THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2021

BASIC APPLIED MATHEMATICS

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CANDIDATES' ITEM RESPONSE ANALYSIS REPORT ON THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2021

141 BASIC APPLIED MATHEMATICS

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Table of Contents

FOREWORD

The National Examinations Council of Tanzania is pleased to issue this report on the Candidates' Item Response Analysis (CIRA) for the 141 Basic Applied Mathematics paper of the Advanced Certificate of Secondary Education Examination (ACSEE) 2021. The aim of this report is to provide feedback to education stakeholders on the performance of the candidates which indicates competences and skills acquired by the candidates in two years of Advanced Secondary Education.

The analysis shows that the performance of the candidates in this examination was generally good. The topics which had good performance included Probability, Calculating Devices and Functions. The candidates' responses to questions from these topics indicated that they were competent in determining sample space and identifying event of experiments; using a scientific calculator to solve mathematical problems; finding range of linear function; as well as turning point and inverse of quadratic function. The only topic which was averagely performed is Statistics.

The candidates' performance in other topics was weak. These topics were: Algebra, Integration, Exponential and Logarithmic Functions, Linear Programming, Differentiation and Trigonometry. This performance was attributed to inability of the many candidates to: find n^{th} term and the sum of the first n terms of the geometrical progression; apply the knowledge of perfect square to solve quadratic equation; and apply knowledge of integration to find area of the region; define exponential function; evaluate integrals; and find compound interest. Other reasons for the weak performance included inability of the candidates to: write the mathematical model of the word problem; find the derivative of expressions; apply concepts of differentiation to solve real life problems; prove trigonometric identities; solve trigonometric equations; and apply cosine rule.

The National Examinations Council hopes that this report will be used by education stakeholders in developing effective strategies for improving the performance in the future examinations.

Finally, the Council would like to thank everyone who participated in preparing this report.

Dr. Charles Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is a result of the analysis of candidates’ responses to the items examined in the 141 Basic Applied Mathematics paper for the Advanced Certificate of Secondary Education Examination (ACSEE) 2021. The paper was set according to the 2019 examination format which is based on the 2010 Basic Applied Mathematics syllabus for Advanced Secondary Education. The paper consisted of 10 compulsory questions with 10 marks each.

A total of 34,976 candidates sat for the 141 Basic Applied Mathematics examination in 2021 and 60.85 percent of them passed. This contrasts with 2020 when 18,483 candidates, equivalent to 59.31 percent passed. Therefore, the performance has increased by 1.54 percent. The performance of candidates in different grades for the years 2020 and 2021 is as shown in Figure 1.

