THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (CSEE) 2018

042 ADDITIONAL MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania has prepared the Candidates' Items Responses Analysis report for Additional Mathematics paper of the Certificate of Secondary Education Examination (CSEE) 2018 in order to provide feedback to students, teachers and other education stakeholders on how candidates responded to the questions.

The report identifies the candidates' strengths and weaknesses, which indicate what the education system was able or unable to offer to the candidates in their four years of ordinary level secondary education.

The analysis of data showed that candidates had good performance in six (6) questions. The questions were set from the following topics: *Geometrical constructions, Sets, Logic, Vectors, Matrices and Linear Transformations, Locus, Permutations and Combinations* and *Probability*. The performance of candidates in the remaining ten (10) questions was average. The questions were set from topics of *Trigonometry, Variations, Equations and Remainder Theorem, Plan and Elevations, Differentiation, Integration, Algebra, Numbers, Statistics* and *Coordinate Geometry*. The factors attributed to weak performance for some candidates included: inability to recall and use correct mathematical facts/theorems/laws and formulae; failure to adhere to the instructions of the questions; incompetence in performing operations (on numbers, sets, logic and vectors); insufficient knowledge and skills in solving equations; and inadequate knowledge and skills in using mathematical instruments to draw diagrams and graphs.

The National Examinations Council of Tanzania believes that this report will be useful to education stakeholders to identify strategies for improving candidates' performance in future examinations of this subject.

The Council would like to thank the examinations officers and all those who were involved in preparing this report.

Dr. Charles Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is a result of the analysis of candidates' responses to the items examined in 042 Additional Mathematics for the Certificate of Secondary Education Examination (CSEE) 2018. The paper was set according to the 2007 Examination format and the 2010 Additional Mathematics Syllabus for Secondary Schools. The report analyses areas in which the candidates faced challenges as well as areas in which candidates performed well.

The paper consisted of two sections namely, A and B, with a total of sixteen (16) questions. Section A comprised 10 questions each carrying 6 marks while section B had 6 questions, each carrying 10 marks. The candidates were required to answer all questions in section A and any 4 questions from section B.

In 2018 a total of 393 candidates sat for the CSEE Additional Mathematics examination, of which 277 (70.48%) candidates passed and 116 (29.52%) candidates failed. In 2017 a total of 424 candidates sat for the CSEE Additional Mathematics examination, of which 296 (70.31%) candidates passed and 128 (29.69%) candidates failed. This indicates that the performance increased by 0.17 percent in 2018.

The analysis of the candidate's performance in all questions is presented in section 2.0. It consists of short descriptions of the requirements of the question and how the candidates responded. It also includes extracts showing the strengths and weaknesses demonstrated by the candidates in attempting each question.

The candidates' performance in each question is categorized by using percentage of candidates who scored 30 percent or more of the total marks allotted to a particular question. The performance was categorized into three groups: 65 to 100 percent for good; 30 to 64 percent for average; and 0 to 29 percent for weak performance. Furthermore, green, yellow and red colors were used to denote good, average and weak performance respectively.

Section 3.0 highlights the factors which contributed to average performance in some topics and presents the recommendations for improvement of the performance in the future. Finally, the analysis of candidates' responses per topic examined is presented.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 Question 1: Numbers

The question had parts (a) and (b). In part (a), the candidates were asked to write the next term in the series $\frac{1\times 2}{-1}+\frac{3\times 4}{-1}+\frac{9\times 8}{1}+\frac{27\times 16}{11}+\ldots$ In part (b), the candidates were required to use the divisibility rules to show that 31752 is divisible by 7 and 9.

The question was attempted by 358 (91.1%) candidates. Of 358 candidates, 138 (38.5%) candidates scored from 2.0 to 6.0 marks, indicating average performance. Figure 1 demonstrates the performance of the candidates in this question.

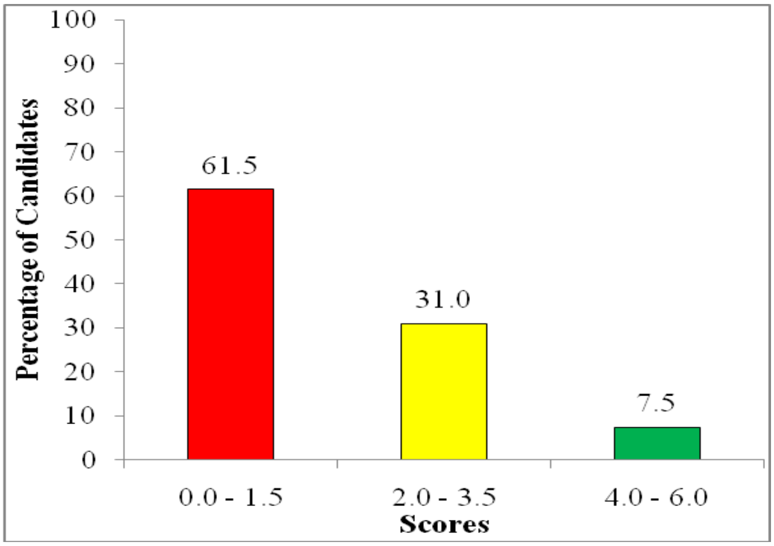


Figure 1: *The Candidates' Performance in Question 1*

The candidates who did well in part (a) were able to develop a formula for general term as $\frac{3^{n-1}\times 2^{n}}{3^{n-1}-2^{n}}$ and used it to obtain a correct next term of the series by substituting $n=5$ to get $\frac{81\times 32}{49}$. The candidates who answered part (b) correctly were able to show that 31752 is divisible by 7 by subtracting the twice of the last digit (2) from the number formed by remained digits (3175). The process was repeated until the result is 0 or multiple of 7. In addition, the candidates showed that 9 divides 31752 by adding the digits of 31752 to get 18, which is divisible by 9. Extract 1.1 is a sample response from one of the candidates who performed well in part (a) of this question.

Extract 1.1

1(b) 31752

For a number to divide seven (7), Substract the last number multiplied by 2 Then check if whether it will divide 7.

Then, 31752

$2 \times 2 = 4$; $3175 - 4 = 3171$

$1 \times 2 = 2$; $317 - 2 = 315$

$5 \times 2 = 10$; $31 - 10 = 21$

Then, $\frac{21}{7} = 3$

$\therefore$ 31752 is divisible by 7.

~~For a number to divide 9~~

For a number to divide 9, the sum of the number should be divisible by 9.

$31752 = 3 + 1 + 7 + 5 + 2$

$= 18$

Then, $\frac{18}{9} = 2$

$\therefore$ 31752 is divisible by 9

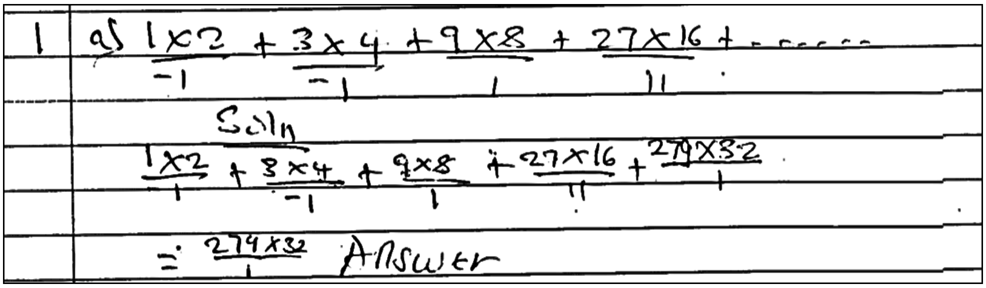
Then

$\therefore$ Hence 31752 is divisible by 7 and 9.

Extract 1.1: A sample of correct response of a candidate who showed that 31752 is divisible by 7 and 9.

However, the analysis of data showed that 220 (61.5%) candidates scored 1.5 marks or less. In part (a), the candidates failed to develop the rule for obtaining the next term. For example, some candidates multiplied the given terms and then added them to get the next term. Also, a notable number of candidates made computational errors in this part. In part (b), many candidates used divisibility rule incorrectly. For instance, some of them omitted 31 from 31752 and then divided 752 by 7, suggesting that they lacked knowledge on divisibility rule. Extract 1.2 shows a sample of incorrect response from one of the candidates.

Extract 1.2


1 a) $\frac{1\times2}{-1} + \frac{3\times4}{-1} + \frac{9\times8}{1} + \frac{27\times16}{11} + \ldots$

Soln

$\frac{1\times2}{1} + \frac{3\times4}{-1} + \frac{9\times8}{1} + \frac{27\times16}{11} + \frac{279\times32}{1}$

$= \frac{279\times32}{1}$ Answer

Extract 1.2: A sample response from one of the candidates who developed incorrect rule for finding the next term of the series.

2.2 Question 2: Sets

The question had parts (a) and (b). In part (a), the candidates were given three sets $A=\{1,2,3,4,5\}$, $B=\{2,4,6\}$ and $C=\{2,3,5\}$, and were required to show the set $A\cup B\cup C$ in Venn diagram by shading it. In part (b), the candidates were asked to show whether (i) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ and (ii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ if $A=\{4,5,7,8,10\}$, $B=\{4,5,9\}$ and $C=\{1,4,6,9\}$.

Analysis of data showed that 387 (98.5%) candidates attempted the question whereby 318 (82.2%) candidates scored the marks ranging from 2.0 to 6.0. Therefore, overall performance of candidates in this question was good. Figure 2 summarizes the performance of the candidates in percentage.

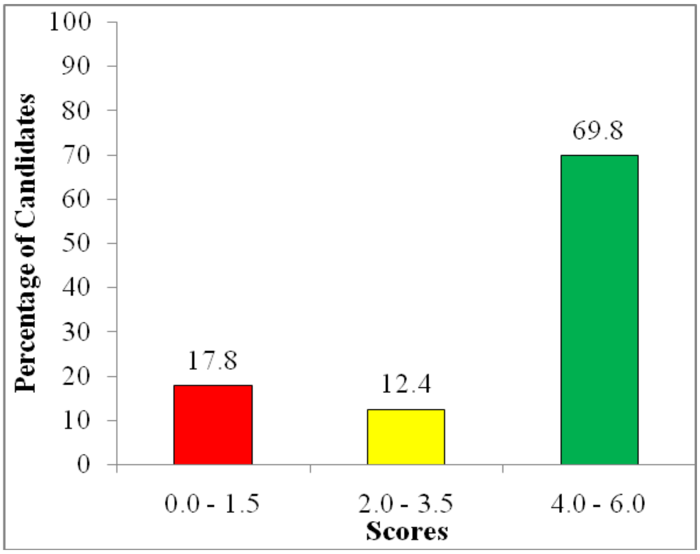
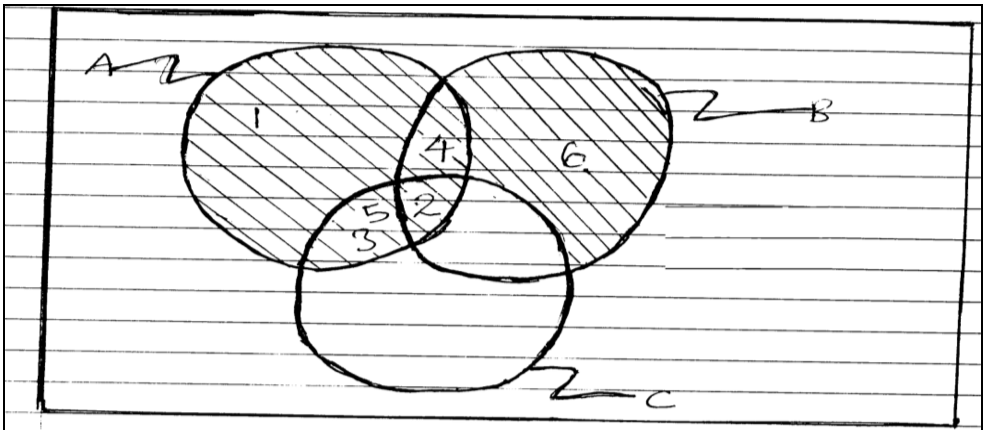


Figure 2: *The Candidates' Performance in Question 2*

Further analysis of data showed that 19 (4.9%) candidates were able to attempt the question as required. In part (a), these candidates identified that $A \cap B \cap C = \{2\}$, $A \cap B \cap C^c = \{4\}$, $A \cap C \cap B^c = \{3, 5\}$, $B \cap C \cap A^c = \phi$, $A \cap B^c \cap C^c = \{1\}$, $B \cap A^c \cap C^c = \{6\}$ and $C \cap A^c \cap B^c = \phi$. Thereafter, they drew Venn diagram consisting of three intersecting circles or ovals enclosed in a rectangle. Finally, these candidates noted the obtained elements in respective regions and shaded them correctly.

In part (b)(i), the candidates determined the sets $B \cap C$, $A \cup (B \cap C)$, $A \cup B$, $A \cup C$ and $(A \cup B) \cap (A \cup C)$ correctly and finally wrote $A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \{4, 5, 7, 8, 9, 10\}$. Also, the candidates did part (b) (ii) in a similar way as part (a) (i). Extract 2.1 is a sample response of a candidate who did question 2 correctly.



(ii) An(Buc) = (AnB) u (Anc) → Given
where,
(B u C) = {1, 4, 5, 6, 9}
(AnB) = {4, 5}.
(An C) = {4}

An (BuC) = {4, 5, 7, 8, 10} n {1, 4, 5, 6, 9}.
= {4, 5}.

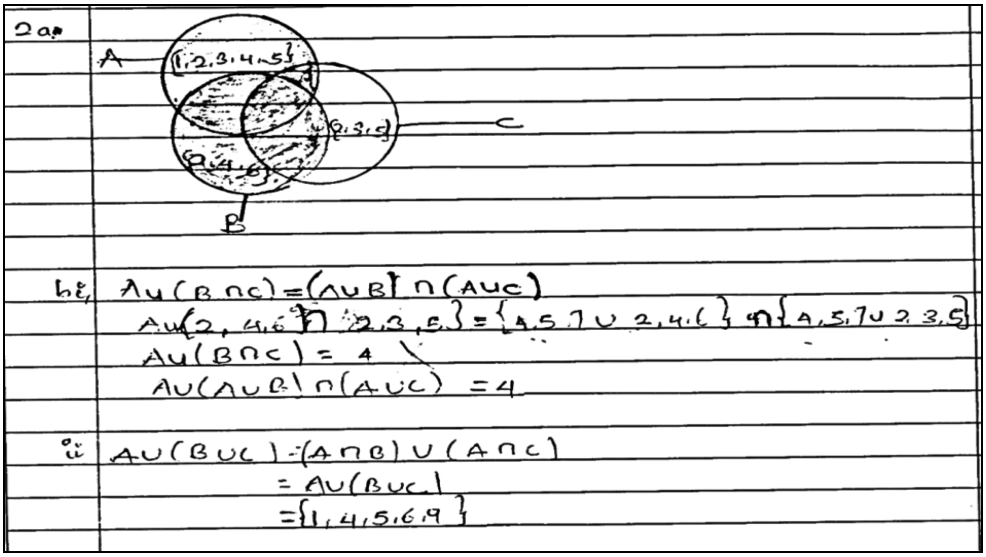
(AnB) u(Anc) = {4, 5} u {4}.
= {4, 5}
Hence An (Buc) = (AnB) u(Anc) = {4, 5} (shown).

Extract 2.1: A sample response of a candidate with good understanding on operation of sets.

