THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2023

ADVANCED MATHEMATICS

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CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2023

142 ADVANCED MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania has prepared this report on the analysis of the candidates' responses for ACSEE 2023 Advanced Mathematics to show how the candidates responded to the questions. The analysis of the data and the candidates' responses are presented so as to point out the strengths and weaknesses of the candidates based on their responses to the items. The report provides feedback for education stakeholders to device some mechanisms to improve the candidates' performance.

Generally, the candidates' performance in the Advanced Mathematics paper was good, as 96.86 per cent of the candidates who sat for the examination passed. The analysis shows that candidates' performance in 13 out of the 18 topics tested was good. Candidates did well in the topics of Functions, Logic, Statistics, Sets, Trigonometry, Differential Equations, Coordinate Geometry I, Algebra, Hyperbolic Functions, Linear Programming, Calculating Devices, Complex Numbers, and Vectors.

It was noted further that the candidates had average performance in the topics of Integration, Coordinate Geometry II, and Differentiation, but they performed weakly in the topics of Probability and Numerical Methods. Failure to understand the concepts of probability distributions, especially Binomial and Poison distributions, and the inability to apply the techniques of partial derivatives and numerical integration contributed to their weak performance.

It is the credence of the National Examinations Council of Tanzania that the analysis presented in this report will pave way for the education stakeholders to adopt the most appropriate measures to improve more candidates' future performance in this subject.

The National Examinations Council of Tanzania would like to express its sincere appreciation to all individuals who participated in preparing this report, including the examination officers who coordinated the whole exercise.

Dr Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Candidates' Items Response Analysis (CIRA) report on ACSEE 2023, Advanced Mathematics subject provides feedback to education stakeholders on how the candidates responded to the examination items. This report is based on the analysis of the data and the candidates' responses.

The report analyses the candidates' performance in all the topics examined in Advanced Mathematics. The Advanced Mathematics examination had two papers: Paper 1, 142/1 Advanced Mathematics 1, which had ten (10) compulsory questions, with each question carrying ten (10) marks. Paper 2, 142/2 Advanced Mathematics 2, comprised of sections A and B. Section A consisted of four (4) compulsory questions which weighed fifteen (15) marks each, while Section B consisted of four (4) optional questions, each weighing twenty (20) marks. The candidates were required to attempt only two (2) questions from section B.

The analysis of each question is presented in Section 2.0. The focus being on the requirements of each question, the analysis of data on candidates' performance in the question, and the candidates' responses. Figures and charts have also been used to show the performance of the candidates. Also, extracts for good and poor responses are included to illustrate the specific cases of candidates' responses to an item. The percentage of candidates' performance in each question is categorised as weak, average, and good in the intervals of 0–34, 35–59, and 60–100, respectively.

In 2023, a total of 13,755 candidates sat for the Advanced Mathematics examination, out of whom 13,308 (96.86%) passed. In comparison, the 2023 performance is 1.03% lower than that of 2022, where a total of 13,105 (97.89%) candidates passed.

The percentages of candidates who passed the Advanced Mathematics examination in different grades are presented in Figure 1.

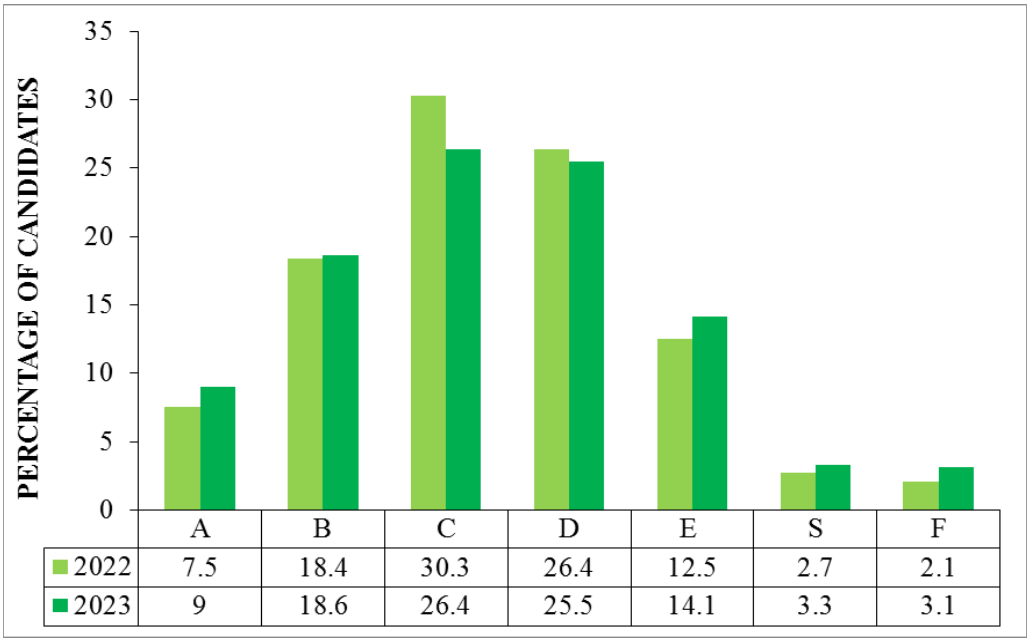


Figure 1: *Overall Performance in the 2022 and 2023 Advanced Mathematics Examination*

Figure 1 illustrates that there is a slight fall in the quality of performance in 2023 as compared to 2022. For example, the percentage of candidates who scored grades F, S, and E has increased compared to 2022, while those who scored grades C and D have decreased by 3.9% and 0.9%, respectively. However, the number of candidates who scored grades A and B increased by 1.5% and 0.2%, respectively.

Section 3.0 presents the analysis of candidates' responses per topic. The factors that contributed to good, average, and weak performance across the topics are highlighted. Finally, the conclusion and recommendations have been given in Section 4.0.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

This section gives an analysis of the candidates' performance in each question. The performance in each question is described into three categories: good, average, or weak under the percentage boundaries of 60 – 100, 35 – 59 and 0 – 34, respectively.

2.1 142/1 ADVANCED MATHEMATICS 1

2.1.1 Question 1: Calculating Devices

This question had parts (a) and (b). The candidates were required to use a scientific calculator to approximate the mean and standard deviation of the constants π, $\sqrt{2}$, e, $\sqrt{3}$, 1.414213, 2.718282, 3.1415, 1.732051 correct to six decimal places, in part (a). In part (b), the candidates were required to compute the value of the following expressions correct to six significant figures using a scientific calculator:

(i) $$(21)\times\begin{vmatrix} 60 & 18 & 49 \\ 50 & 0 & -14 \\ 20 & -6 & -20 \end{vmatrix}\times\left(e^{\pi}\right)\times\left(\log_8{}^{16}\right)\times\left(2.7\times 10^{-8}\right).$$

(ii) $$\left(\frac{\ln 612}{\ln 121+4\ln 2}\right)^{\frac{1}{4}}\ln\left(\frac{\ln\left(\frac{22}{7}\right)}{\log\left(\frac{22}{7}\right)}\right)^{\frac{1}{8}}$$

The analysis shows that 50.4 per cent of the candidates who attempted the question scored from 6 to 10 marks, 17.0 per cent scored from 3.5 to 5.5 marks, and 32.6 per cent scored from 0 to 3 marks. Generally, the candidates' performance in this question was good, as 67.4 per cent of the candidates scored not less than 3.5 marks. Figure 2 illustrates the candidates' performance in this question.

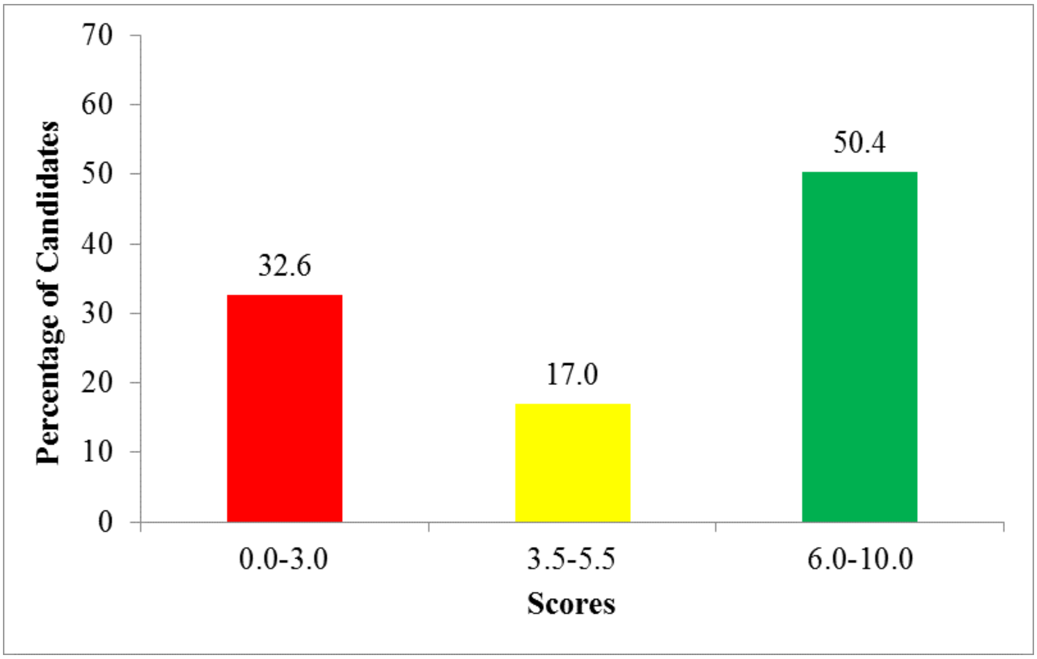


Figure 2: *Candidates' Performance in Question 1 of Paper 1*

The candidates who performed well in this question were able to use the scientific calculators' functions correctly to approximate the mean and standard deviation of the given constants. For example, in part (a), they obtained the value of the mean ($\bar{x}$) equal to 2.251523 and the standard deviation (δ) equal to 0.703730 correct to six decimal places. Also, in part (b), they obtained the correct answers -0.118612 or 1.18612×10^{-1} correct to six significant figures, in (b) (i) and 0.100039 or 1.00039×10^{-1} correct to six significant figures in (b) (ii). Extract 1.1 shows a sample response from a candidate who performed well in this question.

1. (a) Mean = 2.251523

Standard deviation = 0.703730.

(b)

(i) −0.118612.

(ii) 0.100039.

Extract 1.1: A sample of correct responses to question 1of paper 1

In Extract 1.1, the candidate used the scientific calculators' functions appropriately, thus was able to get the correct answers.

Although many candidates performed well, there were a few who showed inadequate skills in tackling the question. In part (a), the candidates failed to use appropriate statistical functions to determine the mean and standard deviation from the given data. For example, some candidates obtained mean as $\bar{x} = 2.035017$, $\bar{x} = 2.425894$, $\bar{x} = 24.358824$, $\bar{x} = 4.457024$ and standard deviation as $\delta = 1.193021$, $\delta = 0.933194$, $\delta = 58.830082$, $\delta = 2.369863$, instead of $\bar{x} = 2.251523$ and $\delta = 0.703730$. Also, in part (b), some candidates failed to compute the required value, while others manipulated the given expressions well but failed to write the resulting answers in appropriate decimal places or significant figures. For example, in part (b) (i), some wrong responses were 0.118612, -0.111055, -7.63400, 0.00577, 0.329165, and in part (b) (ii), samples of incorrect responses are such as 0.967534, 0.0610066, 0.0220975, 01000394, 0.940996. Extract 1.2 is an example of a candidate's response who had inadequate skills in the concepts tested.

1. Solution.

Given:

$\pi, \sqrt{2}, e, \sqrt{3}, 1.414213, 2.718282, 3.1415, 1.732051$

$= 3.1415, 1.4142, 2.7182, 1.732, 1.414213, 2.718282, 3.1415, 1.732051.$

Required to approximate mean:

Mean $(\bar{x}) = 2.2514$

$\therefore$ Mean $(\bar{x}) = 2.25149$

Then Standard deviation $= 0.75230$

$\therefore$ Mean $= 2.2514$

Standard devition $= 0.75230.$

b). Solution

Given.

$21 \times \begin{pmatrix} 60 & 18 & 49 \\ 50 & 0 & -14 \\ 20 & -6 & -20 \end{pmatrix} \times (e^{\pi}) \times \log_8 16 \times (2.7 \times 10^{3}$

Extract 1.2: A sample of incorrect responses to question 1 of paper 1

In Extract 1.2, the candidate was not able to use the scientific calculators' functions appropriately, and therefore failed to get the correct mean and standard deviation.

2.1.2 Question 2: Hyperbolic Functions

This question had parts (a), (b) and (c). In part (a), the candidates were required to express h in terms of m and n if the equation $m\sinh x + n\cosh x = h$ has equal roots. In part (b), the candidates were required to prove that $(\cosh x - \cosh y)^2 - (\sinh x - \sinh y)^2 = -4\sinh^2\left(\frac{x-y}{2}\right)$ and finally in part (c), it required them to integrate the integrand $\frac{1}{\sqrt{x^2+2x+10}}$ with respect to x.

The data analysis shows that 13,755 candidates (100%) attempted this question. 4,125 (30.0%) candidates scored from 0 to 3 marks, while 2,894 (21.0%) candidates scored from 3.5 to 5.5 marks, and 6,736 (49.0%) candidates scored from 6 to 10 marks. Figure 3 shows the percentage of candidates' performance.

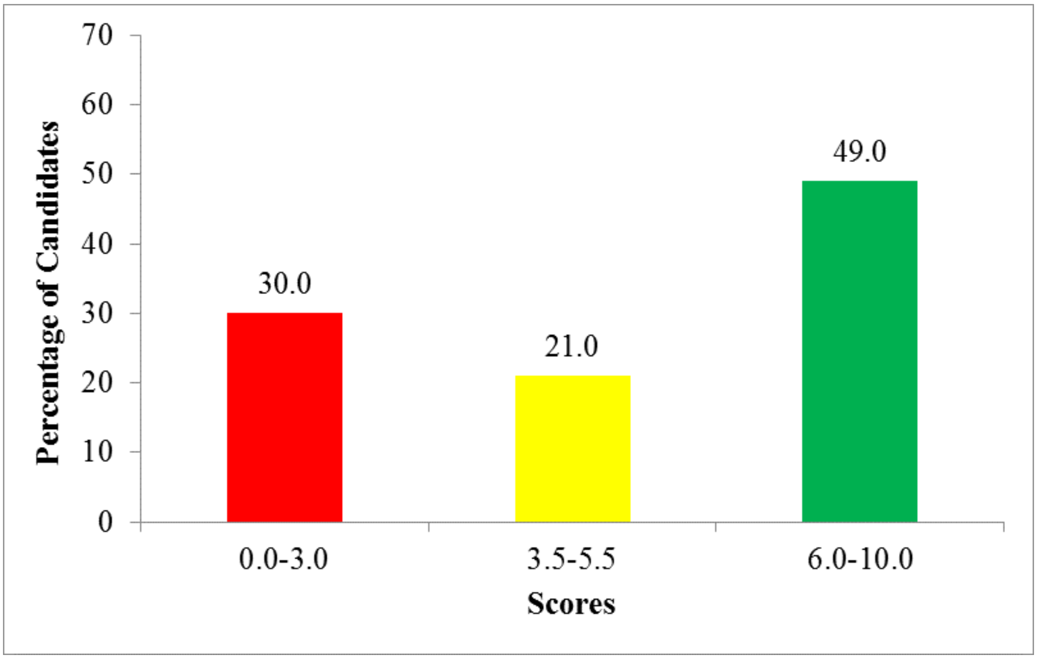


Figure 3: *Candidates' Performance in Question 2 of Paper 1*

Figure 3 shows that only 30 per cent of the candidates scored less than 3.5, indicating good performance. The majority of the candidates showed outstanding performance in this question. In part (a), the candidates defined $\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$ and $\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$ then substituted in $m\sinh x + n\cosh x = h$ which led them to $(m+n)e^{2x} - 2he^{x} + (n-m) = 0$. Thereafter, they rearranged the equation to obtain $-(m+n)e^{2x} + 2he^{x} + (m-n) = 0$ and eventually solved the equation to get $h = \pm\sqrt{n^2 - m^2}$. In part (b), the candidates managed to express $\cosh x - \cosh y$ as $2\sinh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$ and $\sinh x - \sinh y$ as $2\cosh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$ as factor formulae, then simplified the given equation $(\cosh x - \cosh y)^2 - (\sinh x - \sinh y)^2$ to get $-4\sinh^2\left(\frac{x-y}{2}\right)$ as per requirement of the question. In part (c), the candidates were able to factorize $x^2 + 2x + 10$ by completing the square that is, $(x+1)^2 + 9$. Then the candidates used the correct hyperbolic substitution, $(x+1) = 3\sinh\theta$ to evaluate the $\int \frac{1}{\sqrt{x^2 + 2x + 10}}\,dx$ and obtained $\sinh^{-1}\left(\frac{x+1}{3}\right) + c$. Extract 2.1 shows a sample response from a candidate who performed well in this question.

	Consider L.H.S.
2b.	$(\cosh x - \cosh y)^2 - (\sinh x - \sinh y)^2$
	from factor formula.
	$\left(2\sinh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)\right)^2 - \left(2\sinh\left(\frac{x-y}{2}\right)\cosh\left(\frac{x+y}{2}\right)\right)^2$
	$4\sinh^2\left(\frac{x-y}{2}\right)\sinh^2\left(\frac{x+y}{2}\right) - 4\sinh^2\left(\frac{x-y}{2}\right)\cosh^2\left(\frac{x+y}{2}\right)$
	$4\sinh^2\left(\frac{x-y}{2}\right)\left(\sinh^2\left(\frac{x+y}{2}\right) - \cosh^2\left(\frac{x+y}{2}\right)\right)$.
	$-4\sinh^2\left(\frac{x-y}{2}\right)\left(\cosh^2\left(\frac{x+y}{2}\right) - \sinh^2\left(\frac{x+y}{2}\right)\right)$.
	but $\cosh^2 x - \sinh^2 x = 1$.
	$-4\sinh^2\left(\frac{x-y}{2}\right) \times 1$.
	$\underline{-4\sinh^2\left(\frac{x-y}{2}\right)}$ hence proved.
2c.	$\displaystyle\int \frac{1}{\sqrt{x^2+2x+10}}\,dx.$
	by completing the square.
	$x^2+2x+10$
	$a=1,\ b=2,\ c=!$

2c

$c = (b/2)^2$.

$c = (2/2)^2$.

$c = 1^2$

$c = 1$.

$x^2 + 2x + 1 + 10 - 1$.

$(x+1)^2 + 9$.

$$\int \frac{1}{\sqrt{(x+1)^2+9}}\,dx.$$

$$\int \frac{1}{\sqrt{9\left[\left(\frac{x+1}{3}\right)^2+1\right]}}\,dx.$$

$$\int \frac{1}{3\sqrt{\left(\frac{x+1}{3}\right)^2+1}}\,dx.$$

let $\left(\frac{x+1}{3}\right) = \sinh\theta$

$\left(\frac{x+1}{3}\right)^2 = \sinh^2\theta$.

$\frac{d}{dx}\left(\frac{x+1}{3}\right) = \frac{d}{dx}\sinh\theta$

$\frac{3}{9} = \cosh\theta\,\frac{d\theta}{dx}$

$\frac{1}{3} = \cosh\theta\,\frac{d\theta}{dx}$.

$dx = 3\cosh\theta\,d\theta$.

$\sinh\theta = \left(\frac{x+1}{3}\right)$.

$\theta = \sinh^{-1}\left(\frac{x+1}{3}\right)$.

2c. $\int \frac{dx}{3\sqrt{\left(\frac{x+1}{3}\right)^2+1}} = \int \frac{3\cosh\theta\,d\theta}{3\sqrt{\sinh^2\theta+1}}$.

$$\int \frac{\cosh\theta\,d\theta}{\sqrt{\sinh^2\theta+1}}$$

but $\cosh^2\theta - \sinh^2\theta = 1$

$\cosh^2\theta = 1 + \sinh^2\theta$.

$$\int \frac{\cosh\theta\,d\theta}{\sqrt{\cosh^2\theta}} = \int \frac{\cosh\theta\,d\theta}{\cosh\theta}$$

$$\int \frac{\cancel{\cosh\theta}}{\cancel{\cosh\theta}}\,d\theta.$$

$\int d\theta = \theta + c$.

$\theta = \sinh^{-1}\left(\frac{x+1}{3}\right)$.

$$\int \frac{1}{\sqrt{x^2+2x+10}}\,dx = \sinh^{-1}\left(\frac{x+1}{3}\right) + c.$$

$\sinh^{-1}\left(\frac{x+1}{3}\right) = y$.

$\frac{x+1}{3} = \sinh y$.

$\frac{x+1}{3} = \frac{e^y - e^{-y}}{2}$.

$2e^y\frac{(x+1)}{3} = e^{2y} - 1$.

$e^{2y} - 2e^y\left(\frac{x+1}{3}\right) - 1 = 0$.

$e^y = \frac{2\left(\frac{x+1}{3}\right) \pm \sqrt{4\left(\frac{x+1}{3}\right)^2 - 4 \times -1}}{2}$

2c

$$e^y = \left(\frac{x+1}{3}\right) \pm \sqrt{4\left(\frac{x+1}{3}\right)^2 + 4}$$

$$e^y = \frac{x+1}{3} \pm 2\sqrt{\left(\frac{x+1}{3}\right)^2 + 1}$$

$$y = \ln\left(\frac{x+1}{3} \pm 2\sqrt{\left(\frac{x+1}{3}\right)^2 + 1}\right)$$

$$\therefore \int \frac{1}{\sqrt{x^2+2x+10}}\,dx = \ln\left[\frac{x+1}{3} + 2\sqrt{\left(\frac{x+1}{3}\right)^2 + 1}\right] + C$$

Extract 2.1: A sample of correct responses to question 2 of paper 1

In Extract 2.1, the candidate was able to use the correct hyperbolic definitions, factor formulae, and complete the squares correctly, thus arriving at the required answers.

Apart from a good performance in this question, there were fewer candidates who failed to answer this question correctly. In part (a), some candidates used an inappropriate approach to express 'h' in terms of m and n. For example, one candidate solved the value of h by taking $\alpha + \beta = \frac{2h}{m+n}$ and $\alpha\beta = \frac{-(m-n)}{m+n}$ then concluded that for equal roots $\alpha + \beta = \alpha\beta$ then $\frac{2h}{m+n} = \frac{-(m-n)}{m+n}$. The candidate obtained wrong answer of $h = -m + n$. Also, few candidates used inappropriate formulae to get 'h'. For example, one candidate applied the R-formula by letting $h = R\sinh(x+\alpha) = R\sinh x\cosh\alpha + R\cosh x\sinh\alpha$ then compared to $m\sinh x + n\cosh x = h$ so that $\alpha = \tanh^{-1}(\frac{n}{m})$ and $R = \sqrt{m^2 - n^2}$. The candidate concluded that $h = \sqrt{m^2 - n^2}\sinh(x + \tanh^{-1}(\frac{n}{m})$.

In part (b), some candidates failed to prove the given relation appropriately. For example, one of the candidates considered the right hand side of the equation in the process of proving it by taking $-4\sinh^2\left(\frac{x-y}{2}\right)=\frac{-4\sinh^2 x+4\sinh^2 y}{2}$ which resulted to $-2(\sinh^2 x-\sinh^2 y)$. Finally, the candidate concluded that $-4\sinh^2\left(\frac{x-y}{2}\right)=-2(\sinh^2 x-\sinh^2 y)=(\cosh x-\cosh y)^2-(\sinh x-\sinh y)^2$.

In part (c), some candidates failed to apply appropriate technique of integration as they didn't realize that $\sqrt{x^2+2x+10}$ could be factorized to $\sqrt{(x+1)^2+9}$ as an essential step to evaluate the given integral. For example, one candidate considered $\sqrt{x^2+2x+10}=(x^2+2x+10)^{-2}$ then integrated it and got $\frac{(x^2+2x+10)^{-2+1}}{-1+2}=\frac{1}{x^2+2x+10}$. Also, in a few circumstances, some candidates concluded that $\int\frac{1}{\sqrt{x^2+2x+10}}dx=\ln x+2x^2+c$. Extract 2.2 represents a sample response from a candidate who attempted this question incorrectly.

a) Since they have equal roots.

$\alpha+\beta=-b/a=2h/m+n$.

$\alpha\beta=c/a=-(m-n)/m+n$.

Thus $\alpha\beta=\alpha+\beta$.

$\frac{-(m-n)}{m+n}=2h/m+n$.

$2h=\frac{-(m-n)(m+n)}{m+n}$

$2h=n-m$.

$h=n-m/2$.

The value of $h=\frac{n-m}{2}$.

2.(a) $m \sinh x + n \cosh x = h$

let $h = R \sinh(x+\alpha)$

$h = R(\sinh x \cosh\alpha + \cosh x \sinh\alpha)$

$h = R\sinh x \cosh\alpha + R\cosh x \sinh\alpha$

By comparing

$$\frac{m \sinh x}{\sinh x} = \frac{R \sinh x \cosh\alpha}{\sinh x}$$

$$m = R\cosh\alpha$$

$$\frac{n\cosh x}{\cosh x} = \frac{R\cosh x \sinh\alpha}{\cosh x}$$

$$n = R\sinh\alpha$$

$$\frac{R\sinh\alpha}{R\cosh\alpha} = \frac{n}{m}$$

$$\tanh\alpha = n/m$$

$$\alpha = \tanh^{-1}(n/m)$$

$h = R\sinh(x+\alpha)$

$h = \cancel{n/m} R\sinh(x+\alpha)$

$R^2\cosh^2\alpha - R^2\sinh^2\alpha = m^2 - n^2$

$R^2(\cosh^2\alpha - \sinh^2\alpha) = m^2 - n^2$

$R^2(1) = m^2 - n^2$

$R = \sqrt{m^2 - n^2}$

$h = R\sinh(x+\alpha)$

$h = \sqrt{m^2-n^2}\,\sinh(x + \tanh^{-1}(n/m))$

$m\sinh x + n\cosh x = (\sqrt{m^2-n^2})\sinh(x + \tanh^{-1}(n/m))$.

2. b) $(\cosh x - \cosh y)^2 - (\sinh x - \sinh y)^2 = -4\sinh^2\left(\frac{x-y}{2}\right)$

Soln

From R.H.S.

$= -4\sinh^2\left(\frac{x-y}{2}\right)$

$= -\frac{4\sinh^2 x + 4\sinh^2 y}{2}$

$= -\frac{4}{2}(\sinh^2 x - \sinh^2 y)$

$= 2\sinh^2 x - \sinh^2 y$

From $\sinh 2x = 2\sinh\cosh$.

$= 2(\sinh x - \sinh y)^2 = (\cosh x - \cosh y)^2 - (\sinh x - \sinh y)^2$

$\therefore -4\sinh^2\left(\frac{x-y}{2}\right) = (\cosh x - \cosh y)^2 - (\sinh x - \sinh y)^2.$

2 (c) $\int = \frac{1}{\sqrt{x^2+2x+10}}$

Soln.

$= \sqrt{x^2+2x+10} = (x^2+2x+10)^{-2}.$

$= \frac{(x^2+2x+10)^{-2+1}}{+1+2.}$

$= \frac{(x^2+2x+10)^{-1}}{-1}.$

$= \frac{1}{-(x^2+2x+10)}$

$= -\frac{1}{x^2+2x+10}.$

Extract 2.2: A sample of incorrect responses to question 2 of paper 1

In Extract 2.2, the candidate failed to apply appropriate techniques for hyperbolic functions for each part of the question.

2.1.3 Question 3: Linear Programming

The question comprised two parts, (a) and (b). In part (a), the question read as follows: "A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make each ring and 30 minutes to make each chain. The maximum number of hours available per day is 16. The profit on a ring is Shs. 3000 and that on a chain is Shs. 1900. If x and y are the numbers of rings and chains, respectively". From this information, the candidates were required to formulate a linear programming problem.

In part (b), the question stated that "A company owns two mines, A and B. Each day mine A produces 1 ton of high grade ore, 3 tons of medium grade ore and 5 tons of low grade ore, while mine B produces 2 tons of each of the three grades of ore. The company needs 80 tons of high grade ore, 160 tons of medium grade ore and 200 tons of low grade ore. It costs sh. 200,000/= per day to operate each mine". Candidates were told to find the number of days in which each mine would operate.

The analysis of the data shows that a total of 13,755 candidates attempted the question. Out of them, 32.0 per cent scored from 0 to 3 marks, 39.6 per cent scored from 3.5 to 5.5 marks, and 28.4 per cent scored from 6 to 10 marks. The general candidates' performance is good, as 68.0 per cent of the candidates scored more than 3 marks. Figure 4 illustrates the candidates' performance in this question.

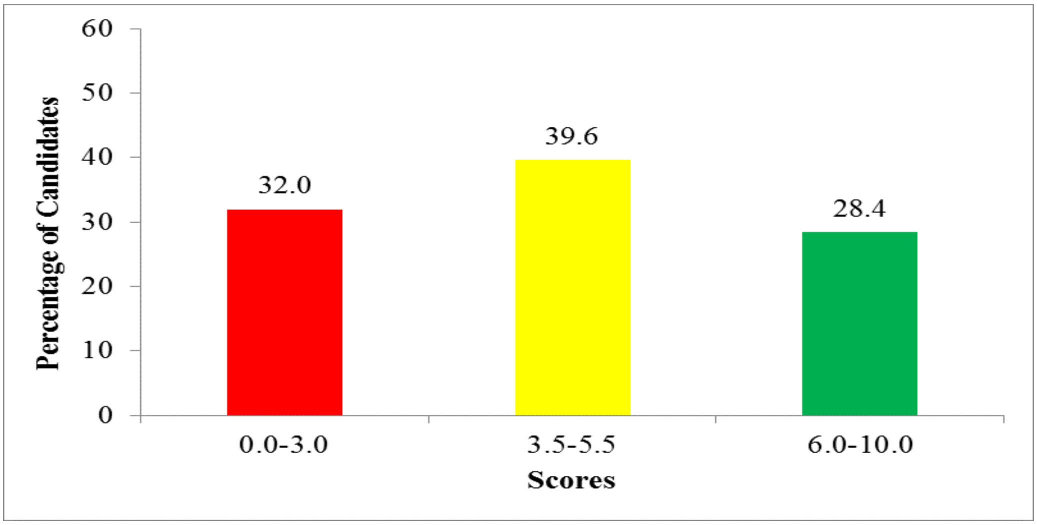


Figure 4: *Candidates' Performance in Question 3 of Paper 1*

Most of the candidates performed well in this question; for instance, in part (a), they were able to formulate the linear programming problem correctly as Maximize: $f(x, y) = 3000x + 1900y$, subject to $x + \frac{1}{2}y \le 16$, $x + y \le 24$, $x, y \ge 0$.

Also, in part (b), most of the candidates were able to interpret the linear programming problem, which helped them to formulate the Mathematical model. The model was Minimize: $f(x, y) = 200{,}000(x + y)$ subject to $x + 2y \ge 80$, $3x + 2y \ge 160$, $5x + 2y \ge 200$, $x, y \ge 0$. Thereafter, they located the inequalities on the $xy-$plane, which enabled them to get $(80, 0)$, $(40, 20)$, $(20, 50)$ and $(0, 100)$ as the corner points. From the points, the candidates realized that mines A and B should operate for 40 and 20 days, respectively with a minimum cost of Tsh 12,000,000. Extract 3.1 is a sample response from a candidate who attempted this question well.

03. (a) x – Numbers of rings
y – Numbers of chains
$x + y \le 24$ —— (i).
$x + \frac{1}{2}y \le 16$ —— (ii)

Objective function f(x, y) is to maximize profit.
$f(x, y) = 3000x + 1900y$
Non zero constraint
$x \ge 0$ —— (iii)
$y \ge 0$ —— (iv).

(b)

Mine	A	B	Resource Demand
High grade ore	1	2	80
Medium grade ore	3	2	160
Low grade ore	5	2	200
Cost	200,000	200,000	

Let x be number of days for operation of mine A
y be number of operating days of mine B.
$x + 2y \ge 80$ —— (i).
$3x + 2y \ge 160$ —— (ii)
$5x + 2y \ge 200$ —— (iii).
Non zero constraint
$x \ge 0$ —— (iv)
$y \ge 0$ —— (v)
Objective function is to minimize cost of operation.
$f(x, y) = 200\,000x + 200\,000y$
to be minimized.

03 Subjected to $x+2y=80$

x	0	80
y	40	0

$3x+2y=160$

x	0	53.3
y	80	0

$5x+2y=200$

x	0	40
y	100	0

Point	$f(x,y) = 200000x + 200000y$
A(0,100)	20,000,000
B(20,50	14,000,000
C(40,20	12,000,000
D(80,0)	16 000 000

Optimal point C(40,20)

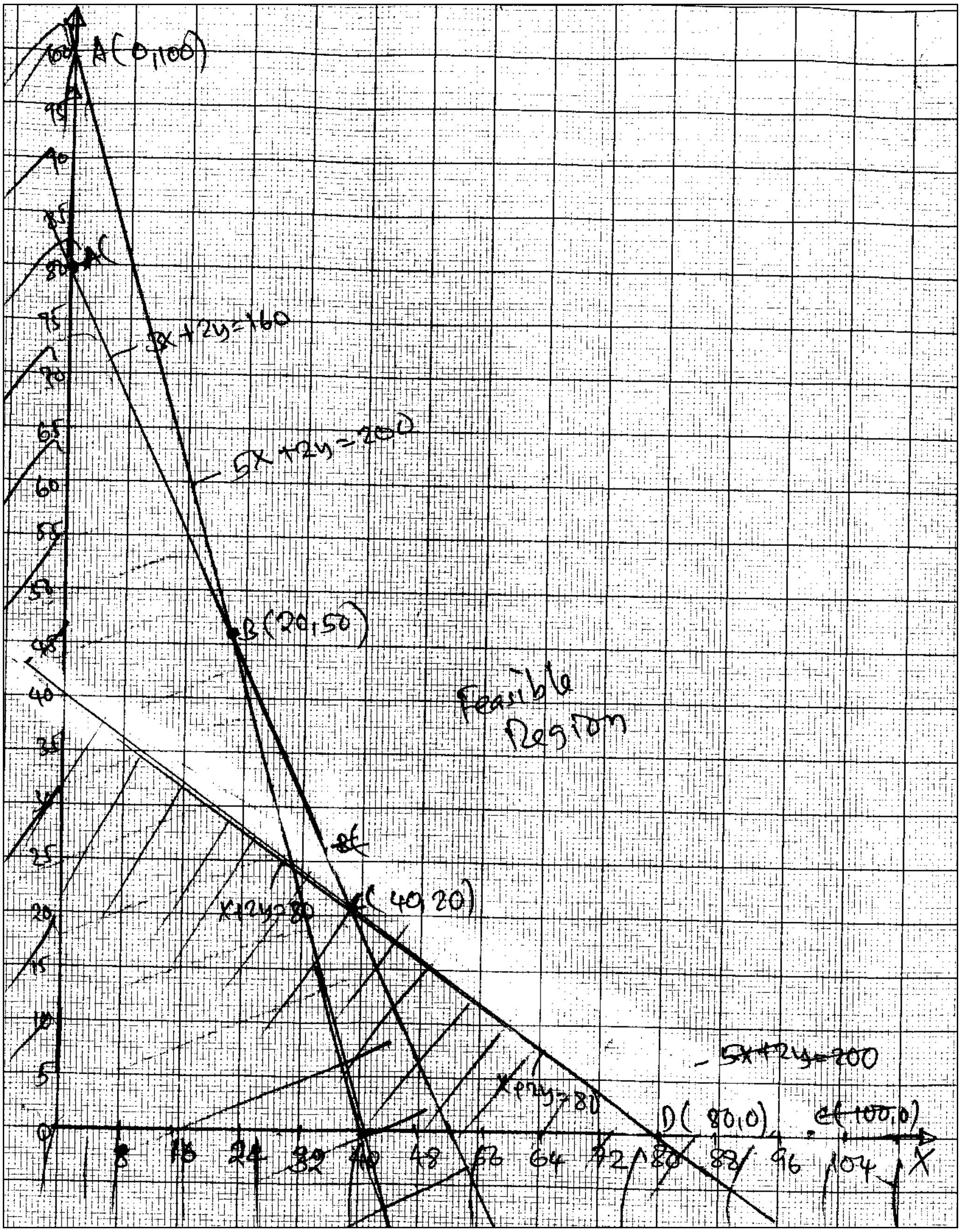
Optimal value (minimum) = 12 000 000F

∴ ~~Each mine should operate~~

∴ Mine A should operate for 40 days

Mine B should operate for 20 days

in order to have a minimum operation cost of 12 000 000F



Extract 3.1: A sample of correct responses to question 3 of paper 1

In Extract 3.1, the candidate applied the appropriate steps for formulating the linear programming problem correctly in part (a). In part (b), the candidate formulated the linear programming model which was used to sketch the graph and obtained the correct answer from the corner points.