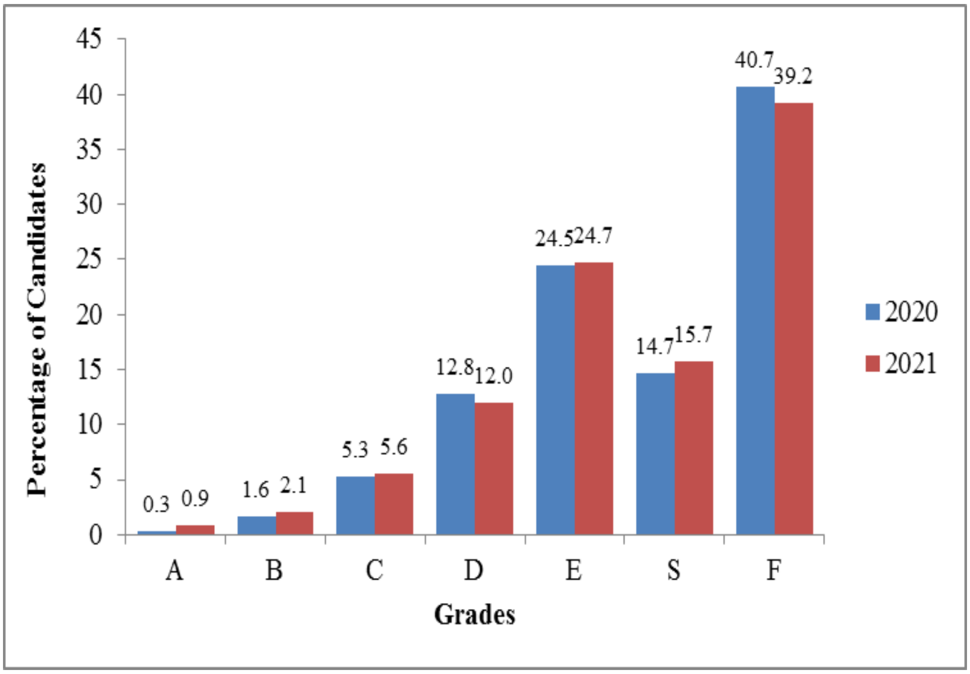


Figure 1: *Comparison of candidates’ performance between 2020 and 2021*

The report also analyses candidates’ performance in each question. The analysis includes statements of the question, performance of the candidates and explanations of strengths and weaknesses observed in their responses. Extracts showing the candidates' works to illustrate their strengths or weaknesses are inserted. The overall candidates' performance in each question is categorized basing on the percentage of candidates who scored

3.5 marks or more. The categories are 60 to 100 percent for good performance; 35 to 59 percent for average performance; and 0 to 34 percent for weak performance. In graphs or charts; green, yellow and red colours are used to denote good, average and weak performance respectively.

The report further shows the topics which had good, average and weak performance. Moreover, the report provides conclusion which summarizes the performance and factors for good and weak performance. It finally provides recommendations for improvement of performance in future.

2.0 ANALYSIS OF CANDIDATES' RESPONSES IN EACH QUESTION

2.1 Question 1: Calculating Devices

This question examined the ability of the candidates to use appropriate technology, particularly scientific calculator to solve mathematical problems. The question required the candidates to; *Use a non-programmable scientific calculator to:*

(a) compute $\dfrac{\sqrt{\sqrt{19e^2}\ \ln 3}}{\sqrt{2}}$ *correct to 5 significant figures.*

(b) evaluate $\int_0^1 \sqrt{1-x^2}\,dx$ *correct to 5 significant figures.*

(c) find the mean and standard deviation of the following data correct to 4 decimal places.

Values	*110*	*130*	*150*	*170*	*190*
Frequency	*10*	*31*	*24*	*2*	*2*

This question was attempted by 34,769 (99.41%) candidates, of which, 25,140 (72.31%) scored from 3.5 to 10 marks. Therefore, the performance of the candidates in this question was good. The percentage of candidates who scored low, average and high marks is presented in Figure 2.

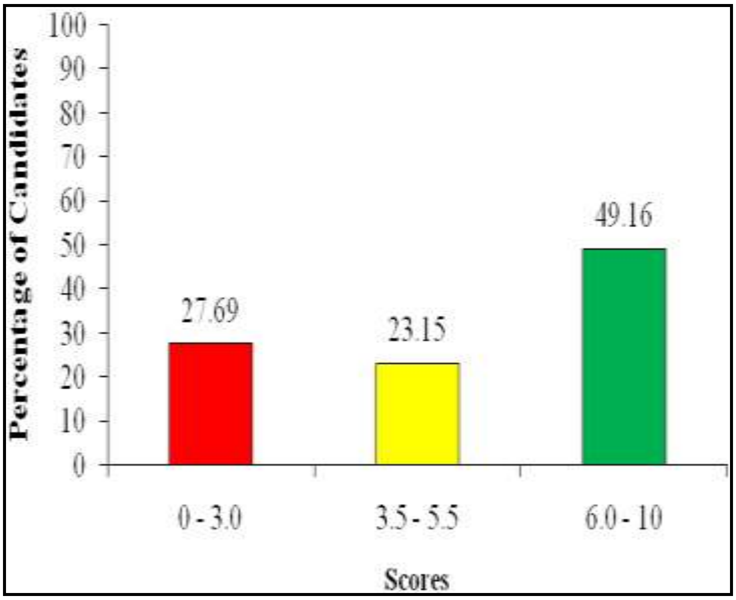


Figure 2: *The candidates' performance in question 1*

A total of 3,197 (9.19%) candidates scored all 10 marks allotted to this question. Generally, these candidates were skillful at using non-programmable scientific calculators to perform basic operations, evaluate integrands, find mean and standard deviation and fix the number to the required significant figures or decimal places. In part (a) the candidates demonstrated competency in using a scientific calculator. They adhered to calculation priority sequence and hence inserted $\frac{\sqrt{\sqrt{19e^2}\ln 3}}{\sqrt{2}}$ correctly into the calculator and fixed the output into five significant figures, ending up with 2.5512. Similarly, in part (b) the candidates inserted $\int_0^1 \sqrt{1-x^2}dx$ correctly into the calculator and fixed the answer to five significant figures and got 0.78540.