On the other hand, 49 (12.7%) candidates did all parts of the question incorrectly. In part (a), some of these candidates drew the three intersecting circles or ovals without enclosing them by a rectangle showing that they lacked knowledge related to universal set. Also, analysis of candidates' responses showed that the candidates inserted the elements of given sets into inappropriate regions. For example, most of them inserted the elements of set A into the region of $A \cap B^c \cap C^c$. The same errors were observed for sets B and C. Moreover, a significant number of candidates did not shade the regions or shaded inappropriate regions.

In part (b), majority of the candidates who scored low marks had inadequate knowledge on operation of sets, particularly union and intersection. Such candidates obtained incorrect answers of the sets $B \cap C$, $A \cup (B \cap C)$, $A \cup B$, $A \cup C$ and $(A \cup B) \cap (A \cup C)$ for part (b) (i) and similarly in part (b) (ii). Other candidates did not understand the language of set. They wrote $A \cup (B \cap C) = 4$ instead of $A \cup (B \cap C) = \{4, 5, 7, 8, 9, 10\}$. Further analysis of candidates' responses showed that several candidates could not conclude that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ or $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, hence their responses lacked a logical connection with the requirements of the question. Extract 2.2 shows a sample solution of a candidate who did question 2 wrongly.

Extract 2.2


2 a)

b i) Au(B∩C) = (AuB) ∩ (AuC)
Au{2,4,6} ∩ {2,3,6} = {4,5,7 ∪ 2,4,6} ∩ {4,5,7 ∪ 2,3,5}
Au(B∩C) = 4
AU(AuB) ∩ (AuC) = 4
ii AU(BUC) = (A∩B) U (A∩C)
= AU(BUC)
= {1,4,5,6,9}

Extract 2.2: A sample solution of a candidate who inserted the elements of given sets in wrong regions and evaluated union and intersection of sets incorrectly.

2.3 Question 3: Equations and Remainder Theorem

The question had parts (a) and (b). In part (a) the candidates were given that; when the polynomial $P(x) = 6x^2 + x + 7$ is divided by $x - a$ the remainder is the same as when it is divided by $x + 2a$. Then, they were required to find the value of a. In part (b), the candidates were required to find the equation whose roots are α^2 and β^2 leaving the answer in terms of K if the roots of the equation $(x+2)^2 - 2Kx = 0$ are α and β.

This question was attempted by 342 (87%) candidates. Amongst, 99 (28.9%) candidates scored marks ranging from 4.0 to 6.0 and 104 (30.4%) candidates scored marks ranging from 2.0 to 3.5 . Therefore, general performance of candidates in this question was average. The summary of candidates' performance in this question is shown in Figure 3.

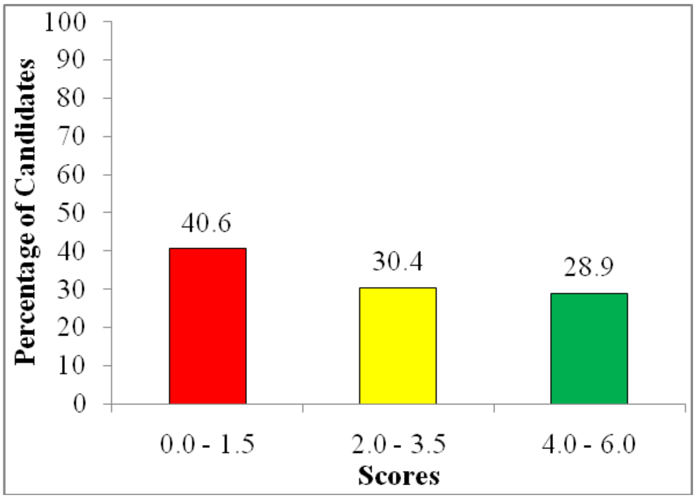


Figure 3: *The Candidates' Performance in Question 3*

The analysis of the candidates' responses showed that in part (a) many candidates were able to use Remainder theorem as they substituted $x = a$ and $x = -2a$ in the polynomial $P(x)$ to produce $P(a) = 6a^2 + a + 7$ and $P(-2a) = 24a^2 - 2a + 7$. Then, they equated the two expressions to get $6a^2 + a + 7 = 24a^2 - 2a + 7$ which leads to $a = 0$ or $a = \frac{1}{6}$.

In part (b), the candidates were able to determine that sum of roots $\alpha + \beta = 2K - 4$ and product of roots $\alpha\beta = 4$ from $(x+2)^2 - 2Kx = 0$. Thereafter, they used the expansion of $(\alpha + \beta)^2$ to obtain

$\alpha^2+\beta^2=4K^2-16K+8$ and $(\alpha\beta)^2=16$ and consequently a quadratic equation $x^2-4(K^2-4K+2)x+16=0$. A sample of a good response of one of the candidates is shown in Extract 3.1

Extract 3.1

3 a) soln:

$P(x) = 6x^2+x+7$

first divisor $x-a$

$x-a=0$

$x=a$

$r = P(x) = 6x^2+x+7$

$r_1 = P(a) = 6a^2+a+7$

$r_1 = 6a^2+a+7$

Second divisor $x+2a$

$x+2a=0$

$x=-2a$

$r = P(x) = 6x^2+x+7$

$r_2 = P(-2a) = 6(-2a)^2-2a+7$

$r_2 = 24a^2-2a+7$

$r_1 = r_2$

$6a^2+a+7 = 24a^2-2a+7$

$24a^2-2a = 6a^2+a$

$18a^2-3a=0$

$\frac{18a^2}{3}-\frac{3a}{3}=\frac{0}{3}$

$6a^2-a=0$

from; $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = \frac{1 \pm \sqrt{1-4\times6\times0}}{12}$

$a = \frac{1 \pm \sqrt{1}}{12}$

$a = \frac{1 \pm 1}{12}$

$a = \frac{2}{12}$ or $a = \frac{0}{12}$

$\therefore a = \frac{1}{6}$ or $a = 0$

b) soln

$(x+2)^2 - 2kx = 0$

$x^2 + 4x + 4 - 2kx = 0$

$x^2 + 4x - 2kx + 4 = 0$

$x^2 + (4-2k)x + 4 = 0$

$a = 1, \quad b = 4-2k, \quad c = 4$

Sum of roots $= \alpha^2 + \beta^2$

from $(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

Sum of roots $= (\alpha+\beta)^2 - 2\alpha\beta$

3. b) then; $\alpha + \beta = \frac{-b}{a}$

$\alpha + \beta = \frac{-(4-2k)}{1}$

$\alpha + \beta = 2k - 4$

$\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{4}{1}$

$\alpha\beta = 4$

then; sum of roots $= (\alpha+\beta)^2 - 2\alpha\beta$

$= (2k-4)^2 - 2\times 4$

$= (2k-4)^2 - 8$

Sum of roots $= 4k^2 - 16k + 16 - 8$

Sum of roots $= 4k^2 - 16k + 8$

Product of roots $= \alpha^2\beta^2$

$= (\alpha\beta)^2$

$= (4)^2$

Product of roots $= 16$

from equation

$x^2 - (\text{sum of roots})x + \text{Product of roots} = 0$

$x^2 - (4k^2 - 16k + 8)x + 16$

$\therefore$ The equation is $x^2 - (4k^2 - 16k + 8)x + 16$.

Extract 3.1: A response of a candidate who applied remainder theorem correctly and managed to obtain the required quadratic equation.

However, further analysis of the candidates' responses showed that, in part (a) most of the candidates did not apply the concept of Remainder theorem as they divided a polynomial $P(x)$ by $x-a$ and $x+2a$ instead of substituting $x = a$ and $x = -2a$ in $P(x)$. In part (b), some candidates approached the question inappropriately by solving the equation $(x+2)^2 - 2Kx = 0$ instead of formulating the new equation from it. Also, many candidates could not obtain the sum of roots $\left(\alpha^2 + \beta^2\right)$ and the product of roots $\left((\alpha\beta)^2\right)$ from $\alpha + \beta = 2K - 4$ and $\alpha\beta = 4$. In addition, a notable number of the candidates could not recall the general form of quadratic equation $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$. These mistakes led to wrong equation such as $x^2 - ((2x-4)^2 - 8)x + 16 = 0$. Extract 3.2 is an example of poor response from one of the candidates.

Extract 3.2

3b Soln.

$(x+2)^2 - 2kx = 0$ are α and β

$x+2^2 - 2kx = 0$

$4x - 2 \cdot kx = 0$

$4x - 2kx$

$(x+2)^2 - 2kx = 0$

$= (x+2)^2 - 2kx$

$\sqrt{(x+2)^2} = \sqrt{kx}$

$= 2x = 2kx$

$= 2kx$

$\therefore = 2kx$

equal α^2 and β^2

$2kx^2 + 4x^2$

$= \sqrt{2kx^2}\ \sqrt{4x^2}$

$\frac{4kx}{4} \pm \frac{12x}{4}$

$kx = 4$

$\therefore k = 4$

Extract 3.2: A sample solution of a candidate who approached the question wrongly by solving the equation instead of formulating the equation.

2.4 Question 4: Algebra

The question consisted of parts (a) and (b). In part (a) the candidates were required to simplify $\frac{3a^2 - 4b^2}{a\sqrt{3} + 2b}$ while in part (b), they were instructed to find $\frac{1}{y^2} + y^2$ from $\frac{1}{y} + y = 2\sqrt{5}$.

A total of 360 (91.6%) candidates attempted this question. Analysis of performance scores revealed that 145 (40.3%) candidates scored marks ranging from 2.0 to 6.0. This implies that candidates' performance in this question was average. Further analysis of data indicated that only 6 (1.7%) candidates scored all allotted marks to this question while 148 (41.1%) candidates were not able to answer the question as required. Figure 4 depicts the percentage of candidates who obtained weak, average and good scores.

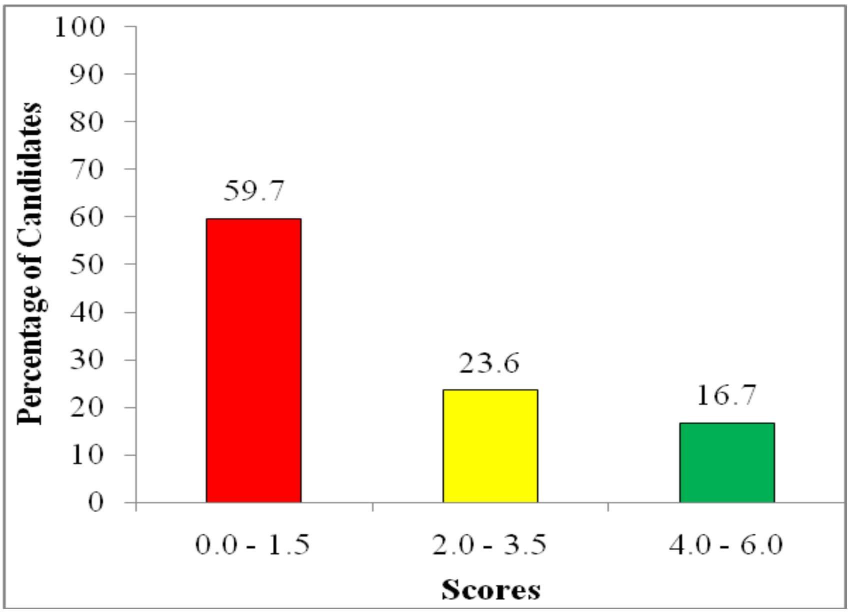


Figure 4: *The Candidates' Performance on Question 4.*

The analysis of the candidates' responses showed that good performance in part (a) was contributed by ability of the candidates to apply the method of difference of two squares. They factorized $3a^2 - 4b^2$ to get $(a\sqrt{3} + 2b)(a\sqrt{3} - 2b)$ which enabled them to obtain $a\sqrt{3} - 2b$ as shown in Extract 4.1. Also, some candidates simplified the expression by rationalizing the denominator. In part (b), good performance was attributed to capability of the candidates to introduce the square of $\frac{1}{y}+y$ so as to develop the expression $\frac{1}{y^2}+y^2$. The candidates squared both sides of the equation $\frac{1}{y}+y=2\sqrt{5}$ to get $\frac{1}{y^2}+2+y^2=20$ which gives out $\frac{1}{y^2}+y^2=18$, as seen in Extract 4.1.

Extract 4.1

4 a) $\frac{3a^2-4b^2}{a\sqrt{3}+2b}$

$3a^2-(2b)^2=(a\sqrt{3}+2b)(a\sqrt{3}-2b)$

$\frac{\cancel{(a\sqrt{3}+2b)}(a\sqrt{3}-2b)}{\cancel{a\sqrt{3}+2b}}=a\sqrt{3}-2b$

$\frac{3a^2-4b^2}{a\sqrt{3}+2b}=a\sqrt{3}-2b$

b) $(y+1/y)^2=(y+1/y)(y+1/y)$

$=y^2+y/y+y/y+1/y^2$

$=y^2+1+1+1/y^2$

$=y^2+1/y^2+2$

$(2\sqrt{5})^2=y^2+1/y^2+2$

$20=y^2+1/y^2+2$

$y^2+1/y^2=20-2$

$y^2+1/y^2=18$

Extract 4.1: A sample solution of a candidate who managed to factorize $3a^2-4b^2$ and introduce the square of $\frac{1}{y}+y$.

Conversely, 215(59.7%) candidates scored 1.5 marks or less. In part (a), majority of these candidates could not factorize $3a^2-4b^2$ correctly whereby $(3a+4b)(3a-4b)$ was frequently observed. The candidates had inadequate knowledge and skills on the factorization by difference of two squares. Furthermore, the analysis of candidates' responses showed that few candidates decided to rationalize the denominator but could not operate radicals correctly.

In part (b), many candidates did not recognize the technique of squaring both sides of the given equation. Instead, some candidates treated $y = 2\sqrt{5}$, which led to $\frac{1}{y^2} + y^2 = \frac{1}{(2\sqrt{5})^2} + (2\sqrt{5})^2$, resulting into wrong answer. Moreover, some candidates did computational errors that led to incorrect solution even to candidates who correctly started with $\left(\frac{1}{y} + y\right)^2 = (2\sqrt{5})^2$.

Extract 4.2 is a sample response of a candidate who did the question wrongly.

Extract 4.2

4 Solution

Given $\frac{3a^2-4b^2}{a\sqrt{3}+2b}$

from $a^2-b^2=(a+b)(a-b)$

$\frac{(3\cancel{a}+\cancel{4b})(3a-4b)}{\cancel{a}\sqrt{3}+\cancel{2b}} = \frac{(3+2)(3a-4b)}{\sqrt{3}}$

$\therefore \frac{5(3a-4b)}{\sqrt{3}} = \frac{15a-20b}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\therefore \frac{15\sqrt{3}a - 20\sqrt{3}a}{3}$.

Extract 4.2: A response of one of the candidates who could not evaluate the square roots of $3a^2$ and $4b^2$ correctly.

2.5 Question 5: Geometric Construction

This question had parts (a) and (b). In part (a), the candidates were given a regular polygon with an exterior angle of 72^0 and required to find: (i) the size of an interior angle and the sum of all interior angles, and (ii) the number of sides the polygon has. In part (b), the candidates were required to use the length of one side as 5cm to draw the regular polygon in (a) above.

This question was attempted by 361(91.9%) candidates. The performance of candidates in this question was good because 304 (84.2%) candidates scored marks ranging from 2.0 to 6.0. Figure 5 provides a summary of candidates' performance in this question.

