candidate calculated the percentages of frequencies instead of determining the central angles to construct the pie chart.

## 2.12 Question 12: The Earth as a Sphere and Three Dimensional Figures

This question had parts (a), (b) and (c). In part (a), the candidates were required to calculate the speed in km/h of the ship which sailed from Pemba ( $4.5^{\circ}\text{S}, 39.5^{\circ}\text{E}$ ) at 11:30 am and arrived in Dar es salaam ( $7.5^{\circ}\text{S}, 39.5^{\circ}\text{E}$ ) at 13:30 pm, given that  $\pi = \frac{22}{7}$  and  $R_E = 6370 \text{ km}$ . In part (b), they were required to sketch a square pyramid whose base is PQRS while the vertex and centre are at W and N respectively, hence use it to: (i) state the projection of  $\overline{RW}$  on PQRS and (ii) name the angle between  $\overline{WS}$  and the plane PQRS. In part (c), the candidates were required to find the height of a square pyramid whose volume is  $28.2 \text{ cm}^3$  and the sides of its base are 4 cm long, giving the answer correct to one decimal place.

This question was attempted by 150,262 (35.4%) candidates, out of which only 32,011 (21.3%) candidates scored from 3 to 10 marks showing that the performance was weak. Figure 12 shows the summary of the candidates' performance in this question.



**Figure 12:** *The summary of candidates' performance in question 12*

Further analysis shows that 118,251 (78.7%) candidates scored from 0 to 2.5 marks, out of which 79,645 (53.0%) candidates scored 0 mark.

In part (a), some of the candidates were able to recall the correct formula as  $d = \frac{\Delta\theta}{360} \times 2\pi R$  and substituted the correct values of  $\Delta\theta$ ,  $\pi$  and  $R$  but

failed to perform the arithmetic operation. Others confused the formula for the distance along the great circles with that along small circles. For instance, they wrote;  $d = \frac{\Delta\theta}{360} \times 2\pi R \cos \lambda^\circ$  or  $d = \frac{2\pi R \cos \theta}{360}$  instead of

$d = \frac{\Delta\theta}{360} \times 2\pi R$  or  $d = \frac{\Delta\theta}{180} \times \pi R$  as the correct formula. Some of them used

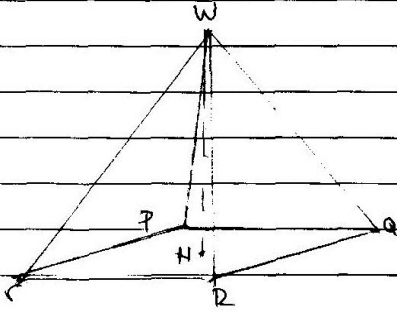
the wrong value of  $\Delta\theta$ , that is, they used  $\Delta\theta$  as  $12^\circ$  or  $39.5^\circ$  instead of  $3^\circ$  and led them to an incorrect value of the distance  $d$  and eventually ended up with wrong value of speed.

In part (b), most of the candidates were unable to correctly sketch the square pyramid PQRS from which they could have stated the projection of W on the plane PQRS, while others drew a rectangular prism and others named the angle between  $\overline{WS}$  and the plane PQRS as: acute,

supplementary or right angles which showed that they lacked knowledge and skills on three dimensional figures.

In part (c), many candidates were unable to write the correct formula for the volume of a square pyramid. For example, there were some candidates who wrote;  $V = \frac{1}{2} \times b \times h$ ,  $V = \frac{1}{3} \pi^2 h$ ,  $V = \pi^2 h$  or  $V = \frac{1}{2} lw$  instead of using the correct formula  $V = \frac{1}{3} \times \text{base area} \times \text{height}$  and this resulted them to an incorrect solution. Some of them had adequate knowledge on writing correctly the formula for finding volume of a square pyramid as  $V = \frac{1}{3} \times \text{base area} \times \text{height}$  and managed to substitute the correct values but failed to perform the arithmetic operation correctly. For instance, they were able to write  $28.2 = \frac{1}{3} \times 16h$  but failed to solve it. Extract 12.1 represents a sample solution of a candidate who failed to answer this question correctly.

12	(a) solution.
	$\alpha = 7.5^\circ$ $\theta = 39.5$
	$\beta = 4.5^\circ$ .
	$S = \frac{(\alpha - \beta) \times 2\pi R \cos \theta}{360} \text{ km}$
	$= \frac{(7.5 - 4.5)}{360} \times \frac{2 \times 22}{7} \times 6370 \text{ km} \times \cos 39.5$
	$= \frac{3.0}{360} \times \frac{44}{7} \times 6370 \times \cos 39.5$
	$= \frac{3}{360} \times 44 \times 910 \times 0.7716 = \frac{30894.8640}{180}$