In part (c), the candidates followed the following steps: (i) Pressing *mode* button of the calculator twice and the button labeled *1* to select single-variable (SD); (ii) inserting the given values and respective frequency and registering each of them by pressing *M+* button which operates as the *DT* key; (iii) pressing the buttons labeled *shift* then *2* which now operates as *S-VAR*; (iv) pressing the button labeled *1* and *2* to get mean and standard deviation respectively; and (v) fixing each answer into four decimal places. Extract 1.1 shows the correct answers for mean and standard deviation.

(c) Mean $= \underline{136.9565}$

Standard deviation $= 17.3041$

Extract 1.1: A sample of correct response to part (c) of question 1

On the other hand, 9,629 (27.69%) candidates scored low marks including 3,728 (10.72%) who got zero. Most of these candidates failed to set calculator into required significant figures or decimal places. For instance, in part (c) many candidates ended up with $mean = 136.9565217$. Also, a significant number of candidates ignored the brackets when answering part (a) and (b). As a result, they got an incorrect answer, $\int_0^1 \sqrt{1-x^2}\,dx = 0.66667$ in particular. Furthermore, some candidates got wrong answers because they inserted approximated values of the terms instead of expressions themselves (see Extract 1.2). This approach is inappropriate because deviation of the term from the exact value has notable effect on the answer after performing operations.

Apart from these, some candidates employed techniques of integration and formulae for calculating mean and standard deviation to answer part (b) and (c) respectively. These approaches are contrary to the requirements of the question which instructed them to use a non-programmable calculator.

1(a) Soln

$\ln 3 = 1.0987$

$\sqrt{19e^2} = 11.8487$

$\sqrt{2} = 1.4142$

$= \sqrt{\frac{(11.8487 \times 1.0987)}{1.4142}}$

$= \sqrt{\frac{13.0181}{1.4142}}$

$= \sqrt{9.205334492}$

$= 3.034029415 \approx 3.03403$

Extract 1.2: A sample of incorrect response to part (a) of question 1

Extract 1.2 shows that the candidate performed operations using approximated values of $\ln 3$, $\sqrt{19e^2}$ and $\sqrt{2}$.

2.2 Question 2: Functions

The question measured knowledge and skills of candidates in finding the range of linear function as well as turning point and inverse of a particular quadratic function. It comprised parts (a), (b) and (c) which were as follows:

(a) *Given that* $g(x) = -3x + 5$. *Find the range of* $g(x)$ *for the domain* $\{x : -2 \le x \le 3\}$.

(b) *Find the turning point of* $h(x) = \frac{1}{2}x^2 - x$.

(c) *Find the inverse of* $f(x) = 2x^2 - 5$.

A total of 32,569 (93.12%) candidates attempted this question, of which, 19,671 (60.40%) scored from 3.5 to 10 marks. This suggests that the overall performance of the candidates in this question was good. Figure 3 provides a summary of the candidates' performance in this question.