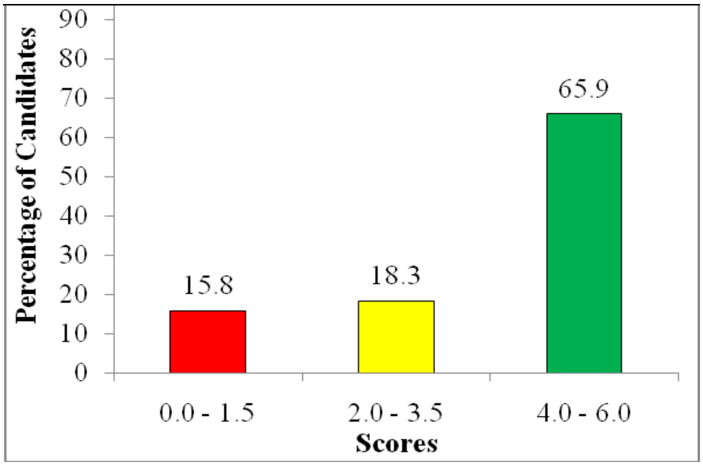
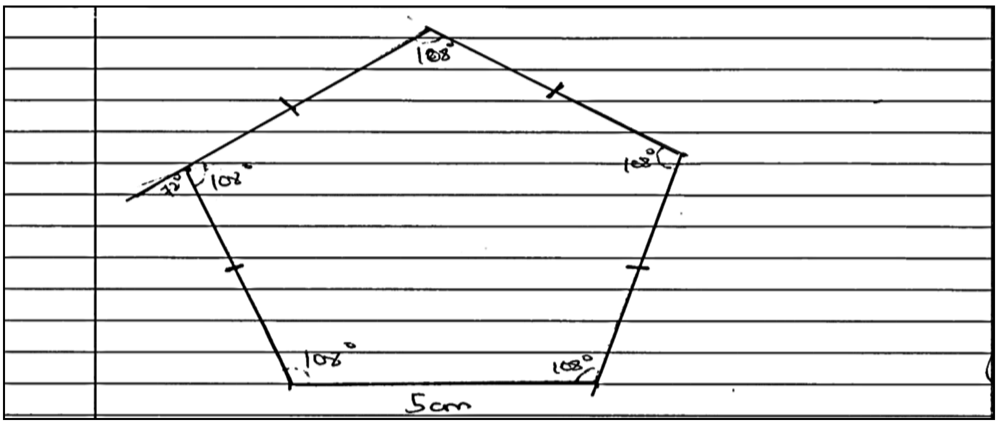


Figure 5: *The Candidates' Performance in Question 5*

The candidates who performed well in part (a) applied the formula $interior\,angle(I) = 180^0 - exterior\,angle(E)$ to get $I = 108^0$. Also, these candidates applied the formula $\mathrm{E} = \frac{360^0}{n}$ or $\mathrm{I} = \frac{(n-2)\times 180^0}{n}$ to get $number\,of\,sides(n) = 5$, which was then substituted to the formula $sum\ of\ all\,interior\,angles(s) = (n-2)\times 180^0$ to get $s = 540^0$. In part (b), the candidates were able to use pair of compasses and protractors to draw a regular polygon whose interior angle is 108^0 and sides of 5cm long each. Extract 5.1 shows a sample of good response from one of the candidates.

Extract 5.1



Extract 5.1: A response of a candidate who used pair of compasses and ruler correctly to draw a regular polygon.

Nevertheless, a total of 57 (15.8%) candidates scored marks ranging from 0 to 1.5. In part (a), the candidates used incorrect formulae such as $360+n^2$ and $2n-4$ taking $n=72^0$ to find size of each interior angle. Consequently, the candidates got wrong answers for sum of all interior angles (*s*) as well as the number of sides (*n*), the polygon has.

In part (b), other candidates sketched the model of polygon instead of drawing. Most of these candidates ignored measurements in their diagrams while other used free hand rather than pair of compass and ruler. These candidates lacked knowledge and skills in using instruments to draw regular polygons. Extract 5.2 is a sample response of one of the candidates who did this question incorrectly.

Extract 5.2

5(a) Sln

(i) Size of interior angle

72° from

Sfn

$= (2n-4)$

$= (2(72°)-4)$

$= (144°-4)$

$= 140°$

$\therefore$ the size of interior angle $= 140°$

Sum of all interior angle is always 180°

(ii) from

$(180°-n)$

$= 180°-72°$

$= 108°$

$\therefore$ There are 180 side in this polygon.

Extract 5.2: A response from one of the candidates who applied the incorrect formulae.

2.6 Question 6: Variations

The question comprised parts (a) and (b). In part (a), the candidates were informed that; "When an object is dropped from a position above the ground, it falls a vertical distance s, which varies directly as the square of the time t. In 10 seconds, the object falls $1{,}600cm$". Then, they were asked to write a formula relating height and time expressing in terms of t.

In part (b), the candidates were also informed that; "A woman invested an amount of money at the rate of 5% in a bank. She also invested twice as much in another bank at the rate of 7%". Then, candidates were required to find the amount of money invested at each rate if her total year amount of simple interest from the two investments is Tsh. 760.

The question was attempted by 356 (90.6%) candidates. The analysis of data also revealed that 75 (21.1%) candidates scored marks ranging from 4.0 to 6.0 and 144 (40.4%) candidates scored marks ranging from 2.0 to 3.5. Therefore, the overall performance of candidates in this question was average. Figure 6 shows candidates' performance in this question.

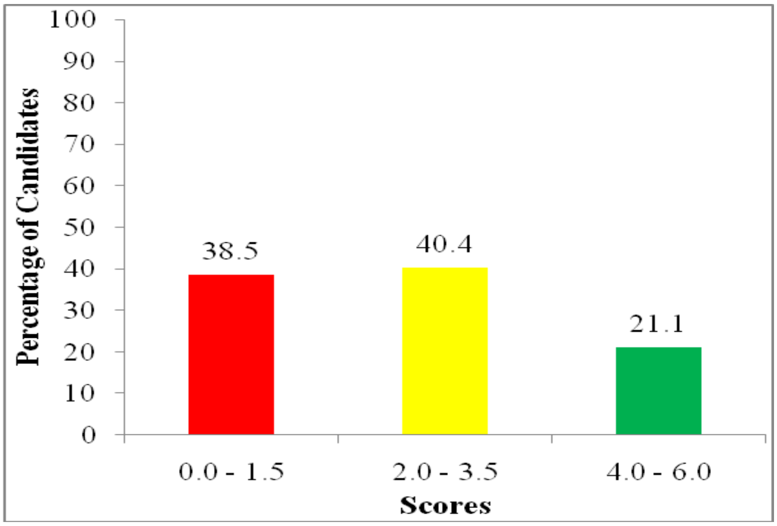


Figure 6: *The Candidates' Performance on Question 6*

Analysis of performance scores revealed that 21 (5.9%) candidates attained all allotted marks to this question. Candidates who did part (a) perfectly realized that the question was a direct variation problem. Therefore, they wrote it into mathematical model, $s \alpha t^2$ and consequently $s = kt^2$ which gives out $k = 16$ after substitution of $s = 1600$ and $t = 10$. The candidates finally replaced k with 16 in $s = kt^2$ to get $s = 16t^2$ as the required formula. In part (b), the candidates wrote the given word problem into mathematical equation. They took x as the amount invested at rate of 5% and $2x$ as the amount invested at rate of 7%. These candidates applied the formula

$Simple Interest(I) = \frac{PRT}{100}$, which led to an equation $\frac{5x}{100} + \frac{7x}{100} = 760$. Finally, they solved it to get $x = 4000$ and interpreted that the amount invested at 5% was Tsh. 4000 and that at 7% was Tsh. 8000 as shown in Extract 6.1.

Extract 6.1

6 b) Soln

let Amount of money be x

First bank — second bank

x — $2x$

$I = \frac{PRT}{100}$

$I_1 = \frac{x \times 5 \times 1}{100}$ — $I_2 = \frac{2x \times 7 \times 1}{100}$

$I_1 = \frac{5x}{100}$ — $I_2 = \frac{14x}{100}$

where $I_1 + I_2 = 760$ Tsh

so; $\frac{5x}{100} + \frac{14x}{100} = 760$ Tsh

$\frac{5x + 14x}{100} = 760$ Tsh

$5x + 14x = 760$ Tsh $\times 100$

$19x = 76000$ Tsh

$\frac{19x}{19} = \frac{76000}{19}$

$x = 4000$ Tsh

$x = 4000$ Tsh — $x_2 = 8000$ Ts

∴ In a bank of rate 5% she invested 4000Tsh and in a bank of rate 7% she invested 8000Tsh.

Extract 6.1: A sample response of a candidate who did part (b) of the question correctly.

Moreover, analysis of performance scores revealed that 36 (10.1%) candidates were not able to use appropriate formulae and performing related operations as expected. The poor performance in this question was greatly attributed to inability of the candidates to transform the word problems into mathematical model.

In part (a), majority of these candidates did not realize that the given word problem was concerned with the concept of direct variation. Some candidates divided $1600cm$ by 10 seconds while others computed the product of $1600cm$ and 10 seconds. Also, some candidates formulated the equation $s = kt^2$ correctly but could not find the numerical value of k hence they treated $s = kt^2$ as final answer instead of $s = 16t^2$. Furthermore, the analysis of candidates' responses showed that there were few candidates who expressed t in terms of s instead of s in terms of t. Such candidates got $t = \sqrt{\frac{s}{k}}$ or $t = \sqrt{\frac{s}{16}}$ or $t = \frac{\sqrt{s}}{4}$, contrary to the requirements of the question.

In part (b), some candidates applied inappropriate formula of the compound interest $An = P\left(1 + \frac{R}{100}\right)^n$ instead of the simple interest formula $I = \frac{PRT}{100}$, as seen in Extract 6.2. Also, few candidates related the problem with concept of ratios. They computed $\frac{5}{12}$ and $\frac{7}{12}$ of 760. Some candidates substituted incorrect data of $I = 760$, $R = 7$ and $T = 5$ into $I = \frac{PRT}{100}$, thus they got wrong answer $P = 2171.4$.

Extract 6.2

6 b) Solution

from $An = P\left(1 + \frac{r}{100n}\right)^{nt}$

$An = P\left(1 + \frac{5}{100}\right)^1$

$760 = P\left(\frac{100+5}{100}\right)$

$\frac{760}{1} = \frac{P}{\times}\left(\frac{105}{100}\right)$

$\frac{760 \times 100}{105} = \frac{P \times 105}{105}$

$P = \frac{76000}{105} = 723.8\text{Tshs}$

$\therefore P = 723.8\text{Tsh}$ was invested at 5% rate.

$760 = P\left(\frac{100+7}{100}\right)$

$\frac{760}{1} \times = P \frac{107}{100}$

$P = \frac{76000}{107}$

∴ 71.21Sh was invested at 7% rate.

Extract 6.2: A sample response of a candidate who applied compound interest formula instead of simple interest formula.

2.7 Question 7: Differentiation and Integration

The question had two parts, from which the candidates were required to;

(a) determine the first derivative in each of the following expressions

(i) $y = \sqrt{x^2 + 3x^3}$

(ii) $2xy\sin y + 2x\cos y\frac{dy}{dx} + y\sin x - \cos x\frac{dy}{dx} = 0.$

(b) find $\int_4^9 \frac{2\sqrt{x}+3}{\sqrt{x}}dx$.

This question was attempted by 300 (76.3%) candidates, of which 116 (38.7%) candidates scored marks ranging from 2.0 to 6.0. This indicates that the overall candidates' performance was average. Figure 7 summarizes the candidates' performance in this question.

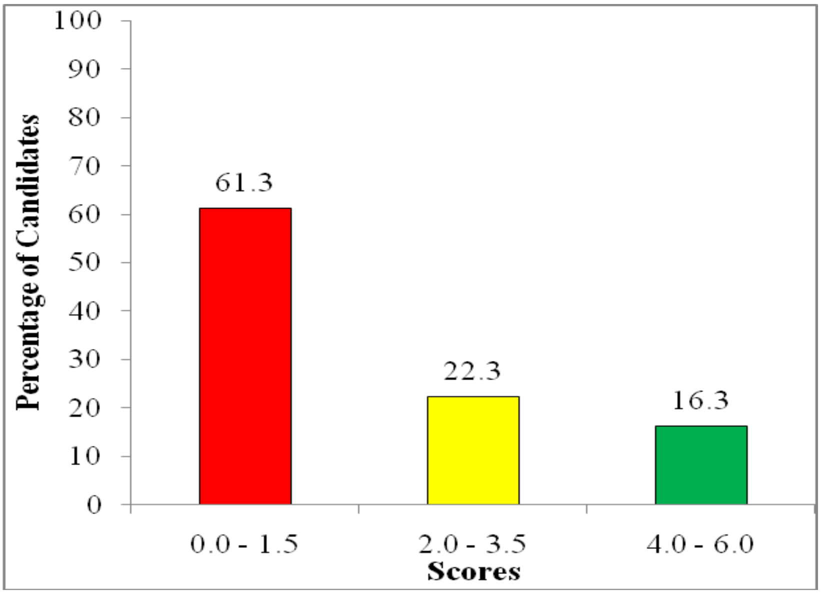


Figure 7: *The Candidates' Performance on Question 7*

Candidates who performed well in part (a) (i) applied chain rule to differentiate $y=\sqrt{x^2+3x^3}$ while in part (a) (ii), the candidate obtained $\frac{dy}{dx}=\frac{-(2\sin y+y\sin x)}{2x\cos y-\cos x}$ after making $\frac{dy}{dx}$ the subject of $2xy\sin y+2x\cos y\frac{dy}{dx}+y\sin x-\cos x\frac{dy}{dx}=0$. In part (b), the candidates computed $\int_4^9 \frac{2\sqrt{x}+3}{\sqrt{x}}dx$ by simplifying $\frac{2\sqrt{x}+3}{\sqrt{x}}$ into $2+3x^{-\frac{1}{2}}$ and evaluating $\int_4^9\left(2+3x^{-\frac{1}{2}}\right)dx$ to get 16. A sample of correct response for this question is shown in Extract 7.1.

Extract 7.1

7 a) i) $y=\sqrt{x^2+3x^3}$

let x^2+3x^3 be u

$y=u^{1/2}$

$\frac{dy}{du}=\frac{1}{2}u^{-1/2}$

$\frac{du}{dx}=2x+9x^2$

$\left(\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}\right)\rightarrow$ chain rule.

$\therefore \frac{dy}{dx}=\frac{1}{2}u^{-1/2}\cdot 2x+9x^2$

$=\frac{1}{2}(x^2+3x^3)^{-1/2}\cdot 2x+9x^2$

$\therefore \frac{dy}{dx}=\frac{2x+9x^2}{2\sqrt{x^2+3x^3}}$

b) $\int_4^9 \frac{2\sqrt{x}+3}{\sqrt{x}}\, dx$

$\int_4^9 \frac{2\sqrt{x}}{\sqrt{x}}dx + \frac{3}{\sqrt{x}}\, dx$

7 b) $= \int_4^9 2dx + 3x^{-1/2}\, dx$

$= \left[2x + 6x^{1/2} + k\right]_4^9$

$= \left(2x + 6\sqrt{x}\right)_4^9$

$= \left(2(9) + 6\sqrt{9}\right) - \left(2(4) + 6\sqrt{4}\right)$

$= (18 + 18) - (8 + 12)$

$= 36 - 20$

$= 16$

$\therefore \int_4^9 \frac{2\sqrt{x}+3}{\sqrt{x}}\, dx = \underline{16}$

Extract 7.1: A sample response from one of the candidates who did the question correctly.