Despite the fact that many candidates who answered this question performed well, there were a few who performed poorly. The reason for the poor performance was associated with incorrect formulation of the inequalities as well as misinterpretation of the terms used in linear programming problems. For example, in part (a), the use of the terms at most and maximum number of hours qualified the question to be the minimum problem, but few candidates treated it as a maximization problem. Thus, coming up with $x+\frac{1}{2}y \geq 16,\ x+y \geq 24$. However, the formulation of non-negative constraints and objective function was done correctly.

In part (b), some candidates treated the problem as a linear transportation problem, hence came up with the wrong inequalities. For example, one candidate formulated $x+y \leq 200000$, $x+y \leq 199800$, $x \leq 80$, $y \leq 160$, $x, y \geq 0$ and objective function of $f(x,y) = -8x+8y+500880$. Then, from the graph, the candidate obtained (0, 199800), (0, 200000), (19980000, 0) (200000,0) as the corner points. Thereafter, the candidate concluded that mine A should operate for 20,000 days and mine B for 0 days. Also, other candidates encountered the problem of incorrect formulation of inequalities due to misinterpretation of the terms. For example, several candidates established $x+2y \leq 80$, $3x+2y \leq 160$, $5x+2y \leq 200$, $x, y \geq 0$ with the objective function $f(x,y) = 200{,}000x + 200{,}000y$ as a mathematical model. Those candidates found that mine A could operate for 40 days and mine B for 0 days. Extract 3.2 is a sample response from the candidate who failed to answer part (b) of this question correctly.

b) Data given

Two mine = A and B

grade = High grad, medium grade and low grade.

A = 1 ton High grade

= 3 ton medium grade.

= 5 ton low grade.

B = 2 tons High grade

= 2 tons medium grade

= 2 ton low grade.

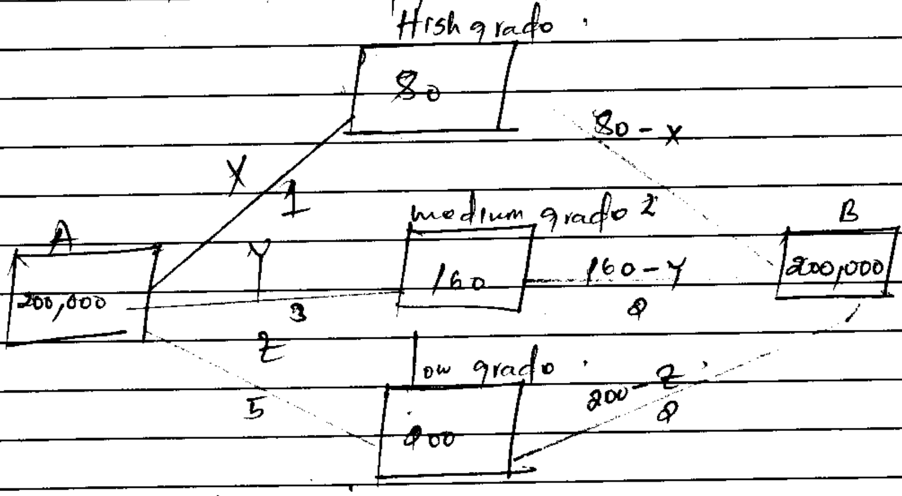
High grade = 80 ton – Needs

Medium grade = 160 ton – Need

low grade = 200 tons – Need

Soln

Consider the figure below



To find inequalities

$X+Y \le 200,000$ — (i)

$X+Y+Z \le 200,000$

$X \le 80$ — (ii)

$Y \le 160$ — (iii)

$Z \le 200$

$X+Y+Z \le 200,000$

03 (b) $X + Y + Z \leq 200{,}000$

$Z \leq 200000 - (X+Y)$

$200 \leq 200{,}000 - (X+Y)$

$X+Y \geq 200{,}000 - 200$

$X+Y \geq 199{,}800$ — (iv)

objective function

$X + 3Y + 5Z + 2(80-X) + 2(160-Y) + 2(200-Z)$

$X + 3Y + 5Z + 160 - 2X + 320 - 2Y + 400 - 2Z$

$X - 2X + 3Y - 2Y + 5Z - 2Z + 160 + 320 + 400$

$-X + Y + 3Z + 880$

$-X + Y + 3(200000 - X - Y)$

$-X + Y + 600\,000 - 3X - 3Y$

$-X - 3X + Y - 3Y + 880 + 600{,}000$

$-4X + 2Y + 600880$

$\therefore$ objective function is equal to $-4X - 2Y + 600880$

To draw the graph

inequalities

$X + Y \leq 200{,}000$ $(200\,000, 200\,000)$

$X + Y \geq 199{,}800$ $(199{,}800, 199\,800)$

$X \leq 80$ $(80, 0)$

$Y \leq 160$ $(0, 160)$

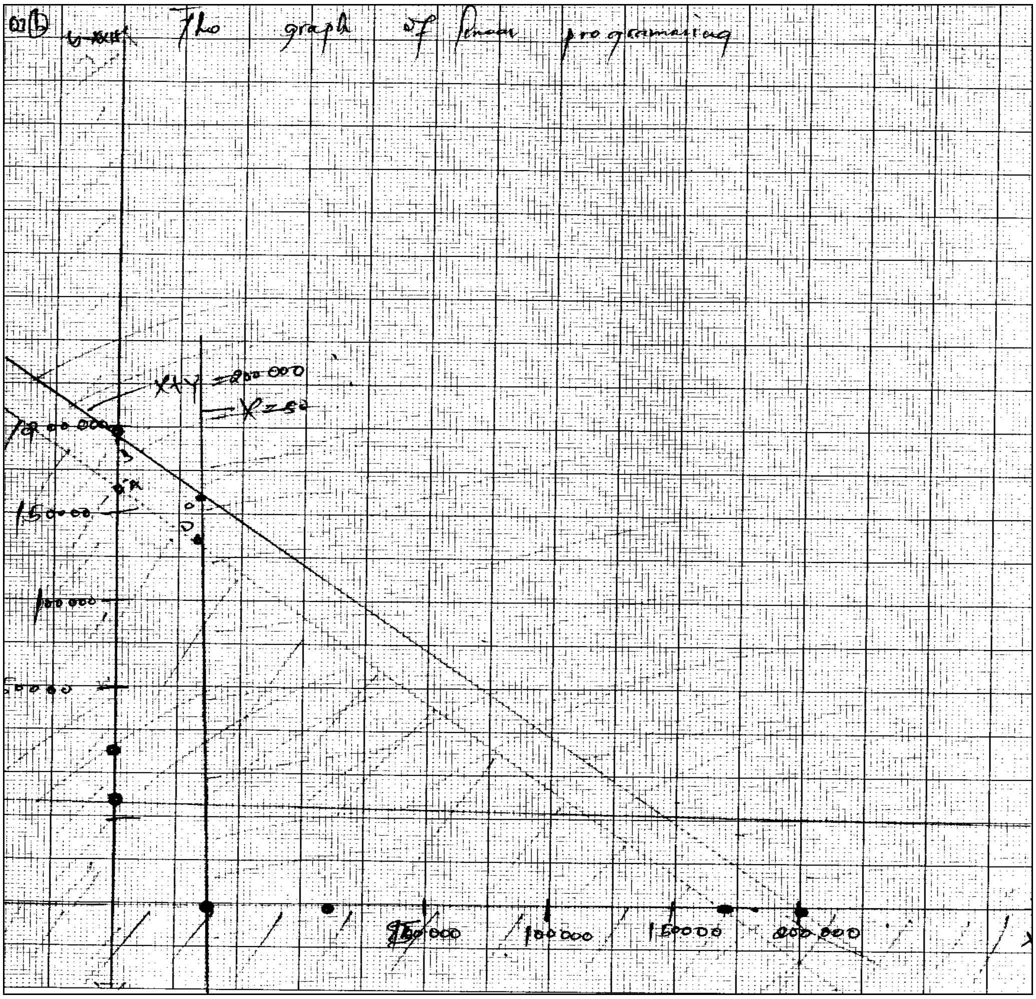
$X \geq 0$

$Y \geq 0$

$A = (0, 199{,}800)$

$B = (0, 200\,000)$

$C = (80,$



Extract 3.2: A sample of incorrect responses to question 3 of paper 1

In extract 3.2, the candidate failed to obtain the correct answer after formulating wrong inequalities and an objective function that led to an incorrect graph.

2.1.4 Question 4: Statistics

The candidates were given the frequency distribution table which has the masses of 36 stones in grams, as shown in the following table:

Mass (g)	50 – 100	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Frequency	3	5	10	8	6	4

Then, they were required to:

(a) Find the mean and variance of the distribution to three (3) decimal places by using the mean deviation method if the assumed mean (A) is 225.
(b) Find the first and third quartiles of the distribution to three significant figures.
(c) Find the 90^{th} percentile of the distribution correct to 3 significant figures

The question was attempted by 13,755 (100%) candidates, 9.0 per cent scored 3.0 marks or less, 21.7 per cent scored from 3.5 to 5.5 marks, and 69.3 per cent scored from 6.0 to 10 marks. In this question, 12,518 (91.0%) candidates scored more than 3.0 marks. Thus, question 4 was among the questions that were well done in this examination. Figure 5 shows the general performance of candidates in this question.

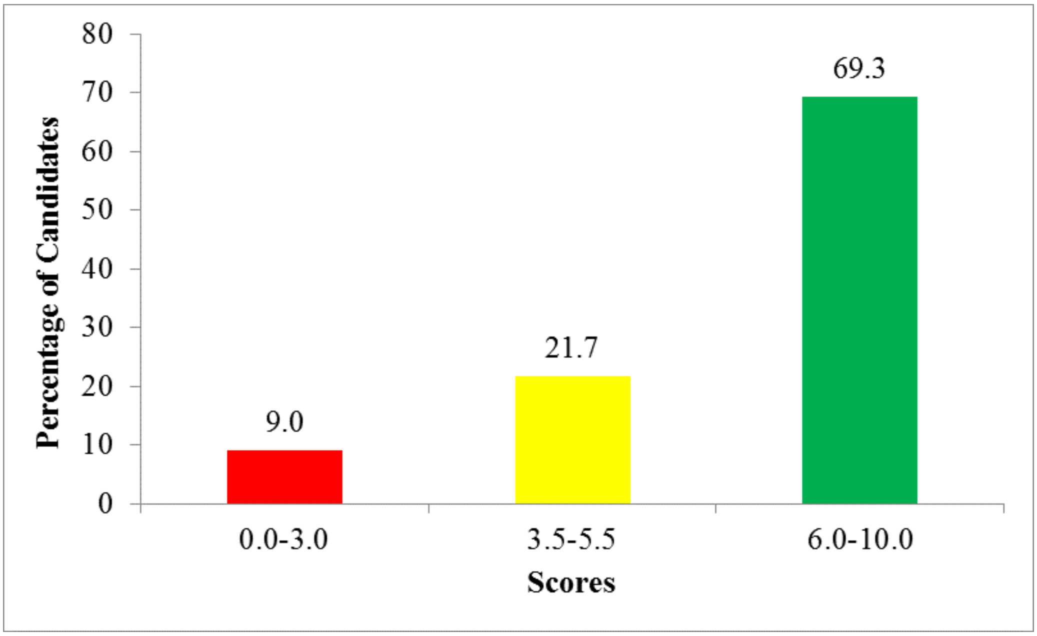


Figure 5: *Candidates' Performance in Question 4 of Paper 1*

Figure 5 shows that the general performance in this question was good. In part (a), the candidates who performed well in this question were able to recall and apply correctly the formula used to find the mean for the deviation method as $\overline{X} = A + \frac{\sum fd}{N}$. Thereafter, they calculated the $\sum fd = -750$ and used it together with $\sum A = 225$ and $\sum N = 36$ to find the mean. Hence, they obtained the mean $\left(\overline{x}\right) = 204.167$. Also, in order to find the variance, the candidates used the formula $\sigma^2 = \frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2$, $N = 36$, $\sum fd^2 = 197500$ and $\sum fd = -750$ leading them to get the variance $(\sigma^2) = 5052.083$.

In part (b), the candidates were able to recall appropriately that $Q_1 = L + \left(\frac{\frac{N}{4} - n_b}{n_w}\right)c$, thereafter identified $L = 150$, $N = 36$, $n_b = 8$ and $n_w = 10$ then inserted the values in the formula to obtain the first quartile, $Q_1 = 155$. Again, they applied formula $Q_3 = L + \left(\frac{\frac{3}{4}N - n_b}{n_w}\right)c$ to find the third quartile by using $L = 250$, $N = 36$, $n_b = 26$, and $n_w = 6$, hence obtained the third quartile, $Q_3 = 258$.

Lastly in part (c), the candidates applied correctly the formula $P_k = L + \frac{\left(K\left(\frac{N}{100}\right) - n_b\right)}{n_w}c$ in order to get the 90$^{\text{th}}$ percentile where $L = 300$, $N = 36$, $k = 90$, $n_w = 4$, $n_b = 32$ and $c = 50$. Thus, they got the 90$^{\text{th}}$ percentile $\left(P_{90}\right) = 305$. Extract 4.1 shows the response of a candidate who had adequate knowledge of the concepts tested in this question, thus performed well.

4. a.

C.interval	c.mark(x)	d = x − A	d²	Frequency	Fd	Fd²	C.size
50-100	75	-150	22500	3	-450	67500	50
100-150	125	-100	10000	5	-500	50000	50
150-200	175	-50	2500	10	-500	25000	50
200-250	225	0	0	8	0	0	50
250-300	275	50	2500	6	300	15000	50
300-350	325	100	10000	4	400	40000	50
				$\Sigma f = 36$	$\Sigma fd = 750$	197500	

$A = 225$

$$\text{Mean } \bar{x} = A + \frac{\Sigma Fd}{N}$$

$$= 225 + \frac{(-750)}{36}$$

$$\bar{x} = 204.1666667 \simeq 204.167$$

$$\bar{x} = 204.167$$

4a i. $$\text{variance} = \frac{\Sigma Fd^2}{N} - \left(\frac{\Sigma Fd}{N}\right)^2$$

$$= \frac{197500}{36} - \left(\frac{-750}{36}\right)^2$$

$$\text{variance} = 5052.083$$

4b.

C.interval	C.mark	C.size	f	C.frequency
50-100	75	50	3	3
100-150	125	50	5	8
150-200	175	50	10	18
200-250	225	50	8	26
250-300	275	50	6	32
300-350	325	50	4	36

$$\text{First Quartile } Q_1 = L + \left(\frac{N/4 - Cb}{f}\right)c$$

The first quartile class $= N/4^{th}$ term

$$= \frac{36}{4}^{th} \text{ term}$$

$= 9^{th}$ term

First quartile class $= 150 - 200$

$$Q_1 = 150 + \left(\frac{36/4 - 8}{10}\right)50$$

4b First quartile $Q_1 = 155$

Third quartile $(Q_3) = L + \left(\frac{3N/4 - Cb}{f}\right)c$

Third quartile class $= \frac{3N}{4}^{th}$ term

$= \frac{3 \times 36}{4}^{th}$ term

$= 27^{th}$ term

Third quartile class $= (250 - 300)$

$Q_3 = 250 + \left(\frac{\frac{3 \times 36}{4} - 26}{6}\right)50$

$Q_3 = 258.33333$

In three significant figures,

(Third quartile) $Q_3 = 258$

4c. 90^{th} percentile $= L + \left(\frac{\frac{90N}{100} - cb}{f}\right)c$

90^{th} percentile class $= \frac{90N}{100}^{th}$ term

$= \frac{90 \times 36}{100}^{th}$ term

$= 32.4^{th}$ term

90^{th} percentile class $= 300 - 350$

4c. 90^{th} percentile $= 300 + \left(\frac{32.4 - 32}{4}\right)50$

90^{th} percentile $= 305$

Extract 4.1: A sample of correct responses to question 4 of paper 1

In Extract 4.1, part (a), the candidate prepared a correct frequency distribution table and then used suitable formulae to correctly obtain the mean and variance. In part (b), the candidate identified the classes for the first and third quartiles and recalled a proper formula to calculate their values correctly. In part (c), the candidate calculated the 90th percentile using the appropriate formula and obtained the correct value.

Though the majority of the candidates showed outstanding performance, a few faced some challenges in responding to this question. In part (a), some candidates failed to recall the formula for finding the mean by the deviation method and quartiles. For instance, they used the wrong formulas, such as $\bar{x} = A + \frac{\sum fx}{N}$, $\bar{x} = A + \frac{\sum fu}{N}$ instead of $\bar{x} = A + \frac{\sum fd}{N}$. Not only that, but also a few candidates failed to abide by the requirement of the question as they computed the mean correctly, but failed to round off the last answer to three decimal places. For example, some wrote the mean as 204.1666667 and others wrote the mean equal to 204.17. In part (b), a few candidates failed to distinguish between the grouped and ungrouped data. That is, some candidates used the formula, $Q_1 = \frac{1}{4}N$ and $Q_3 = \frac{3}{4}N$, then used $N = 36$ to obtain $Q_1 = 9$ and $Q_3 = 27$. In other cases, some candidates used an inappropriate formula to find the quartiles. For example, they used the formula $Q_1 = L + \left(\frac{\frac{N}{2} - nb}{n_w} \right) c$ which is used to find the Median. Not only that, but some candidates applied the wrong formulae for quartiles, such as $Q_1 = L + \frac{\left(\frac{N}{4} + n_b \right)}{n_w} c$ and $Q_3 = L + \frac{\left(\frac{3N}{4} + n_b \right)}{n_w} c$. In these formulas, the insertion of addition (+) instead of minus (-) makes them incorrect. Also, a few candidates used the wrong lower boundary for the lower quartile and upper quartile classes. For example, a candidate used 149.5 and 249.5 instead of 150 and 250 for the lower and upper quartiles, respectively.

In part (c), a few candidates faced some challenges while responding to the question, such as the wrongly determined position of the 90th percentile. Also, other candidates faced the problem of wrong computation while simplifying. Not only that, but there were also some candidates who used the incorrect formula as $p_{90} = L + \frac{\left(\frac{90N}{100} + f_b\right)}{f_w}c$ such that the insertion of addition (+) instead of minus (-) made it incorrect. Likewise, other candidates used improper formulas such as $Q_1 = L + \frac{\left(\frac{N}{4} - f_b\right)}{f_w}c$ which is used to determine the 1st quartile and not the percentile. Extract 4.2 shows a part of the response of a candidate who answered this question incorrectly.

(4) soln

X-interval	Frequ	X	fx	$x-\bar{x}$
50-100	3	75	225	-150
100-150	5	125	625	-100
150-200	10	175	1750	-50
200-250	8	225	1800	0
250-300	6	275	1650	50
300-350	4	325	1300	100
	N=		fx=7350	$\Sigma\bar{x}$-150

from Mean $= A + \frac{\Sigma fx}{N}$

from $\bar{x} = 225 + \frac{7350}{36}$

Mean $\bar{x} = 429.167$

Varence $= \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$

Varence $= \frac{36(22500)}{36}$

Varence $= 36 \quad 22500$

Varience $= 22500.000$

4(b) Soln

from 1^{st} quatiles $= \frac{1}{4}(36)$

$$L + \left(\frac{\frac{N}{2} - f_b}{f_m}\right)c.$$

$\frac{1}{4}N$ $\frac{n+1}{4} = \frac{36}{4} = 9.$

$L = 200.$

$\frac{36}{4} = 9.$

$f_b = 18.$

$f_m = 8.$

$c = 50$

$$200 + \left(\frac{9 - 18}{8}\right)50$$

1^{st} quatile $1.44 \times 10^2.$

third quatiles $= \frac{3N}{4} = \frac{108}{4}$

$27.$

$$200 + \left(\frac{27 - 18}{8}\right)50$$

3^{rd} quatile $= 2.56 \times 10^2$

4(c) 90^{th} percentile

from $\frac{90}{108} =$

$\frac{90^{th}}{100} = 32.4.$

$$200 + \left(\frac{32.4 - 18}{8}\right)50$$

$200 + 90$

90^{th} percentile $= 290$

90^{th} Percentile in 3 sf $= 2.90 \times 10^2$

Extract 4.2: A sample of incorrect responses to question 4 of paper 1

In Extract 4.1, the candidate applied incorrect formulae to determine the mean and variance in part (a). In part (b), the candidate applied the formula used to calculate the median to find the first quartile, which is inappropriate. Also, in part (c), the candidate failed to get the 90th percentile due to the improper class.

2.1.5 Question 5: Sets

The question consisted of parts (a) and (b). In part (a), the candidates were required to simplify the sets' expressions in part (i) $(A \cap B') \cup (A' \cup B')$ and (ii) $(A \cup B)' \cap (A \cap B)'$ by using the laws of algebra of sets. While in part (b), the question stated: "A survey of 500 students shows that 83 students study Economics and Geography, 63 study Geography and Mathematics, 217 study Economics and Mathematics, 295 study Mathematics, 186 study Geography and 329 study Economics. If every student studies at least one course among Economics, Geography and Mathematics" to use Venn diagram to find the number of students who study; (i) all three subjects and (ii) Economics or Mathematics but not Geography.

This question was attempted by 13,755 (100%) candidates. The analysis of the data shows that 1,473 (10.7%) candidates scored from 0 to 3 marks and 12,282 (89.3%) candidates scored more than 3 marks. This indicates that the candidates' performance in the question was good. Figure 6 summarizes the candidates' performance in this question.

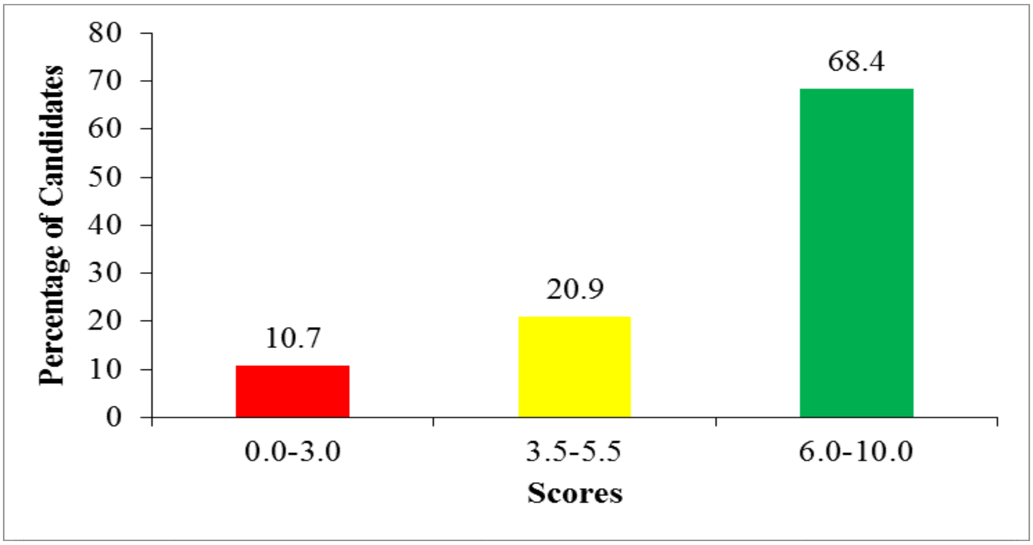


Figure 6: *Candidates' Performance in Question 5 of Paper 1*

The analysis of the responses shows that most of the candidates who answered the question correctly were able to state and apply the laws of algebra of sets to simplify the given expressions in part (a). For instance, in part (a) (i), they obtained $(A\cap B')\cup(A'\cup B')\equiv A'\cup B'$ and in part (a) (ii), they got $(A\cup B)'\cap(A\cap B)'\equiv A'\cap B'$. In part (b), the candidates were able to transform the word problem into Mathematical form and present the information in a Venn diagram correctly. This indicated that the candidates had good knowledge of the concepts tested. Thus, in part (b) (i), they managed to get 53 as the number of students who study all three subjects. In part (b) (ii), they got 314 as the number of students who study Economics or Mathematics but not Geography. Extract 5.1 shows a candidate's response who performed well in this question.

5a. i. $(A\cap B')\cup(A'\cup B')$ given

$\equiv (A\cap B')\cup(B'\cup A')$ commutative law

$\equiv [(A\cap B')\cup B']\cup A'$ associative law

$\equiv B'\cup A'$ abcorption law

$\equiv A(B\cap A)'$ demorgan's law

ii. $(A\cup B)'\cap(A\cap B)'$ given

$\equiv (A'\cap B')\cap(A'\cup B')$ demorgan's law

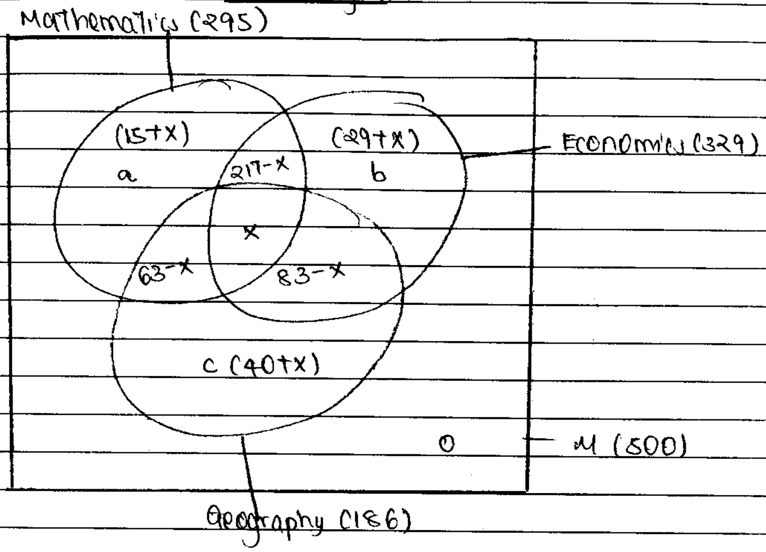
$\equiv A'\cap[B'\cap(A'\cup B')]$ associative law

$\equiv A'\cap[B'\cap(B'\cup A')]$ commutative law

$\equiv A'\cap B'$ abcorption law

$\equiv (A\cup B)'$ demorgan's law

5b.i venn diagram



$a + 217 - x + x + 63 - x = 295$

$a = 15 + x$

$b + 217 - x + 83 - x + x = 329$

$b = 29 + x$

$c + 63 - x + x + 83 - x = 186$

$c = 40 + x$

Total of all students = 500

$295 + 29 + x + 83 - x + 40 + x = 500$

$x = 500 - 447$

$x = 53$

53 students study all subjects.

Extract 5.1: A sample of correct responses to question 5 of paper 1

In Extract 5.1, the candidate was able to use the required laws of algebra of sets and managed to simplify the given expressions in part (a). In part (b), the candidate used the Venn diagram properly to obtain the number of students asked on each part correctly, as seen in part (b) (i).

In spite of the good performance shown by most of the candidates, there were a few who performed poorly on this question. This was due to some factors as explained in the following: In part (a), some of the candidates failed to differentiate the algebraic laws of sets, such as commutative from distributive law, De Morgan's from complement law, and identity from idempotent law. Likewise, other candidates faced challenges due to poor recall of the laws. With these misconceptions, a large number of candidates obtained $(A \cap B') \cup (A' \cup B') \equiv B'$ in part (a) (i), and in part (a) (ii) $(A \cup B)' \cap (A \cap B)' \equiv B$. In part (b), some candidates used the set formulas to solve the problem contrary to the requirement of the question. Also, a few candidates drew an improper Venn diagram as they inserted incorrect elements in regions of the Venn diagram. Extract 5.1 is a sample response from the candidate who performed poorly in this question.

5a) Soln

i) $(A \cap B') \cup (A' \cup B')$ --- given

$(A \cup A') \cap (B' \cup B')$ --- Associative law

$A \cap \emptyset$ ---- Associative

$\underline{A}$ --- Identity

ii) Soln

$(A \cup B)' \cap (A \cap B)'$ --- given

$(A' \cap B') \cap (A' \cup B')$ --- Double compliment

$(A' \cup A') \cap (B' \cup B')$ --- Associative law

$\emptyset \cap \emptyset$ --- Identity

$\underline{\emptyset}$ --- Identity

5b Soln

Data given

Total number of student $(\mathcal{U}) = 500$

Economic and Geography $= 83$

Geography and Mathematic $= 63$

Economic and Mathematic $= 217$

Economic $= 329$

Geography $= 186$

Mathematic $= 295$

Let

student study Geography $= G$

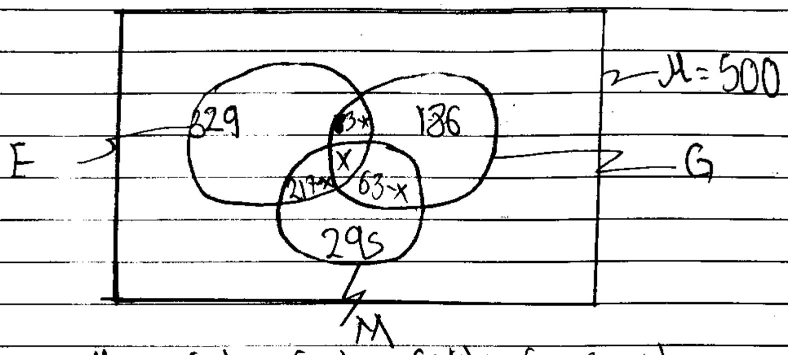
Student study Economic $= E$

student study Mathematic $= M$

Total number of student $= \mathcal{U}$

5 b Consider a venn diagram below

i)



$\mu = n(E) + n(G) + n(M) + n(E \cap G \cap M) + n(G \cap M)$
$+ n(\cancel{M} \cap G) + n(M \cap E)$

$500 = 186 + 295 + 329 - (83 - x + 217 - x +$
$63 - x + x)$

$500 = 976\cancel{0}6 - 2x$
$+ 97104 \,/\, -3x$
$+3x \,/\, -3x$

$500 = 447 - 2x$

$\frac{53}{2} = x$

$x = 26.5$

$\therefore$ Number of student study all three subject $= 27$

Extract 5.2: A sample of incorrect responses to question 5 of paper 1

In Extract 5.1, the candidate failed to use the correct laws in simplifying the set expressions in part (a). In part (b), the candidate was not able to correctly locate the given information in the regions of the Venn diagram, thus getting a fraction of the number of people.

2.1.6 Question 6: Functions

The question was composed of two parts, (a) and (b). In part (a) (i), the candidates were supposed to draw the curve of $f(x)=3^x$ and $g(x)=\log_3 x$ on the same pair of axis and in (ii) required them to state how $f(x)$ relates to $g(x)$ in 6 (a) (i). In part (b), the candidates were given a function $f(x)=\dfrac{x^2-2x-3}{x^2-4}$ and the requirement was to: (i) find the vertical and horizontal asymptotes for $f(x)$ and (ii) sketch the graph of $f(x)$.

The data analysis shows that the question was attempted by 13,755 (100%) candidates, whereby 3.8 per cent scored from 0 to 3 marks, 15.3 per cent scored from 3.5 to 5.5 marks and 80.9 per cent scored from 6 to 10 marks. In this question, 13,229 (96.2%) candidates scored more than 3 marks. Therefore, question 6 was among the three questions that were well done in this examination. Overall, the performance in this question was the highest. Figure 7 shows the general performance of candidates in this question.

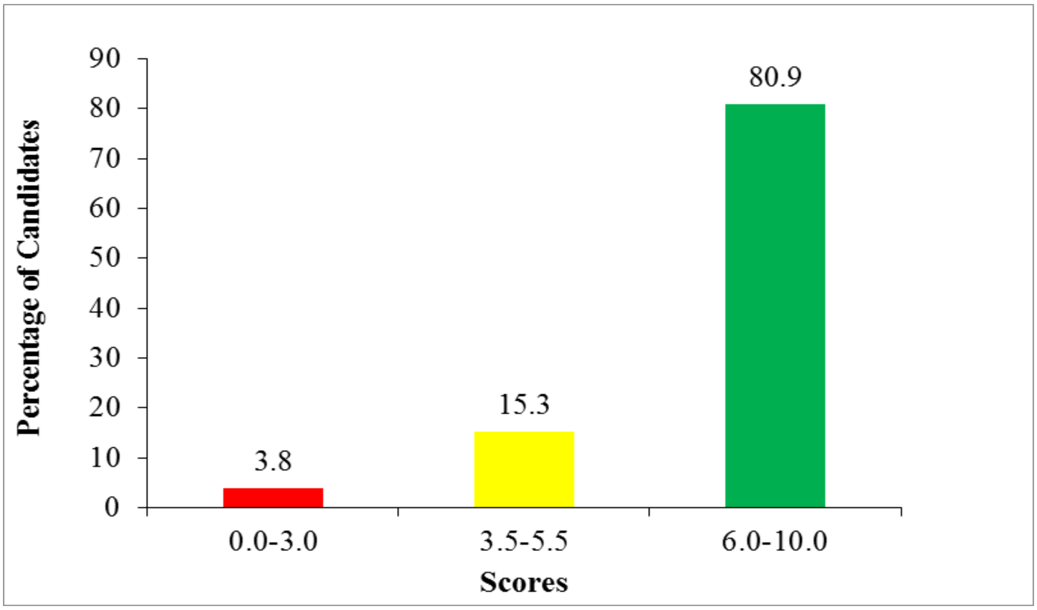


Figure 7: *Candidates' Performance in Question 6 of Paper 1*

The analysis of the candidates' responses showed that the question was answered well by most of the candidates. For instance, in part (a) (i), the candidates drew the curves of $f(x)=3^x$ and $g(x)=\log_3 x$ on the same pair of axes by using suitable coordinates. In part (a) (ii), the candidates were able to realize that the graph of $g(x)=\log_3 x$ is the inverse of $f(x)=3^x$ and $y=x$ is the line of symmetry for the two graphs or $f(x)$ and $g(x)$ both make reflections on the line $y=x$.

Also, in part (b), the candidates were able to use the function $f(x)=\dfrac{x^2-2x-3}{x^2-4}$ to find the vertical and horizontal asymptotes as ± 2 and 1 respectively. Again, they found $x-$intercepts as -1 and 3 also have $y-$intercept equals to $\dfrac{3}{4}$, then they used these information to correctly sketch the graph of $f(x)$. Extract 6.1 shows candidate's good responses in this question.

Soln.

6 (a) i) $f(x)=3^x$. $g(x)=\log_3 x$.

Table value

X	−2	−1	0	1	2	3
$y=3^x$	0·11	0.33	1	3	9	27

$g(x)=\log_3 x$.

$y=\log_3 x$

$x=3^y$.

Table value.

y	−2	−1	0	1	2	3
$x=3^y$	0·11	0.33	1	3	9	27

ii) f(x) is related to g(x) in

g(x) is the inverce function of f(x).

Soln.

(b) $f(x)=\dfrac{x^2-2x-3}{x^2-4}$.

i) V.A

$x^2-4=0$

$x^2=4$

$x=\sqrt{4}$

$x=\pm 2$.

Vertical asymptotes = +2 and −2.