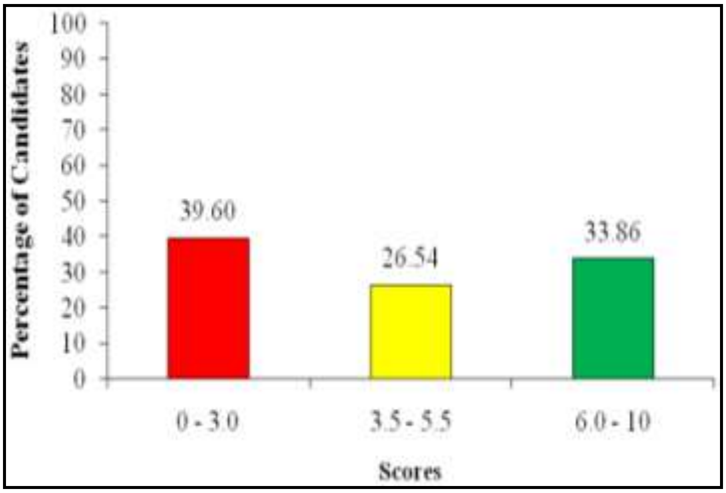


Figure 3: *The candidates' performance in question 2*

The data reveals that 66 (0.20%) candidates answered the question correctly scoring all 10 marks. In part (a), the candidates were aware of the fact that linear function is definite for all real numbers. Therefore, they computed $g(-2)$ and $g(3)$ from $g(x) = -3x + 5$ and got -4 and 11 respectively. These values give the interval of range of the function. Therefore, they wrote $Range = \{y : -4 \le y \le 11\}$. In part (b), many candidates used the formula for calculating turning point of quadratic function $ax^2+bx+c=0$, $\mathrm{T}(x,\ y)=\left(-\frac{b}{2a},\ \frac{4ac-b^2}{4a}\right)$ (Extract 2.1). Other candidates used the concept of differentiation to determine the turning point. These candidates computed $h'(x)$ from $h(x)=\frac{1}{2}x^2-x$ and got $h'(x)=x-1$. Then, they determined the abscissa of the turning point by equating $h'(x)$ to zero and solving the resulting equation to get $x=1$. The candidates also determined the ordinate of the turning point by computing $h(1)$ from $h(x)=\frac{1}{2}x^2-x$ and got $-\frac{1}{2}$. Therefore, they wrote the turning point as $T(x,y)=\left(1,-\frac{1}{2}\right)$. In part (c), the candidates interchanged the variables x and y in $y=2x^2-5$ yielding $x=2y^2-5$. Then, they expressed y in terms of x and got $y=\pm\sqrt{\frac{x+5}{2}}$, implying that $f^{-1}(x)=\pm\sqrt{\frac{x+5}{2}}$.

	(b) $h(x)=\frac{1}{2}x^2-x$
	Value of x.
	from $x=\frac{-b}{2a}$
	$x=\frac{-(-1)}{2(\frac{1}{2})}$
Qn 2	(b) $x=\frac{1}{1}$
	$x=1$.
	Value of y.
	from
	$y=\frac{4ac-b^2}{4a}$

$$y = \frac{(4 \times 1/2 \times 0) - (-1)^2}{4(1/2)}$$

$$y = \frac{0 - 1}{2}$$

$$y = -1/2$$

Hence turning point $(x, y) = (1, -1/2)$

Extract 2.1: A sample of correct response to part (b) of question 2

In Extract 2.1 shows that a candidate replaced a, b and c in the formula $\mathrm{T}(x,\ y) = \left(-\frac{b}{2a},\ \frac{4ac - b^2}{4a}\right)$ with $\frac{1}{2}$, -1 and 0 respectively. Then, they performed basic operations correctly and got correct answer.

Conversely, 12,898 (39.60%) candidates scored low marks. A number of mistakes were observed in their responses. In part (a) several candidates considered the domain $\{x : -2 \le x \le 3\}$ as the set of integers $\{-2, -1, 0, 1, 2, 3\}$ instead of real numbers. These candidates calculated $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, $g(2)$ and $g(3)$ to get 11, 8, 5, 2, -1 and -4 respectively. Therefore, they wrote $Range = \{y : y = 11, 8, 5, 3, 2, -1, -4\}$, which is an incorrect answer because it excludes some real numbers between -4 and 11. Also, some candidates presented the obtained interval incorrectly including $Range = \{y : 11 \le y \le -4\}$ and $Range = \{y : -4 \ge y \ge 11\}$. This shows that the candidates lacked knowledge of mathematical symbols. Furthermore, a significant number of candidates calculated the sum of the values of $g(-2)$ and $g(3)$ ending up with wrong answer, $Range = 7$.

In part (b), some candidates applied the incorrect formula $T(x, y) = \left(\frac{4ac - b^2}{4a}, \frac{-b}{2a}\right)$ instead of $T(x, y) = \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$. These candidates interchanged the components of the formula, which resulted in incorrect turning point $T(x, y) = \left(-\frac{1}{2}, 1\right)$. Moreover, other candidates integrated $h(x)=\frac{1}{2}x^2-x$. Some of these candidates worked correctly, but they were not awarded any marks because the approach that they used is not appropriate to the given question.

In part (c), a significant number of candidates misinterpreted the concept of inverse of a function (Extract 2.2). Also, some candidates did not interchange the variables. Instead, they correctly expressed x as the subject of $y=2x^2-5$, that is $x=\sqrt{\frac{y+5}{2}}$ and commented that $f(x)^{-1}=\sqrt{\frac{y+5}{2}}$.