120)	$= 17.1$
	$\therefore \text{distance} = 17.1 \text{ km}, t = 2 \text{ hrs.}$
	$V = \frac{d}{t} = \frac{17.1 \text{ km}}{2 \text{ hrs.}}$
	$= 8.55 \text{ km/h.}$
	$\therefore \text{The speed of the ship is } 8.55 \text{ km/h.}$
12	<p>(b)</p>  <p>(i) The projection of RW</p> $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin \theta = \frac{NW}{RW}$ <p>when <math>\theta</math> = angle between RW and plane PQR</p> $\theta = \sin^{-1} \left( \frac{NW}{RW} \right)$ <p><math>\therefore</math> Projection is <math>\sin^{-1} \left( \frac{NW}{RW} \right)</math></p>
12	<p>(c) Volume of pyramid = Base area <math>\times</math> height.</p> $28.2 \text{ cm}^3 = 4 \times 4 \times h$ $28.2 \text{ cm}^3 = 16h \text{ cm}^2$ $\frac{28.2 \text{ cm}^3}{16 \text{ cm}^2} = \frac{16h \text{ cm}^2}{16 \text{ cm}^2}$ $\text{height} = 1.7 \text{ cm.}$

**Extract 12.1:** A sample of the candidate's incorrect responses in question 12

In Extract 12.1 the candidate incorrectly applied the formula for finding the distance between two places along parallel of latitudes instead of determining distance along the given meridian. The candidate also applied an incorrect formula for finding volume of a square pyramid.

Contrarily, the analysis shows that 8,058 (5.4%) candidates scored from 6.5 to 10 marks, out of which 363 (0.2%) candidates scored full marks.

The candidates applied the correct formula to find distance between the two places and hence the speed of the ship as well as showing the ability to determine the height of the square pyramid as indicated in Extract 12.2.

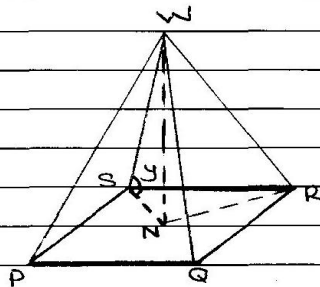
$$\begin{aligned}
 10. \quad a \rightarrow \text{Dist} &= \frac{\Delta\theta}{360^\circ} \times 2\pi R \\
 &= \frac{7.5 - 4.5}{360^\circ} \times 2 \times \frac{22}{7} \times 6370 \\
 &= \frac{3}{360} \times 2 \times \frac{22}{7} \times 6370 \\
 &= \frac{1001 \text{ km}}{3} = 333.67 \text{ km}
 \end{aligned}$$

$$\begin{aligned}
 \text{Time} &= \text{Arrival} - \text{Departure} \\
 &= 1330 \text{ pm} - 1130 \text{ pm} \\
 &= 2 \text{ hrs}
 \end{aligned}$$

$$\begin{aligned}
 \text{Speed} &= \frac{\text{Dist}}{\text{Time}} \\
 &= \frac{333.67 \text{ km}}{2 \text{ hr}} \\
 &= 166.835 \text{ km/hr}
 \end{aligned}$$

$\therefore$  The speed of the sheep was 166.835 km

b.2



- i. The projection of RW on PQRS is  $\overline{RN}$
- ii The angle between  $\overline{WS}$  and the plane is  $\widehat{WSN} = y^\circ$

12.	c.) Volume = $\frac{1}{3} \times \text{base area} \times h$
	$28.2 \text{ cm}^3 = \frac{1}{3} \times 4 \text{ cm} \times 4 \text{ cm} \times h$
	$h = \frac{28.2 \text{ cm}^3 \times 3}{4 \times 4 \text{ cm}^2}$
	$= 5.2875 \text{ cm}$
	$\approx 5.3 \text{ cm}$
	$\therefore$ The height of the pyramid is
	5.3 cm

**Extract 12.2:** A sample of the candidate's correct responses in question 12

Extract 12.2 reveals that the candidate had adequate knowledge and skills to solve the problem in three dimensional figures and spheres using the correct formulae.

### 2.13 Question 13: Matrices and Transformations

The question consisted of parts (a), (b) and (c). In part (a) (i), the candidates were supposed to find  $2P - \frac{1}{3}Q$ , given the matrices  $P = \begin{pmatrix} 2 & -3 \\ 5 & 4 \end{pmatrix}$  and

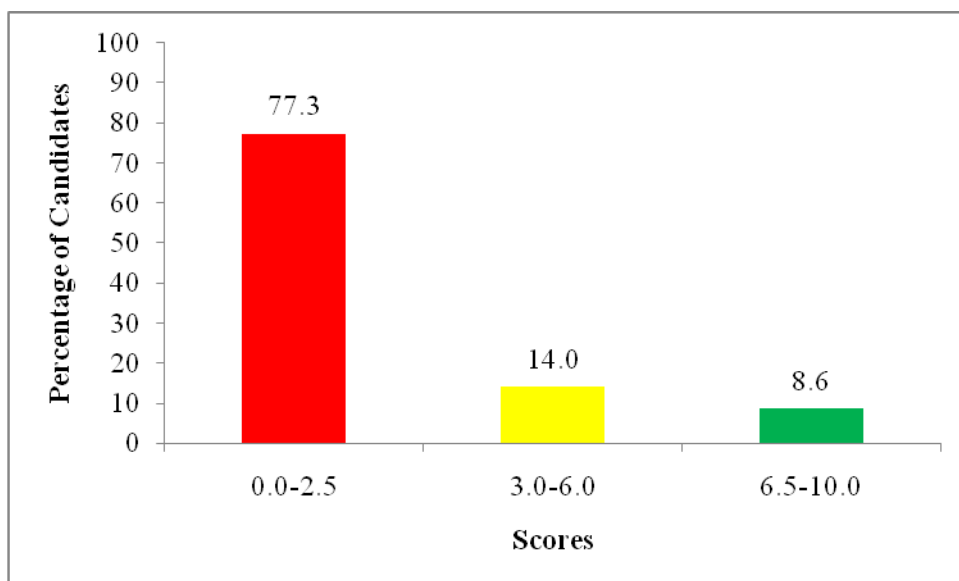
$Q = \begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix}$  while in part (a) (ii), they were required to find the possible