However, a total of 184 (61.3%) candidates scored marks ranging from 0 to 1.5. In part (a) (i), most candidates did not apply chain rule. Instead, some of them added expressions $x^2 + 3x^3$ wrongly to get $y = 3x^5$ as a derivative. In part (a) (ii), many candidates skipped it while some of those who responded to this part could not express $\frac{dy}{dx}$ in terms of x, y, $\sin x$, $\sin y$, $\cos x$ and $\cos y$ from $2xy\sin y + 2x\cos y\frac{dy}{dx} + y\sin x - \cos x\frac{dy}{dx} = 0$. For example, some candidates gave the definitions of sine and cosine. This suggests that majority did not understand the requirements of the question.

In part (b), the great challenge to the candidates was on evaluating the integrals involving negative exponents and limits. Most of them were able to simplify $\frac{2\sqrt{x}+3}{\sqrt{x}}$ into $2+3x^{-\frac{1}{2}}$ but could not get $2x+6x^{\frac{1}{2}}$

from $\int\left(2+3x^{-\frac{1}{2}}\right)dx$. Also, a notable number of candidates had correct procedures up to $\int_4^9\left(2+3x^{-\frac{1}{2}}\right)dx=\left[2x+6x^{\frac{1}{2}}\right]_4^9$ but failed to obtain 16. These candidates could not substitute the limits correctly or did computational errors. Extract 7.2 shows the response from a candidate who answered this question wrongly.

Extract 7.2

6) ii)

b) $\int_4^9 \frac{2\sqrt{x}+3}{\sqrt{x}} \cdot dx$

Soln Simplify first

$\frac{2\sqrt{x}+3}{\sqrt{x}}$

$\frac{2\sqrt{x}}{\sqrt{x}} + \frac{3}{\sqrt{x}}$

7b) $2 + \frac{3}{\sqrt{x}}$

$2 + 3(\sqrt{x})^{-1}$

$2 + 3\cdot(x^{-1/2})$

$\int_4^9 2 + 3\cdot x^{-1/2} \cdot dx$

$\left[2x + 3\cdot\frac{1}{2}x^{1/2}\right]_4^9$

$\left[2x + \frac{3\sqrt{x}}{2}\right]_4^9$

$\left[2(9) + \frac{3\sqrt{9}}{2}\right] - \left[2(4) + \frac{3\sqrt{4}}{2}\right]$

$\left[18 + \frac{9}{2}\right] - \left[8+3\right]$

$\left[18 + 4{\cdot}5\right] - 11$

$22{\cdot}5 - 11$

$11{\cdot}5$

$\therefore$ The answer is $11{\cdot}5$ or $23/2$

Extract 7.2: A sample of poor response from one of the candidates who evaluated the integral incorrectly.

2.8 Question 8: Trigonometry

In this question, the candidates were required to prove whether $\frac{\sin\theta}{1+\cos\theta}-\frac{1+\cos\theta}{\sin\theta}=\frac{-2\cos\theta}{\sin\theta}$.

A total of 264 (67.2%) candidates attempted this question. Analysis of performance scores showed that 97 (36.7%) candidates scored 1.5 marks or less and 167 (63.3%) candidates scored marks ranging from 2.0 to 6.0. This indicates that the performance of candidates was average. Figure 8 summarizes the candidates' performance in this question.

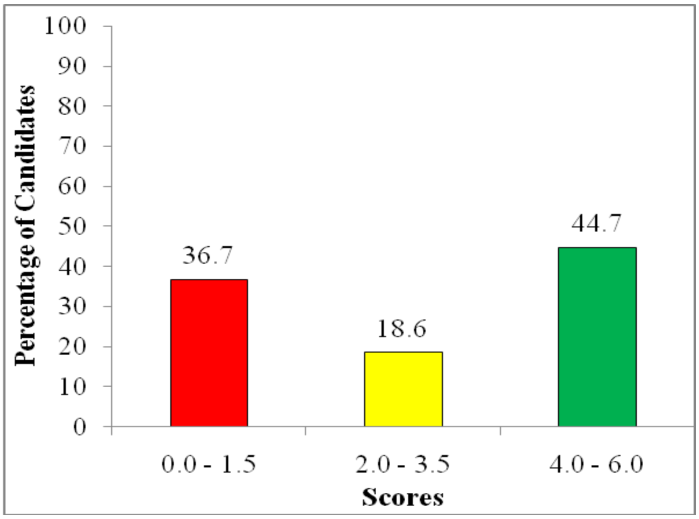


Figure 8: *The Candidates' Performance in Question 8*

Further analysis of data shows that 87 (33.0%) candidates scored all marks allotted to this question. These candidates simplified the expression $\frac{\sin\theta}{1+\cos\theta}-\frac{1+\cos\theta}{\sin\theta}$ while applying the identity $\cos^2\theta+\sin^2\theta=1$ and factorization to get $\frac{-2\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$ which was then simplified to $\frac{-2\cos\theta}{\sin\theta}$. Extract 8.1 is a sample solution showing how some candidates with adequate knowledge and skills on Trigonometry responded to the question.

Extract 8.1

8.a) $\frac{\sin\theta}{1+\cos\theta} - \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta - (1+2\cos\theta+\cos^2\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta - (\cos^2\theta+\sin^2\theta+2\cos\theta+\cos^2\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{\cancel{\sin^2\theta} - \cos^2\theta - \cancel{\sin^2\theta} - 2\cos\theta - \cos^2\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{-2\cos^2\theta - 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{-2\cos\theta(\cos\theta+1)}{(1+\cos\theta)\sin\theta}$

$= \frac{-2\cos\theta\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\sin\theta}$

$= \frac{-2\cos\theta}{\sin\theta}$

$\therefore \frac{\sin\theta}{1+\cos\theta} - \frac{1+\cos\theta}{\sin\theta} = \frac{-2\cos\theta}{\sin\theta}$

Extract 8.1: A sample solution of a candidate with clear knowledge on trigonometric identities and factorization.

A total of 97 (36.7%) candidates scored marks ranging from 0 to 1.5. Most of these candidates opted to evaluate the expression $\frac{\sin\theta}{1+\cos\theta}-\frac{1+\cos\theta}{\sin\theta}$ so as to get $\frac{-2\cos\theta}{\sin\theta}$. However, some of them reciprocated each term to get $\frac{1+\cos\theta}{\sin\theta}-\frac{\sin\theta}{1+\cos\theta}$. This procedure is incorrect and does not give a correct means of getting $\frac{-2\cos\theta}{\sin\theta}$. Also, majority of these candidates were challenged by LCM of $1+\cos\theta$ and $\sin\theta$ whereby incorrect answer $1+\cos\theta+\sin\theta$ was frequently seen in their responses. Furthermore, as Extract 8.2 illustrates, there were a notable number of candidates who performed operations involving positive and negative signs wrongly leading to expressions that could not be simplified to $\frac{-2\cos\theta}{\sin\theta}$.

Extract 8.2

$$\frac{\sin\theta}{1+\cos\theta} - \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta - 1+2\cos\theta+\cos\theta^2}{\sin\theta+\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - 1+2\cos\theta+\cos^2\theta}{\sin\theta+\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta-1+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$\frac{1-1+-2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$\frac{2\cos\theta}{\sin\theta(1+\cos\theta)}$$

Extract 8.2: A response of a candidate who worked with positive and negative signs incorrectly.

2.9 Question 9: Locus

The question had parts (a) and (b). In part (a), candidates were required to find the equation of the locus of a point which is always equidistant from the points $A(1, 2)$ and $B(-2, -1)$. In part (b), the candidates were required to find the equation of a circle with points $(0, 1)$ and $(2, 3)$ as ends of its diameter.

This question was attempted by 326 (83.0%) candidates, out of which 214 (65.6%) candidates scored marks ranging from 2.0 to 6.0. Therefore, performance of candidates in this question was good. Figure 9 gives a summary of candidates' performance in this question.

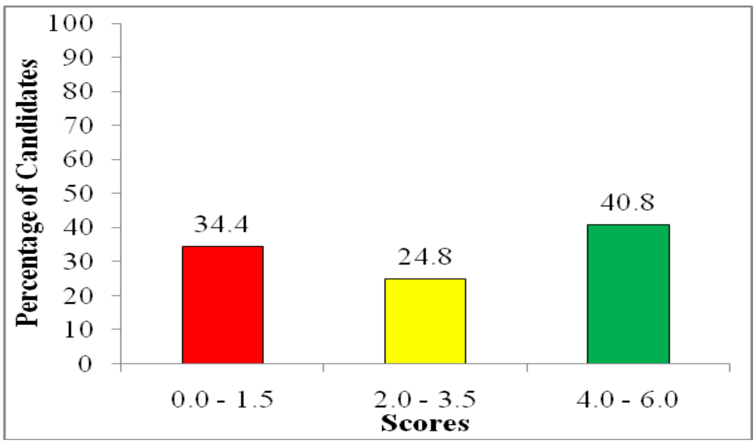
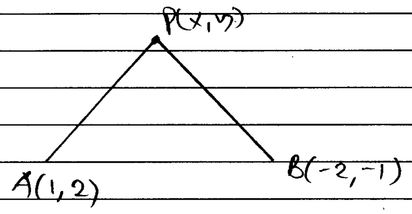


Figure 9: *The Candidates' Performance in Question 9*

Many candidates who did well in part (a) assumed that the point whose locus is asked is $P(x, y)$. Thereafter, they wrote $\overline{PA} = \overline{PB}$ and applied the distance formula $\mathrm{d} = \sqrt{(\mathrm{x}_1 - \mathrm{x}_2)^2 + (\mathrm{y}_1 - \mathrm{y}_2)^2}$ to get the required locus $x + y = 0$. In part (b), majority of the candidates computed the centre of a circle $(C(a,b) = (1, 2))$ by finding the midpoint of the points $(0, 1)$ and $(2, 3)$, and radius $r = \sqrt{2}$ using distance formula involving coordinates of the centre and one of the end points. Then, they substituted the values into $(x-a)^2 + (y-b)^2 = r^2$ to get $x^2 + y^2 - 2x - 4y + 3 = 0$. Apart from this approach, some candidates did the question correctly by using the formula $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ where (x_1, y_1) and (x_2, y_2) are ends of the diameter. Extract 9.1 is a sample of good response from one of the candidates.

Extract 9.1

9. a) $A\ (1,2)$ $B(-2,-1)$



$\overline{AP} = \overline{BP}$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(1-x)^2+(2-y)^2} = \sqrt{(x+2)^2+(y+1)^2}$

$\sqrt{(1-2x+x^2)+(4-4y+y^2)} = \sqrt{(x^2+4x+4)+(y^2+2y+1)}$

$\left(\sqrt{1-2x+x^2+4-4y+y^2}\right)^2 = \left(\sqrt{x^2+4x+4+y^2+2y+1}\right)^2$

$x^2+y^2-2x-4y+5 = x^2+y^2+4x+2y+5$

$\cancel{x^2-x^2}+\cancel{y^2-y^2}-2x-4x-4y-2y+5-5=0$

$-6x-6y=0$

$\therefore$ The locus is $x+y=0$

	b) $(0,1)$ $(2,3)$
	$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
	$(x-0)(x-2)+(y-1)(y-3)=0$
	$x^2-2x+y^2-3y-y+3=0$
	$x^2+y^2-2x-4y+3=0$

Extract 9.1: A sample solution of a candidate who applied the distance formula and general formula for equation of a circle given end points of a diameter.

A total of 76 (23.3%) candidates scored a zero mark. In part (a), majority of the candidates could not interpret the term "equidistant" correctly. Some of them calculated equation of straight line passing the points $A(1, 2)$ and $B(-2, -1)$ as seen in Extract 9.2. Unexpectedly, few candidates perceived the question as sets. Such candidates presented a solution showing Venn diagram of two intersecting circles enclosed in a rectangle. Similarly, in part (b), the candidates computed slope or equation of a straight line using the given ends of a diameter. It is obviously that several candidates lacked of knowledge and skills on concepts of Loci.

Extract 9.2

9.	Soln
	$A(1,2)$ and $B(^-2,^-1)$
	from
	Slope $=\frac{y_1-y_2}{x_1-x_2}$
	$=\frac{2+(^-1)}{1+(^-2)}$
	$=\frac{3}{3}$
	$=1$
	Slope $=\frac{y-y_1}{x-x_1}$
	$\frac{1}{1}=\frac{y-2}{x-1}$
	$(y-2)1=(x-1)1$
	$y-2=x-1$
	$y=x-1+2$
	$\therefore y=x+1$

Extract 9.2: A sample solution of a candidate who wrongly interpreted part (a) of the question by computing equation of a straight line.

2.10 Question 10: Plan and Elevations

The candidates were instructed to use third angle projection to draw the plan, side and front elevations of a rectangular prism.

The question was attempted by 249 (63.4%) candidates. A total of 131 (52.6%) candidates scored marks ranging from 2.0 to 6.0. This implies that overall performance of candidates in this question was average. Figure 10 shows the percentage of candidates who attained weak, average and good scores.

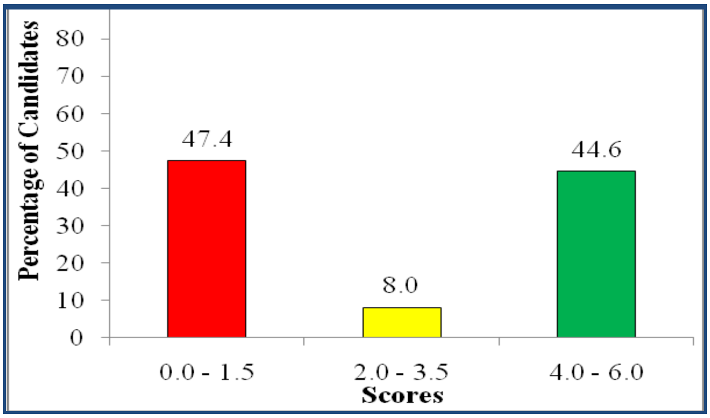
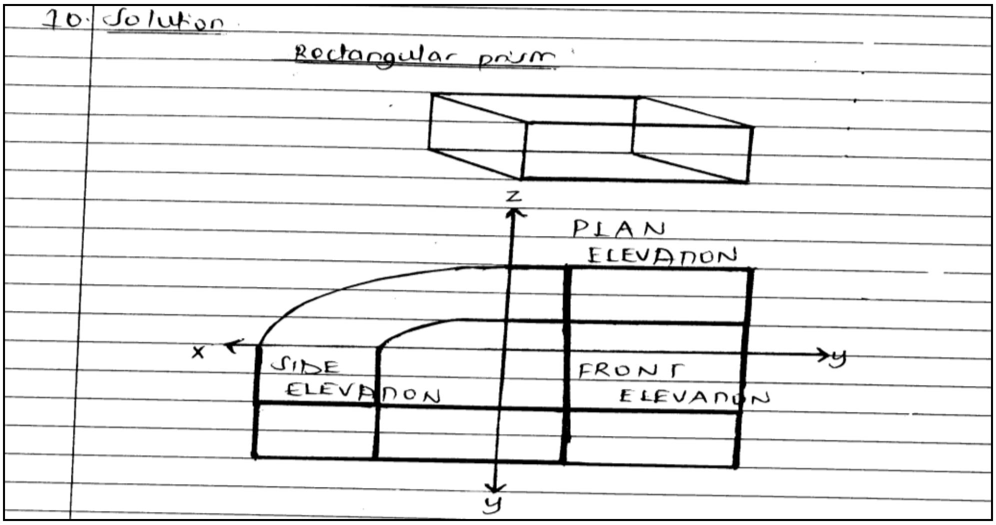


Figure 10: *The Candidates' Performance in Question 10*

The analysis of data showed that 60 (24.1%) candidates scored all six (6) marks. Such candidates presented clear diagram with plan and elevations located at correct positions, as shown in Extract 10.1.