6 (b). $H.A = \frac{x^2-2x-3}{x^2-4}$

$$H.A = \frac{x^2/x^2 - 2x/x^2 - 3/x^2}{x^2/x^2 - 4/x^2}$$

$$H.A = \frac{1-0}{1-0}$$

$$H.A = 1.$$

(ii) Graph of $f(x)$.

$$y = \frac{x^2-2x-3}{x^2-4}$$

y-Intercept $x=0$

$$y = \frac{0-3}{0-4}$$

$$y = 3/4$$

y-Intercept $= 0.8$

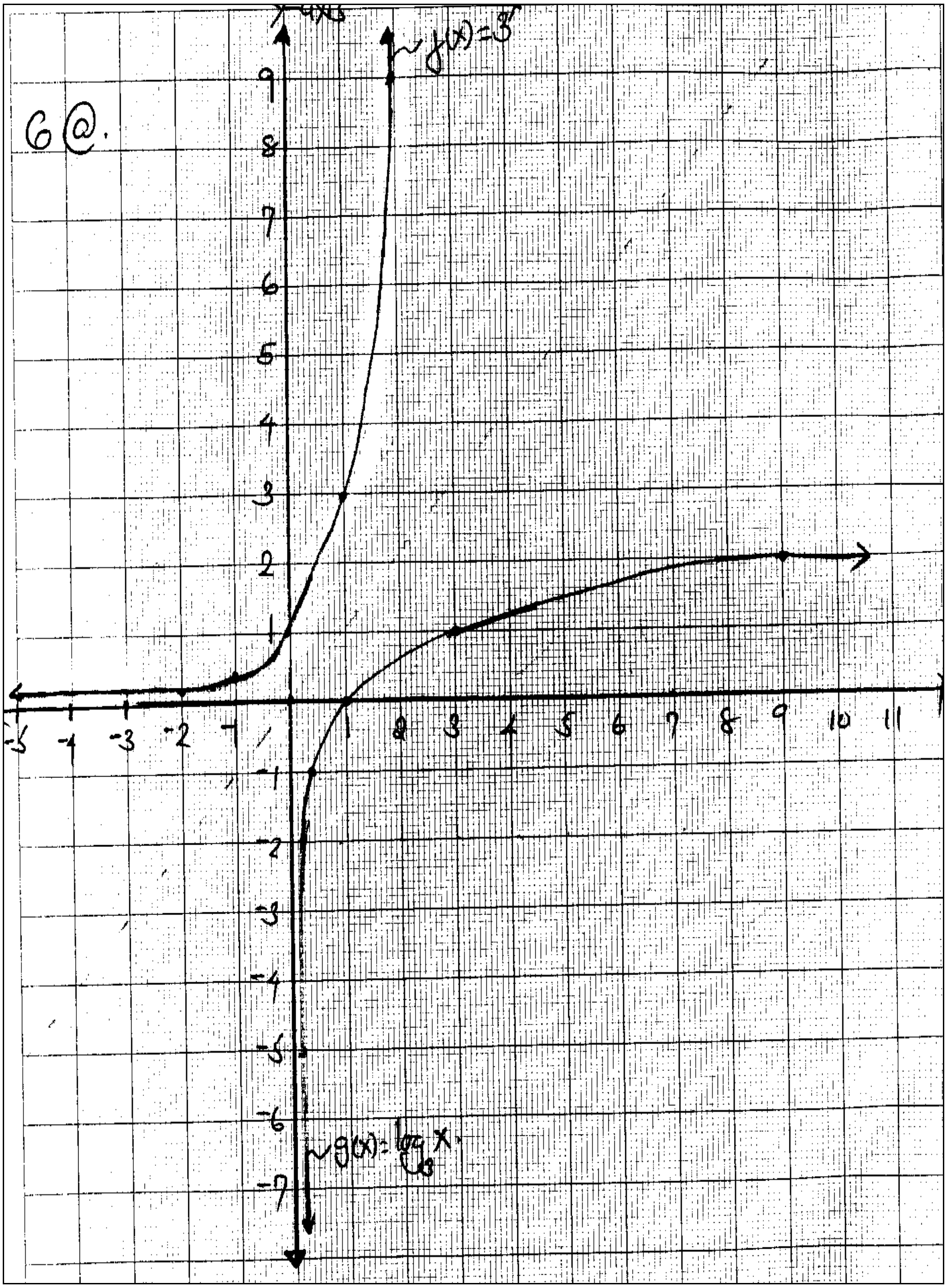
X-Intercept $y=0$.

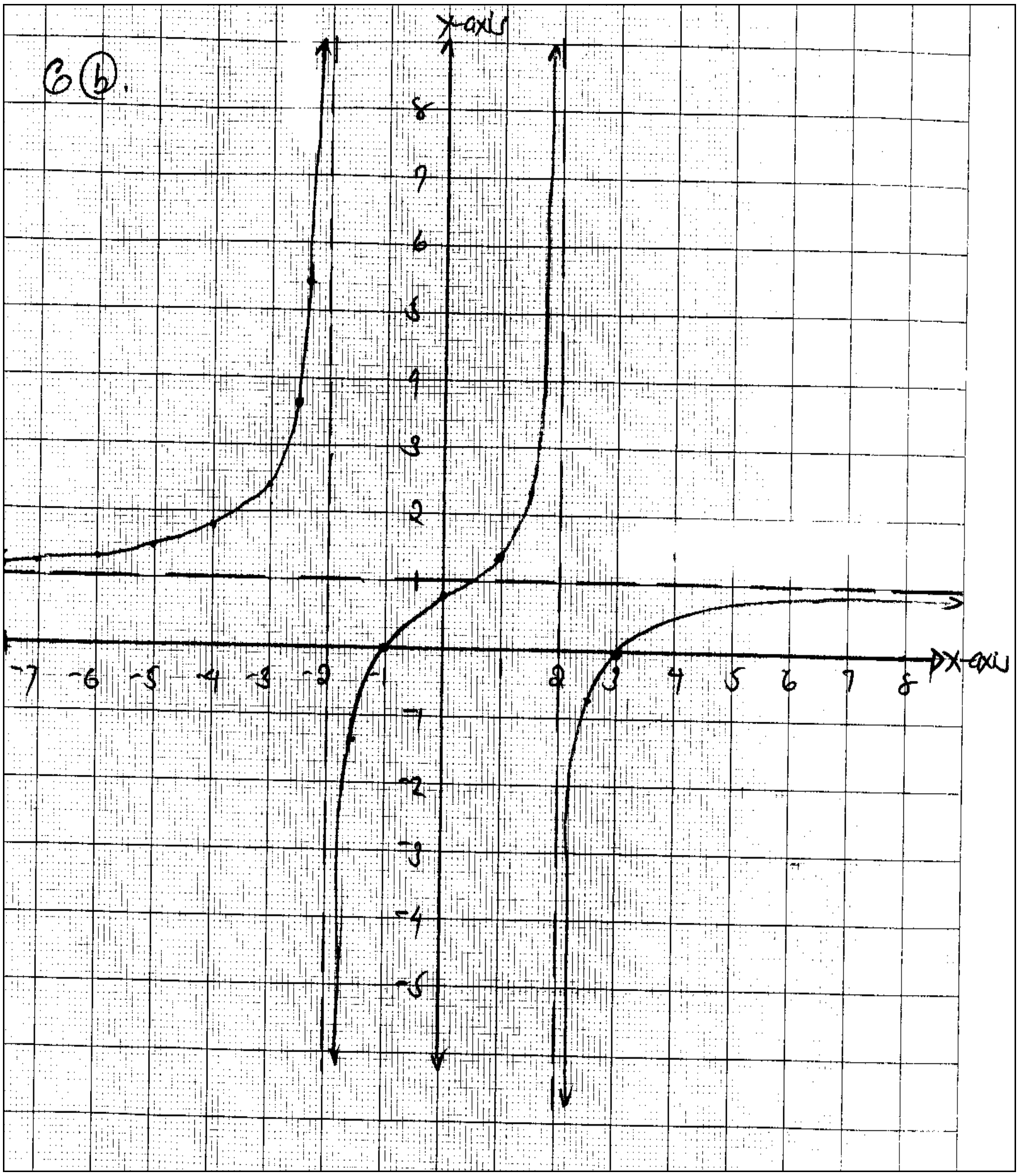
$$0 = \frac{x^2-2x-3}{x^2-4}$$

$$x^2-2x-3=0.$$

$$x_1 = 3 \quad x_2 = -1$$

X-Intercept $= 3$ and -1.


6 @.
y-axis
$f(x)=3^x$
$g(x)=\log_3 x$



Extract 6.1: A sample of correct responses to question 6 of paper 1

In Extract 6.1, in part (a), the candidate was able to sketch the graph of the given functions correctly on the same pair of axes and managed to relate the functions of $f(x)$ and $g(x)$. Likewise, in part (b), they were able to obtain the asymptotes correctly for $f(x)$ and also managed to sketch the graph of $f(x)$.

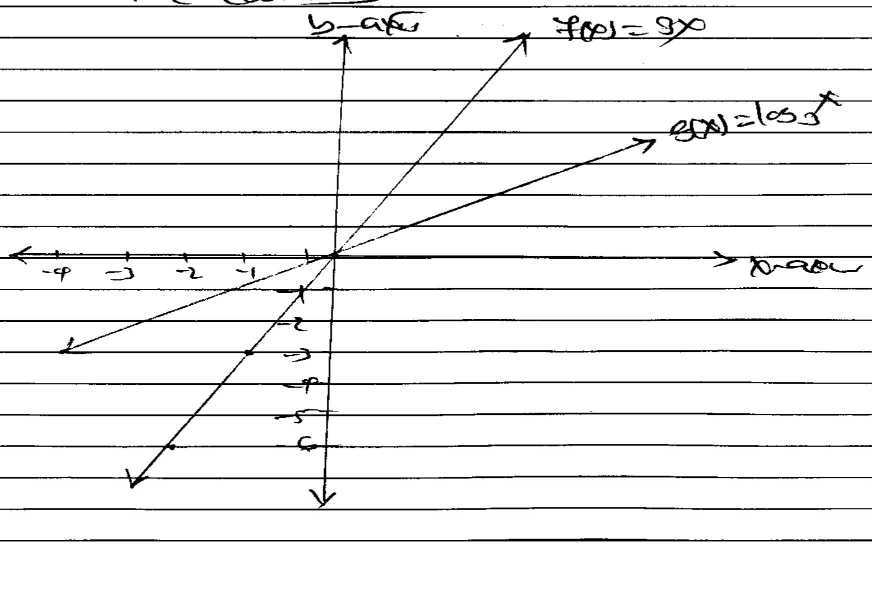
Despite the good performance shown by most of the candidates there were a few who performed poorly in this question. The poor performance was due to several reasons. In part (a) (i), some candidates presented incorrect graphs of $f(x)$ and $g(x)$ as they joined the coordinates of the functions incorrectly, hence producing straight line graphs instead of curves. Others drew curves of the given functions, but they didn't realize that the graph of $f(x)$ has to pass at $(0,1)$ and that of $g(x)$ at $(1,0)$. In part (a) (ii), a notable number of candidates stated that; as the graph of $f(x)$ increases also $g(x)$ increases. Furthermore, some candidates equated $f(x)$ to $g(x)$ therefore, they stated that the functions are related by $g(x) = x\dfrac{\log x}{f(x)}$. In part (b), some candidates calculated wrongly the vertical and horizontal asymptotes. For example, they came up with 4 and -4 as vertical asymptotes and 1 as horizontal asymptote, hence sketched an incorrect graph. Likewise, others had an inadequate knowledge on sketching the graph of the rational function. Extract 6.2 shows a response from one of the candidates who performed poorly in this question.

6(i) $f(x) = 3x$

$g(x) = \log_3 x$

x	−2	−1	0	1	2	3
f(x)	−6	−3	0	3	6	9
gx	−0.95	−0.48	0	0.48	0.95	1.43

The curves



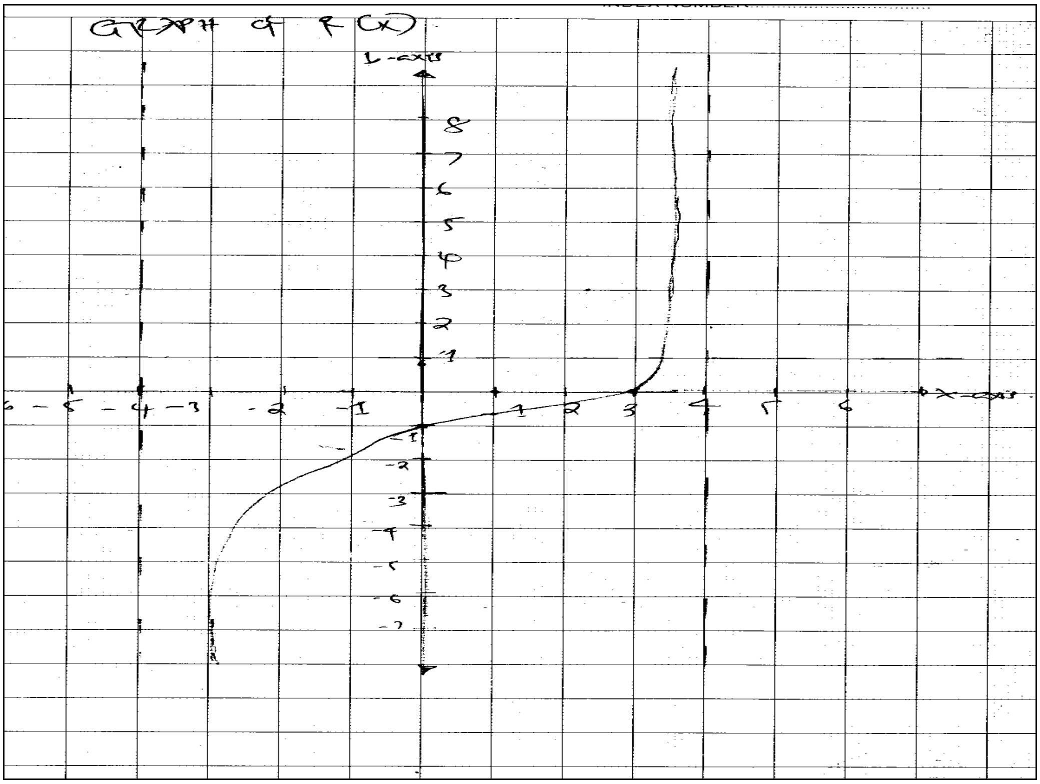
b) i(c) ~~Horz~~

vertical Asymptotes

$x^2-4=0$

$x^2=4$

$x=\pm 4.$



Extract 6.2: A sample of incorrect responses to question 6 of paper 1

In Extract 6.1, the candidate drew line graphs instead of curves for the given functions in part (a). Also, they obtained the wrong vertical asymptotes and rational graph for part (b).

2.1.7 Question 7: Numerical Methods

This question was divided into two parts; (a) and (b). In part (a), the question read as: The area enclosed between $x^2 + y^2 = 100$, $x \geq 0$ and $y \geq 0$ is divided into ten equal intervals. Candidates were required to use the Trapezoidal and

Simpson's rules to approximate the value of π correct to three significant figures. In part (b), the candidates had to state which rule in part (a) gives a better approximation if the actual value of π is 3.14.

This question was attempted by 13,755 (100%) candidates. Figure 8 present the summary of candidates' performance in this question.

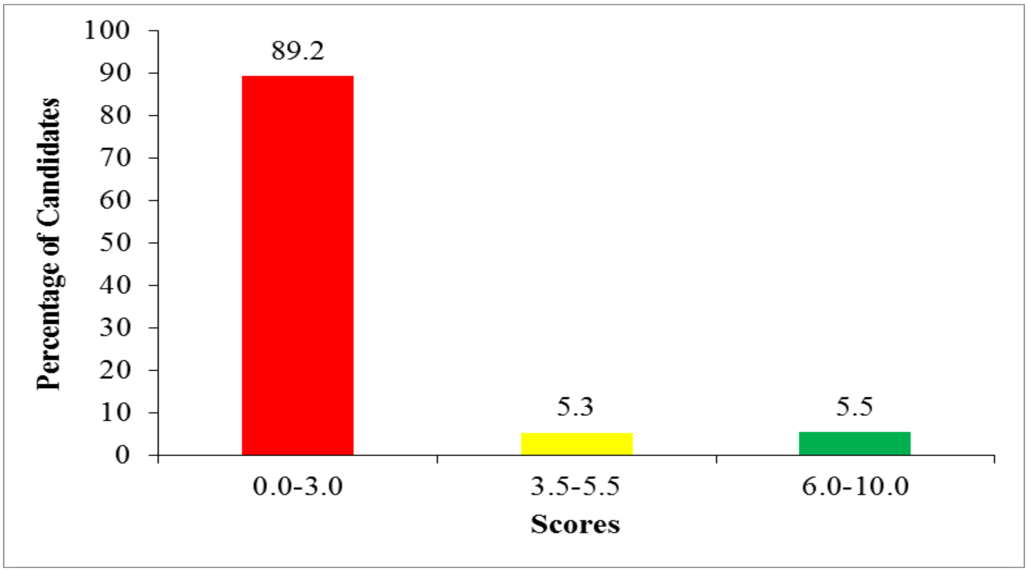


Figure 8: *Candidates' Performance in Question 7 of Paper 1*

Analysis shows that among 13,755 candidates who attempted the question, 12,270 (89.2%) scored 0 to 3 marks. 735 (5.3%) candidates scored 3.5 to 5.5 marks and 750 (5.5%) candidates scored 6 to 10 marks. Therefore, the performance of the candidates in this question was weak, as only 10.8% of the candidates who attempted the question scored 3.5 marks or more.

The analysis of the candidates' responses showed that many candidates performed poorly in this question. The poor performance was due to several factors. In part (a), most candidates failed to identify the lower and upper boundaries of the function $y = \sqrt{100 - x^2}$ hence considering -10 and 10 as the boundaries as well as the value of $h = 2$ instead of $h = 1$. Those candidates didn't consider the fact that $x \geq 0$ and $y \geq 0$, thus obtaining incorrect solutions. Others used inappropriate formulas; for example, some applied the formula used to find the length of an arc, which is $\int_a^b \sqrt{1 + [f'(x)]^2}\,dx$. Then, firstly they differentiated the equation $x^2 + y^2 = 100$ to $\frac{dy}{dx} = \frac{-x}{y}$ and arranged the original equation as $y = \sqrt{100 - x^2}$. Thereafter, continued with $A = \int_0^{10} \sqrt{1 + \left(\frac{-x}{\sqrt{100 - x^2}} \right)^2} dx$. However, the approach was wrong. In part (b), most of the candidates failed to state which rule in part (a) gives a better approximation. Extract 7.1 shows a sample response from a candidate who performed poorly this question.

7 a) $x^2+y^2=100$.

Volume $= \pi\int y^2 dx$.

$y=\sqrt{100-x^2}$

Volume $=\pi\int(100-x^2)\,dx$

Then $h = \frac{10+10}{10} = 2$

Then From trapezoidal

Volume $=\pi\left[\frac{2}{2}\left(y_0+y_n+2\sum \text{remain}\right)\right]$

Then .

x	y	
-10	0	y_0
-8	36	y_1
-6	64	y_2
-4	84	y_3
-2	96	y_4
0	100	y_5
2	96	y_6
4	84	y_7
6	64	y_8
8	36	y_9
10	0	y_{10}

Then Volume $=\pi\left[(0+0+2(36+64+84+96)+100)\right]$

Volume $=\pi(660)$

Volume $=660\pi$

7 a) Then $\pi\int_{-10}^{10}(100-x^2)\,dx = 1333.33\pi$ units cube

Then

Thus $666\pi = \frac{4188.77}{3}$ cube unit

$\pi = \frac{2094.365}{660} = 3.173$

Volume of $\pi = 3.173$ for ~~Simpson~~

For Simpson.

$\pi\left[\frac{h}{3}\left(y_0 + y_n + 2(\text{even}) + 4\ \text{odd}\right)\right]$

$\pi\left[\frac{2}{3}\left(0+0+2(64+96+96+64) + 4(36+84+100+84+36)\right)\right]$

$\frac{2}{3}\times\pi(1200)$ Volume of $1200\pi\times\frac{2}{3} = 666\pi$ cube

Then Volume $~~800~~\pi = \frac{4188.77}{2}$

$\pi =$

Volume $= 666\pi = \frac{4188.77}{2}$

$\pi = \frac{4188.77}{2\times 666}$

$\pi = 3.143$

b) Simpson is more accurate than trapezoidal rule

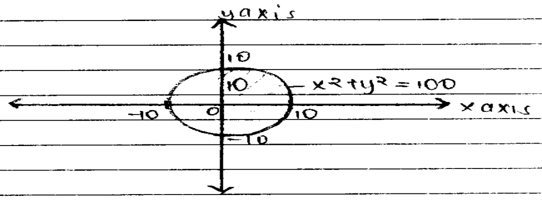
Extract 7.2: A sample of incorrect responses to question 7 of paper 1

In Extract 7.2, the candidate calculated the volume generated by a solid instead of the area under the curve and used the wrong limits.

However, there were a few candidates who performed well in this question. In part (a), the candidates were aware that the actual value was the area of one fourth of the circle, $A = \frac{\pi r^2}{4} = \frac{\pi(10)^2}{4}$ equivalent to 25π (square units).

Thereafter, they rearranged $x^2 + y^2 = 100$ as $y = \sqrt{100 - x^2}$ for $x \geq 0$ and $y \geq 0$. By using the Trapezoidal rule $A_1 = \frac{h}{2}[y_0 + y_{10} + 2(y_1 + y_2 + ... + y_9)]$ with $n = 10$ strips, $a = 0$, $b = 10$ and $h = 1$ obtained $A_1 = 77.6131$ units. But $A = A_1$ then compared the two areas, that is $25\pi = 77.6131$ to obtain $\pi = 3.10$. Again with $n = 10$, $a = 0$, $b = 10$ and $h = 1$ by using the Simpson's rule $A_1 = \frac{h}{3}[y_0 + y_{10} + 4(y_1 + y_3 ... + y_9) + 2(y_2 + y_4 + ... + y_8]$ they got $A_2 = 78.17563$ units. Then, by comparing this area to 25π they obtained $\pi = 3.13$. In part (b), the candidates were able to obtain the absolute errors by using the actual value of π as 3.14. Thus, from the trapezoidal rule, the absolute error was calculated and obtained as $3.14 - 3.10 = 0.04$. By Simpson's rule, the absolute error obtained was $3.14 - 3.13 = 0.01$. Through these steps, the candidates realized that the absolute error of Simpson's rule is less than that obtained by the trapezoidal rule. From this fact, they concluded that, Simpson's rule has a better approximation than the trapezoidal rule. Extract 7.1 shows a sample response from a candidate who performed well in this question.

7a.



$x^2 + y^2 = 100$

$y^2 = 100 - x^2$

$y = \sqrt{100 - x^2}$

$A = \int_0^{10} \sqrt{100 - x^2}\, dx$

$h = \frac{b - a}{n}$

$h = \frac{10 - 0}{10}$

$h = 1$

x	0	1	2	3	4	5	6	7	8	9	10
$y = \sqrt{100 - x^2}$	10	$3\sqrt{11}$	$4\sqrt{6}$	$\sqrt{91}$	$2\sqrt{21}$	$5\sqrt{3}$	8	$\sqrt{51}$	6	$\sqrt{19}$	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_n

7ai Trapezoidal's rule

$$A = \int_a^b f(x)dx$$

where

$$\int_a^b f(x)dx = \frac{h}{2}\left[y_0 + y_n + 2(\Sigma \text{ remaining ordinates})\right]$$

Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}\left[y_0 + y_n + 4\Sigma\text{odd ordinates} + 2\Sigma\text{even ordinates}\right]$$

i. Trapezoidal rule

$$\int_0^{10} \sqrt{100-x^2}\,dx = \frac{1}{2}\left[10 + 0 + 2\left[3\sqrt{11} + 4\sqrt{6} + \sqrt{91} + 2\sqrt{21} + 5\sqrt{3} + \sqrt{51} + 8 + 6 + \sqrt{19}\right]\right]$$

$$\int_0^{10} \sqrt{100-x^2}\,dx = 77.61295816 \text{ square units}$$

but Area $= \frac{\pi r^2}{4}$

$$\frac{\pi r^2}{4} = 77.61295816$$

$$\pi = \frac{4 \times 77.61295816}{(10)^2}$$

$$\pi = 3.104518326 \simeq 3.10$$

$\pi = 3.10$ (According to Trapezoidal rule)

7a. From simpson's rule

$$\int_0^{10} \sqrt{100-x^2}\,dx = \frac{1}{3}\left[10+0+4(3\sqrt{11}+\sqrt{91}+5\sqrt{3}+\sqrt{51}+\sqrt{19}) + 2(4\sqrt{6}+2\sqrt{21}+8+6)\right]$$

$$\int_0^{10} \sqrt{100-x^2}\,dx = 78.17520397 \text{ square units}$$

but Area $= \frac{\pi r^2}{4}$

$$\frac{\pi r^3}{4} = 78.17520397$$

$$\pi r^2 = 4 \times 78.17520397$$

$$\pi = \frac{4 \times 78.17520397}{10^2}$$

$$\pi = 3.127008159 \simeq 3.13$$

$\pi = 3.13$ (from simpson's rule)

7b. Absolute error = |Actual value − measured value|

For Trapezoidal rule

Absolute error $= |3.14 - 3.10|$
$= 0.04$

7b For simpson's rule

Absolute error $= |3.14 - 3.13|$
$= 0.01$

Since simpson's rule has a less value of absolute error, Then it gives a better approximation.

Extract 7.2: A sample of correct responses to question 7 of paper 1

In Extract 7.2, the candidate used the correct formulae to approximate the area under the curve by applying the Trapezoidal and Simpson's rules, thus they were able to approximate the value of Pie.

2.1.8 Question 8: Coordinate Geometry I

This question consisted of three parts: (a), (b) and (c). In part (a), the candidates were given the equation of the circle $2x^2 + 2y^2 + 8x + 12y - 136 = 0$ and asked to find the Centre and radius of the circle. In part (b), the line $12x + 16y + 12 = 0$ was given and the candidates were required to find the perpendicular distance from the centre of the circle calculated in part (a). Part (c) of the question reads as follows: "The coordinates of points P and Q are (x_1, y_1) and (x_2, y_2) respectively". Candidates were required to find the coordinates of a point R which divides the line PQ internally in the ratio $m_1 : m_2$.

The analysis shows that 21.5 per cent of the candidates who attempted this question scored from 0 to 3 marks, 22.9 per cent scored from 3.5 to 5.5 marks and 55.6 per cent scored from 6 to 10 marks. Generally, the candidates' performance in this question was good, as 78.5 per cent of the candidates got more than 3 marks. Figure 9 illustrates the candidates' performance in this question.

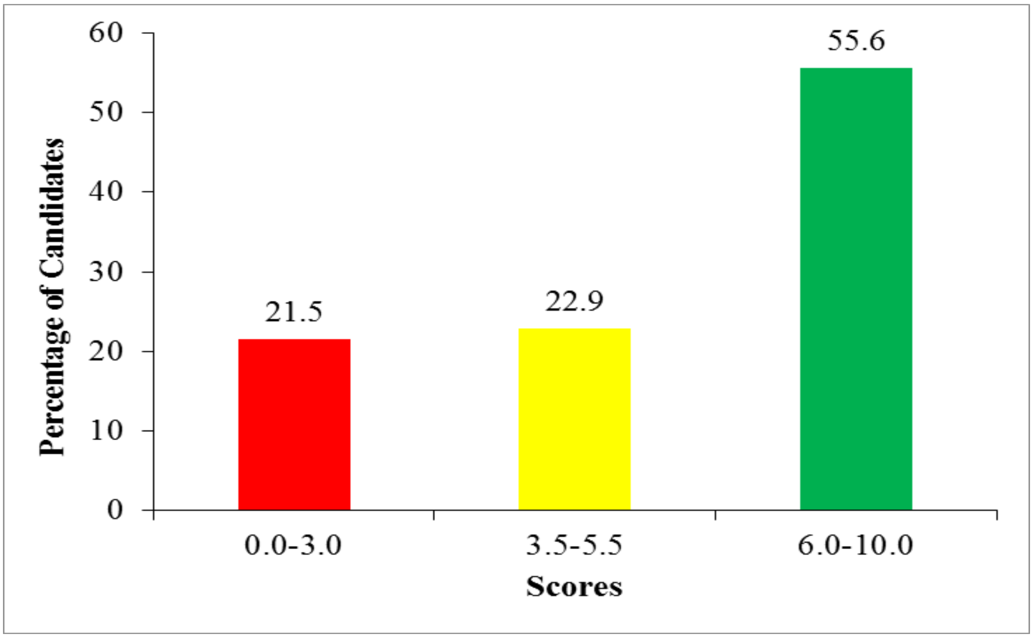


Figure 9: *Candidates' Performance in Question 8 of Paper 1*

The analysis of the candidates' responses shows that the question was answered well by most of the candidates. For instance, in part (a), candidates were able to express the equation of the circle $2x^2 + 2y^2 + 8x + 12y - 136 = 0$ in simple a form, that is, $x^2 + y^2 + 4x + 6y - 68 = 0$ and used appropriate methods to obtain the centre of a circle $(-2, -3)$ and the radius of a circle (9 units). In part (b), they were able to determine the perpendicular distance from the point $(-2, -3)$ obtained in part (a) to a line $12x + 16y + 12 = 0$ by using the formula $d = \left|\frac{Ah + Bk + C}{\sqrt{A^2 + B^2}}\right|$. Therefore, they correctly obtained a perpendicular distance equal to 3 units.

In part (c), the candidates proved that $(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$ at point R, using the concept of similarity of triangles if $P(x_1, y_1)$ and $Q(x_2, y_2)$ are coordinates of a line segment and R divides $\overline{PQ}$ internally in the ratio $m_1 : m_2$. Extract 8.1 is a sample of responses from one of the candidates who performed well in this question.

8. (a) Given circle equation:

$2x^2 + 2y^2 + 8x + 12y - 136 = 0.$

Dividing by 2 throughout;

$\frac{1}{2} \times (2x^2 + 2y^2 + 8x + 12y - 136) = 0 \times \frac{1}{2}$

$x^2 + y^2 + 4x + 6y - 68 = 0.$

Comparing to circle equation:

$x^2 + y^2 + 2gx + 2fy + c = 0.$

Then:

$2gx = 4x.$

$\frac{2gx}{2x} = \frac{4x}{2x}$

$g = 2.$

For centre; $a = -g$

$a = -2.$

Also:

$2fy = 6y.$

$\frac{2fy}{2y} = \frac{6y}{2y}$

$f = 3.$

For centre; $b = -f.$

$b = -3.$

Therefore; Coordinates of the centre $= \underline{(-2, -3)}$.

Also;

radius, $r = \sqrt{a^2 + b^2 - c}$

where; $c = -68.$

Then;

$r = \sqrt{(-2)^2 + (-3)^2 - (-68)}$

$r = \sqrt{4 + 9 + 68}$

$r = \sqrt{81}$

$r = 9.$

Therefore; Radius of the circle is $\underline{9 \text{ units}}$.

8\. (b) Given: $12x + 16y + 12 = 0$.

Centre of the circle: $(a, b) = (^-2, ^-3)$.

Then:

Perpendicular distance, $d = \left| \frac{Ax + By + C}{\sqrt{A^2 + B^2}} \right|$.

Here $Ax + By + C = 12x + 16y + 12$.

and $(x, y) = (^-2, ^-3)$.

Then;

$$d = \left| \frac{12(-2) + 16(-3) + 12}{\sqrt{12^2 + 16^2}} \right|$$

$$d = \left| \frac{-24 - 48 + 12}{\sqrt{144 + 256}} \right|$$

$$d = \left| \frac{-60}{\sqrt{400}} \right|$$

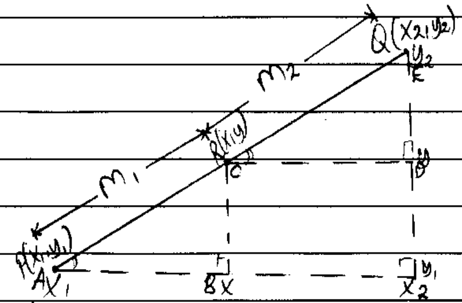
$$d = \left| \frac{-60}{20} \right|$$

$$d = |-3|$$

$$d = 3 \text{ units}.$$

Hence: The perpendicular distance is <u>3 units</u>.

(c)



Consider the diagram above;

$\Delta ABC \sim \Delta CDE$.

Thus; $\frac{AB}{CD} = \frac{AC}{AE}$.

Then;

8 (c) $\dfrac{x - x_1}{x_2 - x} = \dfrac{m_1}{m_2}$.

$m_1(x_2 - x) = m_2(x - x_1)$.

$m_1x_2 - m_1x = m_2x - m_2x_1$.

$m_1x_2 + m_2x_1 = m_2x + m_1x$.

$m_1x_2 + m_2x_1 = (m_2 + m_1)x$.

$\dfrac{(m_1 + m_2)x}{m_1 + m_2} = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$.

$x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$.

Also;

$\dfrac{\overline{BC}}{\overline{DE}} = \dfrac{\overline{AC}}{\overline{CE}}$.

$\dfrac{y - y_1}{y_2 - y} = \dfrac{m_1}{m_2}$.

$m_1(y_2 - y) = m_2(y - y_1)$.

$m_1y_2 - m_1y = m_2y - m_2y_1$.

$m_1y_2 + m_2y_1 = m_2y + m_1y$.

$m_1y_2 + m_2y_1 = (m_2 + m_1)y$.

$\dfrac{(m_1 + m_2)y}{m_1 + m_2} = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$.

$y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$

Therefore;

Coordinates of $R(x, y) = \left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

Extract 8.1: A sample of correct responses to question 8 of paper 1

In Extract 8.1, the candidate was able to use suitable methods to determine the centre and radius of the circle. Also, by using the centre obtained in part (a), they managed to calculate the perpendicular distance from a given line. Finally, in part (c), he/she obtained the coordinates of a point R, which divides the line PQ internally in the given ratio.

Even though the majority of the candidates provided correct responses to this question, a few candidates performed poorly due to some challenges they faced while answering the question. In part (a) , they failed to apply the fundamental concepts of the circle, which means that for the equation to be a circle, the coefficients of x^2 and y^2 must be equal to one. So, those candidates were using directly the coefficients of x and y from the equation

$2x^2 + 2y^2 + 8x + 12y - 136 = 0$ to calculate the centre of the circle. Thus, they ended up getting $(-4, -6)$ as the centre of the circle, which is incorrect.

In part (b), some candidates failed to calculate the perpendicular distance from the centre obtained in part (a) because part (a) was a necessary requirement for part (b) of the question to be solved. Likewise, some candidates didn't understand the requirement of the question. For example, one candidate calculated the equation of the line instead of the perpendicular distance. Started by differentiating the equation $2x^2 + 2y^2 + 8x + 12y - 136 = 0$ to get $\frac{dy}{dx} = \frac{4-2x}{2y+6}$. Then, at $(-2, -3)$ they obtained $\frac{2}{3}$ as the gradient. They finally calculated the equation of the perpendicular line, $y = -\frac{3}{2}x$, by using $-\frac{3}{2}$ and $(-2, -3)$.

In part (c), the candidates failed to provide correct responses due to the lack of knowledge and skills in identifying the point that divided the line segment internally in the ratio $m_1 : m_2$. Thus, some students ended up calculating the point $R(x, y) = \left(\frac{m_1 x_2 - m_1 y_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2} \right)$, which divides the line segment externally. Extract 8.2 is a sample response from one of the candidates who answered this question incorrectly.