values of  $k$  if the matrix  $\begin{pmatrix} 4k & 8 \\ 2 & 9k \end{pmatrix}$  is singular. In part (b), the candidates were required to solve the following system of linear equations using the

inverse matrix method:  $\begin{cases} 2x + 3y = 7 \\ y = \frac{1}{2}x \end{cases}$

In part (c), they were instructed to find the image of the point  $A(-2, 3)$  by using the transformation matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ , hence state the axis in which this point is reflected.

This question was attempted by 287,589 (67.7%) candidates, out of which only 65,160 (22.7%) candidates scored from 3 to 10 marks showing that the performance was weak. Figure 13 shows the candidates' performance summary in this question.



**Figure 13:** *The summary of candidates' performance in question 13*

Further analysis shows that 222,429 (77.3%) candidates scored from 0 to 2.5 marks, out of which 147,099 (51.1%) candidates scored 0 mark.

In part (a) (i), the candidates substituted the values of matrices P and Q in the given matrix expression  $2P - \frac{1}{3}Q$  as  $2\begin{pmatrix} 2 & -3 \\ 5 & 9 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix}$  but

failed to perform multiplication of the matrices by the scalars. For example,

$2\begin{pmatrix} 2 & -3 \\ 5 & 9 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 3 & 4 \\ -5 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -6 \\ 15 & 8 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 3 & 4 \\ -5 & 1 \end{pmatrix}$  was noted from some scripts. Instead, they were supposed to write

$2\begin{pmatrix} 2 & -3 \\ 5 & 9 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -6 \\ 10 & 8 \end{pmatrix} - \begin{pmatrix} 3 & 4 \\ -5 & 1 \end{pmatrix}$  which is a correct step.

This shows that, the candidates lacked knowledge and skills in performing addition, subtraction and scalar multiplication of matrices.

In part (a)(ii), some candidates were able to write the determinant of matrix  $\begin{pmatrix} 4k & 8 \\ 2 & 9k \end{pmatrix}$  as  $(4k \times 9k) - (2 \times 8) = 0$  but failed to solve  $36k^2 - 16 = 0$ . Those candidates lacked knowledge of solving quadratic equations. Others lacked knowledge on how to find the determinant of matrix since they calculated the determinant as;  $(4k \times 9k) + (2 \times 8) = 0$  instead of  $(4k \times 9k) - (2 \times 8) = 0$  which is an incorrect step.

In part (b), the majority of candidates failed to rearrange the given system of equations  $\begin{cases} 2x + 3y = 7 \\ y = \frac{1}{2}x \end{cases}$  in matrix form thus leading to the incorrect answers. For example, they wrote incorrect rearrangements like:  $\begin{pmatrix} 2 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$  instead of  $\begin{pmatrix} 2 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$  before proceeding. Other candidates applied either elimination or substitution method when solving the equations contrary to the given instructions.

Few candidates were able to rearrange the system of linear equations in matrix form but failed to find its determinant and therefore ended up with incorrect answers.

In part (c), some candidates failed to apply the transformation matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  to reflect the point  $A(-2, 3)$  and state the axis in which this point is reflected. They were unable to correctly find the image of the point  $A(-2, 3)$  after failing to determine the order in which the transformation was to take place. They could not realize that the image was to be obtained by pre-multiplying the transformation matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  by the point  $A(-2, 3)$ , that is,  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ . Instead, they wrote  $\begin{pmatrix} -2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ .

Furthermore, other candidates simply determined the image of the point by finding the sum of transformation matrix and the given point such that;  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$  which is an incorrect step. This indicates that the

candidates lacked knowledge and skills on transforming a point by transformation matrix. Extract 13.1 shows a sample response of the candidate who failed to answer this question correctly.