They were supposed to complete the solution by replacing y in $\sqrt{\frac{y+5}{2}}$ with x so as to obtain the correct answer $f(x)^{-1}=\sqrt{\frac{x+5}{2}}$.

c)	$f(x) = 2x^2 - 5$
	Asked $= f(x)^{-1}$
	$F(x) = 2x^2 - 5$
	$f(x)^{-1} = (2x^2-5)^{-1}$
	$f(x)^{-1} = \frac{1}{2x^2-5}$
	$F(x)^{-1} = \frac{1}{2x^2-5}$
	$F(x)^{-1} = f(x)'$
	$\therefore f(x)' = \frac{1}{2x^2-5}$

Extract 2.2: A sample of incorrect response to part (c) of question 2

Extract 2.2 shows that the candidate computed the reciprocal function instead of the inverse of a function.

2.3 Question 3: Algebra

In part (a), the question measured the ability and skills of candidates to find n^{th} term and sum of the first n terms of the geometrical progression. The question was as follows; *In a sequence, the sum of the third and fourth terms is -12, the sum of the third and fifth terms is 60 and the sum of the fourth and fifth terms is 24. (i) Show that the terms form geometrical sequence. (ii) Find the sum of the first ten terms of the sequence.*

In part (b), the question measured competence of the candidates to apply the knowledge of perfect square to solve quadratic equation. The question statement was as follows; "*If the equation* $(2k+3)x^2+2(k+3)x+(k+5)=0$ *has equal roots, find the numerical value(s) of* x".

This question was attempted by 28,311 (80.94%) candidates, of which 3,508 (12.39%) scored from 3.5 to 10 marks. Therefore, the performance of the candidates was generally weak. Table 1 shows a summary of candidates' performance in this question.

Table 1: Number of Candidates and Their Scores in Question 3.

Scores	**0 - 3.0**	**3.5 - 5.5**	**6.0 - 10**
Number of Candidates	24,803	3,132	376
Percentage of Candidates	87.61%	11.06%	1.33%

Table 1 shows that 24,803 (87.61%) candidates got low marks. In part (a) (i), most candidates formulated correct equations from word problem, $x_3+x_4=-12$, $x_3+x_5=60$ and $x_4+x_5=24$, but failed to solve for x_3, x_4 and x_5. The challenge was on how to produce a strategic equation from the formulated equations. They were supposed to eliminate one variable (for example x_3) from two equations ($x_3+x_4=-12$ and $x_3+x_5=60$) to produce the strategic equation ($x_5-x_4=72$). Now, the equations $x_4+x_5=24$ and $x_5-x_4=72$ can be solved by either elimination or substitution method to get $x_4=-24$ and $x_5=48$. Then, substituting either $x_4=-24$ or $x_5=48$ into any equation containing x_3 ($x_3+x_4=-12$ or $x_3+x_5=60$) and solving it results to $x_3=12$. Some candidates misinterpreted the question that x_3+x_4, x_3+x_5 and x_4+x_5 form geometrical progression. Therefore, they approached the required verification inappropriately by computing common ratio from

$r = \frac{x_3 + x_5}{x_3 + x_4} = \frac{60}{-12}$ resulting in $r = -5$ and $x_1 = 0.05$. It was necessary for the candidates to realize that what are expected to form Geometrical Progression is the terms x_3, x_4 and x_5. Therefore, the common ratio could be obtained by performing $\frac{x_4}{x_3}$ or $\frac{x_5}{x_4}$ which leads to $r = -2$. The incorrect answers obtained in part (a) (i) affected negatively the answer of part (a) (ii). For instance, some candidates substituted $r = -5$ and $x_1 = 0.05$ into $S_n = \frac{G_1(r^n - 1)}{r - 1}$ and computed correctly, but obtained incorrect answer $S_{10} = -81380.2$. In addition, few candidates applied incorrect formulae, including $S_n = \frac{G_1(r^n - 1)}{1 - r}$ and $S_n = \frac{G_1(1 - r^n)}{r - 1}$ which led to $S_{10} = 1023$.