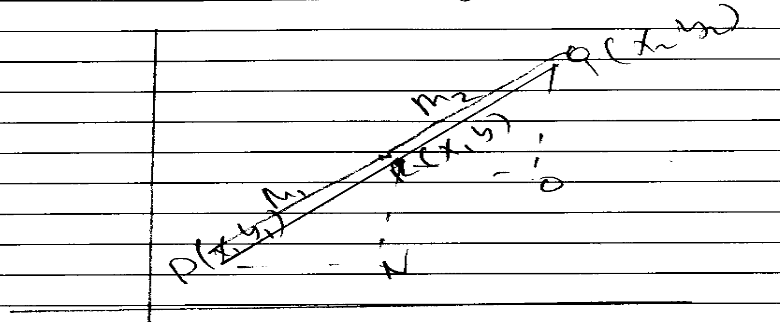
(b)

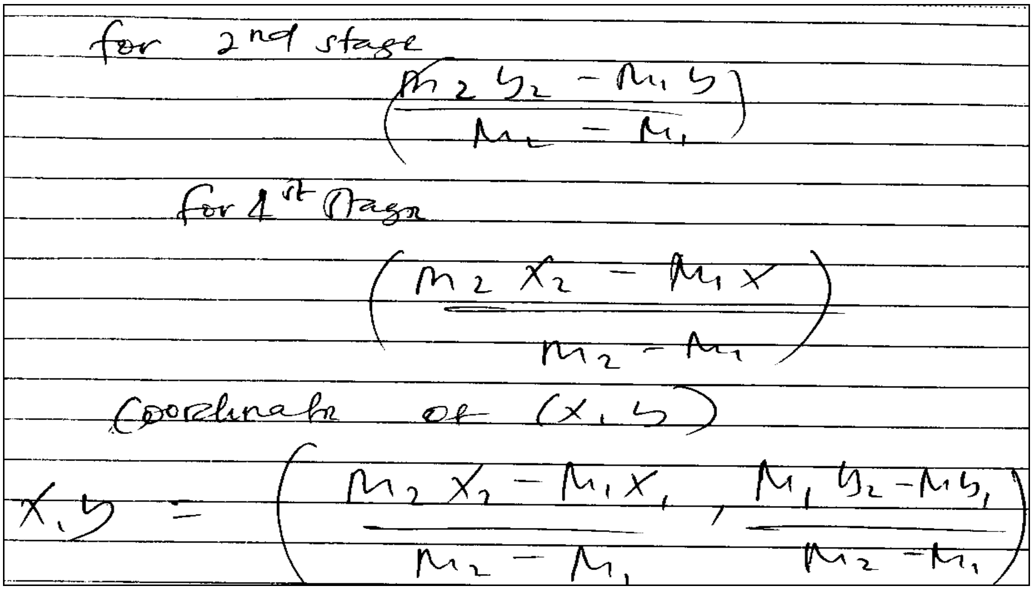
$x^2 + y^2 + 4x + 6y = 68$

$2x + 2y\frac{dy}{dx} + 4 + 6\frac{dy}{dx} = 0$

$2x + 2y\frac{dy}{dx} + 6\frac{dy}{dx} = 4.$

$2x + (2y + 6)\frac{dy}{dx} = 4.$

8	(b)	$\frac{dy}{dx} = \frac{4-2x}{2y+6}$ $(-2, -3)$
		$m = \frac{4+4}{-6+6}$
		$m = 2/3$.
		$m_1 m_2 = -1$
		$2/3\, m_2 = -1$
		$m_2 = -3/2$.
		$-3/2 = \frac{y+3}{x+2}$
		$-3(x+2) = 2(y+3)$
		$-3x - 6 = 2y - 6$
		$2y = -3x$
		$y = -3/2\, x$.
8.	c)	Soln.
		Consider the figure below.
		
		From the figure ~~above~~.
		1st stage $\frac{P}{R} = R/N$
		$\frac{M_1}{M_2} = \frac{x - x_1}{y - y_1}$
		$M_1 y - M_1 y_1 = M_2 x - M_2 x_1$
		for
		2nd stage
		$R/Q = R/Q$
		$\frac{M_2}{M_1} = \frac{y_2 - y}{x_1 - x}$
		$M_2 x_2 - M_2 x = M_1 y_2 - M_1 y$



for 2nd stage

$$\left(\frac{m_2 y_2 - m_1 y}{m_2 - m_1}\right)$$

for 1st stage

$$\left(\frac{m_2 x_2 - m_1 x}{m_2 - m_1}\right)$$

Coordinate of (x, y)

$$x, y = \left(\frac{m_2 x_2 - m_1 x_1}{m_2 - m_1}, \frac{m_1 y_2 - m_1 y_1}{m_2 - m_1}\right)$$

Extract 8.2: A sample of incorrect responses to question 8 of paper 1

In Extract 8.2, the candidate was calculating the equation of normal to the circle instead of the perpendicular distance from the given line, in part (b). In part (c), the candidate obtained the wrong coordinates of a point that divides the line internally.

2.1.9 Question 9: Integration

This question was composed of two parts, namely (a) and (b). Part (a) required the candidates to find the following indefinite integrals: (i) $\int \frac{x}{\sqrt{x+1}}dx$ and (ii) $\int \theta^2 e^{2\theta} d\theta$. In part (b), the candidates were supposed to find the volume of the solid generated by rotating about the $x-$axis the area enclosed by $y^2 - x^3 = 4$ and $y = 0$ between $x = 0$ and $x = 3$.

The analysis of the data shows that the question was attempted by 13,755 (100%) candidates. 49.9 percent of the candidates scored 0 to 3 marks, 14.2 per cent scored from 3.5 to 5.5 marks and 35.9 per cent scored from 6.0 to 10 marks. The data also shows that 1,828 (13.3%) candidates scored 10.0 marks while 3,863 (28.1%) candidates scored 0. A summary of the candidates' performance is presented in Figure 10.

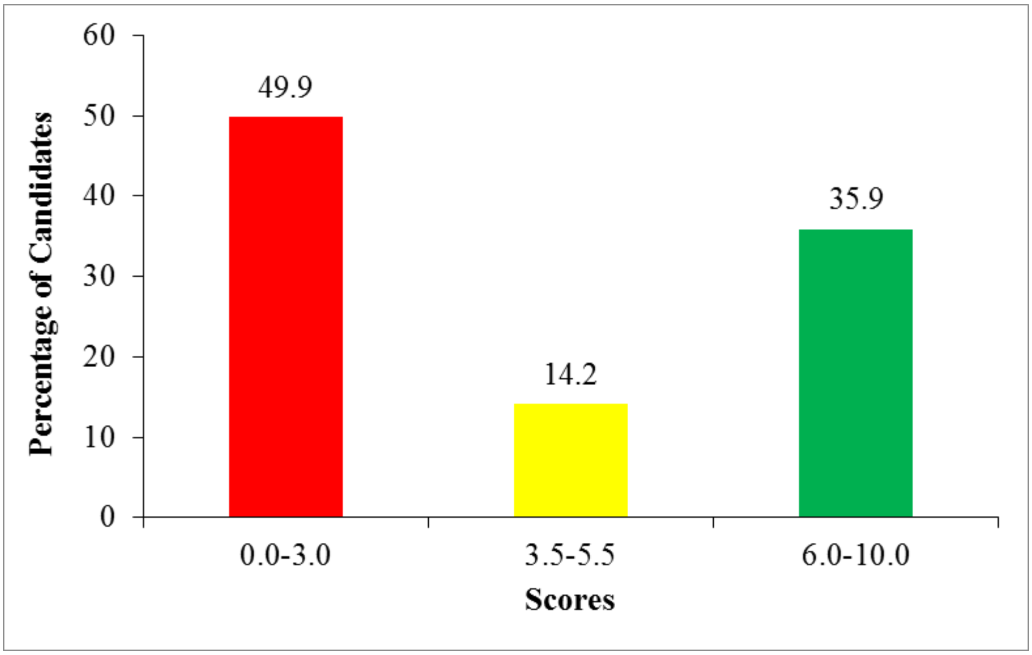


Figure 10: *Candidates' Performance in Question 9 of Paper 1*

Further analysis shows that 50.1 per cent of the candidates scored more than 3 marks. Therefore, the candidates' performance in this question was average.

The candidates who performed well in this question demonstrated the following skills: In part (a) (i), they managed to apply the appropriate substitution techniques by considering $u=\sqrt{x+1}$ thus $u^2=x+1$ or $u=x+1$. Then, by solving the integral, they managed to get $\int \frac{x}{\sqrt{x+1}}dx=\frac{2}{3}(x+1)^{\frac{3}{2}}-2\sqrt{x+1}+c$. In part (a) (ii), candidates were able to use integration by parts, that is, $\int udv=uv-\int vdu$, to reach the desired solution $\int \theta^2 e^{2\theta}d\theta=\frac{1}{4}e^{2\theta}(2\theta^2-2\theta+1)+c$. In part (b), they were able to use $V=\pi\int_a^b y^2dx$ to calculate the volume of the solid generated by rotating about the $x-$axis the area enclosed by $y^2-x^3=4$ and $y=0$ between $x=0$ and $x=3$. Finally, they obtained $V=\pi\int_0^3\left(4+x^3\right)dx=\frac{129}{4}\pi$ cubic units. Extract 9.1 shows a sample of responses from one of the candidates who performed well in this question.

9. (a) (i) $\int \frac{x}{\sqrt{x+1}}\,dx$.

Let $u = x + 1$.

$du = dx$.

Also;

$x = u - 1$.

Then;

$$\int \frac{u-1}{\sqrt{u}} \times du.$$

$$= \int \left(\frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\right) du.$$

$$= \int u^{1-1/2}\,du - \int u^{-1/2}\,du.$$

$$= \int u^{+1/2}\,du - \int u^{-1/2}\,du.$$

$$= \frac{u^{1/2+1}}{1/2+1} - \frac{u^{-1/2+1}}{-1/2+1} + c.$$

$$= \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + c.$$

$$= \frac{2}{3}\sqrt{u^3} - 2\sqrt{u} + c.$$

But $u = x + 1$.

$$= 2/3\sqrt{(x+1)^3} - 2\sqrt{x+1} + c.$$

Thus;

$$\int \frac{x}{\sqrt{x+1}}\,dx = \underline{\frac{2}{3}\sqrt{(x+1)^3} - 2\sqrt{x+1} + c}.$$

9. (a) (ii) Given: $\int \theta^2 e^{2\theta} d\theta$.

By parts method;

Let $u = \theta^2$ and $dv = e^{2\theta} d\theta$.

$du = 2\theta d\theta$.

Also;

$\int dv = \int e^{2\theta} d\theta$

$v = \frac{1}{2} e^{2\theta}$.

By parts;

$\int u dv = uv - \int v du$.

$\int \theta^2 e^{2\theta} d\theta = \theta^2 (\frac{1}{2} e^{2\theta}) - \int \frac{1}{2} e^{2\theta} (2\theta) d\theta$.

$\int \theta^2 e^{2\theta} d\theta = \frac{1}{2} \theta^2 e^{2\theta} - \int \theta e^{2\theta} d\theta$.

Then;

For $\int \theta e^{2\theta} d\theta$.

Let $u = \theta$ and $dv = e^{2\theta} d\theta$.

$du = d\theta$

Also; $\int dv = \int e^{2\theta} d\theta$

$v = \frac{1}{2} e^{2\theta}$.

By parts;

$\int u dv = uv - \int v du$.

$\int \theta e^{2\theta} d\theta = \theta (\frac{1}{2} e^{2\theta}) - \int \frac{1}{2} e^{2\theta} d\theta$.

$= \frac{1}{2} \theta e^{2\theta} - \frac{1}{2} \int e^{2\theta} d\theta$.

$= \frac{1}{2} \theta e^{2\theta} - \frac{1}{4} e^{2\theta}$.

Then;

$\int \theta^2 e^{2\theta} d\theta = \frac{1}{2} \theta^2 e^{2\theta} - [\frac{1}{2} \theta e^{2\theta} - \frac{1}{4} e^{2\theta}] + c$

$\therefore \int \theta^2 e^{2\theta} d\theta = \frac{1}{2} \theta^2 e^{2\theta} - \frac{1}{2} \theta e^{2\theta} + \frac{1}{4} e^{2\theta} + c$.

9. (b) Given: $y^2 - x^3 = 4$.

$y^2 = 4 + x^3$.

$x = 0$ and $x = 3$.

But;

Volume, $V = \pi \int_0^3 y^2 \, dx$

$V = \pi \int_0^3 (4 + x^3) \, dx$.

$V = \pi \left[4x + \frac{x^4}{4} + c \right]_0^3$

$V = \pi \left[4(3) + \frac{3^4}{4} - 0 \right]$.

$V = \pi \left(12 + \frac{81}{4} \right)$.

$V = \frac{129\pi}{4}$ cubic units.

Thus; Volume generated $= \frac{129\pi}{4}$ cubic units

Extract 9.1: A sample of correct responses to question 9 of paper 1

In Extract 9.1, the candidate applied the correct techniques to integrate indefinite integrals given in part (a) and applied the correct formula to find the volume of solid revolution correctly in part (b).

Those who performed poorly in this question had the following weaknesses: In part (a) (i), a few candidates defined $\frac{d}{dx}(\sin^{-1} x) = \frac{x}{\sqrt{x+1}}$ then integrated both sides to get $\int \frac{x}{\sqrt{x+1}} dx = (\sin^{-1} x) + c$. Some of them used the hyperbolic substitution by letting $x = \sinh^2 \theta$. By evaluating the integral they got $\int \frac{x}{\sqrt{x+1}} dx = 2\sinh^3(\sinh^{-1}\sqrt{x}) + 2\sinh(\sinh^{-1}\sqrt{x}) - 2\sinh(\sinh^{-1}\sqrt{x}) + c$.

Others were rationalizing the denominator of the integral $\int \frac{x}{\sqrt{x+1}} dx$. Finally, by manipulating this integral, they obtained $\int \frac{x}{\sqrt{x+1}} dx = \frac{x\sqrt{x+1}}{2} + x^2 + c$. In part (a) (ii), some candidates could not get correct solutions because they were using the technique of substitutions instead of integration by parts. Finally, in part (b), a few candidates calculated the value of $y^2 - x^3$ instead of finding volume. That is, they were integrating $y^2 - x^3$ from $x = 0$ to $x = 3$ hence getting 20.25 as the volume of the solid generated. Those candidates had inadequate skills in the application of integration. Extract 9.2 shows the response of one of the candidates who performed poorly in this question.

9 (a) (i) $\int \frac{x}{\sqrt{x+1}}\, dx$

let $x = \sinh^2\theta$

$dx = 2\cosh^2\theta$

$\int \frac{\sinh^2\theta}{\sqrt{\sinh^2\theta+1}}\, 2\cosh^2\theta\, d\theta$

$\int \frac{\sinh^2\theta}{\sqrt{\cosh^2\theta}}\, 2\cosh^2\theta\, d\theta$

$\int \frac{\sinh^2\theta}{\cosh\theta}\, 2\cosh^2\theta\, d\theta$

$2\int \sinh^2\theta \cosh\theta\, d\theta$

But $\sinh^2\theta = \cosh^2\theta - 1$

$2\int (\cosh^2\theta - 1)\cosh\theta\, d\theta$

$2\int (\cosh^3\theta - \cosh\theta)\, d\theta$

$2\int \cosh^3\theta\, d\theta - 2\int \cosh\theta$

Cons for $2\int \cosh^3\theta = \int \cosh^2\theta \cosh\theta \, d\theta$

$$= \int (\sinh^2\theta + 1)\cosh\theta \, d\theta$$

$$2\int \sinh^2\theta \cosh\theta + 2\int \cosh\theta$$

2 let $\sinh\theta = u$.

$$2\int u^2 \cdot \cancel{\cosh\theta} \cdot \frac{du}{\cancel{\cosh\theta}} + 2\sinh\theta$$

$$2\frac{u^3}{3} + 2\sinh\theta$$

But $u = \sinh\theta$

$$\frac{2\sinh^3\theta}{3} + 2\sinh\theta$$

But $x = \sinh^2\theta$

$$\sqrt{x} = \sinh\theta$$

$$\theta = \sinh^{-1}(\sqrt{x})$$

$$\int \frac{x}{\sqrt{x+1}}\,dx = 2\sinh^3(\sinh^{-1}\sqrt{x}) + 2\sinh(\sinh^{-1}\sqrt{x})$$

$$+ 2\sinh(\sinh^{-1}\sqrt{x}) + C$$

9 b. solution

$$\int_{y,x=0}^{x=3} y^2 - x^3$$

$$\int y^2 - x^3$$

$$\left.\frac{y^3}{3} - \frac{x^4}{4}\right|_{y=0,\,x=0}^{x=3}$$

$$\left(\frac{0^3}{3} - \frac{3^4}{4}\right) - \left(\frac{0^3}{3} - \frac{0^4}{4}\right)$$

$$= (0 + -20.25) - (0 - 0)$$

$$= + -20.25$$

Volume $= -+20.25$

volume $= -20.25$

Extract 9.2: A sample of incorrect responses to question 9 of paper 1

In Extract 9.2, the candidate used a wrong substitution to evaluate the indefinite integral. Also, the candidate used a wrong concept to find the volume of the solid revolution.

2.1.10 Question 10: Differentiation

The question consisted of three parts; (a), (b) and (c). In part (a), the candidates were required to differentiate $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ with respect to x. In part (b), they were required to use the second derivative test to investigate the stationary values of the function $3xe^{-x}$. In part (c), it was given that, "The air pollution index p in a certain city is determined by the amount of solid waste (x) and noxious gas (y) in the air. The index is given by the equation $p = x^2 + 2xy + 4xy^2$". The candidates were required to obtain the following partial derivatives of (i) $\frac{\partial p}{\partial x}$ and (ii) $\frac{\partial p}{\partial y}$ at (10, 5).

This question was attempted by 13,755 (100%) candidates. The analysis of the data shows that 7,065 (51.4%) candidates scored 0 to 3 marks, 3,181 (23.1%) candidates scored 3.5 to 5.5 marks and 3,509 (25.5%) candidates scored from 6 to 10 marks. This indicates that the candidates' performance in the question was average. Figure 11 summarizes the candidates' performance in this question.

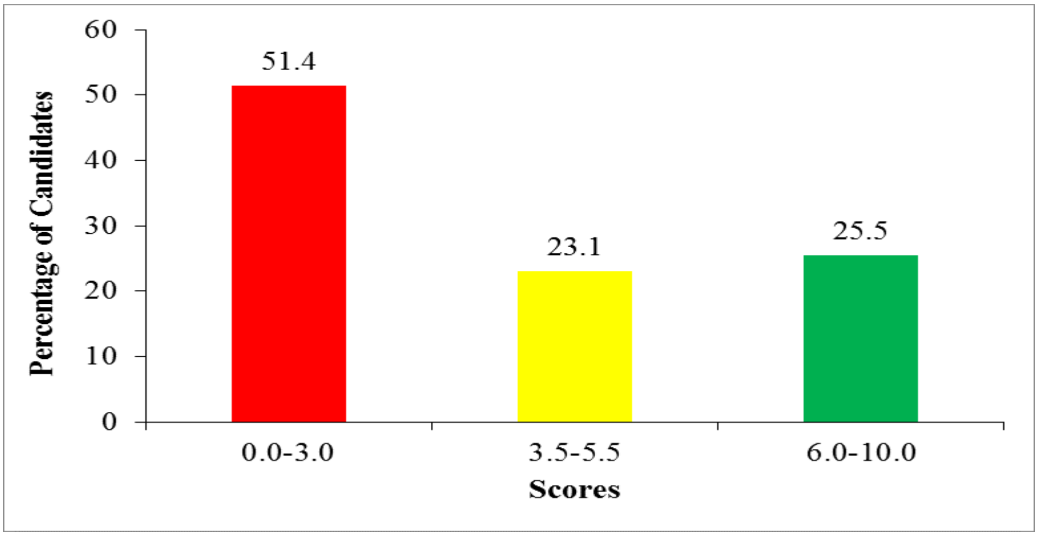


Figure 11: *Candidates' Performance in Question 10 of Paper 1*

Further analysis shows that 6,690 (48.6%) candidates scored from 3.5 to 10 marks. Those candidates who performed well in this question had adequate knowledge and skills of the concepts tested. In part (a), the candidates attempted to differentiate $y=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ with respect to x, by first rationalizing the denominator of $\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$ to obtain $\frac{1-\sqrt{1-x^2}}{x}$. Then by differentiating $y=\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$ they obtained $\frac{dy}{dx}=\frac{1}{2\sqrt{1-x^2}}$. Others applied trigonometric substitution by letting $x=\cos\theta$ or $\cos 2\theta$, then by simplifying the equation $y=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ they obtained $y=\left(\frac{\pi}{4}-\theta\right)$. Finally, they differentiated it to get $\frac{dy}{dx}=\frac{1}{2\sqrt{1-x^2}}$.

Also in part (b), they were able to use the second derivative test to investigate the stationary values of the function $y=3xe^{-x}$. Firstly, they obtained $\frac{dy}{dx}=3e^{-x}-3xe^{-x}=3e^{-x}\left(1-x\right)$.Then, at $\frac{dy}{dx}=0$ they obtained $x=1$ and $y=\frac{3}{e}$. Secondly determined $\frac{d^2y}{dx^2}=-6e^{-x}+3xe^{-x}$. Those candidates realized that at $x=1$, $\frac{d^2y}{dx^2}<0$ hence concluded that the stationery value of the function is maximum. In part (c), the candidates were capable to use the techniques of partial derivatives to the equation $p=x^2+2xy+4xy^2$ at point $(10,5)$ hence getting $\frac{\partial p}{\partial x}=130$ and $\frac{\partial p}{\partial y}=420$. Extract 10.1 shows the sample response of one of the candidates who had adequate skills in the concepts tested.

10a. $$y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$$

$$y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \cdot \frac{(\sqrt{1+x} - \sqrt{1-x})}{(\sqrt{1+x} - \sqrt{1-x})}\right)$$

$$y = \tan^{-1}\left(\frac{(\sqrt{1+x} - \sqrt{1-x})^2}{1+x-(1-x)}\right)$$

$$y = \tan^{-1}\left(\frac{1+x+1-x-2\sqrt{1-x^2}}{2x}\right)$$

$$y = \tan^{-1}\left(\frac{2-2\sqrt{1-x^2}}{2x}\right)$$

$$y = \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$$

10a

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{1-\sqrt{1-x^2}}{x}\right)^2} \cdot \frac{x\left(-\frac{1}{2}(1-x^2)^{-\frac{1}{2}}2x\right)-\left(1-\sqrt{1-x^2}\right)}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+\frac{(1-\sqrt{1-x^2})^2}{x^2}} \cdot \frac{x^2(1-x^2)^{-\frac{1}{2}}-\left(1-\sqrt{1-x^2}\right)}{x^2}$$

$$= \frac{x^2}{x^2+(1-\sqrt{1-x^2})^2} \cdot \frac{x^2(1-x^2)^{-\frac{1}{2}}-\left(1-\sqrt{1-x^2}\right)}{x^2}$$

$$= \frac{\frac{x^2}{\sqrt{1-x^2}}-\left(1-\sqrt{1-x^2}\right)}{x^2+(1-\sqrt{1-x^2})^2}$$

$$= \frac{x^2-\left(\sqrt{1-x^2}-(1-x^2)\right)}{\sqrt{1-x^2}\left(x^2+(1-\sqrt{1-x^2})^2\right)}$$

$$= \frac{x^2-\sqrt{1-x^2}+1-x^2}{\sqrt{1-x^2}\left(x^2+(1-\sqrt{1-x^2})^2\right)}$$

$$= \frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}\left(x^2+1+1-x^2-2\sqrt{1-x^2}\right)}$$

$$= \frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2-2\sqrt{1-x^2}\right)}$$

$$= \frac{1-\sqrt{1-x^2}}{2\sqrt{1-x^2}\left(1-\sqrt{1-x^2}\right)}$$

10a	$dy/dx = \frac{1}{2\sqrt{1-x^2}}$
	$\therefore dy/dx = \frac{1}{2\sqrt{1-x^2}}$
10b.	$y = 3xe^{-x}$
	$\frac{dy}{dx} = -3xe^{-x} + 3e^{-x}$
	$dy/dx = -y + 3e^{-x}$
	$\frac{d^2y}{dx^2} = -dy/dx + -3e^{-x}$
	$\frac{d^2y}{dx^2} = -(-3xe^{-x} + 3e^{-x}) - 3e^{-x}$
	$= 3xe^{-x} - 3e^{-x} - 3e^{-x}$
	$\frac{d^2y}{dx^2} = 3xe^{-x} - 6e^{-x}$
	For stationary values
	$dy/dx = 0$
	$0 = -3xe^{-x} + 3e^{-x}$
	$0 = e^{-x}(3 - 3x)$
	$e^{-x} = 0$ or $3 - 3x = 0$
	$e^{-x} = 0$ (not valid)
	$3 - 3x = 0$
	$3x = 3$
	$x = 1$

10 b	$y = 3xe^{-x}$
	$y = 3(1)e^{-1}$
	$y = \frac{3}{e}$
	$y = 1.104$
	$\frac{d^2y}{dx^2} = 3xe^{-x} - 6e^{-x}$
	at $x = 1$
	$\frac{d^2y}{dx^2} = \frac{3}{e} - \frac{6}{e}$
	$\frac{d^2y}{dx^2} = -\frac{3}{e}$
	since $\frac{d^2y}{dx^2} < 0$
	Then the stationary value of $(1, 3/e)$ or $(1, 1.104)$ is a maximum value
10c.	$p = x^2 + 2xy + 4xy^2$
	$\frac{\partial p}{\partial x} = 2x + 2y + 4y^2$
	at $(10, 5)$
	$\frac{\partial p}{\partial x} = 2(10) + 2(5) + 4(5)^2$
	$\partial p/\partial x = 130$
	$\frac{\partial p}{\partial y} = 2x + 8xy$
	at $(10, 5)$
	$\frac{\partial p}{\partial y} = 2(10) + 8(10)(5)$
	$\partial p/\partial y = 420$

Extract 10.1: A sample of correct responses to question 10 of paper 1

In Extract 10.1, the candidate was able to apply the techniques of differentiation properly to solve the given problem in each part.

The candidates who performed poorly in this question had inadequate knowledge and skills on the topic. For example, in part (a), some of them applied $f(x)' = \lim h \to 0 \dfrac{f(x+h) - f(x)}{h}$ and came up with very complicated expressions, such as $f(x)' = \tan^{-1}(\infty)$. Others were just adding the numbers under the radicals. For example, it was written $y = \tan^{-1}\left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} = \tan^{-1}\left(\dfrac{\sqrt{1+x-1+x}}{\sqrt{1+x+1-x}}\right)\right)$. Finally they obtained $y = \tan^{-1}\left(\dfrac{\sqrt{2x}}{2}\right)$ and could not proceed. However, this approach was wrong. In other cases, the candidates had problems in rationalizing the given radical, thus they could not reach to the required solution. In part (b), some candidates had insufficient skills to differentiate the exponential function. For example, they wrote $y' = 3xe^{-x} + 3e^{-x}$ instead of $y' = -3xe^{-x} + 3e^{-x}$, thus obtaining incorrect responses. Others failed to define the product rule of differentiation because they wrote $y' = -3xe^{-x} \times 3e^{-x}$. With this misconception, it was impossible to reach the required solution.

Lastly, in Part (c), a few candidates lacked the required skills on how to find the derivative of partial functions. For instance, some candidates started by substituting the point $(10, 5)$ into the equation $p = x^2 + 2xy + 4xy^2$ followed by differentiation. For example, in part (c) (i), they wrote $p = x^2 + 110x$ when $y = 5$ then obtained $\dfrac{\partial p}{\partial x} = 2x + 110$. Likewise, in (c) (ii), it was written as $p = 100 + 20y + 40y^2$ when $x = 10$ hence got $\dfrac{\partial p}{\partial y} = 20 + 80y$. In other circumstances, few candidates differentiated the given equation as an implicit differentiation problem. Extract 10.2 shows a sample response from one of the candidates who performed poorly in this question.

10 (a) Soln

$$y = \tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right]$$

$$y = \tan^{-1}\left[\frac{\sqrt{1+x-(1-x)}}{\sqrt{(1+x)+(1-x)}}\right]$$

$$y = \tan^{-1}\left[\frac{\sqrt{2x}}{\sqrt{2}}\right]$$

$$y = \tan^{-1}\left(\frac{\sqrt{2x}}{\sqrt{2}}\right)$$

$$\tan y = \frac{\sqrt{2x}}{\sqrt{2}}$$

$$\sec^2 y \frac{dy}{dx} = \frac{\sqrt{2} \cdot \sqrt{2}(2x)^{-1/2} - \sqrt{2x}\,(2)^{-1/2}}{2}$$

$$\frac{dy}{dx} = \frac{\sqrt{2} \cdot (2x)^{-1/2} - \sqrt{2x} \cdot 2^{-1/2}}{2\operatorname{cosec}^2 x}$$

$$\frac{dy}{dx} = \frac{2^{-1/2}\left(\sqrt{2}\,x^{-1/2}\right) - \sqrt{2x}\,)}{2\operatorname{Cosec}^2 x}$$

$$\frac{dy}{dx} = \frac{2^{-1/2}\left(\sqrt{2}\,x^{-1/2} - \sqrt{2x}\right)}{2\operatorname{cosec}^2 x}$$

10 b) $y = 3xe^{-x}$

$y' = 3x(e^{-x}) \cdot e^{-x}(3)$

$= 3xe^{-x} \cdot 3e^{-x}$

$= 3xe^{-x} \cdot 3e^{-x}$

$y'' = (3xe^{-x} \cdot 3e^{-x})\,(3e^{-x} \cdot (e^{-x}(0)))$

$= 3xe^{-x} \cdot 3e^{-x} \cdot 3e^{-x}$

$= 3xe^{-x} \cdot (3e)^{x+-x}$

$= 3xe^{-x} \cdot (3e)^{0}$

but

any number power zero $= 1$.

$= 3xe^{-x} \cdot 1$.

$= 3xe^{-x}$

$\therefore$ $y = 3xe^{-x}$ remains stationary at the second derivative

10 c) soln.

ii) $P = x^2 + 2xy + 4xy^2$.

at $(10, 5)$.

for $x = 10$.

$P = (10)^2 + 2(10)y + 4(10)y^2$.

$P = 100 + 20y + 40y^2$.

$P \cdot dp/dy = 20 + 20y$

$\therefore dp/dy = 20 + 80y$.

for $y = 5$.

i) $P = x^2 + 2x(5) + 4x(5)^2$

$P = x^2 + 10x + 100x$.

$P = x^2 + 110x$

$dp/dx = 2x + 110$.

Extract 10.2: A sample of incorrect responses to question 10 of paper 1

In Extract 10.2, the candidate added the radicands instead of rationalizing the denominator or numerator as an important step for differentiation, in part (a). In part (b), the candidate was not able to use the product rule to differentiate the given function. Likewise, in part (c), the candidate failed to apply the required techniques of partial differentiation.

2.2 142/2 ADVANCED MATHEMATICS 2

2.2.1 Question 1: Probability

The question comprised three parts: (a), (b), and (c). Part (a) required the candidates to prepare the probability distribution table of obtaining 0, 1, 2, 3, 4, and 5 defective tomatoes in a random batch of 20 tomatoes for which, on average, 15 percent of tomatoes were defective using: (i) Binomial distribution and (ii) Poison distribution. In part (b), the candidates were required to compute the mean and standard deviation of the two cases, in part (a) and in part (c), they were required to find the number of students who scored between 30% and 70% inclusive of 100 students in a senior mathematics contest examination for the year 2014, with the mean and standard deviation of 64 and 16, respectively.

The question was attempted by 13,754 (99.99%) candidates. The summary of candidates' performance in this question is presented in Figure 12.

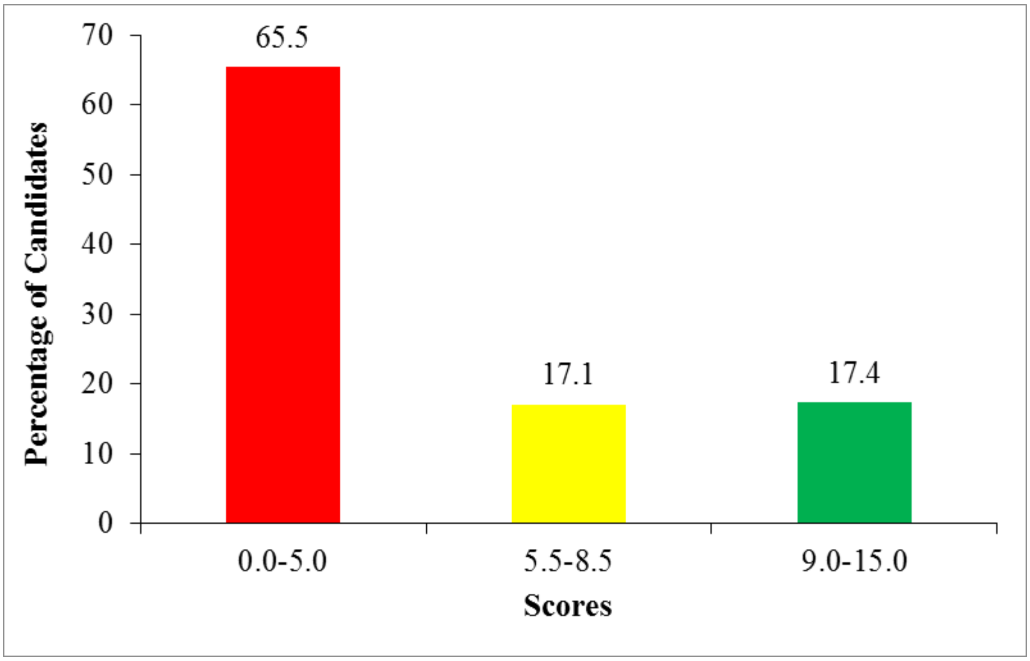


Figure 12: *Candidates' Performance in Question 1 of Paper 2*

The analysis shows that among the 13,754 candidates who attempted the question, 9,011 (65.5%) scored 0 to 5 marks. While 2,355 (17.1%) candidates scored 5.5 to 8.5 marks and 2,388 (17.4%) scored 9 to 15 marks. Therefore, the performance of the candidates in this question was weak since 34.5 per cent of the candidates who attempted this question scored from 5.5 to 15 marks.

Most of the candidates who attempted the question encountered some challenges that led to a weak performance. One of the challenges faced by them is the failure to understand the need of the question, recall it, and write the correct formulae. Others applied the correct formulae but failed to apply them. For example, in part (a), some candidates used $P(X=x)={}^{n}C_{x}p^{x}q^{n-x}$ then, they substituted $n=6$ instead of $n=20$. Using these values, they obtained the wrong probability distribution table:

x	0	1	2	3	4	5
$P(x)$	0.3771	0.3993	0.1762	0.0415	0.0059	0.0004

Also, there were other candidates who used the wrong formula such as $C_{n}p^{x-1}q^{x-1}$, and failed to apply it to obtain the values of probabilities associated with given values of the random variable X. In part (b), some candidates misinterpreted the question and used the wrong formulae to find the mean and standard deviation of the distribution. For example, one of the candidates applied $E(X)=np$ and obtained the wrong answer of 3 instead of applying $E(X)=\sum_{i=0}^{5}x_{i}p(x_{i})$ to obtain the correct answer of 2.5662. Likewise, they applied $E(X)=\sqrt{npq}$ for standard deviation and obtained the wrong answer of 1.5969 instead of applying $\delta=\sqrt{E(x^{2})-\left(E(x)\right)^{2}}$ to obtain the correct answer of 1.4620.

In part (c), most of the candidates applied the wrong approach to compute the probability $P(30\%\le X\le 70\%)$ by using an improper formula. They wrote the formula for the probability of non-mutually exclusive events $P(A)+P(B)-P(A\cap B)=P(A\cup B)$ and substituted $P(A)=\frac{3}{10}$ and

$P(A)=\frac{7}{10}$ to obtain an incorrect answer of 0.79. Extract 11.1 shows a sample of incorrect responses from one of the candidates who attempted the question.

1.

a)
i) Binomial distribution.

x	0	1	2	3	4	5
$P(x=X)$	0	0.13	0.16	0.27	0.24	0.20

ii) Poison distribution.

x	0	1	2	3	4	5
$P(x=X)$	0	0.4	0.1	0.06	0.27	0.17

b) from the formula
→ Binomial distribution.

Mean $(\bar{x}) = E(x)$.