13(a)(i)	$P \begin{pmatrix} 2 & -3 \\ 5 & 4 \end{pmatrix}$ and $Q \begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix}$ find $2P - \frac{1}{3}Q$
	Soln
	Matrix $P \begin{pmatrix} 2 & -3 \\ 5 & 4 \end{pmatrix}$
	let $P = \begin{pmatrix} 2 & -3 \\ 5 & 4 \end{pmatrix}$
	$ P  = 2 \times 4 - 5 \times -3$
	$ P  = 8 - -15$
	$P = 15$
	$P = 15$
	Matrix $Q \begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix}$
	let $Q = \begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix}$
	$ Q  = 9 \times 3 - -15 \times 12$
	$ Q  = 27 - -180$
	$ Q  = 207$
	$Q = 207$
ii)	$\begin{pmatrix} 4k & 8 \\ 2 & 9k \end{pmatrix}$
	for Singular matrix determinant is 0
	$36k^2 - 16 = 0$
	$\frac{36k^2}{36} = \frac{16}{36}$
	$k^2 = \frac{16}{36}$
	$k = \frac{\sqrt{16}}{\sqrt{36}}$
	$\therefore k = \frac{\sqrt{16}}{6}$

b	$2x + 3y = 7$
	$y = \frac{1}{2}x$
	$2x + 3y = 7$
	$\frac{1}{2}x + y = 0$
	$\begin{pmatrix} 2 & 3 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$
	10 find <del>Ans</del> $(2 \times 1) - (\frac{1}{2} \times 3)$
	$2 - \frac{3}{2}$
	$\frac{1}{2}$
13	$\begin{pmatrix} 1 & -3 \\ \frac{1}{2} & 0 \end{pmatrix}$
	$\frac{1}{2} \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$
	$\begin{pmatrix} 1 \times \frac{1}{2} & -3 \times \frac{1}{2} \\ -\frac{1}{2} \times \frac{1}{2} & -2 \times \frac{1}{2} \end{pmatrix}$
	$\begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{4} & -1 \end{pmatrix} \begin{pmatrix} 7 \\ 0 \end{pmatrix}$
	$\begin{pmatrix} \frac{7}{2} & 0 \\ -\frac{7}{4} & 0 \end{pmatrix}$
	$\begin{pmatrix} \frac{7}{2} & + & 0 \\ -\frac{7}{4} & + & 0 \end{pmatrix}$
	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{7}{2} \\ \frac{7}{4} \end{pmatrix}$

**Extract 13.1:** A sample of the candidate's incorrect responses in question 13

Extract 13.1 represents the response from a candidate who calculated the determinants of P and Q contrary to the given instructions. Also the candidate wrote that the determinant of a singular matrix is equal to 1 instead of 0, as a result he/she got the incorrect value of  $k$ . Again, he/she failed to express the given equations in matrix form, which was an important step in arriving at the required answers.

Despite the weak performance, 24,795 (8.6%) candidates scored from 6.5 to 10 marks, out of which 1,473 (0.5%) candidates scored full marks.

The candidates were able to operate the matrices of order  $2 \times 2$ , apply the  $2 \times 2$  matrices in solving the given simultaneous equations and apply correctly the matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  to reflect the given point as shown in Extract 13.2.

13.	a) Solution
	Given: $P = \begin{pmatrix} 2 & -3 \\ 5 & 4 \end{pmatrix}$ and $Q = \begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix}$
	Required: $2P - \frac{1}{3}Q$
	Let $M = 2P - \frac{1}{3}Q$
	$M = 2\begin{pmatrix} 2 & -3 \\ 5 & 4 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 9 & 12 \\ -15 & 3 \end{pmatrix}$
	$M = \begin{pmatrix} 4 & -6 \\ 10 & 8 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ 5 & -1 \end{pmatrix}$
	$M = \begin{pmatrix} 1 & -10 \\ 15 & 7 \end{pmatrix}$
	$\therefore 2P - \frac{1}{3}Q$ is $\begin{pmatrix} 1 & -10 \\ 15 & 7 \end{pmatrix}$

13. a) ii) Solution  
 Given:  $\begin{vmatrix} 4k & 8 \\ 2 & 9k \end{vmatrix} = 0$

Required: k

Let  $\begin{vmatrix} 4k & 8 \\ 2 & 9k \end{vmatrix} = M$

$$M = (4k \cdot 9k) - (8 \times 2)$$

$$M = 36k^2 - 16 = 0$$

$$36k^2 - 16 = 0$$

$$36k^2 = 16$$

$$\frac{36}{36} \frac{k^2}{36} = \frac{16}{36}$$

$$\sqrt{k^2} = \sqrt{\frac{16}{36}}$$

$$k = \pm \sqrt{\frac{16}{36}}$$

$$k = \pm \frac{\sqrt{16}}{\sqrt{36}}$$

$$k = \pm \frac{4}{6}$$

$$k = \pm \frac{2}{3}$$

13. b) Solution  
 Given:  $\begin{cases} 2x + 3y = 7 \\ y = \frac{1}{2}x \end{cases}$

$$2x + 3y = 7$$

$$x - 2y = 0$$

13. b) In matrix form  
 $\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} = *$

Let  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ ,  $X = \begin{pmatrix} x \\ y \end{pmatrix}$  and  $P = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$

$$\text{adj}(A) = \begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = (2 \times -2) - (3 \times 1)$$

$$= -4 - 3$$

$$|A| = -7$$

$$A^{-1} = -\frac{1}{7} \begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & -\frac{2}{7} \end{pmatrix}$$

multiplying  $A^{-1}$  both sides of \* equation

$$A A^{-1} \cdot x = A^{-1} \cdot p$$

But ;  $A \cdot A^{-1} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & -\frac{2}{7} \end{pmatrix} \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{7} \times 7 + 0 \\ \frac{1}{7} \times 7 + 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\therefore (x, y)$  is  $(2, 1)$