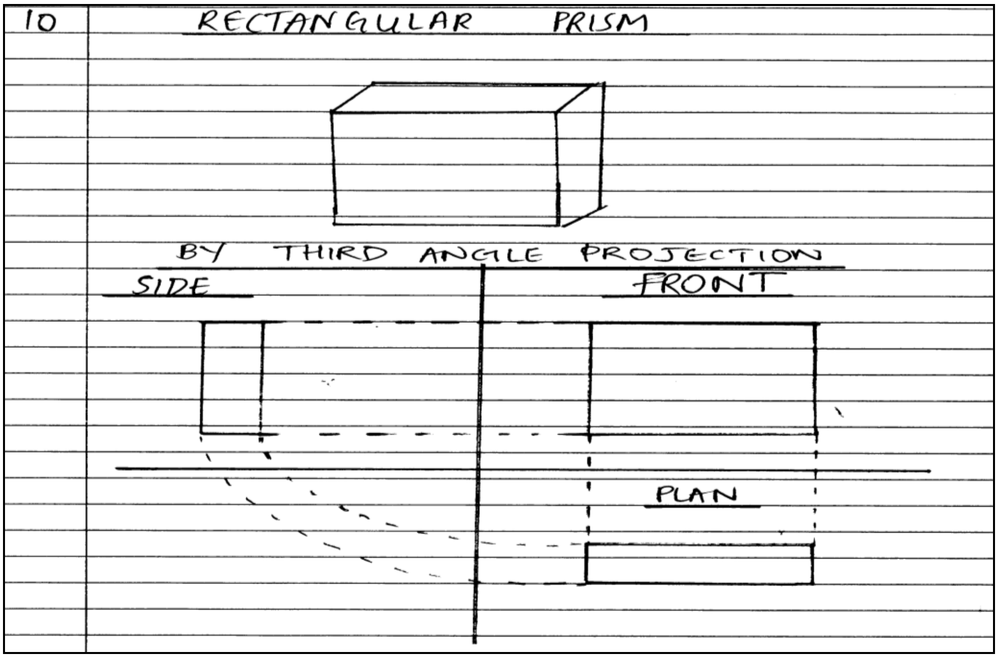
Extract 10.1



Extract 10.1: The candidate was able to draw the plan and elevations correctly.

Conversely, 118 (47.4%) candidates scored 1.5 marks or less. Majority had adequate knowledge and skills on plan and elevations but could not distinguish the terms rectangular prism and triangular prism. Such candidates presented the correct diagram describing the plan and elevations of triangular prim rather than rectangular prism. Also, some candidates drew the plan and elevations independent from xyz axes, lacking supportive concept in their responses. In addition, a significant number of candidates were not aware about the locations for plan, front and side elevations in *xyz* axes when they applied Third Angle Projection. Extract 10.2 shows a sample of a poor response from one of the candidates.

Extract 10.2



Extract 10.2: A sample response of a candidate who located plan, front and side elevations incorrectly.

2.11 Question 11: Coordinate Geometry

The question had parts (a), (b) and (c). The candidates were required to find:

(a) x given that $\mathrm{PA} = \mathrm{PB}$ and the points $P(x,0), A(8,4)$ and $B(6,6)$ are corners of equilateral triangle.

(b) the equation of a circle which passes through points $(1,1)$ and $(2,-1)$ if its centre lies on the line $y = 3x - 7$.

(c) the equation of a line through the point $\mathrm{P}(5,11)$ and parallel to the x- axis.

Analysis of data showed that 246 (62.6%) candidates attempted the question. Of 246 candidates, 31.3 percent scored marks ranging from 3.0 to 10 indicating that the candidates' performance was average. Figure 11 illustrates the performance of the candidates in this question.

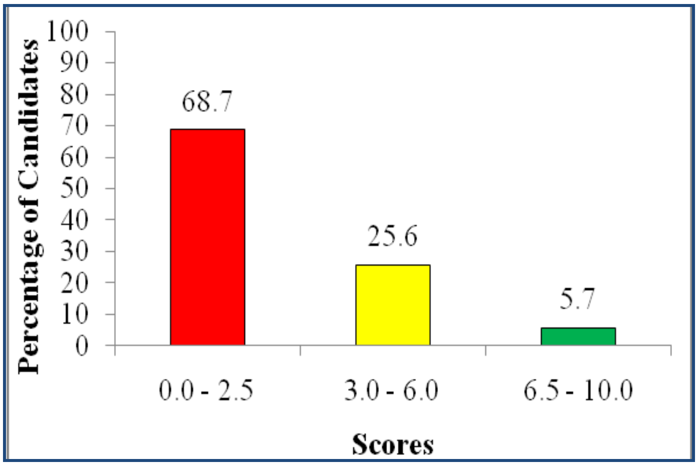


Figure 11: *The Candidates' Performance in Question 11*

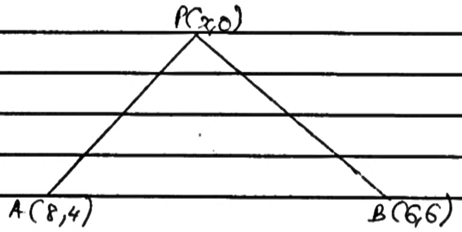
Furthermore, the analysis of data showed that 8 (3.3%) candidates did the question perfectly. In part (a), the candidates applied the distance formula $\mathrm{d} = \sqrt{(\mathrm{x}_1 - \mathrm{x}_2)^2 + (\mathrm{y}_1 - \mathrm{y}_2)^2}$ using the condition PA=PB to get $\sqrt{x^2 - 12x + 72} = \sqrt{x^2 - 16x + 80}$ which resulted to $x = 2$. These candidates had adequate knowledge and skills in calculating distance between two points and equations. Majority of the candidates attempted part (b) of the question as follows: Firstly, they replaced (x, y) in $x^2 + y^2 - 2gx - 2fy + c = 0$ with $(1,1)$ and $(2,-1)$ to formulate the two equations, $2g + 2f + c = -2$ and $4g - 2f + c = -5$ respectively. Secondly, they replaced (x, y) in $y = 3x - 7$ with the coordinates of the centre of required circle ((g, f)), to get $3g - f = 7$ as third equation. Finally, these candidates solved the three equations simultaneously to obtain $g = \frac{5}{2}$,

$f=\frac{1}{2}$ and $c=4$ which were then substituted into $x^2+y^2-2gx-2fy+c=0$ to get $x^2+y^2-5x-y+4=0$.

In part (c), the candidates realized that slope of the required line is zero (0) because it is horizontal line. Thereafter, they substituted $m=0$ and $(x_1, y_1)=(5,11)$ into the formula for finding slope $m=\frac{y-y_1}{x-x_1}$ to get the equation $y=11$. Extract 11.1 is a sample answer from one of the candidates who performed well in this question.

Extract 11.1

11 (a)



Given PA = PB

$PA = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$PA = \sqrt{(x-8)^2 + (0-4)^2}$ $\quad PB = \sqrt{(x-6)^2 + (0-6)^2}$

$PA = \sqrt{x^2 - 16x + 64 + 16}$ $\quad PB = \sqrt{x^2 - 12x + 36 + 36}$

$PA = \sqrt{x^2 - 16x + 80}$

$PA = PB$

$\sqrt{x^2 - 16x + 80} = \sqrt{x^2 - 12x + 72}$

$x^2 - 16x + 80 = x^2 - 12x + 72$

$80 - 72 = -12x + 16x$

$8 = 4x$

$\underline{x = 2}$

(b) Refer General formulae

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(1)^2 + (1)^2 + 2g(1) + 2f(1) + c = 0$

$1 + 1 + 2g + 2f + c = 0$

$2g + 2f + c = -2 \quad \ldots$ (i)

$(2)^2 + (-1)^2 + 2g(2) + 2f(-1) + c = 0$

$4 + 1 + 4g + {}^-2f + c = 0$

$4g - 2f + c = {}^-5 \ldots$ (ii)

11 (b) $y = 3x - 7$ center $C(-g, -f)$.

$-f = {}^-3g - 7$

$f = 3g + 7 \ldots$ (iii)

$$\begin{cases} 2g + 2f + c = {}^-2 \\ 4g - 2f + c = {}^-5 \\ f = 3g + 7 \end{cases}$$

By substitution method

$2g + 2(3g + 7) + c = {}^-2$

$2g + 6g + 14 + c = {}^-2$

$8g + c = -16.$

$4g - 2(3g + 7) + c = {}^-5$

$4g - 6g - 14 + c = {}^-5.$

${}^-2g + c = {}^-5 + 14$

${}^-2g + c = 9$

$c = 9 + 2g$

$8g + 9 + 2g = {}^-16$

$10g = -16 - 9$

$10g = {}^-25$

$g = \frac{{}^-25}{10}$

$g = \frac{{}^-5}{2}$

$f = 3\left(\frac{{}^-5}{2}\right) + 7$ $\qquad c = 9 + 2\left(\frac{{}^-5}{2}\right)$

$f = \frac{-15}{2} + 7$ $\qquad c = 9 - 5$

$c = 4$

$f = \frac{-15 + 14}{2}$

$f = \frac{{}^-1}{2}$

11 (b) $g = \frac{{}^-5}{2}$

$f = \frac{{}^-1}{2}$

$c = 4$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 + 2\left(\frac{-5}{2}\right)x + 2\left(\frac{-1}{2}\right)y + 4 = 0$

$x^2 + y^2 - 5x - y + 4 = 0.$

(c) $m = 0$ $(1,0)$

$Y = m(x - x_1) + Y_1$

$Y = 0(x - 5) + 11$

$Y = 11.$

Equation : $y = 11.$

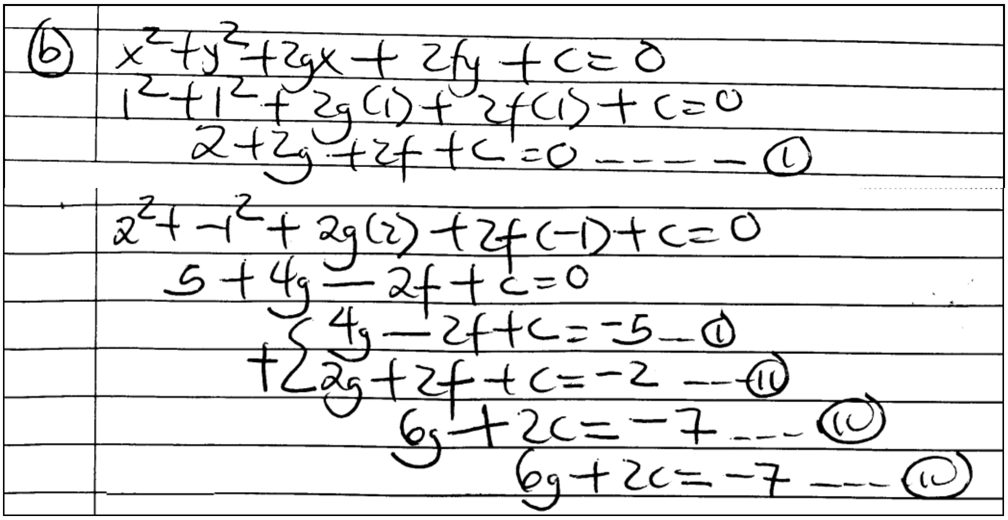
Extract 11.1: A sample response from one of the candidate who did the question correctly.

On the other side, 169 (68.7%) candidates scored marks ranging from 0 to 2.5. In part (a), majority of the candidates applied the distance formula to the condition $\mathrm{PA} = \mathrm{PB}$ to get $\sqrt{x^2 - 12x + 72} = \sqrt{x^2 - 16x + 80}$ correctly but could not evaluate it to get $x = 2$. Such candidates showed weakness in solving the equations. Also, there were few candidates who added the coordinates of the three vertices incorrectly and equated to 180^0 to formulate a wrong equation $144x = 180^0$. These candidates could not understand that PA and PB are sides (not angles).

In part (b), some candidates had a correct idea of forming three equations and solving them simultaneously. However, they made computational errors that led to incorrect equation(s), as shown in Extract 11.2. Moreover, a significant number of candidates used the points $(1, 1)$ and $(2,\text{-}1)$ to find equation of straight line. Such candidates proved that they lacked knowledge and skills on equation of a circle.

In part (c), several candidates could not realize that any line that is parallel to the x-axis is horizontal thus its slope is 0. In the candidates' responses incorrect values of slope $m = 1$ and $m = 2$ were frequently seen. Also, some candidates interpreted wrongly that the equation of a line which is parallel to the x-axis is always $y = 0$. Such candidates used the points $(5, 11)$ and $(x, 0)$ to find the equation.

Extract 11.2



(b) $x^2+y^2+2gx+2fy+c=0$

$1^2+1^2+2g(1)+2f(1)+c=0$

$2+2g+2f+c=0$ ---- (i)

$2^2+-1^2+2g(2)+2f(-1)+c=0$

$5+4g-2f+c=0$

$4g-2f+c=-5$ — (i)

$+\ 2g+2f+c=-2$ — (ii)

$6g+2c=-7$ --- (iii)

$6g+2c=-7$ --- (iii)

Extract 11.2: A sample response from one of the candidates who did computational errors when formulating equations.

2.12 Question 12: Statistics

In this question, the candidates were given the following table showing the distribution of marks of students in mathematics class test at a certain school.

Marks	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	1	3	8	16	40	26	5	2

The candidates were required to:

(a) use coding method with assumed mean $A = 37.5$ to find mean and standard deviation in two decimal places;

(b) interpret the relationship between the obtained mean and standard deviation; and

(c) draw a cumulative frequency curve.

Analysis of data indicated that 277 (70.5%) candidates attempted the question, of which 0.7 percent had good score of 9.0 marks and 5.1 percent scored a zero mark. It was further noted that there were no candidate who scored all 10 marks due to failure of all candidates to attempt part (b) of the question correctly. Figure 12 presents a summary of candidates' performance.

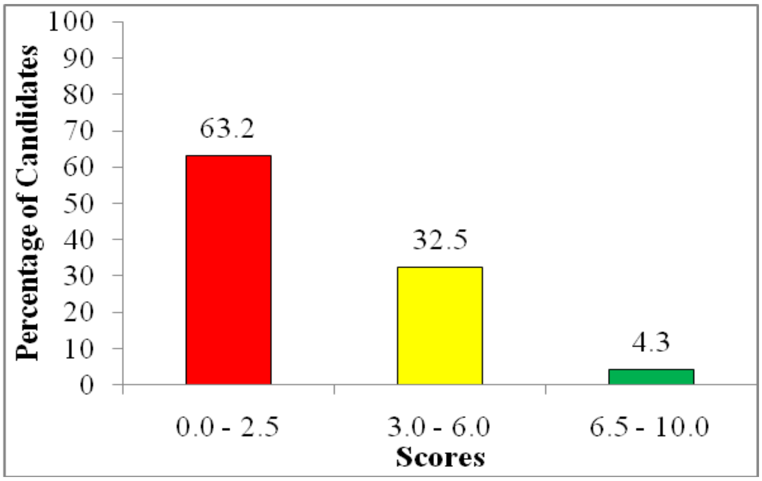


Figure 12: *The Candidates' Performance on Question 12*

As Figure 12 shows, 102 (36.8%) candidates scored marks ranging from 3.0 to 10. Therefore, the overall candidates' performance in this question was average.