$\bar{x} = x\sum P(x=X)$.

standard deviation $(\sigma) = E(x^2) - [E(x)]^2$

1. (i) $\bar{x} = 64$

$\sigma = 16.$

$$\text{Number of students} = \left(\frac{x_1 - \bar{x}}{\sigma}\right) - \left(\frac{x_2 - \bar{x}}{\sigma}\right)$$

$$= \left(\frac{0.7 - 64}{16}\right) - \left(\frac{0.3 - 64}{16}\right)$$

$$= {}^{-}3.96 - {}^{-}3.98$$

$$= -3.96 + 3.98$$

$$= 0.02$$

$$\text{Number of students} = 100 \times 0.02$$

$$\text{Number of students} = 2 \text{ students.}$$

$\therefore$ The number of students who score between 30% and 70% inclusive are 2 students

Extract 11.1: A sample of incorrect responses to question 1 of paper 1

In Extract 11.1, part (a), the candidate obtained the wrong probability distribution tables, and he/she also did not write the formulae used to get those probability distribution tables. In parts (b) and (c), the candidate wrote the wrong formulae, thus obtained incorrect answers.

On the other hand, the candidates who attempted this question correctly had adequate knowledge and skills regarding the topic of probability. In part (a), candidates were able to prepare the probability distribution table using the correct functions, that is, the binomial distribution given by $P(x) = \binom{n}{x} p^x q^{n-x}$

and the Poisson distribution given by $p(x)=\frac{\lambda^x e^{-\lambda}}{x!}$. In part (a) (i), using the binomial distribution function and the values of $p=0.15$, $q=0.85$ and $n=20$, they obtained the required probability distribution table as follows:

x	0	1	2	3	4	5
$P(x)$	0.0388	0.1368	0.2293	0.2428	0.1821	0.1028

In part (a) (ii), using the Poisson distribution function and the value of $\lambda=np=20\times0.15=3$, they obtained the required probability distribution table as follows:

x	0	1	2	3	4	5
$P(x)$	0.0498	0.1494	0.2240	0.2240	0.1680	0.1008

In part (b), the candidates computed the mean and standard deviation by using the probability distribution tables they obtained in parts (a) of Roman (i) and (ii), respectively. Based on the binomial distribution table, they calculated the mean by applying the formula: $\text{Mean}=E(x)=\sum_{x=0}^{5}x_i p(x_i)$ and obtained the correct answer of 2.5662. Then, they calculated the standard deviation (δ) using the formula $\delta=\sqrt{E(x^2)-(E(x))^2}$ and obtained the correct answer of 1.4620. For Poison distribution table, they calculated the mean by applying the formula: $\text{Mean}=E(x)=\sum_{x=0}^{5}x_i p(x_i)$ and obtained the required answer of 2.4454. Then, they calculated the standard deviation (δ) using the formula $\delta=\sqrt{E(x^2)-(E(x))^2}$ and obtained the correct answer of 1.5131.

In part (c), the candidates were able to obtain the required number of students who scored between 30% and 70% inclusive by taking $P(30\%\le X\le 70\%)\times 100$. They converted the random variable X into the Z score using the formula $z_i=\frac{x_i-\mu}{\sigma}$, where $\mu=64$ and $\sigma=16$. Then, they obtained $P(30\%\le X\le 70\%)=P(-2.125\le Z\le 0.375)$. Using a scientific calculator or mathematical table, they obtained $P(-2.125\le Z\le 0.375)=0.62938$. With this value, they calculated the number of students in a senior mathematical contest as 63. Extract 11.2 illustrates the correct responses from one of the candidates who attempted this question.

1 (a) (i) By binomial distribution.

$n = 20$

$X = 0, 1, 2, 3, 4, 5$

$p = \frac{15}{100} = 0.15$ (probability of defective tomatoes)

$q = 1 - p = 1 - 0.15$

$q = 0.85$ (probability of non defective tomatoes)

from,

$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$P(X=0) = {}^{20}C_{0}(0.15)^{0}(0.85)^{20}$

$= 0.0388$

$P(X=1) = {}^{20}C_{1}(0.15)^{1}(0.85)^{19}$

$= 0.1368$

$P(X=2) = {}^{20}C_{2}(0.15)^{2}(0.85)^{18}$

$= 0.2293$

$P(X=3) = {}^{20}C_{3}(0.15)^{3}(0.85)^{17}$

$= 0.2428$

$P(X=4) = {}^{20}C_{4}(0.15)^{4}(0.85)^{16}$

$= \cancel{0.58}\ 0.1821$

$P(X=5) = {}^{20}C_{5}(0.15)^{5}(0.85)^{15}$

$= 0.1028$

table.

X	0	1	2	3	4	5
$P(X=x)$	0.0388	0.1368	0.2293	0.2428	0.1821	0.1028

1 (a) (ii) Poisson distribution.

$n = 20$

$p = \frac{15}{100} = 0.15$

$\lambda = np = 20(0.15)$.

$= 3$.

$x = 0, 1, 2, 3, 4, 5$

from

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}.$$

$$P(X=0) = \frac{e^{-3}(3)^{0}}{0!}$$

$= 0.0498.$

$$P(X=1) = \frac{e^{-3}(3^{1})}{1!}$$

$= 0.1494.$

$$P(X=2) = \frac{e^{-3}(3^{2})}{2!}$$

$= 0.2240.$

$$P(X=3) = \frac{e^{-3}(3^{3})}{3!}$$

$= 0.2240.$

$$P(X=4) = \frac{e^{-3}(3^{4})}{4!}$$

$= 0.1680.$

$$P(X=5) = \frac{e^{-3}(3^{5})}{5!}.$$

$= 0.1008.$

1(a) (i) table.

x	0	1	2	3	4	5
$P(X=x)$	0.0498	0.1494	0.2240	0.2240	0.1680	0.1008

(b) Mean and standard deviation.

for

Binomial (Mean)

$= \sum x P(X=x)$

$= (0 \times 0.03888) + (1 \times 0.1368) + (2 \times 0.2293) + (3 \times 0.2428)$
$+ (4 \times 0.1821) + (5 \times 0.1028).$

$= 2.5662$

$\therefore$ Mean $= 2.5662.$

Standard deviation (σ).

$$\sigma = \sqrt{E(x^2) - (E(x))^2}.$$

$E(x^2) = (0^2 \times 0.0388) + (1^2 \times 0.1368) + (2^2 \times 0.2293)$
$+ (3^2 \times 0.2428) + (4^2 \times 0.1821) + (5^2 \times 0.1028)$

$E(x^2) = 8.7228$

$$\sigma = \sqrt{8.7228 - (2.5662)^2}$$

$= 1.462$

$\therefore$ Standard deviation is 1.462.

for Poison.

Mean.

$= \sum x P(X=x)$

$= (0 \times 0.0498) + (1 \times 0.1494) + (2 \times 0.2240) + (3 \times 0.2240)$
$+ (4 \times 0.1680) + (5 \times 0.1008).$

$= 2.4454.$

1 (b) Mean = 2.4454

Standard deviation (σ)

$\sigma = \sqrt{E(x^2) - (E(x))^2}$

$E(x^2) = (0^2 \times 0.0491) + (1^2 \times 0.1494) + (2^2 \times 0.2240)$
$+ (3^2 \times 0.2240) + (4^2 \times 0.168) + (5^2 \times 0.1001)$
$= 8.2694$

$\sigma = \sqrt{8.2694 - 2.4454^2}$

$= 1.513$

$\therefore$ Standard deviation is 1.513.

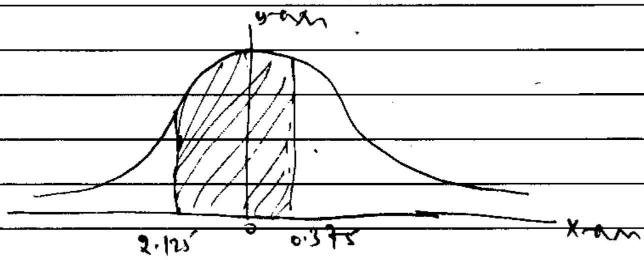
(c) $n = 100$.
$\mu = 64$
$\sigma = 16$.
$30 \le X \le 70$,

from $z = \dfrac{X - \mu}{\sigma}$.

$\dfrac{30-64}{16} \le z \le \dfrac{70-64}{16}$.

$-2.125 \le z \le 0.375$.

$P(-2.125 \le z \le 0.375)$



1 (c)

$\cancel{P(-z \le z}$

$P(-2.125 \le z \le 0.375) = 0.62938$

$P(z) = 0.62938$

$P(z) \approx 0.63.$

$\cancel{\text{from } P(x) = \frac{E(x)}{E(s)}}$

$E(x) = P(x) \cdot E(s)$

$= 0.63 \times 100$

$= 63.$

$\therefore$ 63 students will score between 30% and 70% inclusive

Extract 11.2: A sample of correct responses to question 1of paper 2

Extract 11.2 reveals that the candidate had adequate knowledge and skills on the topic of probability. He/she stated and applied the concepts and formula correctly according to the requirements of the question. In part (a), the candidate prepared the probability distribution tables correctly. Likewise, in part (b), the candidate calculated the mean and standard deviation as required. Lastly, in part (c), the candidate obtained the correct number of students.

2.2.2 Question 2: Logic

This question had three parts: (a), (b), and (c). Part (a) required the candidates to simplify $(P \rightarrow (Q \vee \sim R)) \rightarrow (P \wedge Q)$ using the laws of propositions of algebra. Part (b) required the candidates to use the truth table to verify whether $\sim (P \leftrightarrow Q) \equiv (P \wedge \sim Q)$ or not. Part (c) required the candidates to test the validity of the argument: “Every time we celebrate my mother’s birthday, I always bring her flowers. It is my mother’s birthday or I wake up late. I did not bring her flowers. Therefore, I woke up late”.

The analysis of the candidates' responses shows that 13,755 (100%) candidates attempted the question. Among them, 10,746 (78.1%) candidates scored from 9 to 15 marks, 2,249 (16.4%) candidates scored from 5.5 to 8.5 marks and 760 (5.5%) candidates scored from 0 to 5 marks. Figure 13 illustrates the performance of the candidates.

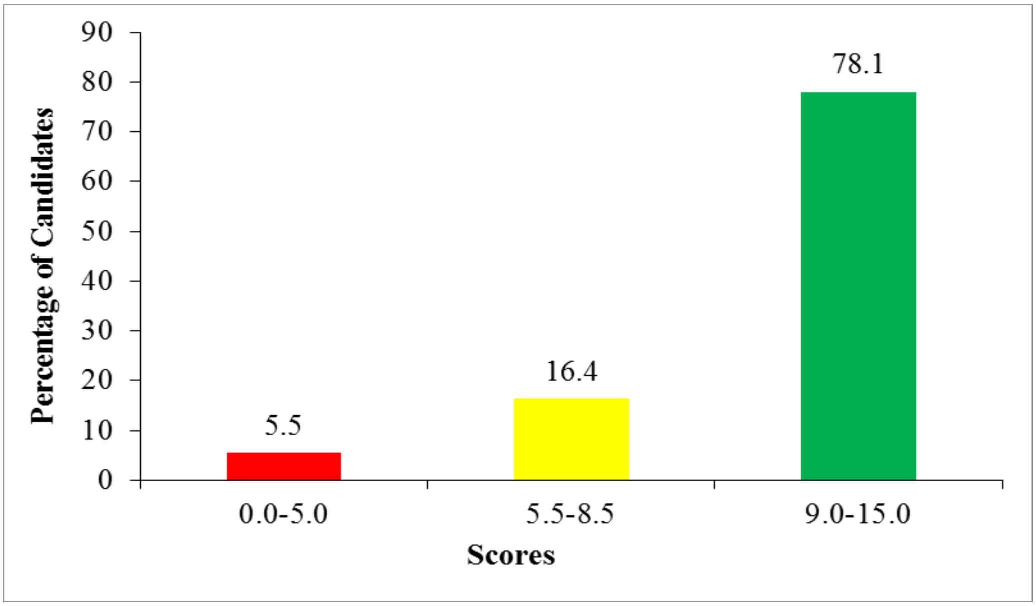


Figure 13: *Candidates' Performance in Question 2 of Paper 2*

The performance of the candidates in this question was good because 94.5 per cent of the candidates who attempted the question scored more than 5 marks.

The candidates who performed well in this question had the appropriate knowledge on the topic of logic. In part (a), the candidates used the laws of propositions of algebra like De-Morgan, Distributive, Identity, and Complement to simplify the compound statement $(P \rightarrow (Q \vee \sim R)) \rightarrow (P \wedge Q)$ to obtain $P \wedge (Q \vee R)$. In part (b), the candidates used the truth table to verify whether $\sim (P \leftrightarrow Q) \equiv (P \wedge \sim Q)$ or not. They drew a truth table and found that the given propositions were not equivalent. In part (c), the candidates were able to change the given argument from verbal form to symbolic form $S(P,Q,R): P \rightarrow Q, P \vee R, \sim Q \mapsto R$. After that, some examined the validity using the truth table, while others used the laws of propositions of algebra, and finally they obtained a valid argument. Extract 12.1 illustrates a correct response from one of the candidates who attempted the question correctly.

Q:(a) $(P \to (Q \vee \sim R)) \to (P \wedge Q)$

$(\sim P \vee (Q \vee \sim R)) \to (P \wedge Q)$ - Definition

$\sim(\sim P \vee (Q \vee \sim R)) \vee (P \wedge Q)$ - Definition.

$(P \wedge \sim(Q \vee \sim R)) \vee (P \wedge Q)$ - Demorgans law.

$P \wedge (\sim(Q \vee \sim R) \vee Q)$ - Distributive law.

$P \wedge ((\sim Q \wedge R) \vee Q)$ - Demorgan's law

$P \wedge ((\sim Q \vee Q) \wedge (R \vee Q))$ - Distributive law.

$P \wedge (T \wedge (R \vee Q))$ - Complement law.

$P \wedge (R \vee Q)$ - Identity law.

(b) $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q)$

$\sim[(P \to Q) \wedge (Q \to P)]$

$\sim[(\sim P \vee Q) \wedge (\sim Q \vee P)]$

				(a)	(b)	c		
P	Q	~P	~Q	~P v Q	~Q v P	a ∧ b	~c	P ∧ ~Q
T	T	F	F	T	T	T	F	F
T	F	F	T	F	T	F	T	T
F	T	T	F	T	F	F	T	F
F	F	T	T	T	T	T	F	F

From the truth table

$\sim(P \leftrightarrow Q)$ is not equivalent to $(P \wedge \sim Q)$

(c) Let

P - Celebrate mothers birthday.

Q - Bring her flowers

R - Wake up early

2 (c) $(p \to q), (q \vee \sim r), \sim Q \longrightarrow \sim r$

$[(p \to q) \wedge (q \vee \sim r) \wedge \sim Q] \longrightarrow \sim r$

p	q	r	$\sim p$	$\sim q$	$\sim r$	(a) $\sim p \vee q$	(b) $p \vee \sim r$	c $a \wedge b$	d $c \wedge \sim q$	$\sim d$	$\sim d \vee \sim r$
T	T	T	F	F	F	T	T	T	F	T	T
T	T	F	F	F	T	T	T	T	F	T	T
T	F	T	F	T	F	F	T	F	F	T	T
T	F	F	F	T	T	F	T	F	F	T	T
F	T	T	T	F	F	T	F	F	F	T	T
F	T	F	T	F	T	T	T	T	F	T	T
F	F	T	T	T	F	T	F	F	F	T	T
F	F	F	T	T	T	T	T	T	T	F	T.

∴ The statement is valid.

Extract 12.1: A sample of correct responses to question 2 of paper 2

In extract 12.1, the candidate was able to apply the laws of propositions of algebra to simplify the given compound statement in part (a). Furthermore, in part (b), he/she used the truth table properly and found that the given propositions were not equivalent. Likewise, in part (c), they were able to use the correct logical connectives to convert the given argument to symbolic form and then test its validity using the truth table.

Despite the good performance obtained by the majority of the candidates, there were a few who performed poorly in this question. In part (a), some of the candidates failed to use the laws of propositions of algebra in simplifying the given compound statement; for example, instead of writing $\big(p \to (Q \vee \sim R)\big) \to (P \wedge Q) \equiv \big(\sim P \vee (Q \vee \sim R)\big) \to (P \wedge Q)$ *definition*, they wrote $\big(p \to (Q \vee \sim R)\big) \to (P \wedge Q) \equiv \big(p \to Q) \vee (P \to \sim R)\big) \to (P \wedge Q)$ *Distributive law*. In part (b), some candidates did not meet the requirements of the question by applying the wrong approach. For example, they used laws instead of a truth table to verify the equivalence between the given compound statements. Furthermore, in part (c), a few candidates failed to understand the appropriate logical connectives in formulating the symbolical form of the given argument. Extract 12.2 shows a sample of an incorrect response from one of the candidates who attempted the question.

Q2. a) Soln.

$(P \rightarrow (\theta \vee \sim R)) \rightarrow (P \wedge \theta)$ – Given

$((P \rightarrow \theta) \vee (P \rightarrow \sim R)) \rightarrow (P \wedge \theta)$ – Distributive law.

$(P \vee (P \rightarrow \sim R)) \rightarrow (P \wedge \theta)$ – Identity law.

$((P \vee P) \rightarrow \sim R) \rightarrow (P \wedge \theta)$ – Distributive law.

$(P \rightarrow \sim R) \rightarrow (P \wedge \theta)$ – Idempotent law.

Let $M = (P \rightarrow \sim R)$.

$M \rightarrow (P \wedge \theta)$ – Set Definition.

$(M \rightarrow P) \wedge (M \rightarrow \theta)$ – Distributive

But $M = P \rightarrow \sim R$

$(P \rightarrow \sim R \rightarrow P) \wedge (P \rightarrow \sim R \rightarrow \theta)$ – Set Definition

$(P \rightarrow \sim R) \wedge (P \rightarrow \sim R)$ – Identity law.

$P \rightarrow \sim R$ – Idempotent law.

$\therefore (P \rightarrow (\theta \vee \sim R)) \rightarrow (P \wedge \theta) = P \rightarrow \sim R$ – Idempotent law.

Q2. b) Soln.

$\sim (P \leftrightarrow \theta) \equiv (P \wedge \sim \theta)$ – Given data

Truth Table: $\sim (P \leftrightarrow \theta)$

P	θ	$P \leftrightarrow \theta$	$\sim (P \leftrightarrow \theta)$
T	F	F	T
T	T	T	F

Truth Table for: $(P \wedge \sim \theta)$

P	θ	$\sim \theta$	$P \wedge \sim \theta$
T	F	T	T
T	T	F	F

Conclusion; Therefore they are equal since in the last Column they resemble or the same

02. Let P = My Mother's Birthday.

Q = Bring her flowers.

R = I wake up late.

$[(P \vee Q) \to (P \wedge R)] \vee (\sim Q \to R)$

Truth Table for; $[(P \vee Q) \to (P \wedge R)] \vee (\sim Q \to R)$

P	Q	R	$\sim Q$	$P \vee Q$	$P \wedge R$	$\overset{A}{(P \vee Q) \to (P \wedge R)}$	$\overset{B}{(\sim Q \to R)}$	$A \vee B$
T	T	T	F	T	T	T	F	T
T	T	F	F	T	F	F	F	F
T	F	T	T	T	T	T	F	T
T	F	F	T	T	F	F	F	F
F	T	T	F	T	F	F	F	F
F	T	F	F	F	F	T	F	T
F	F	T	T	T	F	F	F	F
F	F	F	T	F	F	T	F	T

Conclusion; Therefore the Argument is Contradiction which means it Contains both "T" and "F" in the last column.

Extract 12.2: A sample of incorrect responses to question 2 of paper 2

Extract 12.2 shows that, in part (a), the candidate was not familiar with the laws of propositions of algebra, hence failed to simplify the given compound statement. In part (b), the candidate failed to provide the correct truth table to verify the equivalence between the given compound statements. In part (c), the candidate failed to use the correct logical connectives while changing the given argument into its symbolic form, as a result, failed to test for validity.

2.2.3 Question 3: Vectors

The question consisted of three parts: (a), (b), and (c). Part (a) required the candidates to show the position vectors $2\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 3\underline{j} - 5\underline{k}$ and $3\underline{i} - 4\underline{j} - 4\underline{k}$ mark the vertices of a right-angled triangle. In part (b), the candidates were required to express vector $\overrightarrow{PQ}$ in the form $a\underline{i} + b\underline{j}$ where $a, b \in \Re$, if vector $\overrightarrow{PQ}$ has a magnitude of 5 units and is inclined at an angle of $150°$ to the $x-$axis. In part (c), the candidates were required to determine the work done by forces on the particle displaced by the forces $2\underline{i} - 5\underline{j} + 6\underline{k}$ and $-\underline{i} - 2\underline{j} - \underline{k}$ from point A to point B if the position vectors of points A and B are $4\underline{i} - 3\underline{j} - 2\underline{k}$ and $6\underline{i} - \underline{j} - 3\underline{k}$ respectively.

The data analysis indicates that out of 13,755 (100%) candidates who answered the question, 5,206 (37.8%) candidates scored 0 to 5 marks. While 4,483 (32.6%) candidates scored 5.5 to 8.5 marks and 4,066 (29.6%) candidates scored 9 to 15 marks. A summary of the candidates' performance is given in Figure 14.

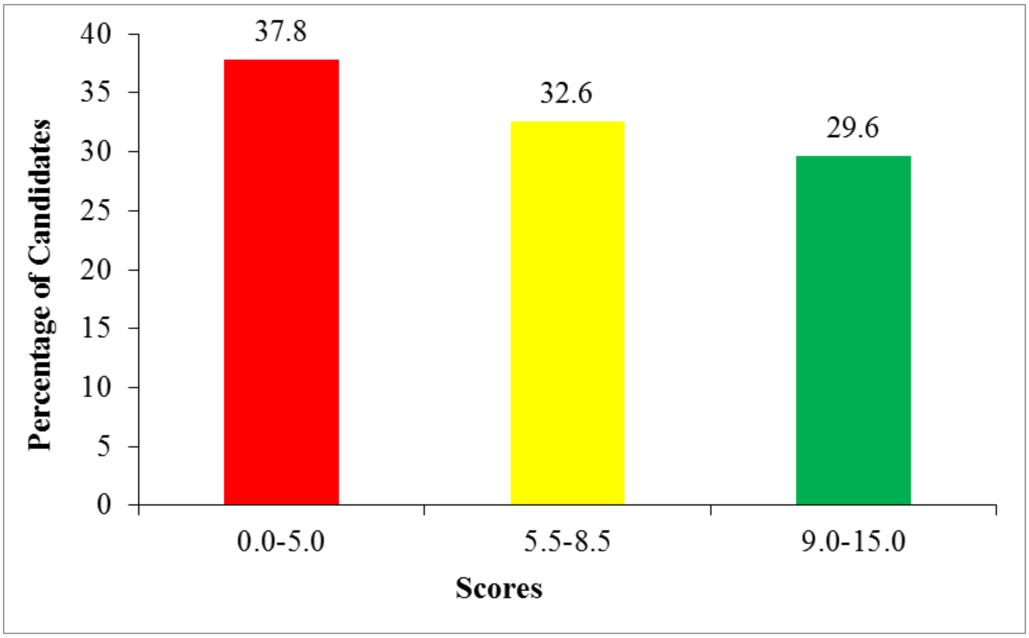


Figure 14: *Candidates' Performance in Question 3 of Paper 2*

The analysis shows further that the performance of the candidates was good, as observed in Figure 14, where 62.2 per cent of the candidates who attempted this question scored more than 5 marks.

The candidates who attempted this question correctly were able to go through all the steps in each part. In part (a), they were able to show the position vectors $2\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 3\underline{j} - 5\underline{k}$ and $3\underline{i} - 4\underline{j} - 4\underline{k}$ mark the vertices of a right-angled triangle using either the concept of Pythagoras' theorem or the concept of dot product. With Pythagoras' theorem, they calculated the distances $\left|\overrightarrow{AB}\right| = \sqrt{41}$,

$\left|\overrightarrow{AC}\right| = \sqrt{35}$ and $\left|\overrightarrow{BC}\right| = \sqrt{6}$, and found that $\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{BC}\right|^2$. In the dot product, they obtained $\overrightarrow{AC} \bullet \overrightarrow{BC} = 0$ which indicates the given triangle is a right-angled triangle.

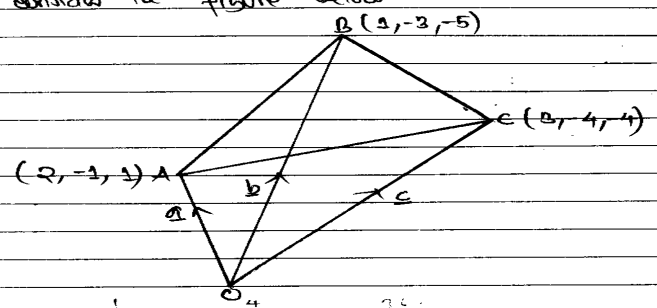
In part (b), the candidates calculated the x and $y-$components of the vector $\overrightarrow{PQ}$ as $x = 5 \times \cos 150^0$ and $y = 5 \times \sin 150^0$ to get $\overrightarrow{PQ} = \frac{-5\sqrt{3}}{2}\underset{\sim}{i} + \frac{5}{2}\underset{\sim}{j}$. In part (c), the candidates determined the work done using the concept of dot product by using the correct formula, $W = \left|\underset{\sim}{F} \bullet \overrightarrow{AB}\right|$. They calculated $\underline{F}$ as a resultant of the forces $2\underline{i} - 5\underline{j} + 6\underline{k}$ and $-\underline{i} - 2\underline{j} - \underline{k}$, and $\overrightarrow{AB}$ as the displacement from $A(4, -3, -2)$ to $B(6, -1, -3)$. Using the values of $\underline{F}$ and $\overrightarrow{AB}$, they obtained the required value of work done, 17 units. Extract 13.1 shows a sample of responses from one of the candidates who attempted this question correctly.

3(a) Let $a = 2i - j + k$

$b = i - 3j + -5k$.

$c = 3i - 4j - 4k$.

Then consider the figure below



$|\overrightarrow{AB}| = \sqrt{(1-2)^2 + (-3-^-1)^2 + (-5-1)^2}$.

$|\overrightarrow{AB}| = \sqrt{41}$.

$|\overrightarrow{BC}| = \sqrt{(3-1)^2 + (-4-^-3)^2 + (-4-^-5)^2}$

$|\overrightarrow{BC}| = \sqrt{6}$.

And

$|\overrightarrow{AC}| = \sqrt{(3-2)^2 + (-4-^-1)^2 + (-4-1)^2}$.

$|\overrightarrow{AC}| = \sqrt{35}$

For a right angled triangle

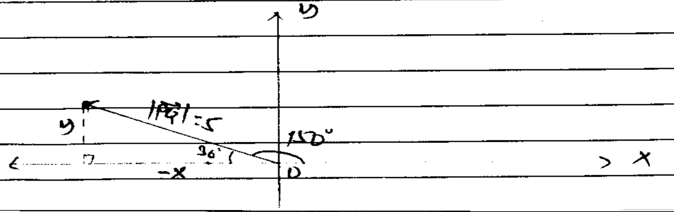
$a^2 + b^2 = c^2$.

$|\overrightarrow{AB}|^2 = |\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2$

$41 = 6 + 35$

$41 = 41$.

$\therefore$ Hence the given position vectors represent the vertices of a right angled triangle

b)	Solution
	Given,
	$\|\overrightarrow{PQ}\| = 5$ units
	Consider a sketch
	
02.b)	ie
	Find x and y
	ie
	$\cos 30° = \frac{-x}{\|\overrightarrow{PQ}\|}$
	$\cos 30° = \frac{-x}{5}$
	$x = -5\cos 30°$
	$x = -\frac{5\sqrt{3}}{2}$
	Also,
	$\sin 30° = \frac{y}{\|\overrightarrow{PQ}\|}$
	$y = \|\overrightarrow{PQ}\| \sin 30°$
	$y = 5(1/2)$
	$y = 5/2$
	ie
	Position vector $\overrightarrow{PQ} = (x, y) = (-5/2\sqrt{3}, 5/2)$
	ie
	$\overrightarrow{PQ} = -5/2\sqrt{3}\, i + 5/2\, j$
	Here,
	expressing in the form $ai + bj$ where
	$a = -5/2\sqrt{3}$ and $b = 5/2$
	$\therefore$
	$\overrightarrow{PQ} = -5/2\sqrt{3}\, i + 5/2\, j$
c)	Solution
	Given forces
	$F_1 = 3i - 5j + 6K$
	$F_2 = -i - 2j - K$
	$\overrightarrow{OA} = 4i - 3j - 2K$
	$\overrightarrow{OB} = 6i - j - 7K$
	Required, To find workdone by forces.

(c) ie

Workdone $= \vec{F_r} \cdot \vec{d}$

ie

But,

Resultant force, $\vec{F_r} = F_1 + F_2$

ie

$\vec{F_r} = (3i - 5j + 6k) + (-i - 2j - k)$

$\vec{F_r} = i - 7j + 5k$

Also,

$\vec{d} = \vec{AB} = \vec{OB} - \vec{OA}$

$= (6i - j - 3k) - (4i - 3j - 2k)$

$\vec{AB} = 2i + 2j - k$

Now,

$Wd = (i - 7j + 5k) \cdot (2i + 2j - k)$

$= \begin{pmatrix} 1 \\ -7 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

$= |2 + -14 + -5|$

$= |2 - 19|$

$= |-17|$

$=$ 17 Joules

$\therefore$ The workdone by the force is 17 Joules.

Extract 13.1: A sample of correct responses to question 3 of paper 2

In Extract 13.1, the candidate applied the correct concepts of vectors. In part (a), the candidate found the lengths of the sides of the triangle using the concept of displacement. Then, he/she applied Pythagoras theorem to prove the given triangle is a right-angled triangle. In part (b), the candidate resolved the given vector into x and $y-$components correctly, and in part (c), he/she found the work done using the correct method.

Apart from those who attempted the question correctly, there were some candidates who performed poorly. The poor performance was due to the following factors: In part (a), some candidates determined the unit vectors in the direction of the position vectors of the vertices of the triangle instead of determining the displacements $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ for which they could use either Pythagoras theorem or the dot product to obtain what was needed. Other candidates considered the position vectors of the vertices of the triangle as the displacement vectors of its sides. Furthermore, a few candidates used the concept of cross product to show that the given position vectors mark the vertices of the given triangle, which is the wrong concept. Others calculated the area of the triangle using the concept of cross product; they took $Area = \frac{1}{2}|\underline{a} \times \underline{b}| = 14.49$. This answer couldn't help to show whether the triangle is a right angle triangle.

In part (b), a few candidates used the concept of dot product to find the components of the vector $\overrightarrow{PQ}$. This concept was wrong as they were supposed to resolve $|\overrightarrow{AB}| = 5$ into x and $y-$components. Others approached this question by formulating the equation, $|\overrightarrow{AB}| = \sqrt{x^2 + y^2}$ which equal to $10 = \sqrt{x^2 + y^2}$ and $\tan\alpha = \frac{y}{x}$ that is $\tan 150 = \frac{y}{x}$. They solved these equations, ending up with the wrong values, $x = 2.74$ and $y = -1.59$.

In part (c), some candidates calculated the work done by using the given forces while others applied the cross product instead of the dot product when calculated the work done, such as $W = |\underline{F} \times \underline{d}| = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -7 & 5 \\ 4 & 2 & -1 \end{vmatrix}$ and obtained an incorrect answer of 36.74 units. Also, some candidates wrote the correct formula $W = |\underset{\sim}{F} \bullet \overrightarrow{AB}|$ but they failed to obtain the correct values of the resultant force $\underline{F}$ and the displacement $\overrightarrow{AB}$. They found the difference between the two forces as $\underline{F} = \underline{F}_1 - \underline{F}_2 = -3\underline{i} + 3\underline{j} - 7\underline{k}$ instead of adding them, that is $\underline{F} = \underline{F}_1 + \underline{F}_2$. Likewise, they calculated displacement as $\overrightarrow{AB} = \underline{a} + \underline{b} = 10\underline{i} - 4\underline{j} - 5\underline{k}$ instead of $\overrightarrow{AB} = \underline{b} - \underline{a}$. Other candidates calculated the resultant force $\underline{F}$ using the dot product of $\underline{F}_1$ and $\underline{F}_2$. One of the candidates obtained $\underline{F} = 2$ and the dot product of $\underline{a}$ and $\underline{b}$ to find the displacement $\overrightarrow{AB}$ which resulted in $\overrightarrow{AB} = 33$. Then calculated the work done by taking $W = \left|\underline{F} \bullet \overrightarrow{AB}\right| = 2 \times 33 = 66$ units. Extract 13.2 illustrates incorrect responses from one of the candidates who performed poorly in this question.

3 a) $2i - j + k$ vectors a.
$i - 3j - 5k$ vectors b.
$3i - 4j - 4k$ vectors c.

Area of a triangle:
$\frac{1}{2}|AB \cdot BC|$

3 a) where AB
$= B - A$
$\begin{pmatrix}1\\-3\\-9\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix}$

$= (-1, -2, -6)$

BC
$= (C - B)$
$\begin{pmatrix}3\\-4\\-4\end{pmatrix} - \begin{pmatrix}1\\-3\\-5\end{pmatrix}$

$= \begin{pmatrix}2\\-1\\1\end{pmatrix}$

$AB \cdot BC$
$\begin{pmatrix}-1\\-2\\-6\end{pmatrix} \cdot \begin{pmatrix}2\\-1\\1\end{pmatrix}$

$= -2 + 2 - 6$
$= -6$.

Area $= \frac{1}{2}|-6|$

Area $= |-3|$
Area $= 3$ units.

3 b) PQ = 5units
Q = 150°
To express PQ in form of ai + bj

soln:

$\theta = \cos^{-1}\left(\frac{P \cdot Q}{|P||Q|}\right)$

where $\theta = 150°$
$|P||Q| = 5$

$150 = \cos^{-1}\left(\frac{P \cdot Q}{5}\right)$

$\cos 150 = \frac{P \cdot Q}{5}$

$-0.87 = \frac{P \cdot Q}{5}$ $\quad -\frac{87}{100} = \frac{P \cdot Q}{5}$

3 b) $\frac{-435}{100} = P \cdot Q \times \frac{100}{100}$

$P \cdot Q = -4.35 = -4\frac{7}{20}$

$P \cdot Q = \frac{-87}{20}$

$PQ = ai + bj$

3 c) $F_1 = 2i - 5j + 6k$

$F_2 = -i - 2j - k$.

$A = (4i - 3j - 2k)$

$B = 6i - j - 3k)$

Workdone = $f \cdot d$.

$\begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix} = (-2 + 10 - 6)$

$= 2$

force = 2 Newtons

Distance

$= A \cdot B$

$\begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ -3 \end{pmatrix} = (24 + 3 + 6)$

$= 33$ m

3 c) Workdone = $f \cdot d$

$= 2 \times 33$

$= 66$

$\therefore$ Workdone = 66 joules.

Extract 13.2: A sample of incorrect responses to question 3 of paper 2

Extract 13.2 is evidence that the candidate lacked enough skills on this topic. In part (a), the candidate applied the wrong concept and the wrong formula in showing whether the given triangle is a right angled triangle. He/she used the formula $Area = \frac{1}{2}|AB.BC|$ instead of applying either Pythagoras theorem or the dot product. In part (b), the candidate obtained wrong results due to the application o the inappropriate concept of a dot product. In part (c), the candidate applied the dot product to find the resultant force as well as the displacement.

2.2.4 Question 4: Complex Numbers

The question had three parts; (a), (b), and (c). In part (a), the candidates were asked to show that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ if $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, hence verify that $\left|\frac{z-2}{z+3i}\right| = 4$ represents a circle. In part (b), the candidates were asked to express $\frac{\alpha + \beta + 4i}{\alpha\beta + 8i}$ in its simplest form if α and β are two roots of the equation $z^2 + 4z + 8 = 0$ without solving the equation. Lastly, in part (c), the candidates were asked to find all the complex roots of $z^3 = 1$.