13. c) Solution

Given :  $A(-2, 3)$  and  $T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Required :  $A' (x', y')$

$$A' \begin{pmatrix} x' \\ y' \end{pmatrix} = T \cdot A \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 + 0 \\ 0 - 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

$\therefore$  The image is  $A'(-2, -3)$  and it is reflected on the x-axis

**Extract 13.2:** A sample of the candidate's correct responses in question 13

Extract 13.2 shows the work of a candidate who correctly performed required operations on the given matrices, solved the given equations using matrix method and applied the properties of transformations to get the required image.

## 2.14 Question 14: Relations, Functions and Linear Programming

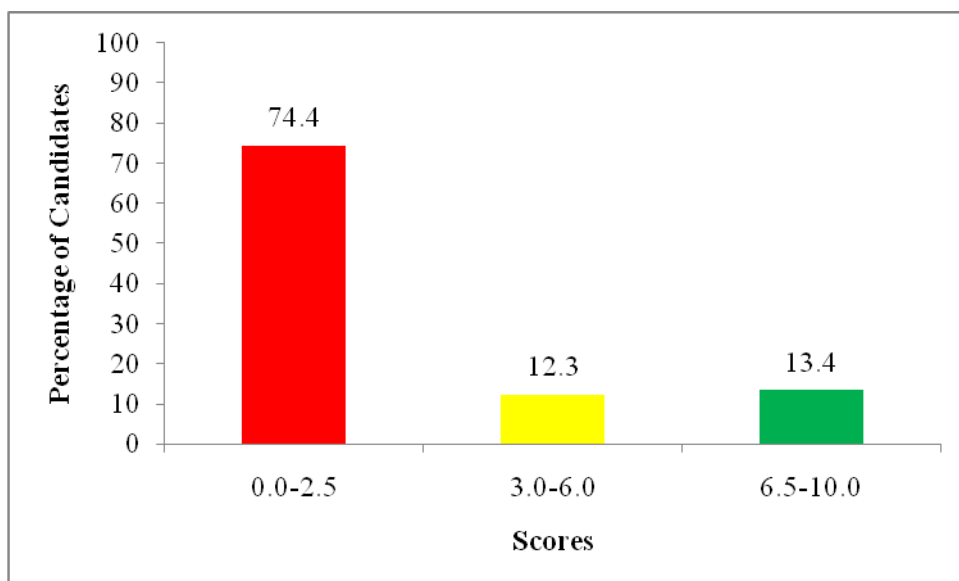
This question had parts (a), (b) and (c). In part (a), the candidates were required to find the values of  $f(1)$  and  $f(-\pi)$  for a function  $f(x)$

$$\text{defined by } f(x) = \begin{cases} x+2 & \text{for } x < 0 \\ 2 & \text{for } 0 \leq x \leq 2 \end{cases}$$

In part (b), the candidates were required to find  $f^{-1}(-1)$  by using the information given in part (a).

In part (c), it was given that; a trader has a space for 5 refrigerators. The trader plans to spend 2,400,000 shillings to buy refrigerators of two brands, Hitachi and Sony. Each Hitachi refrigerator costs 600,000 shillings whereas each Sony refrigerator costs 400,000 shillings. The unit profits for Hitachi and Sony refrigerators are 200,000 shillings and 150,000 shillings respectively. The candidates were able to determine the number of refrigerators for each brand that maximizes the profit, denoting  $x$  and  $y$  as the number of Hitachi and Sony refrigerators respectively.

This question was attempted by 224,334 (52.8%) candidates, out of which only 57,519 (25.6%) candidates scored from 3 to 10 marks showing that the performance was weak as summarised in Figure 14.



**Figure 14:** The summary of candidates' performance in question 14

Further analysis shows that 166,815 (74.4%) candidates scored from 0 to 2.5 marks, out of which 114,304 (51.0%) candidates scored 0 mark.

The candidates were unable to identify the correct limits in which the values  $f(1)$  and  $f(-\pi)$  are found. For example, they substituted the value of  $x=1$  in the function  $f(x)=x+2$  and got the value of  $f(1)=1+2=3$  which is incorrect, instead of  $f(1)=2$  as  $0 \leq 1 \leq 2$ . Moreover, there were candidates who drew the graph of the functions,  $f(x)=x+2$  or step function,  $f(x)=\begin{cases} x+2 & \text{for } x < 0 \\ 2 & \text{for } 0 \leq x \leq 2 \end{cases}$  and stated the domain and range of the functions contrary to the requirement of the question.

In part (a) (ii), when finding  $f^{-1}(-1)$ , some candidates failed to identify the proper function to use, others used  $f(x)=2$  and did as follow;  $f(x)=2 \Rightarrow y=2$  and interchanged the variables to get  $x=2$  and hence concluded that  $f^{-1}(-1)=2$ . Others identified the correct function that is,  $f(x)=x+2$  and interchanged the variables to get  $x=y+2$  but failed to make  $y$  the subject of the function, as a result they got  $f^{-1}(x)=2-x$ , which is an incorrect step. Although few candidates managed to get the inverse function correctly as  $f^{-1}(x)=x-2$ , yet they committed errors when computing for the final answer.