Candidates with good performance in this question used the given class intervals to determine both class size (c) and class marks (x). Then, they used the given assumed mean $(A = 37.5)$ and frequency (f) to develop a general distribution table for part (a) and (c) by computing the values in columns of $u = \frac{x - A}{c}$, fu, fu^2, cumulative frequency (cf) and upper real limits (URL). To answer part (a), these candidates extracted the required data for the formulae $\bar{x} = A + c\frac{\sum fu}{\sum f}$ and $\delta = c\sqrt{\frac{\sum fu}{\sum f} - \left(\frac{\sum fu}{\sum f}\right)^2}$ from the table and evaluated correctly to get $\bar{x} = 37.5$ and $\delta = 6.19$.

In part (c), the candidates drew horizontal and vertical axes labelled Upper Real Limits and Cumulative frequency respectively. Thereafter, they plotted the points using corresponding values read from the table and joined them using a smooth line. Extract 12.1 illustrates a correct response for part (a) of the question from one of the candidates.

Extract 12.1

12. Solution.

FREQUENCY DISTRIBUTION TABLE

marks	f	x	X-A	u	fu	C.F	u^2	fu^2	U.R.L
15-20	1	17.5	-20	-4	-4	1	16	16	15
20-25	3	22.5	-15	-3	-9	4	9	27	20
25-30	8	27.5	-10	-2	-16	12	4	32	25
30-35	16	32.5	-5	-1	-16	28	1	16	30
35-40	40	37.5	0	0	0	68	0	0	35
40-45	26	42.5	5	1	26	94	1	26	40
45-50	5	47.5	10	2	10	99	4	20	45
50-55	2	52.5	15	3	6	101	9	18	50

$$\text{mean}(\bar{x}) = A + c\frac{\Sigma fu}{\Sigma f}.$$

$$= 37.5 + \frac{5 \times -3}{101}$$

$$= 37.5 + -0.15$$

$$= 37.35$$

12 ∴ the mean is 37.35 ...

$$S.D = c\sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$$

$$S.D = 5\sqrt{\frac{155}{101} - \left(\frac{-3}{101}\right)^2}$$

$$S.D = 5\sqrt{1.53 - 0.0009}$$

$$S.D = 5\sqrt{1.5291}$$

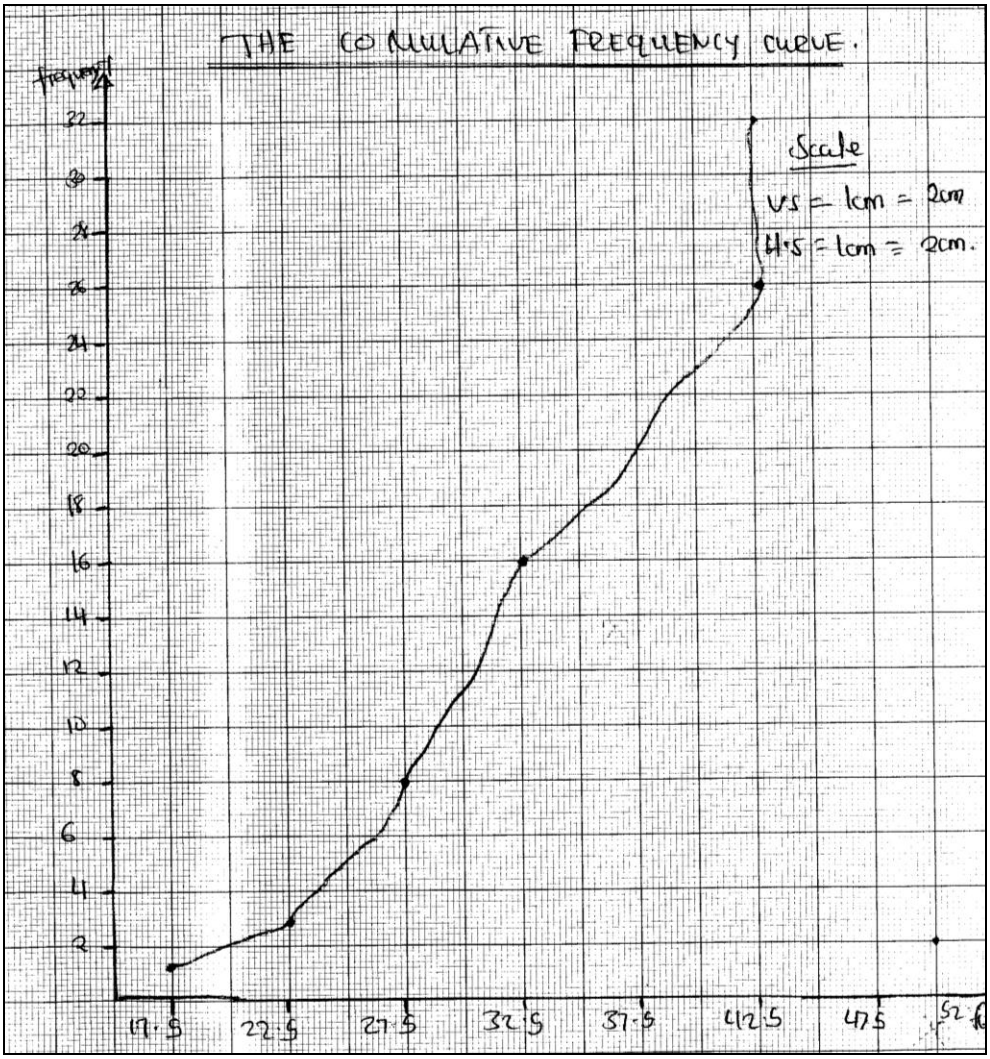
	S·D = 5 × 1·237
	S·D = 6·185 ≈ 6·19.
	∴ the standard deviation is 6·185 ≈ 6·19 (2 d.places).

Extract 12.1: A sample response of candidate who computed mean and standard deviation correctly using coding method.

On the other hand, 175 (63.2%) candidates scored marks ranging from 0 to 2.5. In part (a), most of the candidates applied inappropriate formulae $\bar{x} = \frac{\sum fx}{\sum f}$ or $\bar{x} = A + \frac{\sum fd}{\sum f}$ and $\delta = \sqrt{\frac{\sum f\left(x - \bar{x}\right)^2}{\sum f}}$. These formulae are correct but do not involve coding. Besides, the candidates' responses showed that many candidates made computational errors indicating the weaknesses in basic operations particularly those involving decimals.

In part (b), all candidates were unable to interpret the relationship between the obtained mean and standard deviation. Majority skipped this part and few candidates who responded had incorrect statements for the relationship. For instance, some candidates wrote, *"mean square equals to the standard deviation"*. These candidates were supposed to explain, *"since the standard deviation is small compared to mean, it means there is a small degree of dispersion, indicating that the observations are closer to the mean"*.

In part (c), several candidates indicated wrong upper class boundaries such as 20.5, 25.5, 30.5, etc instead of 20, 25, 30, etc. Such candidates did not realize that they were given class boundaries (real class limits) and not class limits. Furthermore, there were candidates who used the given frequencies instead of cumulative frequencies to draw cumulative frequency curve, hence presented incorrect graph. Apart from this, a considerable number of candidates connected the points using a straight line. With such misconceptions, they presented polygon rather than curve. Extract 12.2 is a sample of response from one of the candidates who performed poorly part (c) of the question.



Extract 12.2: A sample response of a candidate who presented incorrect upper real limits.

2.13 Question 13: Logic

The question had three parts. Candidates were required to:

(a) use the laws of algebra to show that $p \to q \wedge \sim q \to \sim p$ is a tautology;

(b) construct the truth table of the proposition $[(p \to \sim q) \wedge (r \to q) \wedge r] \to \sim p$;

(c) test the validity of the argument; "If I like algebra, then I will study Mathematics. Either I study Mathematics or I play dance music. Therefore I play dance music which implies that I do not like algebra".

This question was attempted by 330 (84.0%) candidates. Out of 330 candidates, 97 (29.4%) candidates scored 2.5 marks or less and 233 (70.6%) candidates scored marks ranging from 3.0 to 10. Therefore, overall performance of candidates in this question was good. The summary of the performance is shown in Figure 13.

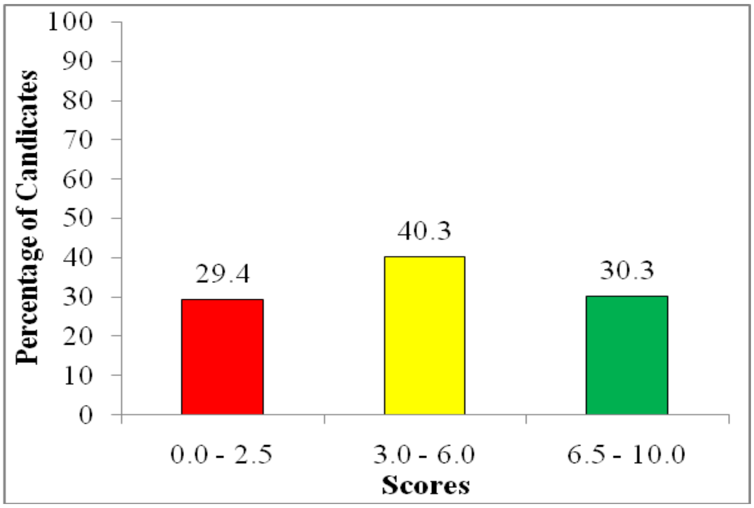


Figure 13: *The Candidates' Performance in Question 13*

Analysis of candidates' performance scores revealed that 100 (30.3%) candidates attained good scores. In part (a), the candidates recalled the order of precedence by starting to evaluate $(q \wedge \sim q)$ to get $p \to f \to \sim p$ using negation law. Then, they used the definition $p \to q \equiv \sim p \vee q$ to obtain $p \to t$ and finally t using negation and identity laws. In part (b), the candidates presented a truth table with columns p, q, r, $\sim p$, $\sim q$, $p \to \sim q$, $r \to q$, $(p \to \sim q) \wedge (r \to q) \wedge r$ and $[(p \to \sim q) \wedge (r \to q) \wedge r] \to \sim p$ correctly. In part (c), the candidates identified three simple propositions that are; I like Algebra (*p*), I study Mathematics (*q*) and I play dance music (*r*). Thereafter, they wrote the symbolic form of the given argument, $[(p \to q) \wedge (q \vee r)] \to (r \to \sim p)$. Finally, they constructed a truth table for the argument that gave both truth-values T and F in its column and concluded that the argument is not valid. Extract 13.1 is an example of a correct response in this question.

Extract 13.1

13. (b) $[(p \to \sim q) \wedge (r \to q) \wedge r] \to \sim p$

Soln,

TRUTH TABLE :

p	q	r	$\sim p$	$\sim q$	$p \to \sim q$ (A)	$r \to q$ (B)	$A \wedge B$ (C)	$C \wedge r$ (D)	$D \to \sim p$
T	T	T	F	F	F	T	F	F	T
T	T	F	F	F	F	T	F	F	T
T	F	T	F	T	T	F	F	F	T
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	T	T	T	T	T
F	T	F	T	F	T	T	T	F	T
F	F	T	T	T	T	F	F	F	T
F	F	F	T	T	T	T	T	F	T
1	2	3	4	5	6	7	8	9	10

∴ Column 10 shows that the statement is tautology

(c) Soln.

Let

p : I like algebra.

q : I will study mathematics

r : I play dance music

TRUTH TABLE FOR : $(p \to q) \wedge (q \vee r) \to (r \to \sim p)$

$p \to q, \ (q \vee r) \ . \vdash \ r \to \sim p$

p	q	r	$\sim p$	$p \to q$	$q \vee r$	$(p \to q) \wedge (q \vee r)$ (A)	$r \to \sim p$ (B)	$A \to B$
T	T	T	F	T	T	T	F	F
T	T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F	T
T	F	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	T	T
1	2	3	4	5	6	7	8	9

13(c) ∴ Column 9 shows that the argument is not valid

Extract 13.1: A sample of candidate's response showing correct truth tables of compound proposition and argument.

However, 25 (7.6%) candidates scored a zero mark. In part (a), the candidates could not follow the order of precedence. They started to evaluate $p \to q$ or $\sim p \to \sim q$ from $p \to q \wedge \sim q \to \sim p$ instead of $q \wedge \sim q$. The candidates were supposed to adhere to the order of precedence; negation (~), conjunction $(\wedge)$, disjunction $(\vee)$, implication $(\to)$ and double implication $(\leftrightarrow)$. Also, some candidates stated incorrect names of applied laws. Furthermore, few candidates ignored the instruction of the question by presenting truth tables instead of simplifying the proposition using laws.

In part (b), majority of the candidates presented incorrect truth tables. They constructed the truth table with number of columns less than ten (10). This indicates that the candidates had inadequate knowledge and skills on determining propositions involved in a compound statement. Similarly, some candidates had correct columns of truth table but could not determine the correct truth-values for all rows.

In part (c), most of the candidates were unable to identify the simple propositions from the given argument. For example, some candidates wrote, "let *r* be I will play dance music or I will study Mathematics". Such candidates did not understand that it is compound proposition whose simple propositions are *I play dance music* and *I study Mathematics*. In a like manner, several candidates presented incorrect symbolic form of the argument due to failure to identify the premises and conclusion. In addition, few candidates had correct symbolic form of the argument but could not present a correct truth table whereby the incorrect truth-values were frequently observed. Extract 13.2 is an incorrect response from one of the candidates.

Extract 13.2

a/ $p \to q \wedge \sim q \to \sim p$

$(\sim p \vee q) \wedge (q \vee \sim p)$ ... $x \to y = \sim x \vee y$

$\sim p \vee (q \wedge q)$ ... Distributive law

$\sim p \vee q$ ... Idempontent

$\therefore p \to q \wedge \sim q \to \sim p$ is not Tautology

c. Soln.

Let.

p = I likes Algebra.

q = I will study mathematics.

r = Either I study Mathematics or I play dance music

$p \to q \vee p \vee s \wedge s \longrightarrow \sim p$

∴ validity of the argument. is.

$p \longrightarrow q \vee p \vee s \wedge s \longrightarrow \sim p$,

Extract 13.2: A sample response of a candidate who could not follow the order of precedence and could not identify premises and conclusion.

2.14 Question 14: Permutations, Combinations and Probability

The question had three parts (a), (b) and (c). In part (a), the candidates were asked as follows;

(i) In how many ways can the letters of the word BARAZA be arranged?
(ii) Find the number of ways of selecting a committee of 3 teachers and 2 students from 5 teachers and 15 students.

In part (b), the candidates were informed that; "A bag contains 5 red counters and 7 black counters. A counter is drawn from the bag, the color is noted and the counter is replaced. A second counter is then drawn". Then, they were required to find the probability that the first counter is red and the second counter is black.

In part (c), the candidates were also informed that; "The probability that Husna and Ally will be selected for further studies are 0.4 and 0.7 respectively." Then, they were required to calculate the probability that one of them will be selected.

The question was attempted by 260 (66.2%) candidates. A total of 167 (64.2%) candidates scored marks ranging from 3.0 and 10 implying that the candidates' performance was good. Figure 14 shows the candidates' performance in this question.