The question was attempted by 13,755 (100%) candidates, 34.9 per cent scored from 0 to 5 marks, 25.7 per cent scored from 5.5 to 8.5 marks and 39.4 per cent scored from 9 to 15 marks. The candidates' performance in this question was good, as 65.1 per cent of the candidates scored from 5.5 to 15 marks. Figure 15 provides a summary of the candidates' performance in this question.

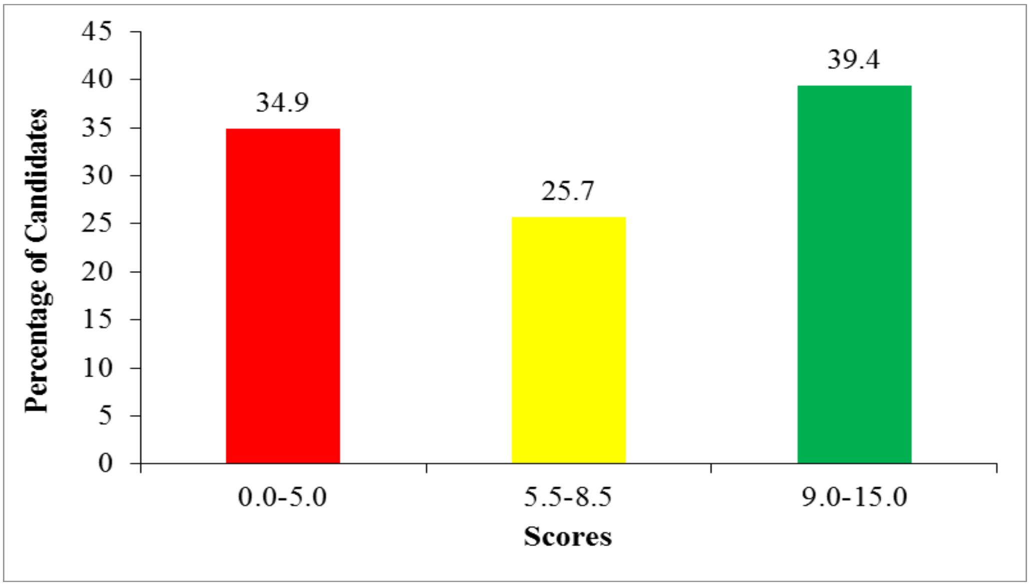


Figure 15: *Candidates' Performance in Question 4 of Paper 2*

The analysis revealed further that 4.0 per cent of the candidates who attempted this question scored 15 marks. They showed a good understanding of the topic as well as how to approach the question according to the requirement. In part (a), the candidates were able to apply the concept of modulus of complex numbers to show that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ where $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. They computed $\left|\frac{z_1}{z_2}\right|$ and obtained $\frac{\sqrt{x_1{}^2 + y_1{}^2}}{\sqrt{x_2{}^2 + y_2{}^2}}$, which is equal to $\frac{|z_1|}{|z_2|}$. Then they used the concept of $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ to prove that $\left|\frac{z-2}{z+3i}\right| = 4$ represents the equation of the circle. On simplifying $\left|\frac{z-2}{z+3i}\right| = 4$, they obtained $15x^2 + 15y^2 + 4x + 96y + 140 = 0$, which is an equation for a circle. In part (b), the candidates were able to determine the sum of the roots, $\alpha + \beta = -4$ and product of the roots, $\alpha\beta = 8$ of the equation $z^2 + 4z + 8 = 0$ and use them to express $\frac{\alpha + \beta + 4i}{\alpha\beta + 8i}$ in its simplest form which is $\frac{i}{2}$. In part (c), they applied the concept of De-Moivre's theorem, $Z_k = \cos\left(\frac{2\pi k}{3}\right) + i\sin\left(\frac{2\pi k}{3}\right)$ to find all the complex roots of the equation $z^3 = 1$ which were $z_1 = 1$, $z_2 = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $z_3 = \frac{-1}{2} - i\frac{\sqrt{3}}{2}$. Extract 14.1 shows a sample of responses from one of the candidates who correctly answered this question.

04(a) $\left|\frac{z_1}{z_2}\right| = \sqrt{\frac{x_1^2x_2^2 + y_1^2y_2^2 + x_2^2y_1^2 + x_1^2y_2^2}{(x_2^2+y_2^2)^2}}$

$= \sqrt{\frac{(x_1^2x_2^2 + x_1^2y_2^2) + (x_2^2y_1^2 + y_1^2y_2^2)}{(x_2^2+y_2^2)^2}}$

$= \sqrt{\frac{x_1^2(x_2^2+y_2^2) + y_1^2(x_2^2+y_2^2)}{(x_2^2+y_2^2)^2}}$

$= \sqrt{\frac{(x_1^2+y_1^2)(x_2^2+y_2^2)}{(x_2^2+y_2^2)^2}}$

$= \sqrt{\frac{x_1^2+y_1^2}{x_2^2+y_2^2}}$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}}$

But $\sqrt{x_1^2+y_1^2} = |z_1|$

$\sqrt{x_2^2+y_2^2} = |z_2|$

Now $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ (Proved)

Now $\left|\frac{z-2}{z+3i}\right| = 4$ Let $z = x+iy$

$\frac{|(x+iy)-2|}{|(x+iy)+3i|} = 4$

$\left|\frac{(x-2)+iy}{x+(y+3)i}\right| = 4.$

04(a) $|(x-2)+iy| = 4|x+(y+3)i|$

$$\sqrt{(x-2)^2+y^2} = 4\sqrt{x^2+(y+3)^2}$$

square both sides.

$$(x-2)^2+y^2 = 16\left(x^2+(y+3)^2\right)$$

$$x^2-4x+4+y^2 = 16(x^2+y^2+6y+9)$$

$$x^2+y^2-4x+4 = 16x^2+16y^2+96y+144$$

$$15x^2+15y^2+4x+96y+140=0$$

Hence it is a circle since it obey general equation of a circle

$$\underline{x^2+y^2+\tfrac{4}{15}x+\tfrac{96}{15}y+\tfrac{140}{15}=0}$$

$$\text{centre} = \left(\tfrac{-2}{15}, \tfrac{-48}{15}\right)$$

$$\text{radius} = \sqrt{g^2+f^2-c}$$

$$= \sqrt{\left(\tfrac{2}{15}\right)^2+\left(\tfrac{48}{15}\right)^2-\tfrac{140}{15}}$$

$$r = 0.946$$

04(b) $z^2+4z+8=0$

compare with the equation

$$z^2-(\alpha+\beta)z+\alpha\beta=0$$

Now, $\alpha+\beta = -4$

$\alpha\beta = 8$

Then $\dfrac{\alpha+\beta+4i}{\alpha\beta+8i} = \dfrac{-4+4i}{8+8i} =$

Q4(b)

$$\frac{\alpha+\beta+4i}{\alpha\beta+8i} = \frac{-4+4i}{8+8i} = \frac{1}{2}\left(\frac{-1+i}{1+i}\right)$$

$$= \frac{1}{2}\left(\frac{(-1+i)(1-i)}{(1+i)(1-i)}\right)$$

$$= \frac{1}{2}\left(\frac{-1+i+i-i^2}{1-i^2}\right)$$

$$= \frac{1}{2}\left(\frac{2i}{2}\right)$$

$$= \frac{1}{2}i$$

$$\frac{\alpha+\beta+4i}{\alpha\beta+8i} = \frac{i}{2}$$

Q4(a)

$z^3 = 1$

$z = (1+0i)^{1/3}$

from $Z_{k+1} = r^{1/n}\left(\cos\left(\frac{2\pi k+\theta}{n}\right) + i\sin\left(\frac{2\pi k+\theta}{n}\right)\right)$

But

$r = \sqrt{1^2+0^2}$

$r = 1$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{0}{1}\right)$

$\underline{\theta = 0^\circ}$

Now

$Z_{k+1} = 1^{1/3}\left(\cos\left(\frac{2\pi k+0}{3}\right) + i\sin\left(\frac{2\pi k+0}{3}\right)\right)$

$Z_{k+1} = 1\left(\cos\left(\frac{2\pi k}{3}\right) + i\sin\left(\frac{2\pi k}{3}\right)\right).$

04(c) for $k=0$

$z_1 = \cos 0 + i\sin 0$

$z_1 = 1$

for $k=1$

$z_2 = \cos\left(2\pi/3\right) + i\sin\left(2\pi/3\right)$

$= -1/2 + \sqrt{3}/2\, i$

for $k=2$

$z_3 = \cos\left(4\pi/3\right) + i\sin\left(4\pi/3\right)$

$= -1/2 - \sqrt{3}/2\, i$

The roots are 1, $-1/2 + \sqrt{3}/2\, i$, and $-1/2 - \sqrt{3}/2\, i$

Extract 14.1: A sample of correct responses to question 4 of paper 2

In this Extract 14.1, the candidate applied all the necessary concepts and steps in approaching each part of the question to obtain the correct answers. For example, in part (a), he/she applied the concept of operations on complex numbers to obtain a simplified form of the expression $\frac{z_1}{z_2}$, while in part (b), he/she determined the values of $\alpha + \beta$ and $\alpha\beta$ using the concepts of sum and product of roots of quadratic equations. In part (c), the candidate applied the concept of De-Moivre's theorem correctly to obtain the required roots.

However, the analysis indicates that 5.2 per cent of the candidates attempted this question incorrectly due to some challenges they faced with complex numbers. For instance, in part (a), the candidates failed to show that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ because they did wrong operations on $\frac{z_1}{z_2}$ and wrong computations of $\left|\frac{z_1}{z_2}\right|$. For example, they wrote $\left|\frac{z_1}{z_2}\right| = \sqrt{\frac{x_1 + iy_1}{x_2 + iy_2}} = \sqrt{\left(\frac{x_1}{y_1}\right)^2 + \left(\frac{x_2}{y_2}\right)^2}$ and finally provided the wrong conclusion, $\left(\frac{x_1}{y_1}\right)^2 + \left(\frac{x_2}{y_2}\right)^2 = \frac{x_1^{\ 2} + y_1^{\ 2}}{x_2^{\ 2} + y_2^{\ 2}}$. Other candidates applied the wrong operations to $\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$ and obtained an incorrect result, $x_2 - y_2 i + ix_2 y_1$. Not only that, but also they failed to use the relation $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ to verify that $\left|\frac{z-2}{z+3i}\right| = 4$ represents the equation of a circle. They used the wrong computation and ended up with the wrong answer, such as $\left|\frac{z-2}{z+3i}\right| = \frac{6i}{3i}$. Others wrote $\frac{z-2}{z+3i} = 4$ and solved this equation, to obtain the wrong result $3z + 12i = 2 = 0$ instead of $15x^2 + 15y^2 + 4x + 96y + 140 = 0$ to verify a circle.

In part (b), some candidates failed to determine the values of $\alpha + \beta$ and $\alpha\beta$ that could be used to express $\frac{\alpha + \beta + 4i}{\alpha\beta + 8i}$ in its simplest form. They applied the concept of the operation of complex numbers to the expression $\frac{\alpha + \beta + 4i}{\alpha\beta + 8i}$; thus they wrote $\frac{\alpha + \beta + 4i}{\alpha\beta + 8i} = \frac{\alpha + \beta + 4i}{\alpha\beta + 8i} \times \frac{\alpha\beta - 8i}{\alpha\beta - 8i}$ and obtained the wrong result, $\frac{\alpha^2\beta + \alpha\beta^2 + 4\alpha\beta - 8\alpha i - 8\beta i - 32}{\alpha^2\beta^2 + 64}$. Others applied inappropriate approach; they solved the given equation $z^2 + 4z + 8 = 0$ and obtained $z = 1.46$ or $z = -5.46$.

In part (c), the candidates applied the wrong concept in finding the roots of the equation $z^3 = 1$; they substituted $z = x + iy$ into $z^3 = 1$ and obtained $x^3 + 3ix^2y - 3y^2 - iy^3 - 1 = 0$. Other candidates applied De-Moivre's theorem wrongly and ended up with wrong results. For example, they wrote $z_{k+1} = 1^{\frac{1}{3}}\left[\cos\left(\frac{2\pi k + n}{2}\right) + i\sin\left(\frac{2\pi k + n}{2}\right)\right]$ and obtained $\cos\left(\frac{3}{2}\right) + i\sin\left(\frac{3}{2}\right)$,

$\cos\left(\frac{2\pi+3}{2}\right)+i\sin\left(\frac{2\pi+3}{2}\right)$ and $\cos\left(\frac{4\pi+3}{2}\right)+i\sin\left(\frac{4\pi+3}{2}\right)$. Some candidates solved the equation $z^3=1$ as a normal cubic equation without taking into consideration the concept of roots of complex numbers, thus obtained $z=1$. Extract 14.2 is a sample of responses from one of the candidates who used the wrong concepts in solving the question.

4 a/ $4z+12i = z-2$

$4z-z+12i=-2$

$3z+12i=-2$

$3z+12i+2=0$

for Quadro equation, equation verify the complex root represent a circle

$\therefore$ The $\left|\frac{z-2}{z+3i}\right| = 4$ represent a circle

b/ Given

$-z^2+4z+8=0$

$-\frac{\alpha+\beta+4i}{\alpha\beta+8i}$

soln

Required to express $\frac{\alpha+\beta+4i}{\alpha\beta+8i}$ in its simplest form

thus

from equation above

$z^2+4z+8=0$

$z=-2+2i$ or $z=-2-2i$

make $\alpha+\beta$ be the z

thus

$=\frac{z+4i}{\alpha\beta+8i}$

$=\frac{-2+2i+4i}{-2-2i+8i}$

4 b/ $= \frac{-2+2i+4i}{-2-2i+8i}$

$= \frac{-2+6i}{-2-6i}$

∴ the simplest form of $\frac{\alpha+\beta+4i}{\alpha\beta+8i}$ ~~is~~ is

$\frac{-2+6i}{-2-6i}$

c/ Given

- Required to find $z^3=1$ all complex roots

Data Given

$z^3=1$

soln

$z^3=1$

from main formula

$z^3=1$

z	1	2	3	4	5	6	7	8	9	10
z^3	1	8	27	64	125	216	343	512	729	1000

∴ The complex root of of $z^3=1$ are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, . . .

Extract 14.2: A sample of incorrect responses to question 4 of paper 2

Extract 14.2 shows that in part (a), the candidate failed to verify that the given equation is a circle when solving the equation $\left|\frac{z-2}{z+3i}\right|=4$ as $\frac{z-2}{z+3i}=4$. In part (b), the candidate solved the equation $z^2+4z+8=0$ instead of finding the values of $\alpha+\beta$ and $\alpha\beta$ without solving it. In part (c), he/she prepared the table of results of the equation $z^3=1$ instead of applying De-Moivre's theorem to find the required roots, as a result he/she performed poorly in this question.

2.2.5 Question 5: Trigonometry

The question consisted of five parts: (a), (b), (c), (d), and (e). Part (a) required the candidates to factorize the expression $\cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta$. Part (b) required the candidate to show that, $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$ and then find $\tan y$ if $\tan(2x+y) = 2$ and $\tan 2x = \dfrac{3}{2}$. Part (c) required the candidates to find the exact values of $\cos(x+y)$ and $\cot(x-y)$ given that $\sin x = \dfrac{3}{5}$ and $\cos y = \dfrac{24}{25}$, where x is an obtuse angle and y as acute angle. Part (d) required the candidates to express $3\sin x - 2\cos x$ in the form of $R\sin(x-\beta)$. Part (e) required the candidates to use the results obtained in Part (d) to solve the equation $3\sin x - 2\cos x = 1$.

The analysis of the data shows that 11,381 (82.7%) candidates responded to this question, while 2,374 (17.3%) candidates did not answer it. Further analysis shows that 13.2 per cent of the candidates scored from 0 to 6.5 marks, 32.9 per cent scored from 7 to 11.5 marks, and 53.9 per cent scored from 12 to 20 marks. Generally, the performance in this question was good, as summarized in Figure 16.

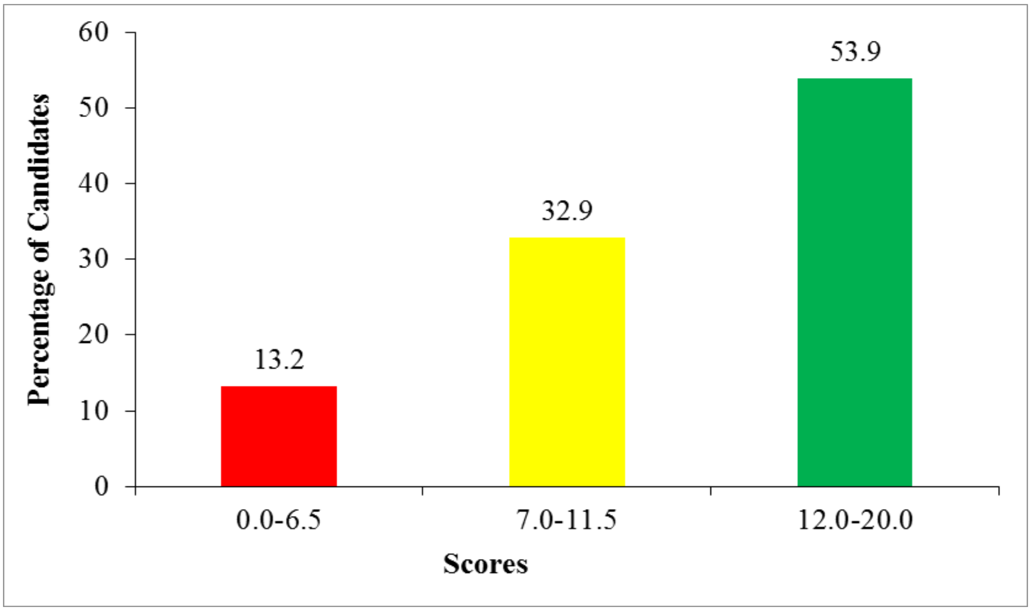


Figure 16: *Candidates' Performance in Question 5 of Paper 2*

The candidates who attempted this question well demonstrated necessary knowledge and skills on the topic of trigonometry. In part (a), they used the concept of factor formulae to factorize the expression $\cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta = (\cos 7\theta + \cos\theta) - (\cos 5\theta + \cos 3\theta)$ as $\left[2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)\right] - \left[2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)\right]$ and finally simplify it to $-4\cos 4\theta \sin 2\theta \sin\theta$. In part (b), they were able to use the concept of compound angle formulae by expressing $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$ to show that $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$. Then, they used this result to find the value of $\tan y$ if $\tan(2x+y) = 2$ and $\tan 2x = \frac{3}{2}$. That is, they formulated the equation $\tan y = \tan((2x+y) - 2x)$ to compare it with the derived identity. Using this equation, they obtained the correct answer, $\tan y = \frac{1}{8}$.

In part (c), the candidates were able to compute the values of $\cos(x+y)$ and $\cot(x-y)$ using the appropriate concepts. From $\sin x = \frac{3}{5}$ and $\cos y = \frac{24}{25}$, taking into consideration that x is an obtuse angle and y is an acute angle, they calculated the values of $\cos x = -\frac{4}{5}$, $\sin y = \frac{7}{25}$, $\tan x = -\frac{3}{4}$ and $\tan y = \frac{7}{24}$. Using these values, they managed to obtain the correct values of $\cos(x+y) = -\frac{117}{125}$ and $\cot(x-y) = -\frac{3}{4}$. In part (d), the candidates were able to express $3\sin x - 2\cos x$ in the form of $R\sin(x-\beta)$. They expanded $R\sin(x-\beta)$ to $R\sin x\cos\beta - R\sin\beta\cos x$. On equating this expansion to $3\sin x - 2\cos x$, they obtained $R = \sqrt{13}$, $\beta = 33.69^\circ$ and finally they obtained $3\sin x - 2\cos x = \sqrt{13}\sin(x - 33.69^\circ)$ as it was required. In part (e), they used the result obtained in part (d) to solve the equation $3\sin x - 2\cos x = 1$. They wrote $\sqrt{13}\sin(x - 33.69^\circ) = 1$ and solved this equation using the general solution of the sine function to obtain $x = 180^\circ n + 16.10^\circ(-1)^n + 33.69^\circ$, where $n \in \mathrm{Z}$. Extract

15.1 shows a sample of the correct responses from one of the candidates who had sufficient knowledge and skills on this topic.

5a.	$\cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta$ —— given.
	$= \cos\theta + \cos 7\theta - \cos 3\theta - \cos 5\theta$
	$= \cos\theta + \cos 7\theta - (\cos 5\theta + \cos 3\theta)$
	$= \left(2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)\right) - \left(2\cos\frac{5\theta+3\theta}{2}\cos\frac{2\theta}{2}\right)$
	$= 2\cos 4\theta\cos 3\theta - 2\cos 4\theta\cos\theta$
	$= 2\cos 4\theta(\cos 3\theta - \cos\theta)$.
	$= 2\cos 4\theta\left(-2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)\right)$.
	$= 2\cos 4\theta(-2\sin 2\theta\sin\theta)$
	$= -4\cos 4\theta\sin 2\theta\sin\theta$
	$\therefore \cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta = -4\cos 4\theta\sin 2\theta\sin\theta$
5b.	to show, $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$
	consider, LHS.
	$\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$. — — — (i)
	$\sin(A-B) = \sin A\cos B - \cos A\sin B$.
	$\cos(A-B) = \cos A\cos B + \sin A\sin B$.

5b

$$\tan(A-B) = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B + \sin A\sin B}$$

divide by

$\cos A\cos B$ to All terms

$$\tan(A-B) = \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{1 + \frac{\sin A\sin B}{\cos A\cos B}}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Hence shown!!

$\tan(2x+y) = 2$, $\tan 2x = 3/2$.

$\tan y = ?$

from; $$\tan(2x+y) = \frac{\tan 2x + \tan y}{1 - \tan(2x)\tan y)}$$

$$2 = \frac{\tan 2x + \tan y}{1 - \tan(2x)\tan y}.$$

but;

$\tan 2x = 3/2$.

$$2 = \frac{3/2 + \tan y.}{1 - 3/2\tan y.}$$

$$2 - 3\tan y = 3/2 + \tan y.$$

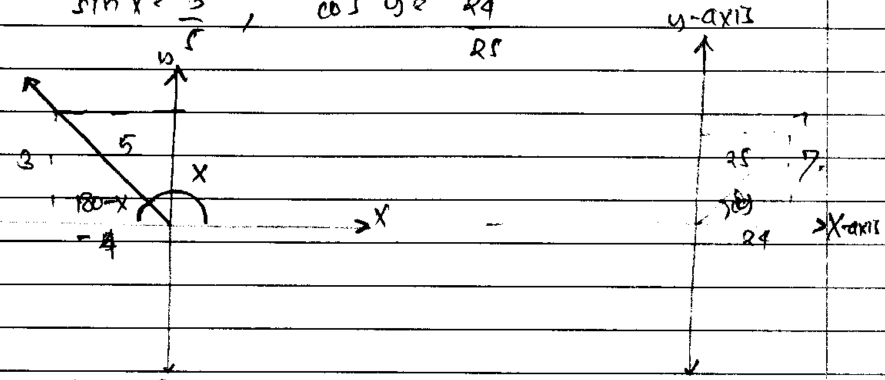
$$2 - 3/2 = 4\tan y.$$

5b. $4\tan y = \frac{1}{2}$

$\tan y = \frac{1}{8}$.

$\therefore \tan y = \frac{1}{8}$

5c. $\sin x = \frac{3}{5}$, $\cos y = \frac{24}{25}$



$\sin x = 3/5$ $\qquad$ $\cos y = \frac{24}{25}$

$\cos x = \frac{-4}{5}$ $\qquad$ $\sin y = \frac{7}{25}$

Required;

$\cos(x+y) = \cos x \cos y - \sin x \sin y.$

$$= \left(\frac{-4}{5}\right)\left(\frac{24}{25}\right) - \left(\frac{3}{5}\right)\left(\frac{7}{25}\right).$$

$\cos(x+y) = -0.936$

$= \frac{-117}{125}$

$\therefore \cos(x+y) = -0.936$

5c. $\cot(x-y) = \dfrac{1}{\tan(x-y)}$

$$= \frac{1}{\dfrac{\tan x - \tan y}{1 + \tan x \tan y}}$$

$$\cot(x-y) = \frac{1 + \tan x \tan y}{\tan x - \tan y}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-3}{4}$$

$$\tan y = \frac{\sin y}{\cos y} = \frac{7}{24}.$$

$$\cot(x-y) = \frac{1 + (7/24)(-3/24)}{-3/4 - 7/24}.$$

$$= \frac{0.78125}{-25/24}.$$

$$= \frac{25}{32} \times \frac{-24}{25}$$

$$= \frac{-24}{32}$$

$$= \frac{-3}{4}$$

$$\therefore \cot(x-y) = -\frac{3}{4}.$$

5d.	$3\sin X - 2\cos X$
	$R\sin(X-\beta) = R\sin X\cos\beta - R\cos X\sin\beta.$
	$R\cos\beta = 3$
	$R\sin\beta = +2.$
	$\tan\beta = \frac{+2}{3}.$
	$\beta = \tan^{-1}(2/3).$
	$R^2 = 3^2 + 2^2$
	$R = \sqrt{9+4}$
	$R = \sqrt{13}$
	$3\sin X - 2\cos X = \sqrt{13}\sin(X - \tan^{-1}(2/3)).$
	$= \sqrt{13}\sin(X - 33.69°).$
5e.	$3\sin X - 2\cos X = 1.$
	but $3\sin X - 2\cos X = \sqrt{13}\sin(X - 33.69°).$
	$\sqrt{13}\sin(X - 33.69°) = 1.$
	$\sin(X - 33.69°) = \frac{1}{\sqrt{13}}$
	from, General solution of sine
	$\theta = \pi n + (-1)^n \alpha$
	$\alpha = \sin^{-1}(1/\sqrt{13})$

5 e	$x - 33.69^\circ = \pi n + (-1)^n \sin^{-1}\left(\frac{1}{\sqrt{13}}\right)$.
	$= \pi n + (-1)^n (16.1^\circ)$.
	$x = 180^\circ n + (-1)^n (16.1^\circ) + 33.69^\circ$.
	$\underline{x = 180^\circ n + (-1)^n 16.1^\circ + 33.69^\circ}$

Extract 15.1: A sample of correct responses to question 5 of paper 2

In Extract 15.1, the candidate attempted each part according to the requirements of the question. In part (a), the candidate applied the factor formulae correctly to factorize the given expression, while in part (b), he/she applied the concept of the compound angle formula to prove the given identity and solved for $\tan y$. In part (c), the candidate applied the appropriate concept to find the values of $\cos(x+y)$ and $\cot(x-y)$. In parts (d) and (e), the candidate applied the concept of compound angle formulae to the expression $R\sin(x-\beta)$ and use it to solve the equation $3\sin x - 2\cos x = 1$ correctly.

Although many candidates had good performance in this question, there were 1.6 per cent of candidates who scored 0 marks. These candidates lacked some skills and adequate knowledge on the topic of trigonometry. In part (a), the candidates failed to use the correct concept of factor formulae to factorize the given expression. For example, they wrote $\cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta = 2\cos\left(\frac{\theta - 3\theta}{2}\right)\cos\left(\frac{\theta - 3\theta}{2}\right) - 2\cos\left(\frac{5\theta + 7\theta}{2}\right)\cos\left(\frac{5\theta - 7\theta}{2}\right)$ then simplified to get $2\cos\theta\cos\theta - 2\cos 6\theta\cos\theta$. Others factorized the given expression by applying the concept of compound angle formulae, such as $\cos\theta - \cos(\theta + 2\theta) - \cos(2\theta + 3\theta) + \cos(3\theta + 4\theta) =$
$\cos\theta - (\cos\theta\cos 2\theta - \sin\theta\sin 2\theta) - (\cos 2\theta\cos 3\theta - \sin 2\theta\sin 3\theta) +$
$\cos 3\theta\cos 4\theta - \sin 3\theta\sin 4\theta$, but they failed to finish. In part (b), the candidates failed to express $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$, which they could use to prove the identity $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$; instead, they expanded $\tan(A-B)$

directly into $\frac{\tan A-\tan B}{1+\tan A\tan B}$. They also failed to use the identity $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$ to find the value of $\tan y$. Instead, they used incorrect identity and wrong manipulation; For example, they wrote $\tan(2x+y)=\frac{\tan 2x+\tan y}{1+\tan 2x\tan y}=\frac{\frac{\tan 2x}{\tan 2x\tan y}+\frac{\tan y}{\tan 2x\tan y}}{1+\tan 2x\tan y}=\frac{1}{\tan y}+\frac{1}{\tan 2x}$ and finally obtained the wrong result, $\tan y=\frac{\tan 2x}{\tan 2x-1}$.

In part (c), the candidates failed to use the concepts of obtuse and acute angles and compound angle formulae to find the exact values of $\cos(x+y)$ and $\cot(x-y)$. They wrote $\cos(x+y)=\cos x\cos y+\sin x\sin y=\frac{24}{25}\cos x+\frac{3}{5}\sin y$. Others found the value of $\cos(x+y)$ as $\cos(x+y)=\cos\left(\frac{4}{5}+\frac{24}{5}\right)=0.9952274$ and $\cot(x-y)=\frac{1}{\tan(x-y)}=\frac{1}{\tan\left(\frac{4}{5}-\frac{-23.47338999}{24}\right)}=33.14611513$.

Likewise, a few candidates approached the question by evaluating the angles x and y; that is, $y=\cos^{-1}\left(\frac{24}{25}\right)=16.26^\circ$ and $x=\sin^{-1}\left(\frac{3}{5}\right)=36.87^\circ$. Then, they used these values to find the values of $\cos(x+y)$ and $\cot(x-y)$ for which they got the wrong values of 0.6 and 2.659, respectively. In part (d), the candidates failed to obtain the correct expansion of $R\sin(x-\beta)$; they wrote $R\sin(x-\beta)=R\sin x\cos\beta-\sin\beta\cos x$ and on comparing, they got the wrong results, $\sin x=3$ and $\cos\beta=-2$. In part (e), the candidates did not use the result obtained in part (d) to solve the equation $3\sin x-2\cos x=1$. Some used Pythagoras identities to transform the equation $3\sin x-2\cos x=1$ into $3\sin x-2\sqrt{1-\sin^2 x}=1$, to obtain $13\sin^2 x-6\sin x-3=0$. They solved this equation to get $x=49.5^\circ$ and $x=-17.46^\circ$ while, others used t-formulae to transform the equation into $3\left(\frac{2t}{1+t^2}\right)-\left(\frac{1-t^2}{1+t^2}\right)=1$ then simplified it to get $t^2+6t-3=0$. Thereafter, they solved the equation to obtain $t=0.46$ or

$t = -6.46 \Rightarrow \tan\frac{x}{2} = 0.46$ or $\tan\frac{x}{2} = -6.46 \Rightarrow x = 49.4^{\circ}$ and -162.4°. Extract 15.2 shows a sample of parts of incorrect responses from one of the candidates.

5.a)

$$\text{Cos}5\theta + \text{Cos}7\theta = \text{Cos}(3\theta+2\theta) + \text{Cos}(5\theta+2\theta)$$

$$\text{Cos}3\theta\text{Cos}2\theta - \text{Sin}3\theta\text{Sin}2\theta + \text{Cos}5\theta\text{Cos}2\theta - \text{Sin}5\theta\text{Sin}2\theta$$

$$\text{Cos}3\theta(\text{Cos}^2\theta - \text{Sin}^2\theta) - \text{Sin}3\theta(2\text{Sin}\theta\text{Cos}\theta) + \text{Cos}5\theta(\text{Cos}^2\theta - \text{Sin}^2\theta) - \text{Sin}5\theta(2\text{Sin}\theta\text{Cos}\theta)$$

$$\text{Cos}3\theta = \text{Cos}^3\theta - 3\text{Sin}^2\theta\text{Cos}\theta$$

$$\text{Cos}^3\theta - 3\text{Sin}^2\theta\text{Cos}\theta(\text{Cos}^2\theta - \text{Sin}^2\theta) - 3\text{Cos}^2\theta\text{Sin}\theta - \text{Sin}^3\theta(2\text{Sin}\theta\text{Cos}\theta) + \text{Cos}5\theta(\text{Cos}^2\theta - \text{Sin}^2\theta) - \text{Sin}5\theta(2\text{Sin}\theta\text{Cos}\theta)$$

$$\therefore \text{Cos}\theta - \text{Cos}3\theta - \text{Cos}5\theta + \text{Cos}7\theta$$
$$= \text{Cos}(2\theta + \theta + 3\theta + 4\theta)$$

$$\text{Cos}3\theta = \text{Cos}(\theta+2\theta),\ \text{Cos}5\theta = \text{Cos}(3\theta+2\theta),$$
$$\text{Cos}7\theta = \text{Cos}(5\theta+2\theta)$$

$$\therefore \text{Cos}\theta - \text{Cos}\theta - \text{Cos}5\theta + \text{Cos}7\theta = \text{Cos}(\theta+2\theta+3\theta+4\theta)$$

05 b. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

consider L.H.S.

$\tan(A-B)$

$= \cancel{\tan A + \tan B}$

$\tan(A-B) = \dfrac{\sin(A-B)}{\cos(\cancel{A}-B)}$.

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 - \tan A \tan B}$.

Divide by $\tan A$ both side.

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 - \tan A \tan B}$.

$\therefore$ Hence shown.

$\tan(2x+y) = 2$

$\tan 2x = 3/2$.

$\tan 2x + \tan y = 2$.

but $\tan 2x = 3/2$.

$3/2 + \tan y = 2$.

$\tan y = \dfrac{2}{1} - \dfrac{3}{2}$

$\tan y = \dfrac{4-3}{2}$

$\tan y = 1/2$.

$\therefore \tan y = 1/2$.

5c. $\cos y = \frac{24}{25}$

$y = \cos^{-1}\left(\frac{24}{25}\right)$

$y = 16.26$

$\cos(x+y) = \cos A\cos B - \sin A\sin B$

$= \cos(36.9)\cos(16.26) - \sin(36.9)\sin(16.26)$

$\cos(x+y) = 0.7996846 58 \times 0.960001 - (0.60042\ 0.225$

$0.27999657.$

$\cos(x+y) = 0.76798071 \ast 0.168115603$

$\cos(x+y) = 0.599582468$

$\therefore \cos(x+y) = 0.599582468$

$\cot(x-y) = \frac{1}{\tan(x-y)}$

$\cot(x-y) = \frac{1}{\frac{\tan A - \tan B}{1+\tan A\tan B}}$

$\cot(x-y) = \frac{1+\tan A\tan B}{\tan A - \tan B}$

$\cot(x-y) = \frac{1-\tan(36.9)\tan(16.26)}{\tan(36.9) - \tan(16.26)}$

$\cot(x-y) = \frac{0.781013383}{0.459158448}$

$\cot(x-y) = 1.700967033$

Extract 15.2: A sample of incorrect responses to question 5 of paper 2

In Extract 15.2 part (a), the candidate failed to use the correct concept of factor formulae to factorize the given expression. Likewise in part (b), the candidate failed to prove the identity $\tan(A-B)$, thus obtained wrong value of $\tan y$. In part (c), he/she recalled correctly the expansion of $\cos(x-y)$ and $\cot(x-y)$ but failed to manipulate, which resulted to the wrong answer.

2.2.6 Question 6: Algebra

This question comprised five parts: (a), (b), (c), (d), and (e). In part (a), the candidates were required to use the Binomial theorem to expand $\frac{1}{(4-x)^2}$ in ascending powers of x up to the term containing x^3. In part (b), the candidates were required to find the partial fractions of the expression $\frac{x^2-2x+1}{(x+1)^2}$. In part (c), the candidates were required to find the equation whose roots are $\alpha+1$, $\beta+1$ and $\mu+1$ if α, β and μ are the roots of the equation $2x^3-x^2+1=0$. In part (d), the candidates were required to find the inverse of $A=\begin{pmatrix}1 & 1 & 1\\ 2 & -3 & 4\\ 3 & -2 & -2\end{pmatrix}$ without using a scientific calculator. In part (e), the candidates were required to use the inverse obtained in part (d) to solve the following system of simultaneous equation: $\begin{cases} x+y+z=2\\ 2x-3y+4z=-4\\ 3x-2y+4z=-9\end{cases}$.