In part (b), most candidates formulated the incorrect linear inequalities like;  $x+y \geq 5$ ,  $3x+2y \geq 12$ ,  $x \leq 5$  and  $y \leq 5$  instead of  $x+y \leq 5$  and  $3x+2y \leq 12$ . Furthermore, others failed to identify the objective function correctly as they used the given costs to get the function  $f(x,y)=600,000x+400,000y$ . However, other candidates used the number of refrigerators when formulating the objective function such that  $f(x,y)=x+y$ . There were other candidates who formulated correctly the linear inequalities and objective function but failed to draw the correct graph, hence they ended up with incorrect feasible region. These candidates lacked knowledge and skills in: formulating linear inequalities in two unknowns, identifying the correct -objective function and locating the corner points of the feasible region when solving a linear programming problem. Extract 14.1 illustrates the work of one of the candidates who failed to answer this question.

14. a) ~~f(1)~~

Given  $f(x) = \begin{cases} x+2 & \text{for } x < 0 \\ 2 & \text{for } 0 \leq x \leq 2 \end{cases}$

$$f(x) = x+2$$

$$= 1+2$$

$$f_1 = 3$$

(b)  $y = x+2$

$$\therefore x = y+2$$

$$2-x = y$$

But  $y = -1$

$$2-x = -1$$

$$-x = -1-2$$

$$-(-x = -3)$$

$$x = 3$$

$$\therefore f^{-1}(-1) = 3$$

14. c) Let  $x$  be Hitachi

$y$  be Sony

	$x$	$y$	Total
Cost	600,000	400,000	2,400,000
Profit	200,000	150,000	350,000
Price	800,000	550,000	1,350,000

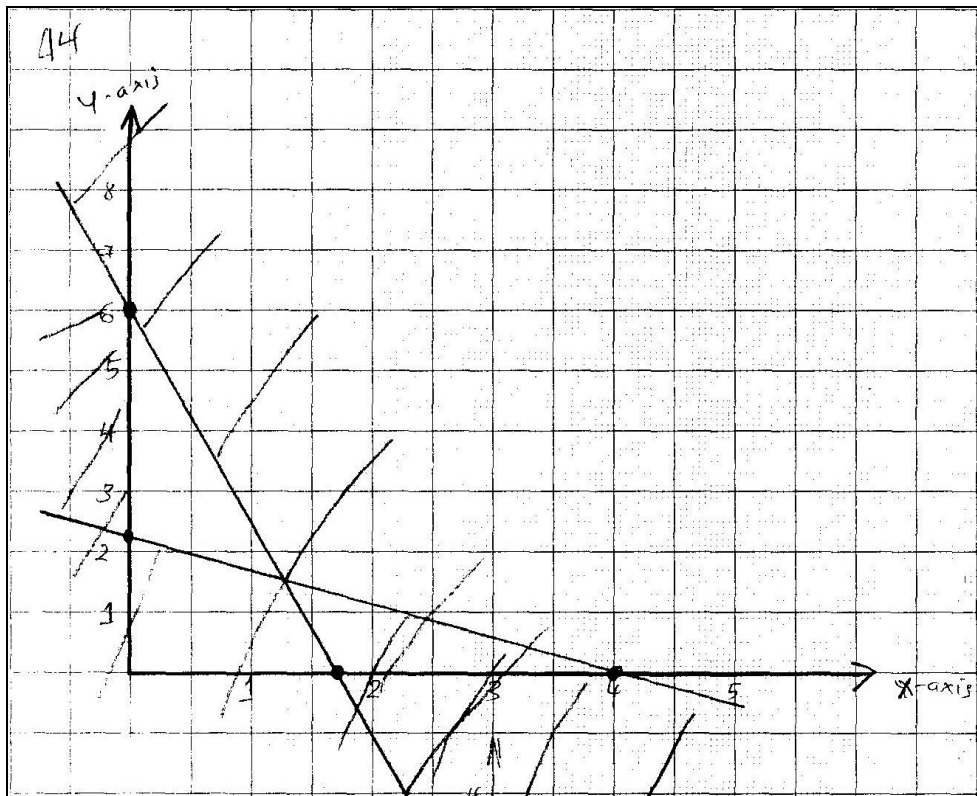
Constraints = 60

$$f(x) = 200,000x + 150,000y \geq 350,000$$

$$600,000x + 400,000y \geq 2,400,000$$

$$x \geq 0$$

$$y \geq 0$$



**Extract 14.1:** A sample of the candidate's incorrect responses in question 14

In extract 14.1 the candidate failed to: apply the given conditions/domains to get the value of  $f(1)$  and identify the linear inequalities and objective function which was a necessary step to get the required solution.

Oppositely, 29,976 (13.4%) candidates scored from 6.5 to 10 marks, out of which 3,335 (1.5%) candidates scored full marks.