In part (b), a number of candidates failed to recall the condition for quadratic equation to have equal roots. They used incorrect approaches to answer the question. Some candidates wrongly interpreted that the equation $(2k+3)x^2 + 2(k+3)x + (k+5) = 0$ has equal roots if $k + 5 = 0$ which implies that $k = -5$. Therefore, they replaced k in $(2k+3)x^2 + 2(k+3)x + (k+5) = 0$ with -5 resulting in $7x^2 + 4x = 0$ and solved it to get $x = 0$ or $x = -\frac{4}{7}$. Extract 3.1 illustrates another incorrect response presented by some candidates. The candidates were supposed to recall that the condition for $ax^2 + bx + c = 0$ to have equal roots is $b^2 = 4ac$ and its solution is $x = -\frac{b}{2a}$ and therefore, considering the given equation, $a = 2k + 3$, $b = 2(k+3)$ and $c = k + 3$. When these expressions are substituted into the condition result in the equation which is equivalent to $k^2 + 7k + 6 = 0$ and therefore gives $k = -1$ or $k = -6$. Again, substituting $a = 2k + 3$ and $b = 2(k+3)$ into $x = -\frac{b}{2a}$ leads to $x = -\frac{k+3}{2k+3}$ giving $x = -2$ for $k = -1$ and $x = -\frac{1}{3}$ for $k = -6$.

3b)	Given,
	$(2k+3)x^2 + 2(k+3)x + (k+5) = 0$
	$(2kx^2 + 3x^2) + (2kx + 6x) + (k+5) = 0$
	Since have equal roots, then $k = 0$.
	$(2(0)x^2 + 3x^2) + (2(0)x + 6x) + (0+5) = 0$.
	$3x^2 + 6x + 5 = 0$.
	$\therefore$ Values of x are $x_1 = -1$ and $x_2 = -1$

Extract 3.1: A sample of incorrect response to part (b) of question 3

The candidate whose work is presented in Extract 3.2 used incorrect condition $k = 0$ for $(2k+3)x^2 + 2(k+3)x + (k+5) = 0$ to have equal roots.

Despite weak performance, 47 (0.17%) candidates scored all 10 marks allotted to this question. In part (a), the candidates interpreted the given word problem using the equations $x_3 + x_4 = -12$, $x_3 + x_5 = 60$ and $x_4 + x_5 = 24$. Then, they solved these equations simultaneously and obtained $x_3 = 12$, $x_4 = -24$ and $x_5 = 48$. Using the numerical values of x_3, x_4 and x_5, these candidates showed that the sequence is geometrical progression by verifying that $\frac{x_4}{x_3} = \frac{x_5}{x_4} = -2$, hence the common ratio $r = -2$. Also, the candidates used the value of either x_3, x_4 or x_5 and the formula $x_n = x_1 r^{n-1}$ to get $x_1 = 3$. Thereafter, they inserted $r = -2$, $x_1 = 3$ and $n = 10$ into the formula for calculating the sum of first n terms of geometrical progression $S_n = G_1\left(\frac{r^n - 1}{r - 1}\right)$ and computed correctly to get $S_{10} = -1023$. The candidates who performed well part (b) were knowledgeable about the condition for perfect square. These candidates replaced a, b and c in the condition $b^2 = 4ac$ with $2k+3$, $2(k+3)$ and $k+3$ and simplified the resulting equation into $k^2 + 7k + 6 = 0$ which was solved to get $k=-1$ or $k=-6$. Thereafter, they substituted k in the given equation with -1 and -6; and produced $x^2+4x+4=0$ and $9x^2+6x+1=0$ respectively. These candidates were also competent in methods of solving quadratic equations, as Extract 3.2 illustrates.

3. (b) given:

$$(2k+3)x^2+(2k+6)x+(k+5)=0 \quad \text{—(i)}$$

$a=2k+3$, $b=2k+6$, $c=k+5$

For equal roots:

$$b^2=4ac$$

$$(2k+6)(2k+6)=4(2k+3)(k+5)$$

$$4k^2+24k+36=4(2k^2+13k+15).$$

$$4k^2+24k+36=8k^2+52k+60$$

$$k^2+7k+6=0$$

$$k^2+1k+6k+6=0$$

$$k(k+1)+6(k+1)=0$$

$$(k+1)(k+6)=0$$

Either:

$k+1=0$ or $k+6=0$

$k=-1$ $\quad$ $k=-6.$

Returning to equation (i)

EITHER: $k=-1$

$$x^2+4x+4=0$$

$$x^2+2x+2x+4=0$$

$$x(x+2)+2(x+2)=0$$

$$(x+2)(x+2)=0$$

Either: $x+2=0$ or $x+2=0$

$x=-2$ $\quad$ $x=-2.$

$$\Rightarrow x_1=-2.$$

3. (b) OR: $k = -6$

$(-9x^2 - 6x - 1 = 0) \times -1$

$9x^2 + 6x + 1 = 0$

$a = 9$, $b = 6$ and $c = 1$.

From:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6 \pm \sqrt{(6)^2 - (4 \times 9 \times 1)}}{2(9)}$

$= \dfrac{-6 \pm \sqrt{0}}{18}$

$= -6/18$

$\Rightarrow x_2 = -1/3$.

$\therefore$ values of x are -2 and $-1/3$.

Extract 3.2: A sample of correct response to part (b) of question 3

Extract 3.2 shows that the candidate used factorization method to get $x = -2$ from $x^2 + 4x + 4 = 0$ and general quadratic formula to get $x = -\frac{1}{3}$ from $9x^2 + 6x + 1 = 0$.

2.4 Question 4: Differentiation

The question comprised parts (a) and (b). In part (a) the question measured ability of the candidates to find derivative of Cartesian and parametric expressions presented in the following subparts: *(i) Find* $\frac{dy}{dx}$ *of* $y = \sqrt{x^2 + 1}$. *(ii) Given the parametric equations* $x = (t^2 - 1)^2, \quad y = t^3$. *Find* $\frac{dy}{dx}$ *in terms of* t.

In part (b) the question measured competence of the candidates to apply the concepts of differentiation in solving real life problems particularly maximization problems. This part read as follows; *A metal wire whose* *length is 600 m is bent to make a rectangular fence. Calculate the dimensions of the fence that could give the maximum area.*

This question was attempted by 29,602 (84.64%) candidates. Amongst, 7,676 (25.93%) candidates scored from 3.5 to 10 marks. Therefore, the candidates had weak performance in this question. The percentages of candidates who scored low, average and high marks are shown in Figure 4.

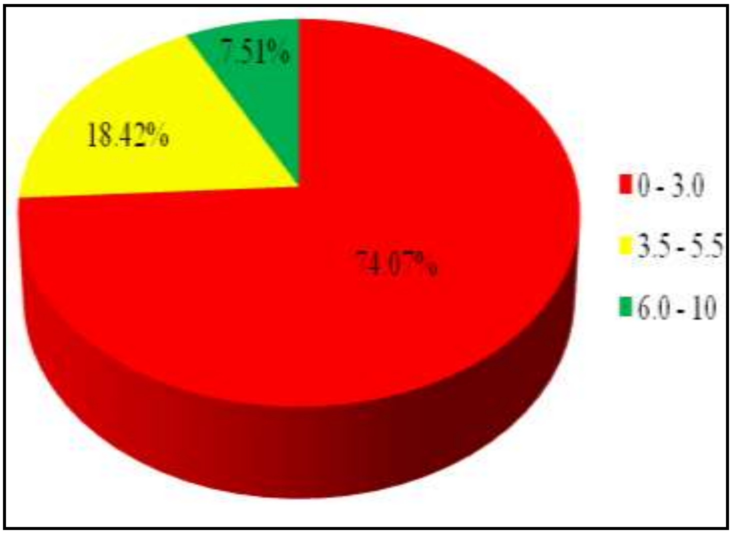


Figure 4: *The candidates' performance in question 4*

As Figure 4 shows, 74.07 percent, equivalent to 21,926 candidates scored 3.0 marks or less. Most candidates faced difficulty in using chain rule to find derivatives. For example, in part (a) some candidates got $\frac{dy}{dx} = \frac{1}{2\sqrt{x^2+1}}$ from $y = \sqrt{x^2+1}$ instead of $\frac{dy}{dx} = \frac{x}{\sqrt{x^2+1}}$. These candidates dealt with the derivative of square root as whole while ignoring the expression x^2+1 as shown in Table 2.

Table 2: Comparison of the Required Response and the Candidates' Responses.