The question was attempted by 10,288 (74.8%) candidates, whereby 23.1 per cent of the candidates scored from 0 to 6.5 marks, 33.8 per cent scored from 7 to 11.5 marks, and 43.1 per cent scored from 12 to 20 marks. The candidates' performance in this question was good, as 76.9 per cent of the candidates scored from 7 to 20 marks. Figure 17 provides a summary of the candidates' performance in this question.

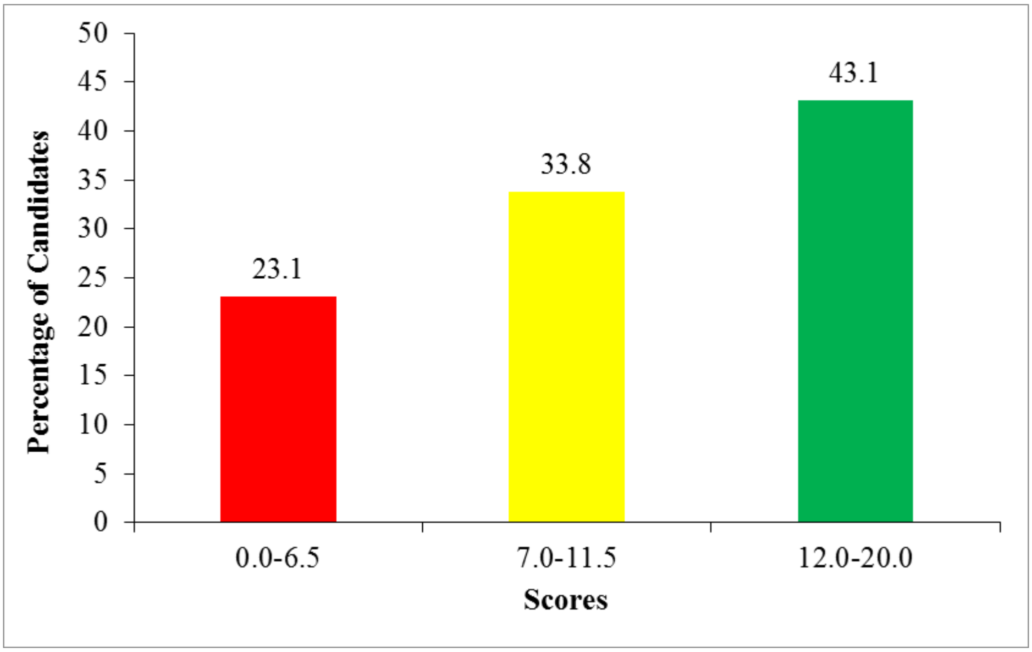


Figure 17: *Candidates' Performance in Question 6 of Paper 2*

The candidates who performed well in this question had appropriate knowledge on the topic of algebra. In part (a), the candidates transformed the expression $\frac{1}{(4-x)^2}$ into $\frac{1}{16}\left(1-\frac{x}{4}\right)^{-2}$. Thereafter, they expanded $\frac{1}{16}\left(1-\frac{x}{4}\right)^{-2}$ as far as the term in x^3 by applying Binomial theorem $(1+x)^n = 1+nx+\frac{n(n-1)x^2}{2!}+\frac{n(n-2)x^3}{3!}+\ldots$ and obtained

$\frac{1}{(4-x)^2}=\frac{1}{16}+\frac{x}{32}+\frac{3x^2}{256}+\frac{x^3}{256}+\ldots$ .Likewise, others expanded the expression $(4-x)^{-2}$ directly by applying Binomial theorem of the form $(a+x)^n = a^n + na^{n-1}x+\frac{n(n-1)a^{n-2}x^2}{2!}+\frac{n(n-1)(n-2)a^{n-3}x^3}{3!}+\ldots$, where they replaced a by 4 and obtained the same answer.

In part (b), the candidates transformed $\frac{x^2-2x+1}{(x+1)^2}$ into $1-\frac{4x}{(x+1)^2}$. Then, they applied the appropriate concept of partial fractions to partialize $\frac{4x}{(x+1)^2}$ by writing $\frac{4x}{(x+1)^2} = 1 - \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$. Finally, they obtained the required partial fractions as $\frac{4x}{(x+1)^2} = 1 - \frac{4}{(x+1)} + \frac{4}{(x+1)^2}$. Others used an alternative method by letting $y = x+1$ and substituting this into the equation $\frac{x^2 - 2x + 1}{(x+1)^2}$ to obtain $\frac{(y-1)^2 - 2(y-1) + 1}{y^2}$. Then, they simplified this to $1 - \frac{4}{y} + \frac{4}{y^2}$ and replaced y by $x+1$ to obtain the required partial fractions $1 - \frac{4}{x+1} + \frac{4}{(x+1)^2}$.

In part (c), the candidates managed to apply the correct steps in formulating the equation whose roots are $\alpha + 1$, $\beta + 1$ and $\mu + 1$ where α, β and μ are the roots of the equation $2x^3 - x^2 + 1 = 0$. They first determined the sum of roots, $\alpha + \beta + \mu = \frac{1}{2}$, the sum of product of roots, $\alpha\beta + \beta\mu + \alpha\mu = 0$ and the product of roots, $\alpha\beta\mu = -\frac{1}{2}$ of the equation $2x^3 - x^2 + 1 = 0$ and then used these to find the sum, the sum of product, and the product of roots of the required equation as $\frac{7}{2}$, 4, and 1 respectively. Using these values, they obtained the equation $2x^3 - 7x^2 + 8x - 2 = 0$. In part (d), the candidates applied the proper procedures for finding the inverse of the given matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & -2 \end{pmatrix}$. They calculated the determinant, $|A| = 35$, matrix of cofactors, $Cof(A) = \begin{pmatrix} 14 & 16 & 5 \\ 0 & -5 & 5 \\ 7 & -2 & -5 \end{pmatrix}$ and adjoint matrix, $Adj(A) = \begin{pmatrix} 14 & 0 & 7 \\ 16 & -5 & -2 \\ 5 & 5 & -5 \end{pmatrix}$. From these values, they obtained the required inverse matrix, $A^{-1} = \begin{pmatrix} \frac{2}{5} & 0 & \frac{1}{5} \\ \frac{16}{35} & -\frac{1}{7} & -\frac{2}{35} \\ \frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \end{pmatrix}$. In part (e), in solving the system of equations using the inverse matrix obtained in part (d), the candidates arranged the equations in matrix form as follows: $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -9 \end{pmatrix}$. The candidates found that the coefficient matrix from these equations is not the same as the matrix A given in part (d). Therefore, they commented that the inverse matrix A^{-1} could not solve the system of simultaneous equations provided. Extract 16.1 shows a sample of responses from one of the candidates who performed well in question 6.

6a. $\dfrac{1}{(4-x)^2}$

$\dfrac{1}{(4-x)^2} = (4-x)^{-2}$.

6a. $(4-x)^{-2}$.

$[4(1-x/4)]^{-2}$

$4^{-2}(1-x/4)^{-2}$.

from.

$(1+x)^{-n} = 1 + nx + \dfrac{n(n-1)x^2}{2!} + \dfrac{n(n-2)(n-1)x^3}{3!} \ldots$

$(1-x/4)^{-2} = 1 + (-2)(-x/4) + \dfrac{-2x-3x(-x/4)^2}{2!} + \dfrac{-2x-4x-3x(-x/4)^3}{3!}$

$(1-x/4)^{-2} = 1 + x/2 + \dfrac{6x^2}{2x16} + \dfrac{24x^3}{64x6}$

$(1-x/4)^{-2} = 1 + \dfrac{x}{2} + \dfrac{3x^2}{16} + \dfrac{x^3}{16}$.

$\dfrac{1}{(4-x)^2} = \dfrac{1}{4^2}\left(1 + \dfrac{x}{2} + \dfrac{3x^2}{16} + \dfrac{x^3}{16}\right)$.

$\dfrac{1}{(4-x)^2} = \dfrac{1}{16} + \dfrac{x}{32} + \dfrac{3x^2}{256} + \dfrac{x^3}{256}$.

$\therefore \dfrac{1}{(4-x)^2} = \dfrac{1}{16} + \dfrac{x}{32} + \dfrac{3x^2}{256} + \dfrac{x^3}{256}$.

6b. $x^2 - 2x + 1$

$(x+1)^2$

$x^2 - 2x + 1$

$x^2 + 2x + 1$

$$\begin{array}{r|l} & 1 \\ \hline x^2+2x+1 & x^2 - 2x + 1 \\ & -\underline{x^2 + 2x + 1} \\ & -4x \end{array}$$

$$\frac{x^2 - 2x + 1}{(x+1)^2} = 1 + \frac{-4x}{(x+1)^2}$$

by partial fraction.

$$\frac{-4x}{(x+1)^2} = \frac{A}{(x+1)^1} + \frac{B}{(x+1)^2}$$

$$\frac{-4x}{(x+1)^2} = \frac{A(x+1) + B}{(x+1)^2}$$

$A(x+1) + B = -4x$.

let $x = -1$.

$A(-1+1) + B = -4(-1)$

$B = 4$.

let $x = 0$.

$A(0+1) + B = -4(0)$.

$A + B = 0$.

$A = -B$.

$A = -4$.

6b $\frac{-4x}{(x+1)^2} = \frac{-4}{(x+1)} + \frac{4}{(x+1)^2}$.

$\therefore \frac{x^2-2x+1}{(x+1)^2} = 1 - \frac{4}{x+1} + \frac{4}{(x+1)^2}$

c. $2x^3 - x^2 + 1 = 0$.

$ax^3 + bx^2 + cx + d = 0$.

$\alpha + \beta + \mu = -b/a$.

$\alpha\beta + \beta\gamma + \alpha\gamma = c/a$.

$\alpha\beta\mu = -d/a$.

$\alpha + \beta + \gamma = -b/a$.

$\alpha + \beta + \mu = -(-1/2)$.

$\alpha + \beta + \mu = 1/2$.

$\alpha\beta + \beta\mu + \mu\alpha = 0$.

$\alpha\beta\mu = -1/2$.

Sum of the roots.

$\alpha+1 + \beta+1 + \mu+1 = \alpha + \beta + \mu + 3$

$= 1/2 + 3$.

$~~= 3/2~~$

$= 3\frac{1}{2}$

$= 7/2$.

$\alpha+1 + \beta+1 + \mu+1 = 7/2$.

6c. $(\alpha+1)(\beta+1)+(\alpha+1)(\mu+1)+(\beta+1)(\mu+1)=$

$\alpha\beta+\alpha+\beta+1+\alpha\mu+\alpha+\mu+1+\beta\mu+\beta+\mu+1.$

$=(\alpha\beta+\alpha\mu+\beta\mu)+2\alpha+2\beta+2\mu+3.$

$=(\alpha\beta+\alpha\mu+\beta\mu)+2(\alpha+\beta+\mu)+3.$

$=0+2(1/2)+3.$

$=1+3$

$=4.$

product of the roots.

$\alpha\beta\mu$

$(\alpha+1)(\beta+1)(\mu+1)=(\alpha\beta+\alpha+\beta+1)(\mu+1)$

$=\alpha\beta\mu+\alpha\beta+\alpha\mu+\alpha+\beta\mu+\beta+\mu+1$

$=\alpha\beta\mu+(\alpha\beta+\alpha\mu+\beta\mu)+(\alpha+\beta+\mu)+1$

but $\alpha\beta\mu=-1/2.$

$\alpha+\beta+\mu=1/2.$

$\alpha\beta+\alpha\mu+\beta\mu=0.$

$(\alpha+1)(\beta+1)(\mu+1)=-1/2+1/2+1+0.$

$=1.$

equation.

$x^3-x^2(\cancel{\mu}+\beta+\alpha)+x(\mu\beta+\mu\alpha+\alpha\beta)-\alpha\beta\mu=0.$

$x^3-x^2(7/2)+x(4)-1=0$

$x^3-7/2x^2+4x-1=0.$

$2x^3-7x^2+8x-2=0.$

$\therefore$ equation is

$2x^3-7x^2+8x-2=0.$

6d. $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & -2 \end{bmatrix}$

$A_{minor} = \begin{bmatrix} (6+8) & (-4-12) & (-4+9) \\ (-2+2) & (-2-3) & (-2-3) \\ (4+3) & (4-2) & (-3-2) \end{bmatrix}$

$A_{minor} = \begin{bmatrix} 14 & -16 & 5 \\ 0 & -5 & -5 \\ 7 & 2 & -5 \end{bmatrix}$

$A^{cofactor} = \begin{bmatrix} 14 & 16 & 5 \\ 0 & -5 & 5 \\ 7 & -2 & -5 \end{bmatrix}$

$A_{adjoint} = \begin{bmatrix} 14 & 0 & 7 \\ 16 & -5 & -2 \\ 5 & 5 & -5 \end{bmatrix}$

$|A| = 1(6+8) - 1(-4-12) + 1(-4+9)$

$|A| = 14 + 16 + 5$

$|A| = 35$

$A^{-1} = \dfrac{A\ adjoint}{|A|}$

$A^{-1} = \frac{1}{35}\begin{bmatrix} 14 & 0 & 7 \\ 16 & -5 & -2 \\ 5 & 5 & -5 \end{bmatrix}$

6d $A^{-1} = \begin{bmatrix} 2/5 & 0 & 1/5 \\ 16/35 & -1/7 & -2/35 \\ 1/7 & 1/7 & -1/7 \end{bmatrix}$

e. $x + y + z = 2$.
$2x - 3y + 4z = -4$
$3x - 2y + 4z = -9$.

$\begin{cases} x + y + z = 2 & \text{--- (i)} \\ 2x - 3y + 4z = -4 & \text{--- (ii)} \\ 3x - 2y + 4z = -9 & \text{--- (iii)} \end{cases}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -9 \end{bmatrix}.$$

The matrix obtained from eqn is not the same to that of part (d) above

As the instruction of the question need to use the inverse of part (d) hence the values of x, y and z will not satisfy the equation(iii) given.

Extract 16.1: A sample of correct responses to question 6 of paper 2

Extract 16.1 shows that the candidate had enough knowledge and skills in algebra. He/she attempted each part of the question using the appropriate concepts and steps.

On the other hand, there are some candidates who performed poorly in this question. This was contributed by the following factors: In part (a), some candidates applied the wrong binomial theorem in expanding $\frac{1}{(4-x)^2}$. They stated the binomial theorem as:

$$(x+y)^n = x^n + \frac{(n-1)x^{n-1}y^0}{1!} + \frac{(n-1)(n-2)x^{n-2}y^1}{2!} + \frac{(n-1)(n-2)(n-3)x^{n-3}y^3}{3!}.$$

They used this statement and ended up with the wrong expansion $\frac{1}{(4-x)^2} = \frac{1}{16} + \frac{x}{192} + \frac{x^2}{1536} - \frac{x^3}{10240}$. Likewise, other candidates failed to obtain a correct transformation of $\frac{1}{(4-x)^2}$ into $\frac{1}{16}\left(1-\frac{x}{4}\right)^{-2}$. They wrote $\frac{1}{(4-x)^2} = (4-x)^2$ and then applied the binomial theorem to expand $(4-x)^2$, from which they obtained the wrong answer $16+8x+x^2+0.0833x^3$. In part (b), the candidates did not use the correct concepts of partialization. They equated the $\frac{x^2-2x+1}{(x+1)^2}$ directly to $\frac{A}{x+1}+\frac{B}{(x+1)^2}$ instead of first transforming $\frac{x^2-2x+1}{(x+1)^2}$ into $1-\frac{4x}{(x+1)^2}$. This led them to wrong results, $\frac{4}{(x+1)^2}-\frac{3}{x+1}$. Others did not have enough knowledge of partialization; they wrote $\frac{x^2-2x+1}{(x+1)^2} = \frac{A^2}{(x+1)} + \frac{Bx+C}{(x+1)}$, for which they ended up getting the wrong expression $\frac{1}{(x+1)} + \frac{x+2}{(x+1)}$.

In part (c), some candidates equated the expression $2x^3 - x^2 + 1$ to ax^3+bx^2+cx+d; this couldn't help in getting the required equation with roots $\alpha+1$, $\beta+1$ and $\mu+1$. They ended up writing the values of $a=2$, $b=-1$, $c=0$ and $d=1$. Others approached this problem by solving the equation $2x^3-x^2+1=0$ and got the value of $x=0.66$ or $x=0.58\pm0.65i$. In part (d), some candidates failed to abide to the instructions of the question. They did not go through the correct steps for determining the inverse of the given matrix A instead, they used a calculator to obtain it. Other candidates tried to find the inverse through the wrong concepts and steps; they transposed the given matrix A to obtain $Transpose = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & 5 \\ 1 & -2 & -5 \end{pmatrix}$. Then, they wrote

$$Adj(A) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & 5 \\ 1 & -2 & -5 \end{pmatrix}$$ and finally obtained the wrong answer,

$$A^{-1} = \begin{pmatrix} \frac{1}{35} & -\frac{2}{35} & \frac{3}{35} \\ -\frac{1}{35} & -\frac{3}{35} & \frac{2}{35} \\ \frac{1}{35} & -\frac{4}{35} & -\frac{2}{35} \end{pmatrix}$$. In part (e), the candidates failed to recognise that the inverse of the matrix A^{-1} could not solve the given system of equations; hence, they solved it and got the wrong answer. Extract 16.2 illustrates incorrect responses from one of the candidates who attempted this question.

6 a) soln

$\frac{1}{(4-x)^2} \Rightarrow (4-x)^{-2}$

let $(4-x)^{-2}$ in form of $(1+x)^n$

$4\left(1-\frac{x}{4}\right)^{-2}$

Recall the formula

$1 + nx + \frac{n(n+1)x^2}{2} + \frac{n(n+1)(n+2)x^3}{3}$

But $n = -2$ and $x = -1/4$

$\Rightarrow 1 + (-2)(-1/4) + \frac{-2}{2}(-2+1)(1/4)^2 + \frac{-2}{3}(-2+1)(-2+2)(1/4)^3$

$\Rightarrow 1 + 1/2 + 1/16 + 0$

$\frac{16+8+1}{16} = 25/16$

6 b) soln

$\frac{x^2-2x+1}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$

$\frac{x^2-2x+1}{\cancel{(x+1)^2}} = \frac{A(x+1)+B}{\cancel{(x+1)^2}}$

$x^2-2x+1 = A(x+1)+B$

$x^2-2x+1 = Ax + A + B$

$\therefore A = -2$ ---- (i)

$A + B = 1$ ---- (ii)

But $A = -2$

$-2 + B = 1$

$B = 2+1$

$\therefore B = 3$

$$\frac{x^2-2x+1}{(x+1)^2} = \frac{-2}{(x+1)} + \frac{3}{(x+1)^2}$$

6 c) Soln

from

$2x^3 - x^2 + 1 = 0$

$\frac{\cancel{2}x^3}{\cancel{2}} - \frac{x^2}{2} + \frac{0x}{2} + \frac{1}{2} = \frac{0}{2}$

$x^3 - \frac{1}{2}x^2 + 0x + \frac{1}{2} = 0$

But

$\mu + \alpha + \beta = -b/a = +1/2$ --- (i)

$\alpha\beta + \mu\beta + \alpha\mu = c/a = 0/2$ --- (ii)

$\alpha\beta\mu = -d/a$ --- (iii)

$\cancel{x^3 + (\alpha+\beta+\mu)x^2 + 0 + \mu\beta\alpha = 0}$

$\cancel{x^3 + (\alpha+1+\beta+1+\mu+1)x^2 + (\alpha+1)(\beta+1)(\mu+1) = 0}$

$\cancel{x^3 + (3 + \alpha+\beta+\mu)x^2 +}$

$x^3 + (\alpha+\beta+\mu)x^2 + \mu\beta\alpha = 0$

$x^3 + (\alpha+1+\beta+1+\mu+1)x^2 + (\alpha+1)(\beta+1)(\mu+1) = 0$

$x^3 + (3+\alpha+\beta+\mu)x^2 + (\alpha\beta\mu + \alpha\beta + \beta\mu + \alpha\mu + \alpha+\beta+\mu+1)$

But $\alpha\beta\mu = -1/2$

$\alpha\beta + \alpha\mu + \beta\mu = 0$

$\alpha+\beta+\mu = 1/2$

$x^3 + (3 + 1/2)x^2 + (-1/2 + 0 + 1/2 + 1) = 0$

$x^3 + \left(\frac{6+1}{2}\right)x^2 + \left(\frac{-1+1+2}{2}\right) = 0$

$x^3 + \frac{7}{2}x^2 + \frac{2}{2} = 0$

$\therefore 2X^3 + 7X^2 + 2 = 0$

6 a) solu

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 4 \end{pmatrix}$$

$\therefore$ Find determinants firstly

$$1\begin{pmatrix} -3 & 4 \\ -2 & 4 \end{pmatrix} - 1\begin{pmatrix} 2 & 4 \\ 3 & 4 \end{pmatrix} + 1\begin{pmatrix} 2 & -3 \\ 3 & -2 \end{pmatrix}$$

$$(-12 + 8) - (8 - 12) + (-4 - -9)$$

$$-4 + +4 + (-4 + 9)$$

$$|\det| = 5$$

Find minor factor

$$\begin{pmatrix} \begin{pmatrix} -3 & 4 \\ -2 & 4 \end{pmatrix} & \begin{pmatrix} 2 & 4 \\ 3 & 4 \end{pmatrix} & \begin{pmatrix} 2 & -3 \\ 3 & -2 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix} & \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} & \begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 \\ -3 & 4 \end{pmatrix} & \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix} & \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} -4 & -4 & 5 \\ 6 & 1 & -5 \\ 7 & 2 & -5 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -9 \end{pmatrix}$$

6e) For Cofactor

$$\begin{pmatrix} +(-4) & -(-4) & +(5) \\ -(6) & +(1) & -(-5) \\ +(7) & -(2) & +(-5) \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -9 \end{pmatrix}$$

$$\begin{pmatrix} -4 & +4 & 5 \\ -6 & 1 & 5 \\ 7 & -2 & -5 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -9 \end{pmatrix}$$

Transpose Cofactor

$$\begin{pmatrix} -4 & -6 & 7 \\ 4 & 1 & -2 \\ 5 & 5 & -5 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -9 \end{pmatrix}$$

Transpose $\times$ $|\text{det}|^{-1}$

$$\begin{pmatrix} -4/5 & -6/5 & 7/5 \\ 4/5 & 1/5 & -2/5 \\ 5/5 & 5/5 & -5/5 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -9 \end{pmatrix}$$

For X

$$\left(-4/5\right) \times (2) + \left(-6/5\right) \times -4 + \left(7/5\right) \cdot (-9)$$

$$-\frac{8}{5} + \frac{24}{5} - \frac{63}{5} = \frac{-71+24}{5} = \frac{-47}{5}$$

$\therefore X = -47/5$ or -9.4

→ For Y

$$\left(\frac{4}{5} \times 2\right) + \left(\frac{1}{5} \times -4\right) + \left(\frac{-2}{5} \times -9\right)$$

$$\frac{8}{5} - \frac{4}{5} + \frac{18}{5} = \frac{22}{5}$$

$Y = 22/5$ or 4.4

For Z

$$\left(\frac{5}{5} \times 2\right) + \left(\frac{5}{5} \times -4\right) + \left(\frac{-5}{5} \times -9\right)$$

$$\frac{10}{5} - \frac{20}{5} + \frac{45}{5} = \frac{35}{5} = 7$$

$\therefore Z = 7$

$\therefore$ So that value of $X = -9.4$, $Y = 4.4$ and $Z = 7$

(d) Soln

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & -2 \end{pmatrix}$$

Find $|A|$

$$1\begin{pmatrix}-3 & 4\\ -2 & -2\end{pmatrix} - 1\begin{pmatrix}2 & 4\\ 3 & -2\end{pmatrix} + 1\begin{pmatrix}2 & -3\\ 3 & -2\end{pmatrix}$$

6 d)

$$6 + 8 - (-4 - 12) + (-4 - -9)$$

$$14 + 16 + 5$$

$$|A| = 35$$

from $|A|^{-1}$

$$|A|^{-1} = \frac{1}{|A|} = \frac{1}{35}$$

$\therefore$ Inverse of A is $\frac{1}{35}$

1 a) soln

I) Binomial distribution

$${}^{n}C_{x}\, p^{x}\, q^{n-x}$$

where

$x = 0$

$n = 20$

$p = 15/100$ or 0.15

$q = 1 - p$

$q = 1 - 0.15$

$q = 0.85$

$${}^{20}C_{0}\ 0.15^{0}\ 0.85^{20}$$

$\Rightarrow 0.038759531$

When

$x = 1$

$${}^{20}C_{1}\ 0.15^{1}\ 0.85^{19}$$

Extract 16.2: A sample of incorrect responses to question 6 of paper 2

In Extract 16.2, the candidate failed to expand the given expression using the binomial theorem. He/she failed to partialize the given expression using the concepts of partialization. Also, the candidate failed to formulate the expression using the roots given as well as determine the inverse of the given matrix.

2.2.7 Question 7: Differential Equations

The question consisted of four parts: (a), (b), (c), and (d). Part (a) required the candidates to show that $y = Ae^{2x}\cos(3x+\varepsilon)$ where ε is the arbitrary constant is a solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$. In part (b), the candidates were required to find the general solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 6e^x + \sin x$. In part (c), the candidates were required to solve the differential equation $(2x-1)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$, given that when $x = 0$, $y = 2$ and $\frac{dy}{dx} = 3$. In part (d), the candidates were required to determine the population of the village in 2009 if the rate of increase in the population of the village was proportional to the number of its inhabitants present at any time, if the population of the village in the year 1999 was 20,000 and in the year 2004 was 25,000.

The analysis of the data shows that 17.8 per cent of the candidates opted for this question. Among them, 85.9 per cent of the candidates scored from 7 to 20 marks. Likewise, the data shows that 14.1 per cent of the candidates scored below 7 marks, 21.7 per cent scored from 7 to 11.5 marks, and 64.2 per cent scored from 12.0 to 20.0 marks. Generally, the candidates' performance in this question was good, as shown in Figure 18.

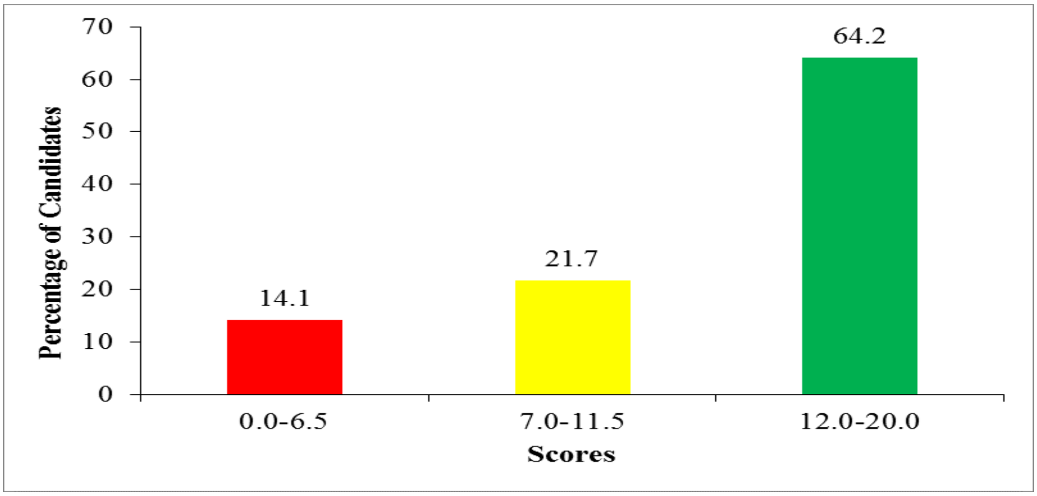


Figure 18: *Candidates' Performance in Question 7 of Paper 2*

Further analysis shows that, among 2,457 (17.8%) of the candidates who answered the question, 288 (11.7%) scored 20 marks. The candidates who had high performance attempted each part of the question using the appropriate concepts. For example, in part (a), they applied the concept of calculus to show that $y = Ae^{2x}\cos(3x+\varepsilon)$ is the solution of a differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$. Some candidates differentiated $y = Ae^{2x}\cos(3x+\varepsilon)$ to obtain $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$ and others integrated $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$ to obtain $y = Ae^{2x}\cos(3x+\varepsilon)$. In part (b), the candidates solved the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 6e^x + \sin x$ by resolving it into two parts, $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ and $f(x) = 6e^x + \sin x$. For the first part, the candidates converted $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ into the auxiliary quadratic equation $m^2 + 3m + 2 = 0$. They solved this equation to obtain $m = -1$ or $m = -2$ and used these values to obtain the complementary solution $y_c = C_1e^{-x} + C_2e^{-2x}$. For the second part, they wrote the particular integral as $y_p = Ae^x + p\cos x + q\sin x$. On differentiating and substituting y_p and its derivatives y_p' and y_p'' into the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 6e^x + \sin x$, they obtained $y_p = e^x + \frac{1}{10}(-3\cos x + \sin x)$. Finally, they gave the general solution as $y = y_c + y_p = C_1e^{-x} + C_2e^{-2x} + e^x + \frac{1}{10}(-3\cos x + \sin x)$.

In part (c), the candidates applied the proper substitutions to reduce the differential equation $(2x-1)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$ into a first order differential equation $(2x-1)\frac{dP}{dx} - 2P = 0$, where $P = \frac{dy}{dx}$. Then, they solved this equation by using an integrating factor to obtain $y = Ax^2 + Bx + C$. Using the given condition $x = 0$ and $y = 2$, they obtained the particular solution

$y = -3(x^2 - x) + 2$. In part (d), the candidates were able to formulate the differential equation $\frac{dy}{dt} = ky$ from the given word problem, where y represents the population of the village at time t. Thereafter, they solved this differential equation to obtain $\ln y = kt + c$. Using the values of $t = 0$ and $y = 20{,}000$, they obtained $C = \ln(20{,}000)$ and again with the values of $t = 5$ and $y = 25{,}000$, they obtained $k = \frac{1}{5}\ln\left(\frac{5}{4}\right)$. On substituting the values of C and k into $\ln y = kt + c$, they obtained the equation $\ln y = \frac{10}{5}\ln\frac{5}{4} + \ln 20{,}000$. Then, by using this equation, they got the required population of the village, which is 31,250. Extract 17.1 shows a sample of the responses from one of the candidates who answered the question correctly.

7(a) Sol

$y = Ae^{2x}(\cos(3x+\varepsilon))$ --- (i)

$y' = 2Ae^{2x}\cos(3x+\varepsilon) + -3Ae^{2x}\sin(3x+\varepsilon)$.

$y' = 2Ae^{2x}\cos(3x+\varepsilon) - 3Ae^{2x}\sin(3x+\varepsilon)$ --- (ii)

$y'' = 4Ae^{2x}\cos(3x+\varepsilon) + 6Ae^{2x} - \sin(3x+\varepsilon) - 6Ae^{2x}\sin(3x+\varepsilon) - 9Ae^{2x}\cos(3x+\varepsilon)$.

$y'' = -5Ae^{2x}\cos(3x+\varepsilon) - 12Ae^{2x}\sin(3x+\varepsilon)$ --- (iii)

7 (a) from eqn (i)

$y' = 2y - 3Ae^{2x}\sin(3x+\varepsilon)$.

~~$y - 2y =$~~

$3Ae^{2x}\sin(3x+\varepsilon) = 2y - y'$.

$Ae^{2x}\sin(3x+\varepsilon) = \frac{2y-y'}{3}$ —— (iii)

$y'' = -5y - 12\left(\frac{2y-y'}{3}\right)$.

$y'' = -5y - 4(2y-y')$.

$y'' = -5y - 8y + 4y'$

$y'' = -13y + 4y'$.

$y'' - 4y' + 13y = 0$.

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

hence proved.

(b) Sln

$m^2 + 3m + 2 = 0$.

$m_1 = -1$.

$m_2 = -2$

$y_c = Ae^{-x} + Be^{-2x}$.

$y_p = Ke^x$.

$y' = Ke^x$

$y'' = Ke^x$.

7(b) $\sin$

$Ke^x + 3ke^x + 2ke^x = 6e^x$.

$K + 3K + 2K = 6$

$6K = 6$.

$K = 1$.

$y_{P_1} = e^x$

$y_{P_2} = A\cos x + B\sin x$.

$y'_{P_2} = -A\sin x + B\cos x$.

$y''_{P_2} = -A\cos x - B\sin x$.

$y''_{P_2} = -(A\cos x + B\sin x)$.

$y''_{P_2} = -y$.

$-y + 3(-A\sin x + B\cos x) + 2y = \sin x$.

$y + 3B\cos x - 3A\sin x = \sin x$

$A\cos x + B\sin x + 3B\cos x - 3A\sin x = \sin x$

$(3B + A)\cos x + (B - 3A)\sin x = \sin x$

$(3B + A)\cos x = 0\cos x$

$3B + A = 0$ -- (i)

$(B - 3A)\sin x = \sin x$

$B - 3A = 1$ -- (ii)

$B = 1/10$

$A = -3/10$.

$y_{P_2} = \frac{\cos x}{10} - \frac{3\sin x}{10}$.

$y = y_c + y_{P_1} + y_{P_2}$.

$\therefore\ y = Ae^{-x} + Be^{-2x} + e^x + \frac{\cos x}{10} - \frac{3\sin x}{10}$.

7(c) Sln

$$(2x-1)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0.$$

$$\frac{d^2y}{dx^2} - \frac{2}{(2x-1)}\frac{dy}{dx} = 0$$

let $\frac{dy}{dx} = P$

$$\frac{dP}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{dP}{dx} - \frac{2}{2x-1}P = 0$$

$$\frac{dP}{dx} = \frac{2P}{2x-1}.$$

$$\int \frac{dP}{2P} = \int \frac{dx}{2x-1}.$$

$$\frac{1}{2}\ln P = \frac{1}{2}\ln(2x-1) + c.$$

let $c = \ln A$.

$$\tfrac{1}{2}\ln P = \tfrac{1}{2}\ln(2x-1) + \ln A.$$

$$\ln P = \ln(2x-1) + 2\ln A.$$

$$\ln P = \ln(2x-1) + \ln A^2$$

$$\cancel{\ln} P = \cancel{\ln}(A^2(2x-1)).$$

$$P = A^2(2x-1).$$

7(c) $dy/dx = A^2(2x-1)$.

$\int dy = \int A^2(2x-1)\,dx$

$y = A^2 \int 2x-1\,dx$

$y = A^2\int 2x\,dx - A^2\int dx$.

$y = \frac{2A^2x^2}{2} - A^2x + c$.

$y = A^2x^2 - A^2x + c$.

$y = A^2(x^2-x) + c$

$2 = A^2(0^2-0) + c$.

$2 = A^2(0) + c$.

$c = 2$.

$dy/dx = A^2(2x-1)$

$3 = A^2(2(0)-1)$.

$3 = A^2 \times -1$

$A^2 = -3$.

$y = -3(x^2-x) + 2$.

$\therefore y = -3(x^2-x) + 2$.

7(d) $\frac{dN}{dt} \underset{\alpha}{} N$ (SK)

$\frac{dN}{dt} \propto N$

$\frac{dN}{dt} = KN$

$\int_{N_0}^{N} \frac{dN}{N} = \int_0^t K\,dt$

$[\ln N]_{N_0}^{N} = [Kt]_0^t$

$[\ln N - \ln N_0] = Kt$

$\ln\left(\frac{N}{N_0}\right) = Kt$

$e^{kt} = \frac{N}{N_0}$

$N = N_0 e^{kt}$

$25000 = 20{,}000\, e^{k5}$

$\frac{25000}{20000} = e^{k5}$

$\ln\left(\frac{25}{20}\right) = 5k$

$\ln(5/4) = 5K$

7(d) $K = \frac{\ln(5/4)}{5}$

~~$K = 0.00$~~

$K = 0.04462871$

$N = N_0 e^{kt}$ $\quad t = 10$

$N = 20{,}000\, e^{0.04462871 \times 10}$

$N = 31250$

$\therefore$ The population in 2004 is 31,250

Extract 17.1: A sample of correct responses to question 7 of paper 2

In Extract 17.1, the candidate approached each part of the question correctly. For example, in part (d), the candidate formulated the correct differential equation from the given word problem and solved it using the appropriate concepts.