The candidates correctly computed the values of  $f(1)$ ,  $f(-\pi)$  and  $f^{-1}(-1)$  and managed to formulate the linear inequalities, objective function  $f(x, y)$  and used them to draw the graph from which the feasible region and corner points were identified. Finally, they determined the number of refrigerators to be bought in order to maximize the profit. Extract 14.2 shows a correct response from one of the candidates.

14

Soln:

(a) Given

$$f(x) = \begin{cases} x+2 & \text{for } x < 0 \\ 2 & \text{for } 0 \leq x \leq 2 \end{cases}$$

$$f(1) = ? \quad f(-\pi) = ?$$

$$\therefore f(1) = 2$$

$$f(-\pi) = x+2$$

$$= -\pi+2$$

$$= -3.14+2$$

$$= -1.14$$

$$\therefore f(-\pi) = -1.14$$

(b)

$$f^{-1}(-1) = ?$$

$$\text{from } f(x) = x+2$$

$$y = x+2$$

$$x = y+2$$

$$y = x-2$$

$$f^{-1}(x) = x-2$$

$$f^{-1}(-1) = -1-2$$

$$= -3$$

$$\therefore f^{-1}(-1) = -3$$

14

(c)

Soln:

Let  $x$  = number of Hitachi refrigerators

$y$  = number of Sony refrigerators

	Number	Cost (Tshs)	Profit (Tshs)
Hitachi	$x$	600,000	200,000
Sony	$y$	400,000	150,000
Total	5	2,400,000	

Objective function: Maximize  $f(x, y) = 200000x + 150000y$

Subject to  $x + y \leq 5$

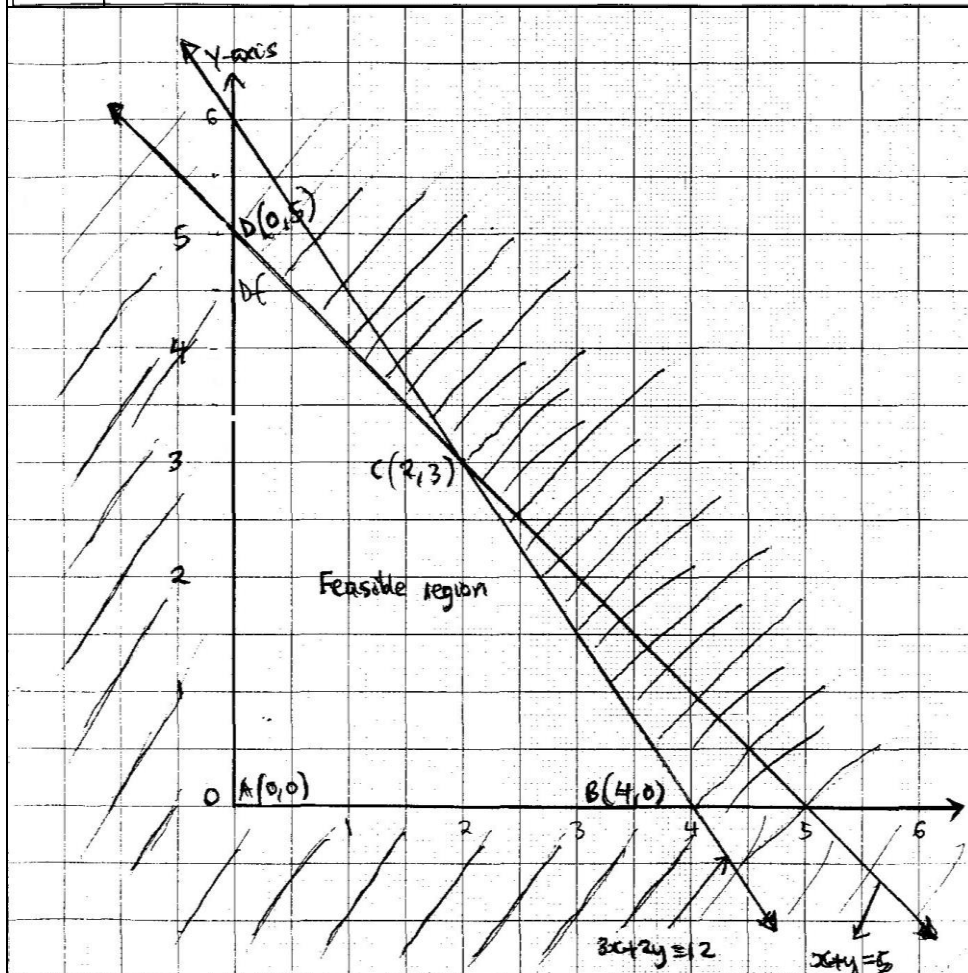
$$600000x + 400000y \leq 2400000$$

$$\Rightarrow 3x + 2y \leq 12$$

Non-zero constraints  $x \geq 0$

$$y \geq 0$$

Corner point	$f(x,y) = 200,000x + 150,000y$	Profit (Tshs)
A (0,0)	$0 + 0$	0
B (4,0)	$800,000 + 0$	800,000
C (2,3)	$400,000 + 450,000$	850,000
D (0,5)	$0 + 750,000$	750,000
Optimal point = C (2,3)		
∴ For maximum profit to be obtained 2 Hitachi re refrigerators and 3 Sony refrigerators are required		



**Extract 14.2:** A sample of the candidate's correct responses in question 14

In Extract 14.2 the candidate determined correctly the value of  $f(1)$ ,  $f(-\pi)$  and  $f^{-1}(-1)$ ; and represented correctly the linear inequalities graphically and finally got the correct number of refrigerators for each brand that maximizes the profit.