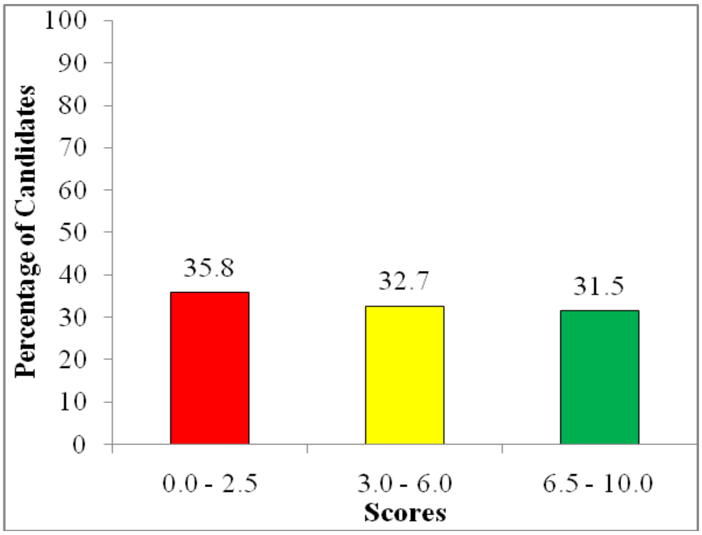


Figure14: *The Candidates' Performance on Question 14*

Analysis of data indicated that 20 (7.7%) candidates did the question perfectly scoring 10 marks. In part (a) (i), the candidates realized that the word BARAZA has 6 letters of which 3 letters (A) are alike. Therefore, they computed $\frac{6!}{3!}$ to get 120 ways. In part (a) (ii), the candidates identified the two steps that complete the experiment of forming a committee. Firstly, selecting 3 teachers from a group of 5 whose ways were given by ${}^{5}C_{3}$ and secondly, selecting 2 students from group of 15 whose ways is given by ${}^{15}C_{2}$. Finally, the candidates applied the Fundamental Principle of Counting to get ${}^{5}C_{3} \times {}^{15}C_{2} = 120 ways$.

In part (c), the candidates determined the probability of drawing first counter that is red, $P(R) = \frac{5}{12}$ and the probability of drawing the second counter that is black, $P(B) = \frac{7}{12}$ correctly. Thereafter, the candidates calculated the probability that the first counter is red and the second counter is black, $P(R \cap B) = \frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$ because the two events are independent. The candidates who answered part (c) correctly computed $P(H') = 0.6$ from $P(H) = 0.4$ and $P(A') = 0.3$ from $P(A) = 0.7$. These candidates were aware of the fact that, $A \cup B = (A \cap B') \cup (B \cap A')$ for mutually exclusive events and $P(A \cap B) = P(A) \times P(B)$ for independent events. The two facts enabled them to have $P(H \cup A) = P(H) \times P(A') + P(A) \times P(H')$ and consequently $P(H \cup A) = 0.54$. Extract 14.1 shows a sample of correct response from one of the candidates.

Extract 14.1

14 (a)(i) BARAZA

$n = 6$ 3A's

$$\frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}}$$

$= 120$ ways

$\therefore$ It can be arranged in 120 ways

(ii) $= {}^{5}C_{3} \times {}^{15}C_{2}$

$$= \frac{5!}{(5-3)!3!} \times \frac{15!}{(15-2)!2!}$$

$$\frac{5!}{2! \times 3!} \times \frac{15}{13! \times 2!}$$

14 (a) $= \left(\frac{5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{2} \times 1 \times \cancel{3} \times \cancel{2} \times \cancel{1}}\right) \times \left(\frac{15 \times \cancel{14}^{7} \times \cancel{13!}}{\cancel{2} \times 1 \times \cancel{13!}}\right)$

$= 10 \times 105$

$= 1050$ ways

$\therefore$ They can be selected in 1050 ways.

(b) Data:

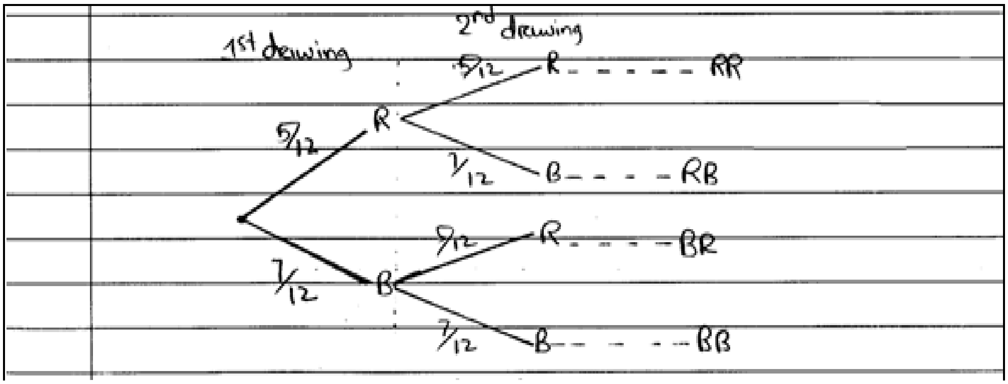
5 red counters

7 black counters.

$n(S) = 12.$

$P(R) = \frac{5}{12}$

$P(B) = \frac{7}{12}$



$P(R \cap B) = P(R) \times P(B)$

$= \frac{5}{12} \times \frac{7}{12}$

$= \frac{35}{144}$

∴ The probability that first counter is red and second counter is black is $\frac{35}{144}$

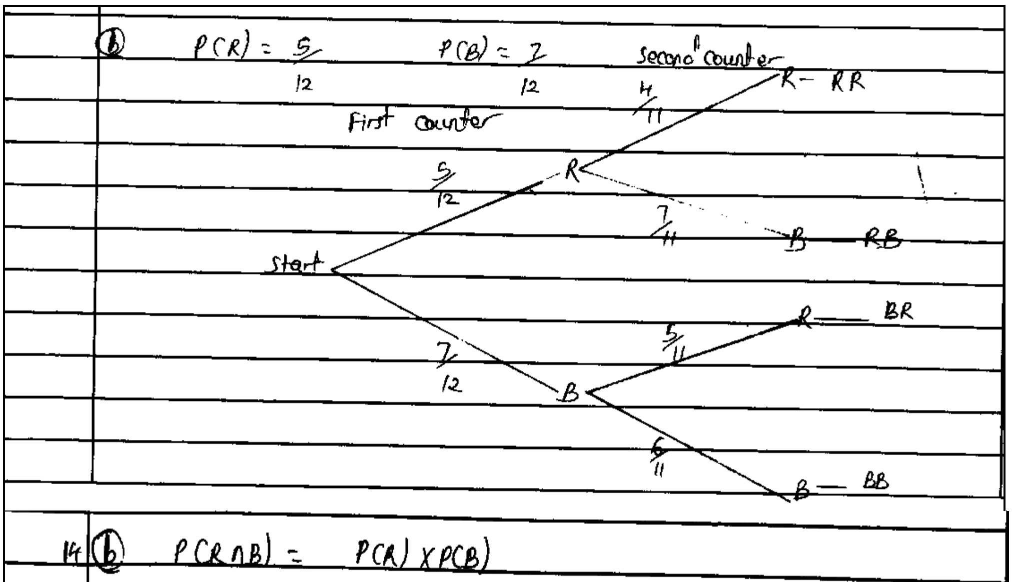
Extract 14.1: A sample solution of candidate with correct understanding on permutations, combinations and probability of independent events.

In contrast, 26 (10.0%) candidates scored a zero mark. In part (a) (i), several candidates could not apply technique of permutations. Some of these candidates presented tree diagram that distributed the six letters. Also, there were few candidates who wrote inappropriate formula ${}^6P_3$ instead of $\frac{6!}{3!}$. It should be noted that the formula ${}^nP_r$ applies for arrangement of *r* objects taken from *n* unlike objects but the word BARAZA has three letters (A) which are alike. In part (a) (ii), many candidates had incorrect solution ${}^5C_3 + {}^{15}C_2 = 115\ ways$ instead of ${}^5C_3 \times {}^{15}C_2$. These candidates were not familiar with the Fundamental Principle of Counting. Further analysis of candidates' responses revealed that some candidates treated $n(S) = 5 + 15 = 20$ and $n(E) = 5$ to get $P(E) = \frac{1}{4}$. Such candidates had not only wrong approach but also did not adhere to the requirements of the question.

In part (b), majority of the candidates could not determine the number of sample space when drawing the second counter. Most of them considered number of sample space as 11 instead of 12 because the first counter was replaced. This error resulted into wrong data on tree diagrams as well as final answer, as shown in Extract 14.2. Further analysis of candidates' responses showed that some candidates evaluated $P(RB)+P(BR)$ instead of $P(RB)$ only. These candidates did not adhere to the given condition that the first counter is red and the second one is black.

In part (c), a common challenge to candidates was inability to identify that the two events are mutually exclusive and independent. It was noted that some candidates applied incorrect formula $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ instead of $P(A\cup B)=P(A\cap B')+P(B\cap A')$, as seen in Extract 14.2. Moreover, there were few candidates who applied the wrong formula $P(A\cup B)=P(A)\times P(B)$ which led to incorrect answer.

Extract 14.2



14 (b) $P(R\cap B) = P(R) \times P(B)$

$= \frac{5}{12} \times \frac{7}{11}.$

$= \frac{35}{132}$

14 (c) Let Husna = H

Given that Ally = A.

$P(H) = 0.4$

$P(A) = 0.7.$

$P(A \cap H) = 0.4 \times 0.7$

$= 0.28.$

$P(A \cup H) = ?$

From

$P(A \cup B) = P(A) + P(B) - P(A \cap B).$

$\Rightarrow P(A \cup H) = P(A) + P(H) - P(A \cap H)$

$= 0.7 + 0.4 - 0.28$

$= 1.1 - 0.28$

$= 0.82.$

$\therefore$ Probability that one of them will be selected is 0.82

Extract 14.2: A sample response of a candidate who had incorrect data in tree diagram and applied incorrect formula for mutually exclusive events.

2.15 Question 15: Vectors, Matrices and Linear Transformation

The question had three parts (a), (b) and (c). The candidates were required to find:

(a) the relationship between x and y given that the vectors $\underline{a} = x\underline{i} + y\underline{j}$ and $\underline{b} = 2\underline{i} + \underline{j}$ are perpendicular,

(b) the vector $\frac{1}{2}\underline{a} \times \underline{b}$ if $\underline{a} = 4\underline{i} + 2\underline{j} + \underline{k}$ and $\underline{b} = 3\underline{i} + 4\underline{j} + 5\underline{k}$,

(c) the image of $3x + 4y + 6 = 0$ under reflection on the line $y = -x$.

The question was attempted by 267 (67.9%) candidates. A total of 182 (68.2%) candidates scored marks ranging from 3.0 to 10, indicating that the general performance of candidates in this question was good. Figure 15 summarizes the candidates' performance in this question.

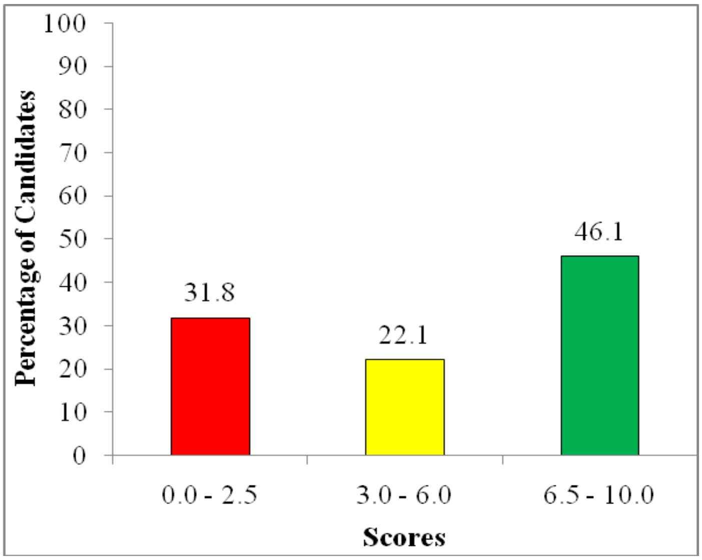


Figure 15: *The Candidates' Performance in Question 15*

Analysis of data showed that 40 (15.0%) candidates scored all allotted marks to this question. In part (a), the candidates were able to recall the condition $\underset{\sim}{a} \bullet \underset{\sim}{b} = 0$ for two perpendicular vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ and evaluated $(\underset{\sim}{a} = x\underset{\sim}{i} + y\underset{\sim}{j}) \bullet (2\underset{\sim}{i} + \underset{\sim}{j}) = 0$ correctly to get the relation $y = -2x$. In part (b), the candidates evaluated $\frac{1}{2}\underset{\sim}{a} \times \underset{\sim}{b} = \frac{1}{2}\begin{vmatrix} i & j & k \\ 4 & 2 & 1 \\ 3 & 2 & 5 \end{vmatrix}$ from $\underset{\sim}{a} = 4\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j} + 5\underset{\sim}{k}$ correctly to get $3i - \frac{17}{2}j + 5k$ as shown in Extract 15.1. In part (c), the candidates were able to find the image of any two points lying on the line $3x + 4y + 6 = 0$ using the formula $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ where $\alpha = -45^0$ or $\alpha = 135^0$ and $\begin{pmatrix} x \\ y \end{pmatrix}$ is the point. Finally, they used the images of the two points to calculate the slope $m = -\frac{4}{3}$ and the required equation which is $4x + 3y - 6 = 0$. A sample of good response from one of the candidates is presented in Extract 15.1.

Extract 15.1

15a) For perpendicular vectors

$\underline{a} \cdot \underline{b} = 0$

$(xi + yj) \cdot (2i + j) = 0$

$2x + y = 0$

$2x = -y$

$\therefore$ The relationship between x and y is that

$2x = -y$.

b) $\frac{1}{2}\underline{a} \times \underline{b}$

$\underline{a} = 4i + 2j + k$

$\frac{1}{2}\underline{a} = \frac{1}{2}(4i + 2j + k)$

$= 2\underline{i} + \underline{j} + \frac{1}{2}\underline{k}$

$\frac{1}{2}\underline{a} \times \underline{b}$

$(2i + j + \frac{1}{2}k) \times (3i + 4j + 5k)$

15b) $\frac{1}{2}\underline{a} \times \underline{b}$

$\begin{vmatrix} i & j & k \\ 2 & 1 & \frac{1}{2} \\ 3 & 4 & 5 \end{vmatrix}$

$\begin{vmatrix} 1 & \frac{1}{2} \\ 4 & 5 \end{vmatrix} i - \begin{vmatrix} 2 & \frac{1}{2} \\ 3 & 5 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} k$

$[(1\times5) - (4\times\frac{1}{2})]i - [(2\times5) - (3\times\frac{1}{2})]j + [(4\times2) - (3\times1)]k$

$[5-2]i - [10 - \frac{3}{2}]j + [8-3]k$

$3i - \frac{17}{2}j + 5k$

$\therefore \frac{1}{2}\underline{a} \times \underline{b} = 3i - \frac{17}{2}j + 5k$

c) $3x + 4y + 6 = 0$

$y = -x$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = 135°$

$3x + 4y + 6 = 0$

x intercept; $y = 0$

$3x + 4y + 6 = 0$

$3x + 4(0) + 6 = 0$

$3x + 6 = 0$

$x = -2$

$(-2, 0)$

y intercept; $x = 0$

$3x + 4y + 6 = 0$

$0 + 4y + 6 = 0$

$4y = -6$

$y = \frac{-6}{4}$

$= -\frac{3}{2}$

$(0, -\frac{3}{2})$

For reflection

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 270 & \sin 270 \\ \sin 270 & -\cos 270 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0 \\ 2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} \cos 270 & \sin 270 \\ \sin 270 & -\cos 270 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 + \frac{3}{2} \\ 0 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}$$

The two coordinates are $(0, 2)$ and $(\frac{3}{2}, 0)$

Slope $= \frac{\Delta y}{\Delta x}$

$= \frac{0-2}{\frac{3}{2}-0}$

$= \frac{-2}{\frac{3}{2}}$

Slope $= \frac{-4}{3}$

Equation

$M = \frac{y - y_1}{x - x_1}$

$\frac{-4}{3} = \frac{y-2}{x-0}$

$3(y-2) = -4x$

$3y - 6 = -4x$

$3y = -4x + 6$

$4x + 3y - 6 = 0$

$\therefore$ The equation of the line $3x + 4y + 6 = 0$ is $4x + 3y - 6 = 0$ after reflection in the line $y = -x$

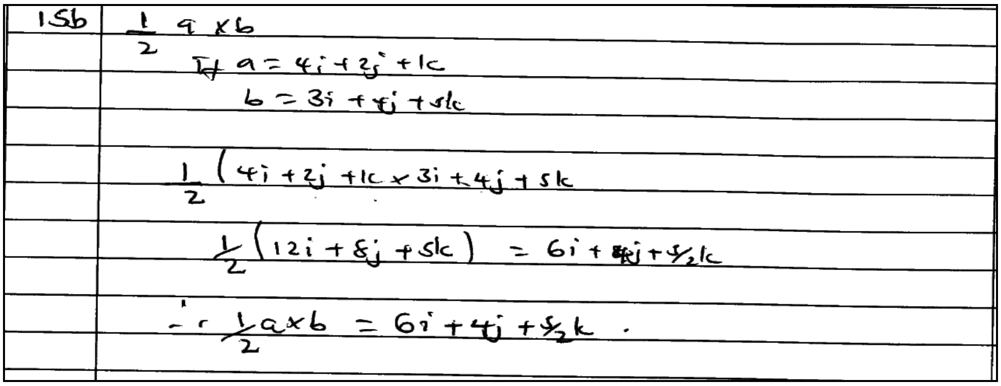
Extract 15.1: A sample of correct response from one of the candidates.