In spite of the good performance for the majority of the candidates, 157 (6.4%) candidates scored less than 3.5 marks. These candidates had low performance due to wrong interpretations of the questions and a lack of knowledge and skills on differential equations. In part (a), some candidates applied the wrong concepts of differentiation in showing that $y = Ae^{2x}\cos(3x+\varepsilon)$ is the solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$. For example, from $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$, they wrote $\frac{dy}{dx} + \frac{dy}{dx} - 4\frac{dy}{dx} = -13y$ thereafter simplified it to get $-2\frac{dy}{dx} = -13y$. Lastly, they concluded that $y = Ae^{2x}\cos(3x+\varepsilon)$ is the solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$. In part (b), some candidates did not have sufficient knowledge of solving second order linear differential equations. They wrote $L.H.S = \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y$ and concluded $6e^{2x} + \sin 45$ was the general solution of the differential equation. Others applied the wrong concept to find the particular integral; they wrote the particular integral for the exponential function as $y_1 = kxe^x$ instead of $y_1 = ke^x$.

In part (c), some candidates failed to reduce the differential equation $(2x-1)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$ to first order differential equation $(2x-1)\frac{dP}{dx} - 2p = 0$, where $P = \frac{dy}{dx}$. Others confused the differential equation $(2x-1)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$ with a linear second order differential equation, and they wrote $(2x-1)D^2 - 2D = 0$. Then neglected $(2x-1)$ and write $D^2 - 2D = 0$. They solved it to obtain $D = 0$ or $D = 2$ and concluded the solution to be $y = A + Be^{2x}$. In part (d), some candidates did not formulate the differential equation from the given word problem; they approached the question by recalling the formula $N = N_0 e^{kt}$. Others confused the given word problem with population decay, thus they used the formula of $N = N_0 e^{-kt}$ instead of $N = N_0 e^{kt}$. Then, they solved the problem using $N = N_0 e^{-kt}$ and obtained the wrong answer (1280.3). Extract 17.2 shows a sample of the responses from one of the candidates who attempted the question wrongly in part (d).

d. Data

$N_o = 20,000$

time, $= 1999$

$N = 25,000$

time, $= 2004$

$N = ?$

$t = 2009$

For first case

$N = N_o e^{-kt}$

$20,000 = N_o e^{-1999t}$ ---- (i)

For second case

$25,000 = N_o e^{-2004t}$ ---- (ii)

Divide equation i and ii

$$\frac{20,000}{25,000} = \frac{N_o e^{-1999t}}{N_o e^{-2004t}}$$

$$\frac{20}{25} = \frac{e^{-1999t}}{e^{2004t}}$$

$$\frac{20}{25} = e^{-1999t+2004t}$$

$$\frac{20}{25} = e^{5k}$$

Apply ln both sides

$$\ln\left(\frac{20}{25}\right) = \ln(e^{-5t})$$

$\ln\left(\frac{20}{25}\right) = +5t \ln e.$

$\frac{0.22314}{5} = \frac{+5t}{5}$

$k = 0.0446$

from first case

$N = N_0 e^{kt}$

$20{,}000 = N_0 e^{-1999(0.0446)}$

$\frac{20{,}000}{1.9067\times10^{-39}} = \frac{(1.9067\times10^{-39})N_0}{1.9067\times10^{-39}}$

$N_0 = 1.0488\times10^{43}.$

At 2009.

$N = N_0 e^{-kt}$

$N = (1.0488\times10^{43})e^{-(0.0446\times2009)}.$

$N = 1280.2.62686$

$\therefore$ The population of the village in 2009 = 1280.2.68

Extract 17.2: A sample of the incorrect responses to question 7 of paper 2

Extract 17.2 shows that in part (d) of the question, the candidate did not formulate the differential equation from the given word problem, and he/she applied an inappropriate formula used to find population decay.

2.2.8 Question 8: Coordinate Geometry II

This question had four parts, (a), (b), (c) and (d). In part (a), the candidates were required to (i) show that the locus of the point is a circle and (ii) find the centre and radius of the circle obtained in (i) if the point moves so that its distance from the point $(3, 2)$ is half its distance from the line $2x + 3y = 1$. In part (b), the candidates were required to show that, the equation of a normal at the point $(a\cos\theta, b\sin\theta)$ to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is $ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$. In part (c), the candidates were required to find the greatest value of the area of the triangle OQR where O is the origin, if the normal at P in part (b) meets the $x-$axis at Q and the $y-$axis at R. In part (d), the candidates were required to sketch the graph of $r^2 = a^2\sin 2\theta$.

The question was opted for by 3,385 candidates. Analysis indicates that 49.0 per cent of the candidates passed. Further analysis shows that 1,727 (51.0%) candidates scored from 0 to 6.5 marks, 1,270 (37.5%) scored from 7 to 11.5 marks, and 388 (11.5%) scored from 12 to 20 marks. Moreover, the analysis shows that 124 (3.7%) candidates scored 0 marks, but 2 (0.1%) candidates scored all the 20 marks. Generally, the candidates' performance was average as summarized in Figure 19.

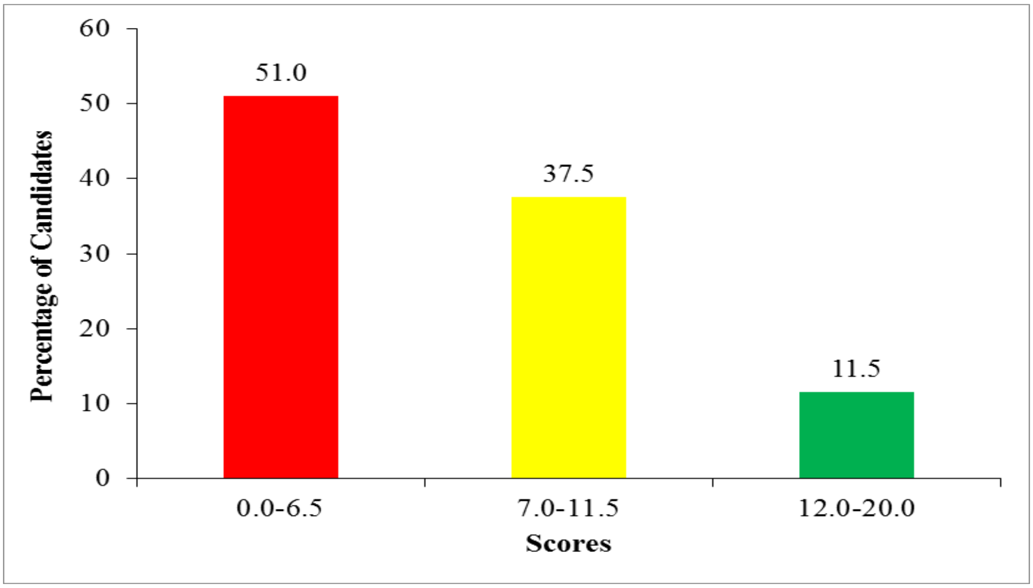


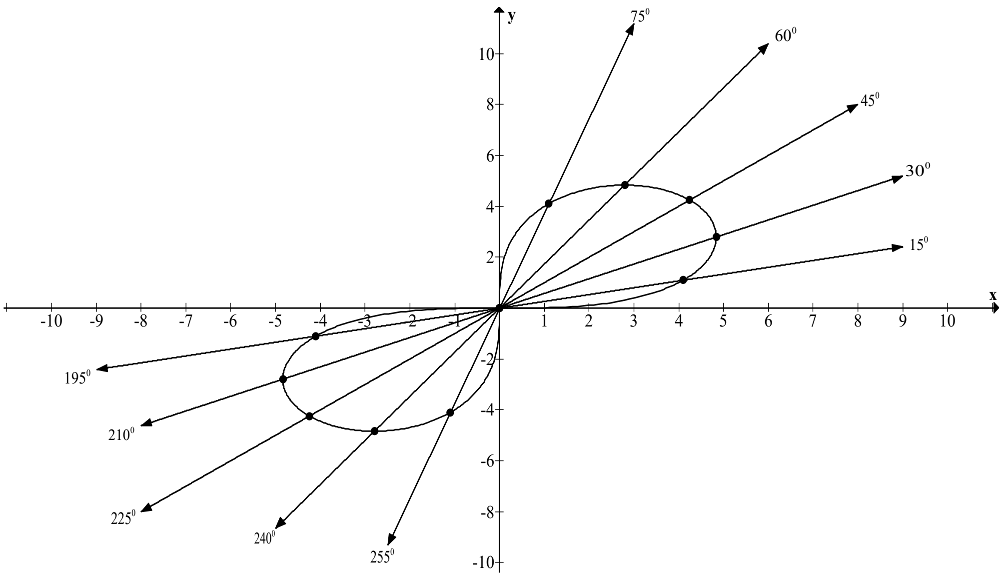
Figure 19: *Candidates' Performance in Question 8 of Paper 2*

The candidates who performed well in this question were able to apply all the necessary concepts to each part of the question. In part (a), they formulated the equation of a locus by applying the concepts of a distance between two points and a perpendicular distance from a point to a line. Using the mentioned concepts and the condition given, they obtained $\sqrt{(x-3)^2+(y-2)^2}=\frac{1}{2}\left(\frac{|2x+3y-1|}{\sqrt{2^2+3^2}}\right)$. On simplifying this equation, they ended up with $48x^2+43y^2-308x-202y-12xy+675=0$. Using this equation, the candidates concluded that, in (i) the locus of the point is not a circle, and (ii) the locus has neither centre nor a radius. In part (b), the candidates were able to show that the equation of the normal at a point to the ellipse is $ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta$ by applying the concept of differentiation to find the gradient of the tangent and then the normal to the ellipse. Some candidates differentiated the equation of the ellipse, $b^2x^2+a^2y^2=a^2b^2$ to obtain $\frac{dy}{dx}=-\frac{b^2x}{a^2y}$ and then substituted the point $(a\cos\theta, b\sin\theta)$ into the derivative to obtain $\frac{dy}{dx}=-\frac{b\cos\theta}{a\sin\theta}$. They used this to obtain the gradient of the normal and its equation, $ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta$. However, others differentiated the parametric equations $x=a\cos\theta$ and $y=b\sin\theta$ to obtain the gradient of the tangent $\frac{dy}{dx}=-\frac{b\cos\theta}{a\sin\theta}$. After that, they went through similar steps to obtain the required equation, $ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta$.

In part (c), the candidates were to find the area of the triangle OQR using its vertices $O(0,0)$, $Q\left(\frac{(a^2-b^2)\cos\theta}{a},0\right)$ and $R\left(0,-\frac{(a^2-b^2)\sin\theta}{b}\right)$ which were obtained by finding the x and $y-$intercepts of the normal $ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta$. Thereafter, they calculated the area of the triangle by taking $Area=\frac{1}{2}\left(\overline{OQ}\times\overline{OR}\right)$ and obtained $Area=\frac{\left(a^2-b^2\right)\sin 2\theta}{4ab}$, but for maximum value, $\sin 2\theta=1$, they obtained the maximum area as $\frac{\left(a^2-b^2\right)}{4ab}$. In part (d), they prepared the appropriate table of values for the equation $r^2 = a^2 \sin 2\theta$ which equals to $r = \pm\, a\sqrt{\sin 2\theta}$. That is:

θ^0	0^0	30^0	45^0	60^0	90^0	180^0
r	0	$\pm 0.9a$	$\pm a$	$\pm 0.9a$	0	0

Then, they correctly used the ordered pairs (r,θ) to sketch the polar graph as follows:



Extract 18.1 shows a sample of correct responses from one of the candidates who attempted this question.

8 (a) Let point be (x, y)

~~Recall,~~

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x-3)^2 + (y-2)^2}$$

From the question,

$$2\sqrt{(x-3)^2 + (y-2)^2} = \left| \frac{2x+3y-1}{\sqrt{2^2+3^2}} \right|$$

$$\frac{2\sqrt{13((x-3)^2 + (y-2)^2)}}{2} = |2x+3y+1|$$

$$52(x^2 - 6x + 9 + y^2 + 4y + 4) = 4x^2 + 9y^2 - 4x - 6y + 12xy + 1$$

$$48x^2 + 43y^2 - 308x + 214y - 12xy + 207 = 0$$

(i) Its not a circle

(ii) No centre and radius

8 (b) From equation of ellipse,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Differentiate,

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-xb^2}{ya^2} \text{ (Slope of tangent)}$$

~~At~~ but,

$$m_1m_2=-1$$

$$m_2=\frac{ya^2}{xb^2} \text{ (Slope of normal)}$$

at point $(a\cos\theta, b\sin\theta)$

$$m_2=\frac{a^2b\sin\theta}{ab^2\cos\theta}=\frac{a\sin\theta}{b\cos\theta}$$

From,

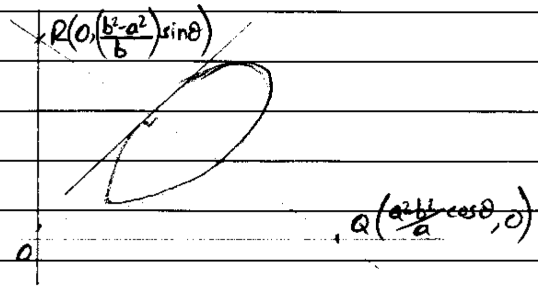
$$y=m(x-x_1)+y_1$$

$$y=\frac{a\sin\theta}{b\cos\theta}(x-a\cos\theta)+b\sin\theta$$

$$yb\cos\theta=ax\sin\theta-a^2\sin\theta\cos\theta+b^2\sin\theta\cos\theta$$

$$ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta \text{ (shown)}$$

(c)



$$A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} 0 & 0 \\ 0 & \left(\frac{b^2-a^2}{b}\right)\sin\theta \\ \frac{a^2-b^2}{a}\cos\theta & 0 \\ 0 & 0 \end{vmatrix}$$

8(c) $\text{Area} = \frac{1}{2}\left| -\left(\frac{a^2-b^2}{a}\right)\cos\theta \times \left(\frac{b^2-a^2}{b}\right)\sin\theta \right|$

$\text{Area} = \frac{1}{2}\left| a^2 b^2 \left(\frac{\sin\theta}{b} \times \frac{\cos\theta}{a}\right)\right|$

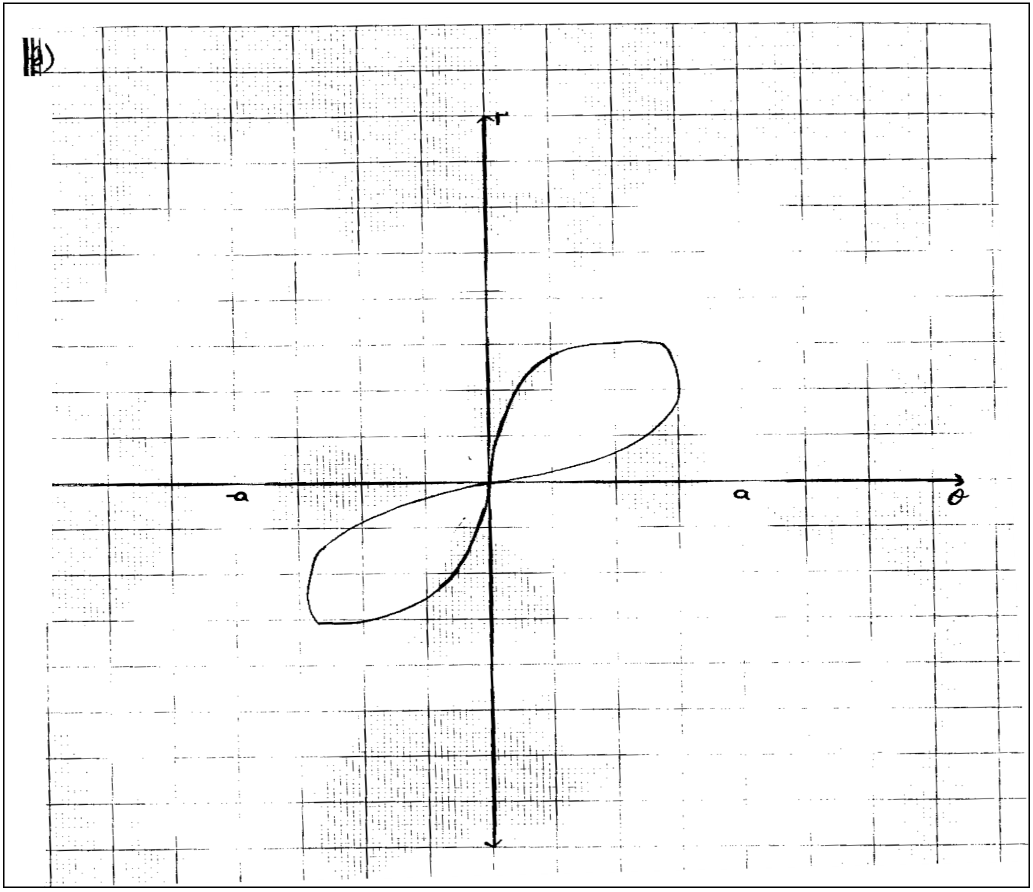
$$\text{Area} = \frac{(a^2-b^2)\sin\theta\cos\theta}{2ab}$$

(d) $r^2 = a^2 \sin 2\theta$

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°
r	0	0.931a	a	0.931a	0	Img	Img	Img	0	0.931a	a	0.931a

270°	300	315°	330°	360°
0	Img	Img	Img	0

where, Img - Imaginary



Extract 18.1: A sample of correct responses to question 8 of paper 2

Extract 18.1 shows the response of a candidate who attempted the question correctly. In part (a), the candidate correctly determined the locus of the point and concluded that it is not a circle, hence there is no radius. In part (b), the candidate was able to show that the given equation is a normal at a point $(a\cos\theta, b\sin\theta)$ using the approach of differentiating the equation of the ellipse. Likewise, in parts (c) and (d), he/she calculated the area correctly and sketched the required graph.

Apart from the performance described above, there were some candidates who failed to answer this question correctly due to some challenges. For instance, in part (a), some candidates failed to interpret the condition regarding the formulation of the equation of the locus. Some wrote $\sqrt{(x-3)^2+(y-2)^2}=\frac{1}{2}\sqrt{\left(x-\frac{1}{2}\right)^2+(y-0)^2}$ where the point $\left(\frac{1}{2},0\right)$ is the $x-$intercept of the line $2x+3y=1$. Then, they simplified this to obtain the equation of the circle, $3x^2+3y^2-23x-16y+\frac{207}{4}$. Other candidates managed to write the correct formula but failed to identify the correct substitution, that is $\sqrt{(x-3)^2+(y-2)^2}=\frac{1}{2}\left|\frac{ax+by+c}{\sqrt{a^2+b^2}}\right|$ then substituted the values as $\sqrt{(x-3)^2+(y-2)^2}=\frac{1}{2}\left|\frac{2(3)+3(2)+1}{\sqrt{2^2+3^2}}\right|$. Thereafter, they simplified this to obtain the equation of the circle, $(x-3)^2+(y-2)^2=\frac{361}{52}$. In part (b), some candidates calculated the gradient of the tangent and used it as the gradient of the normal, so they obtained the equation of the tangent and not the normal as the question instructed. For example, one candidate obtained the gradient of the tangent as $\frac{dy}{dx}=-\frac{b\cos\theta}{a\sin\theta}$ and the corresponding equation $bx\cos\theta+ay\sin\theta=ab$. Then it was concluded that the equation of the normal is $bx\cos\theta+ay\sin\theta=ab$ and not $ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta$. Also, a few candidates calculated the gradient of the normal by taking the given point $(a\cos\theta, b\sin\theta)$ and the origin $(0,0)$; they obtained $gradient=\frac{b\sin\theta}{a\cos\theta}$ then ended up with the equation $ax\sin\theta-by\cos\theta=0$. Likewise, a few candidates calculated the equation of the normal by substituting $(a\cos\theta, b\sin\theta)$ into $b^2x^2 + a^2y^2 = a^2b^2$ to obtain $a^2b^2\cos^2\theta + b^2a^2\sin^2\theta = a^2b^2$.

In part (c), the candidates obtained the wrong area of the triangle OQR because they failed to find the coordinates of its vertices. For example, some wrote $Area = \frac{1}{2} \times base \times height$ then, they took $base = \overrightarrow{OQ} = (Q,0) - (0,0) = Q$ and $height = \overrightarrow{OR} = (R,0) - (0,0) = R$ which led them to obtain $Area = \frac{1}{2}QR$ units.

In part (d), the candidates drew the wrong graph of $r^2 = a^2\sin 2\theta$ because they failed to obtain the correct table of values of r against θ; some prepared the table by ignoring negative values of r and the constant a. Likewise, others prepared the table with wrong values of r as illustrated in the following table:

θ^0	0^0	30^0	45^0	60^0	90^0	180^0
r	0	0.9	1	0.9	0	0

Extract 18.2 shows a sample of incorrect responses from one of the candidates who attempted the question.

8 (a)(i) locus of the point is a circle

is

$$(x-3)^2+(y-2)^2=\frac{121}{13}$$

or

$13(x-3)^2+13(y-2)^2=121$, with radius $r=\sqrt{\frac{121}{13}}=\frac{11}{\sqrt{13}}$ units and centre (3,2)

(ii) Required centre of the circle

Soln

eqn of locus $(x-3)^2+(y-2)^2=\frac{121}{13}$.

Compare with

$$(x-a)^2+(y-b)^2=r^2.$$

Where (a, b) is the centre

$a=3$

$b=2$

$(a,b)=(3,2)$.

Radius of circle.

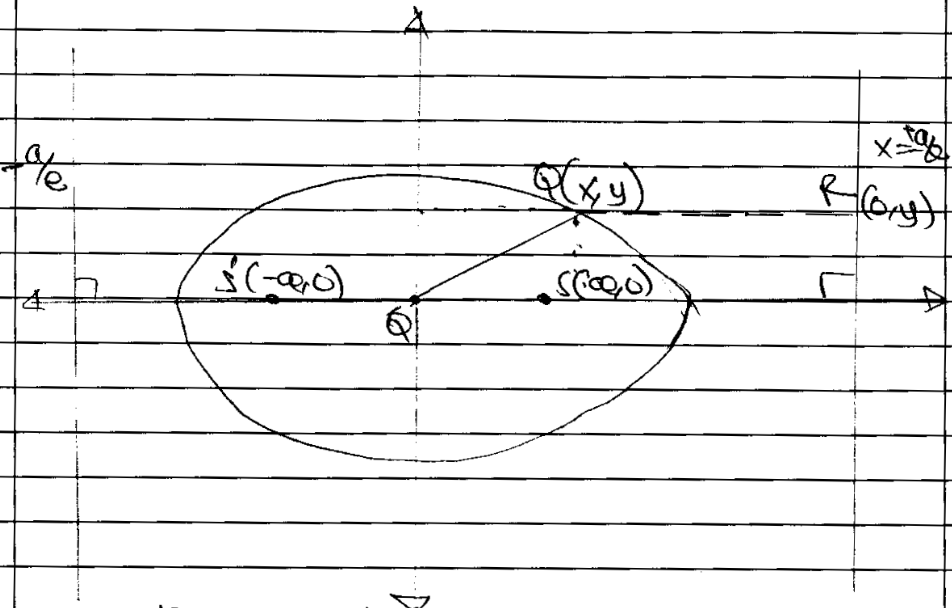
$$r^2=\frac{121}{13}$$

$$r=\sqrt{121/13}$$

$$r=\frac{11}{\sqrt{13}}\text{ units}$$

∴ The centre of circle is $(a,b)=(3,2)$ and radius is $11/\sqrt{13}$ units.

9 (c) Consider the diagram below.



Required the greatest value of Area of triangle

from

Area = $\frac{1}{2}$ base × height

Area = $\frac{1}{2}bh$.

For the height:

$$\text{height} = \sqrt{(x-ae)^2 + y^2}$$

$$h = \sqrt{(x-ae)^2 + y^2}$$

For the base

$$b = y^2$$

$$b = \sqrt{(y-0)^2 + 0^2}$$

$$b = \sqrt{y^2}$$

(c) $\text{Area} = \frac{1}{2} y^2 \sqrt{(x-ae)^2 + y^2}$

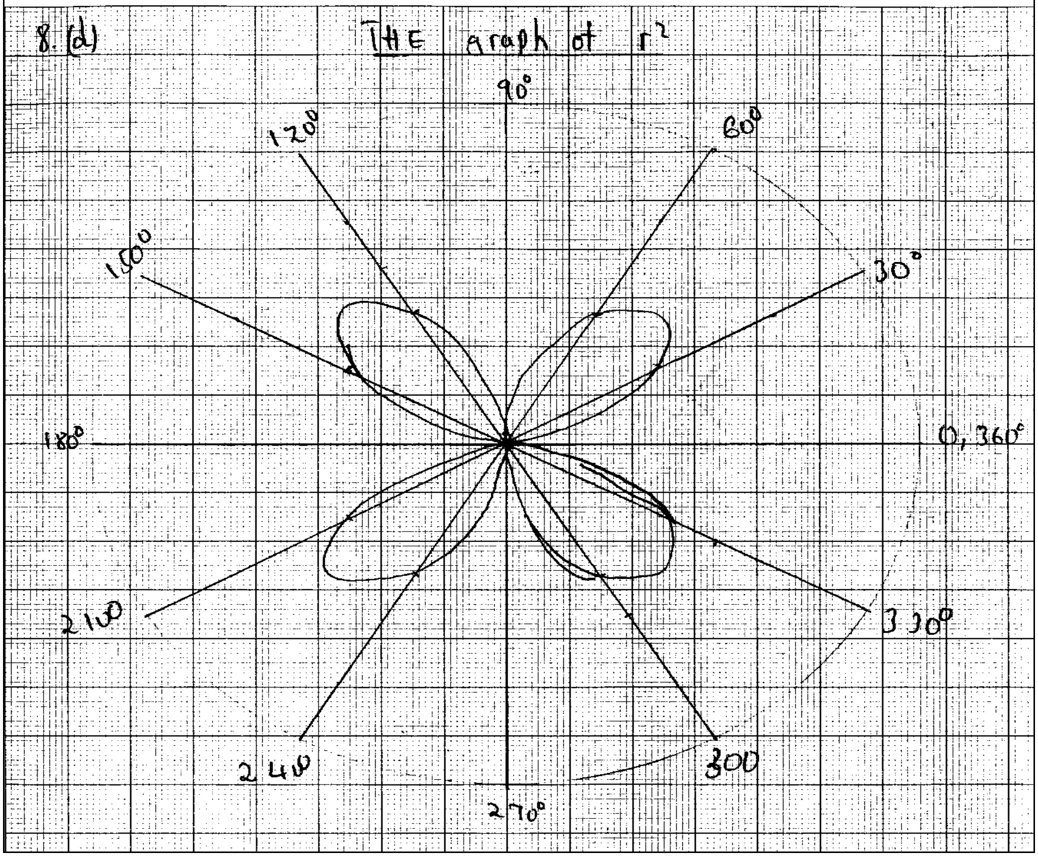
$$A = \frac{1}{2} y^2 \sqrt{(x-ae)^2 + y^2}$$

$$A = \frac{1}{4} y^4 (x-ae)^2 + y^2$$

$$A = \frac{1}{4} b^4 \sin^4\theta \, (a\cos\theta - ae)^2 + b^2 \sin^2\theta$$

(d)

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r^2	0	0.9a	0.9a	0	-0.9a	-0.9a	0	0.9a	0.9a	0	-0.9a	-0.9a	0



Extract 18.2: A sample of incorrect responses to question 8 of paper 2

Extract 18.2 shows that in part (a), the candidate failed to interpret the condition given in the word problem, thus determining the wrong equation of the locus and obtaining the centre and radius of a circle wrongly. In part (c), the candidate failed to interpret the question and ended up considering the triangle in the ellipse. In part (d), the candidate obtained the wrong table of values, which led to the wrong sketch of the graph.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The candidates were tested in 18 topics, out of which 10 topics were from Advanced Mathematics 1 and 8 topics from Advanced Mathematics 2. There were ten (10) questions in Advanced Mathematics 1, and eight (8) in Advanced Mathematics 2. In Advanced Mathematics 1, the following topics were covered: *Calculating Devices*, *Hyperbolic Functions*, *Linear Programming*, *Statistics*, *Sets*, *Functions*, *Numerical Methods*, *Coordinate Geometry I*, *Integration and Differentiation*. In Advanced Mathematics 2, the topics covered were *Probability*, *Logic*, *Vectors*, *Complex Numbers*, *Trigonometry*, *Algebra*, *Differential Equations* and *Coordinate Geometry II*.

The topic wise analysis of the performance in the Advanced Mathematics papers for 2023 shows that thirteen (13) topics were well performed, three (3) topics had average performance, and two (2) topics were poorly performed. The topics with good performance in the examination were *Functions* (96.2%), *Logic* (94.5%), *Statistics* (91.0%), *Sets* (89.3%), *Trigonometry* (86.8%), *Differential Equations* (85.9%), *Coordinate Geometry I* (78.5%), *Algebra* (76.9%), *Hyperbolic Functions* (70.0%), *Linear Programming* (68.0%), *Calculating Devices* (67.4%), *Complex Numbers* (65.1%), and *Vectors* (62.2%). The 03 topics that had average performance were *Integration* (50.1%), *Coordinate Geometry II* (49.0%), and *Differentiation* (48.6%). (See Appendix I)

The good and average performance in these topics were attributed to candidates' ability on the following:

(i) Using scientific calculators to perform computations of various mathematical expressions involving statistics, matrices, exponential and logarithmic functions.

(ii) Solving problems related to hyperbolic functions, identities and integration.

(iii) Formulating mathematical models and solving linear programming problems graphically.

(iv) Applying the concepts of statistics to determine the measures of central tendency and dispersion.

(v) Simplifying set expressions using laws of algebra of sets and solve related problems using Venn diagrams.

(vi) Drawing graphs of logarithmic, exponential and rational functions and determining the vertical and horizontal asymptotes of rational functions.

(vii) Using the concepts of coordinate geometry in solving related problems such as centre and radius of the circle, equations of tangent and normal to conic sections, the perpendicular distance of a point from a line, the ratio theorem, as well as curve sketching of polar equations.

(viii) Using the concepts of calculus to solve related problems such as: differentiating some expressions, evaluating indefinite integrals, formulating and solving differential equations and applying calculus in solving the real life problems.

(ix) Simplifying logical expressions using the laws of propositions of algebra, determine the equivalence between two compound statements using the truth tables and testing the validity of an argument by using the truth tables or laws of propositions of algebra.

(x) Using the vector concepts to prove whether the vertices form the right angled triangle, resolve the given vector into its components and finding the work done by force.

(xi) Applying the concept of complex numbers as well as solving roots of complex numbers.

(xii) Using the trigonometric identities to simplify, prove and solve the trigonometric equations.

(xiii) Applying the concept of binomial theorem, partializing, formation of cubic equation and determining the inverse of matrix and its application in solving linear equations.

Although many candidates had good or average performance in most of the topics, further analysis revealed that there were two topics that the candidates performed poorly. These topics are *Probability* (34.5%) and *Numerical methods* (10.8%), see Appendix I.

The analysis indicates that weak performance in the topics of *Probability* and *Numerical Methods* was due to the following:

(i) Failure to use the binomial and poison distribution functions to prepare the corresponding probability distribution tables and computing the mean and standard deviation from the probability distribution tables. Also, the inability to convert the random variable X to the standardized random variable Z and obtain the corresponding probabilities by either a calculator or mathematical tables.

(ii) Failure to identify the lower and upper limits of the area to be calculated and applying incorrect numerical integration formulae.

Primarily, the analysis of candidates' performance per topic reveals that in the year 2023, the performance has dropped in eleven (11) topics as compared to the year 2022, as shown in Appendix II.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

Generally, in 2023 the performance in the Advanced Mathematics examination has dropped by 1.01 per cent because 96.86 per cent of the candidates passed the examination compared to the 2022 performance in which 97.89 per cent of the candidates passed. The Candidates' Item Response Analysis (CIRA) report on ACSEE 2023 basically aims to reveal the strengths and weaknesses of the candidates' responses on various items from the topics examined.

It is hopeful that education stakeholders will make use of the recommendations presented in this report to improve the future performance of the candidates in Advanced Mathematics examinations.

4.2 Recommendations

In order to improve the candidates' performance in Advanced Mathematics examinations, the following are recommended:

(a) Students should have prior knowledge, basic concepts, techniques, and skills on the topics of Advanced Mathematics. Teachers are advised to familiarize students with the basics of the topics in real life situations.

(b) All topics and subtopics should be taught effectively and assessed frequently, with feedback to learners, to enhance smooth teaching and learning processes.

(c) Topics such as Calculating Devices, Differentiation, Integration, Trigonometry, and Algebra are the key topics to be emphasized more in students learning because they are connected to other topics.

(d) Students should be cultivated with the spirit of self-motivation and higher determination of learning future working fields such as planning, economics, science, and management.

(e) Parents, guardians, and non-governmental organizations such as financial institutions such as banks are advised to support the government's efforts in improving the teaching and learning processes.

Appendix I: Analysis of Candidates' Performance on each Topic

S/N	Topic	Question Number	The Percentage of Candidates who Passed	Remarks
1.	Functions	6 (P_1)	96.2	Good
2.	Logic	2 (P_2)	94.5	Good
3.	Statistics	4 (P_1)	91.0	Good
4.	Sets	5 (P_1)	89.3	Good
5.	Trigonometry	5 (P_2)	86.8	Good
6.	Differential Equations	7 (P_2)	85.9	Good
7.	Coordinate Geometry I	8 (P_1)	78.5	Good
8.	Algebra	6 (P_2)	76.9	Good
9.	Hyperbolic Functions	2 (P_1)	70.0	Good
10.	Linear Programming	3 (P_1)	68.0	Good
11.	Calculating Devices	1 (P_1)	67.4	Good
12.	Complex Numbers	4 (P_2)	65.2	Good
13.	Vectors	3 (P_2)	62.2	Good
14.	Integration	9 (P_1)	50.1	Average
15.	Coordinate Geometry II	8 (P_2)	49.0	Average
16.	Differentiation	10 (P_1)	48.6	Average
17.	Probability	1 (P_2)	34.5	Weak
18.	Numerical Methods	7 (P_1)	10.8	Weak

Appendix II: Analysis of candidates' Performance on each Topic in the 2022 and 2023 Advanced Mathematics Examinations

S/N	Topic	Question Number	2022		2023	
			The Percentage of Candidates who Passed	Remarks	The Percentage of Candidates who Passed	Remarks
1.	Functions	6	60.4	Good	96.2	Good
2.	Logic	2	98.4	Good	94.5	Good
3.	Statistics	4	82.3	Good	91.0	Good
4.	Sets	5	91.4	Good	89.3	Good
5.	Trigonometry	5	92.0	Good	86.8	Good
6.	Differential Equations	7	64.6	Good	85.9	Good
7.	Coordinate Geometry I	8	54.1	Average	78.5	Good
8.	Algebra	6	81.9	Good	76.9	Good
9.	Hyperbolic Functions	2	70.9	Good	70.0	Good
10.	Linear Programming	3	87.8	Good	68.0	Good
11.	Calculating Devices	1	79.8	Good	67.4	Good
12.	Complex Numbers	4	63.6	Good	65.2	Good
13.	Vectors	3	71.9	Good	62.2	Good
14.	Integration	9	22.6	Weak	50.1	Average
15.	Coordinate Geometry II	8	92.8	Good	49.0	Average
16.	Differentiation	10	36.2	Average	48.6	Average
17.	Probability	1	55.0	Average	34.5	Weak
18.	Numerical Methods	7	66.9	Good	10.8	Weak