### **3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC**

The CIRA report for CSEE 2019 showed that out of fourteen (14) questions, none had good performance. Questions 11 and 6 had average performance while the remaining questions had weak performance. It was further noted that, the candidates had the highest performance in the questions that were set from the topics of *Statistics* and *Circles* followed by *Rates and Variations*. Contrarily, the candidates performed poorly in the rest of the questions which were set from the *Relations, Functions, Linear Programming, Ratio, Profit and Loss, Accounts, Numbers, Approximations, Decimals and Percentages, Matrices and Transformations, The Earth as a Sphere, Three Dimensional Figures, Sets, Probability, Trigonometry, Pythagoras Theorem, Algebra, Quadratic Equations, Coordinate Geometry, Vectors, Exponents, Radicals and Logarithms, Congruence, Similarity, Geometry and Sequences and Series* as summarized in the Appendix.

### **4.0 CONCLUSION AND RECOMMENDATIONS**

#### **4.1 Conclusion**

Generally, the analysis showed that the performance of the candidates in the CSEE 2019 has increased compared to the performance of CSEE 2018 in the Basic Mathematics subject. According to data analysis, only three topics, namely: *Statistics, Circles and Rates and Variations* had an average performance in 2019. The question that had the best performance was set from the topics of *Statistics and Circles*. On the other hand, the question that had the weakest performance was set from the topic of *Sequence and Series* (See the Appendix). The candidates' weak performance in those topics was contributed by several reasons including: inability to derive and apply formulae in solving word problems related to arithmetic mean, geometric mean, general term, sum of terms in arithmetic progression and geometric progression; failure to apply Venn diagrams and solve problems related to sets; inability to use mathematical tables correctly in evaluating logarithm of numbers and laws of logarithms and exponents in finding the

product, squares and square roots of numbers; failure to factorize quadratic expressions and inability to solve quadratic equations by method of completing the square; failure to correctly apply the sine rule and cosine rule to solve related problems; inability to prepare the trading, profit and loss account by using trial balance; and failure to find the Lowest Common Multiples (LCM) and Greatest Common Factor (GCF) by using repeated division and listing method and apply them to solve related word problems.

## **4.2 Recommendations**

In order to improve the candidates' future performance in Basic Mathematics the following are recommended:

- (a) Teachers should lead students to derive and apply formulae in solving word problems related to arithmetic mean, geometric mean, general term, sum of terms in arithmetic progression and geometric progression using different teaching and learning resources.
- (b) Teachers should guide students to use Venn diagrams to solve problems involving two sets using various teaching and learning resources like Venn diagrams, team of players and playing cards.
- (c) Teachers should insist on the correct and proper use of mathematical tables in finding the logarithm of numbers, including the laws of logarithms and exponents in determining the product, square and square root of numbers through group discussions.
- (d) Teachers should instruct students on how to factorize quadratic expressions by inspection, splitting the middle term, difference of two squares and perfect squares; and solve quadratic equations by completing the square using various teaching and learning resources like multiplication charts and factor tree.
- (e) Teachers should demonstrate to students in small groups on how to derive and apply the sine rule and cosine rule to solve related problems using various teaching and learning resources like ruler, charts, trigonometric tables and coloured chalks.
- (f) Teachers should lead students' discussion on how to prepare the trading, profit and loss account by using the trial balance using

various teaching and learning resources like ruler, ledger books, voucher samples and different business transactions.

- (g) Teachers should guide students to find the LCM and GCF by using repeated division and listing method and solve word problems related to LCM and GCF in small groups.

## Appendix

### ANALYSIS OF THE CANDIDATES' PERFORMANCE TOPIC - WISE CSEE 2019

S/N	Topics	Question Number	Percentage of Candidates who Passed	Remarks
1	Statistics and Circles	11	53.4	Average
2	Rates and Variations	6	33.9	Average
3	Relations, Functions and Linear Programming	14	25.6	Weak
4	Ratio, Profit and Loss and Accounts	7	25.1	Weak
5	Numbers, Approximations, Decimals and Percentages	1	23.9	Weak
6	Matrices and Transformations	13	22.7	Weak
7	Spheres and Three Dimensional Figures	12	21.3	Weak
8	Sets and Probability	3	18.5	Weak
9	Trigonometry and Pythagoras Theorem	9	18.1	Weak
10	Algebra and Quadratic Equations	10	16.6	Weak
11	Coordinate Geometry and Vectors	4	15.9	Weak
12	Exponents, Radicals and Logarithms	2	12.8	Weak
13	Congruence ,Similarity and Geometry	5	12.1	Weak
14	Sequences and Series	8	5.7	Weak