On the other hand, 85 (31.8%) candidates scored marks ranging from 0 to 2.5. In part (a), some candidates could not apply the condition $\underset{-}{a} \bullet \underset{-}{b} = 0$. Instead, they attempted the question wrongly by computing the equation of a straight line using the points (x, y) and $(2,1)$ derived from the vectors $\underset{-}{a} = x\underset{-}{i} + y\underset{-}{j}$ and $\underset{-}{b} = 2\underset{-}{i} + \underset{-}{j}$ respectively. Some candidates approached the question incorrectly by adding the two vectors $\underset{-}{a}$ and $\underset{-}{b}$ incorrectly to get $-x\underset{-}{i} + y\underset{-}{j}$.

In part (b), many candidates correctly arranged the cross product as $\frac{1}{2}\underset{-}{a} \times \underset{-}{b} = \frac{1}{2}\begin{vmatrix} i & j & k \\ 4 & 2 & 1 \\ 3 & 2 & 5 \end{vmatrix}$ but could not evaluate it to get $3i - \frac{17}{2}j + 5k$. Such candidates made a number of computational errors that led to incorrect answer. Apart from this, some candidates applied cross product incorrectly by taking product of corresponding coefficients of *i*, *j* and *k* to get incorrect answer $6i - 4j + \frac{5}{2}k$.

In part (c), several candidates approached the question wrongly by making y the subject of $3x + 4y + 6 = 0$ to get the equation of the reflected line $y = \frac{-3x}{4} - \frac{3}{2}$. Also, a notable number of candidates had correct procedures at the beginning, though they were unable to operate matrices correctly. Extract 15.2 is a sample of a poor response presented by one of the candidates.

Extract 15.2


1.5b $\frac{1}{2} a \times b$

If $a = 4i + 2j + 1k$

$b = 3i + 4j + 5k$

$\frac{1}{2}(4i + 2j + 1k \times 3i + 4j + 5k$

$\frac{1}{2}(12i + 8j + 5k) = 6i + 4j + 5/2k$

$\therefore \frac{1}{2} a \times b = 6i + 4j + 5/2k$.

Extract 15.2: A sample response of a candidate who multiplied the corresponding coefficients of *i*, *j* and *k*.

2.16 Question 16: Differentiation and Integration

The question instructed candidates to:

(a) find (i) $\int (x+1)\sqrt{x+3}dx$ (ii) $\int \tan^2 x \sec^2 x dx$ by using the substitution of $k = \tan x$;

(b) determine the area of region bounded by the curve $y = \frac{x^2}{2}$ and the line $y = x$; and

(c) find the volume of the solid of revolution formed by rotating the area enclosed by the curve $y = x + x^2$, the *x*- axis and the coordinates $x = 2$ and $x = 3$ through about *x*- axis.

This was the most skipped question in this examination whereby only 85 (21.6%) candidates attempted it. Out of these candidates, 50 (58.8%) candidates scored marks ranging from 3.0 to 9.5 signifying average performance of the candidates. Figure 16 summarizes candidates' performance in this question.

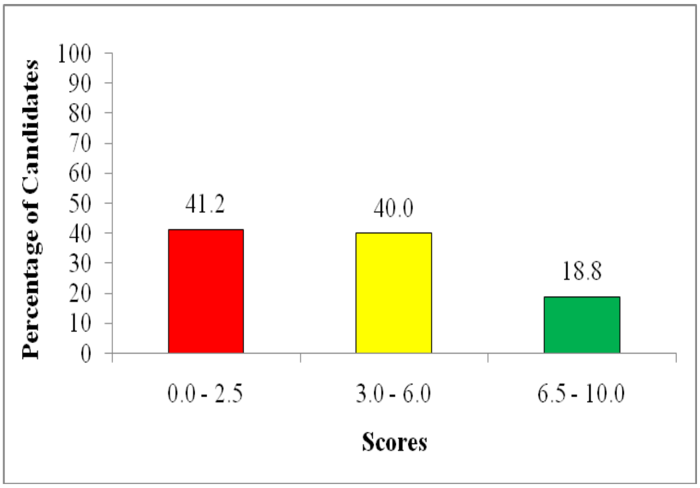


Figure 16: *The Candidates' Performance in Question 16*

In part (a), good performance was due to candidates' ability to use the technique of changing the variables through substitution. In part (a) (i), the candidates wrote $u = \sqrt{x+3}$, the idea which enabled them to have $x = u^2 - 3$ and $dx = 2udu$. After substitution, $\int (x+1)\sqrt{x+3}dx$ changed to

$\int(2u^4 - 4u^2)du$ which led to $\frac{2}{5}u^5 - \frac{4}{3}u^3 + C$. Finally, they got required integral $\frac{2}{5}(x+3)^{\frac{5}{2}} - \frac{4}{3}(x+3)^{\frac{3}{2}} + C$ after replacing u with $\sqrt{x+3}$ from $\frac{2}{5}u^5 - \frac{4}{3}u^3 + C$. Similarly, in part (a) (ii) the candidates got the integral $\frac{\tan^3 x}{3} + C$ after evaluating $\int \tan^2 x \sec^2 x dx$ by substituting $k = \tan x$ and $dk = \sec^2 x dx$.

In part (b), the candidates determined the limits $x = 0$ and $x = 2$ by either solving $y = \frac{x^2}{2}$ and $y = x$ simultaneously or drawing the graphs. Later on, they correctly treated $y = x$ as upper function and $y = \frac{x^2}{2}$ as lower function to obtain $Area = \int_0^2 \left(x - \frac{x^2}{2}\right)dx$ and finally $Area = \frac{2}{3}$ *square units*.

In part (c), the candidates realized that $x = 2$ and $x = 3$ are the limits hence they substituted $y = x + x^2$ into the formula $V = \pi\int_2^3 y^2 dx$ to get $V = \frac{27}{10}\pi$ *cubic units* as seen in Extract 16.1.

Extract 16.1

16 c · $\text{Volume} = \int_a^b \pi y^2 \cdot dx.$

$b = 3, \quad a = 2.$

$y^2 = (x + x^2)^2.$

$\text{Volume} = \int_2^3 \pi (x + x^2)^2 . dx.$

$= \int_2^3 \pi (x^2 + 2x^3 + x^4) dx.$

$= \left[\pi\left(\frac{x^3}{3} + \frac{2x^4}{4} + \frac{x^5}{5}\right)\right]_2^3.$

$= \left(\frac{3^3}{3} + \frac{2(3)^4}{4} + \frac{3^5}{5}\right)\pi - \left(\frac{2^3}{3} + \frac{2(2)^4}{4} + \frac{2^5}{5}\right)\pi$

$= \left(9 + \frac{81}{2} + \frac{243}{5}\right)\pi - \left(\frac{8}{3} + 8 + \frac{32}{5}\right)\pi$

$$= \left(\frac{90+405+486}{10}\right)\pi - \left(\frac{40+120+96}{15}\right)\pi$$

$$= \frac{981\pi}{10} - \frac{256\pi}{15}$$

$$= \left(\frac{14715 - 2560}{150}\right)\pi$$

$$= \frac{2431\pi}{30}$$

$\therefore$ Volume of revolution is $\frac{2431\pi}{30}$.

Extract 16.1: A sample response of a candidate who managed to compute volume of solid of revolution.

On the other hand, the analysis of data showed that no candidate who scored all the allotted marks to this question. Analysis of candidates' responses revealed that 9 (10.6%) candidates scored a zero mark. In part (a) (i), most of the candidates wrote $u = x + 3$, which was actually correct. However, these candidates failed to evaluate the integral of terms involving radicals, $\int (u-2)\sqrt{u}\,du$. They were supposed to write $u = \sqrt{x+3}$ which eliminates the radical sign after correct substitution.

The poor performance in part (a) (ii) for some candidates resulted from failure to adhere to instruction of using $k = \tan x$. For example, there were candidates who used substitution $u = \sec^2 x$, which did not lead to correct solution. Also, majority of the candidates could not make a distinction between the concepts of differentiation and integration as they got $\frac{du}{dx} = \tan x$ from $u = \sec^2 x$.

In part (b), several candidates could not consider both curves $y = \frac{x^2}{2}$ and $y = x$ when calculating area of bounded region. Instead, they considered the curve $y = \frac{x^2}{2}$ only that led to $Area = \int_0^2 \frac{x^2}{2}\,dx$ which resulted into wrong answer, $\frac{4}{3}$.

In part (c), a notable number of candidates applied incorrect formula $V = \int_a^b y dx$ instead of $V = \pi \int_a^b y^2 dx$. Such candidates obtained incorrect answer, $\frac{53}{6}$. Extract 16.2 is a response of a candidate who did the question incorrectly.

Extract 16.2

16. (ii)

$\int \tan^2 x \cdot \sec^2 x \; \delta x.$

let $U = \sec^2 x.$

$\frac{\delta U}{\delta x} = \tan x.$

but $\tan x = k.$

$\int k^2 \cdot U \; \frac{\delta U}{k}$

$\int k \cdot k \cdot U \; \frac{\delta U}{k}$

$= \int k \cdot U \; \delta U.$

but $k = \tan x$

$\int \tan x \cdot U \; \delta U.$

$\left[\sec^2 x \cdot \frac{U^2}{2}\right] + C$

$\sec^2 x \cdot \frac{(\sec^2 x)^2}{2} + C.$

$\sec^2 x \cdot \frac{\sec^4 x}{2} + C.$

$= \frac{\sec^6 x}{2} + C.$

$\therefore = \frac{\sec^6 x}{2} + C.$

16 b.

$$\int_0^2 \frac{x^2}{2}\, \delta x.$$

$$\frac{1}{2}\int_0^2 x^2\, \delta x$$

$$\frac{1}{2}\left[\frac{x^3}{3}\right]_0^2$$

$$\frac{1}{2}\left[\frac{x^3}{3}\right]^2 - \frac{1}{2}\left[\frac{x^3}{3}\right]^0.$$

$$\frac{1}{2}\left(\frac{2^3}{3}\right) - \frac{1}{2}\left(\frac{0^3}{3}\right)$$

$$= \frac{8}{6}$$

$$= \frac{4}{3}.$$

$\therefore$ Area of the region bounded by the curve $y = \frac{x^2}{2}$ and the line $y = x$ is $\frac{4}{3}$ square unit

Extract 16.2: A sample response of a candidate who did not adhere to the instruction of part (a) (ii) and applied incorrect formula in part (b).

3.0 CONCLUSION AND RECOMMENDATIONS

3.1 Conclusion

In general, the analysis of candidates' performance in each question showed that 18 topics were examined. Candidates' performance in eight (8) topics examined in questions 2, 5, 9, 13, 14 and 15 was good. These topics were: *Geometrical Constructions, Sets, Logic, Vectors, Matrices and Linear Transformations, Locus, Permutations and Combinations* and *Probability*. Candidates' performance in the remaining ten (10) topics examined in questions 1, 3, 4, 6, 7, 8, 10, 11, 12 and 16 was average. These topics included; *Trigonometry, Variations, Equations and Remainder Theorem, Plan and Elevations, Differentiation, Integration, Algebra, Numbers, Statistics* and *Coordinate Geometry*. Furthermore, the analysis showed that the overall performance in this examination was good because 70.48 percent of the candidates passed.

Good performance in some topics was greatly contributed by ability of candidates to use mathematical instruments, perform operations, use of correct mathematical facts/theorems/laws and formulae as well as adhering to the instructions of the questions.

The weak performance for some candidates was partly attributed to: candidates' inability to recall and use correct mathematical facts/theorems/laws and formulae; failure to adhere to the instructions of the questions; incompetence in performing operations (on numbers, sets, logic and vectors); insufficient knowledge and skills in solving equations and using mathematical instruments to present information using diagrams and graphs. A summary of analysis of the candidates' performance in each topic is shown in Appendix I.

3.2 Recommendations

In order to improve the candidates' performance in the future, this report presents the following recommendations:

(a) Students should do many exercises in order to improve their ability to apply facts/theorems/laws and formulae in solving questions.

(b) Teachers and students should use mathematical instruments during teaching and learning process to improve their competencies in answering questions.

(c) Teachers should allocate enough time in discussing the correctness of the various mathematical facts/theorems/laws/formula during teaching and learning process.

(d) Teachers should employ mnemonics, teaching aids and examples that reflect the daily life of the students for easier grasp of the concepts.

(e) Teachers should set revision questions and tests, which measure competence of the students rather than content only. This will help the students to be familiar with competence-based questions, which is an important component in assessment of learners by the National Examinations Council of Tanzania.

(f) The Government, through their education authorities, should strive to improve the performance by making consultations with other education stakeholders.

Appendix

Analysis of Candidates' Performance per Topic in Additional Mathematics 2018

S/N	Topic	Questions Number	Percentage of Candidates who Scored an Average of 30% or Above	Remarks
1	Geometrical Constructions	5	84.2	Good
2	Sets	2	82.2	Good
3	Logic	13	70.6	Good
4	Vectors, Matrices and Linear Transformation	15	68.2	Good
5	Locus	9	65.9	Good
6	Permutation , Combination and Probability	14	64.2	Good
7	Trigonometry	8	63.3	Average
8	Variations	6	61.5	Average
9	Equations and Remainder Theorem	3	59.4	Average
10	Plan and Elevations	10	52.6	Average
11	Differentiation and Integration	7,16	48.8	Average
12	Algebra	4	40.3	Average
13	Numbers	1	38.5	Average
14	Statistics	12	36.8	Average
15	Coordinate Geometry	11	31.3	Average