Steps	Required response	Candidates' response
1^{st}	$y = \sqrt{x^2+1}$	$y = \sqrt{x^2+1}$ correct
2^{nd}	$y = (x^2+1)^{\frac{1}{2}}$	$y = (x^2+1)^{\frac{1}{2}}$ correct

Steps	Required response	Candidates' response
3rd	$\frac{dy}{dx}=\left(\frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}\right)(2x)$	$\frac{dy}{dx}=\frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}$ incorrect, ignored derivative of x^2+1
4th	$\frac{dy}{dx}=\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)$	$\frac{dy}{dx}=\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}$ incorrect, effect of 3rd step
5th	$\frac{dy}{dx}=\frac{x}{\sqrt{x^2+1}}$	$\frac{dy}{dx}=\frac{1}{2\sqrt{x^2+1}}$ incorrect, effect of 3rd step

This error was also observed in the responses of the candidates when finding the derivative of $x=\left(t^2-1\right)^2$ in part (a) (ii) as most of them wrote $\frac{dx}{dt}=2\left(t^2-1\right)$ instead of $\frac{dx}{dt}=2\left(t^2-1\right)(2t)=4t\left(t^2-1\right)$. Also, some candidates reduced the equations $x=\left(t^2-1\right)^2$ and $y=t^3$ into $y=\left(\sqrt{x}+1\right)^{\frac{3}{2}}$ by eliminating t. However, most of them faced the same challenge of using chain rule, and consequently failed to get $\frac{dy}{dx}=\frac{3}{4}\left(\frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}}\right)$. Few candidates ended at $\frac{dy}{dx}=\frac{3}{4}\left(\frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}}\right)$ which is against the requirements of the question. They were supposed to make substitution accordingly so as to express the derivative in terms of t.

In part (b), many candidates realized that the length of a wire equals to the perimeter of the formed rectangle, and therefore, they wrote $P=2(l+w)=600$. Also, they wrote the correct formula for calculating area of the rectangle as $A=l\times w$. However, most of these candidates failed to formulate a strategic equation $A=w(300-w)$ or $A=l(300-l)$ from $2(l+w)=600$ and $A=l\times w$. They also used incorrect condition for maximizing a function, as Extract 4.1 shows. The correct condition is

$\frac{dA}{dw}=0$ or $\frac{dA}{dl}=0$ where $\frac{dA}{dw}$ or $\frac{dA}{dl}$ is derived from either $A=w(300-w)$ or $A=l(300-l)$.

4b For a rectangular fence;

Perimeter = (L+W)2.

600 = (L+W) 2

$L+W = \frac{600}{2}$

L+W = 300 ···(i)

Area of rectangle; A

A = L X W.

A = LW.

Area maximum when Perimeter = 0.

~~k =~~

P = 2(L+W).

2(L+W) = 0.

L+W = 0.

L = -W.

A = L X W

$A = \frac{\cancel{k} \times 300}{\cancel{k}}$ A = 300.

∴ Length of the fence, L = (300-W)m.

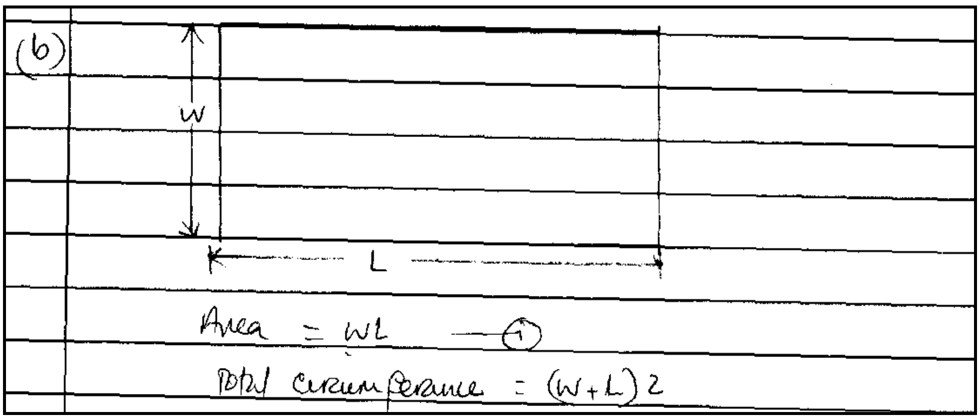
Width of the fence, W = (300-L) m.

Extract 4.1: A sample of incorrect response to part (b) of question 4

As Extract 4.1 shows, the candidate used incorrect condition $perimeter = 0$ as the basis for finding dimensions that maximize area of the rectangle.

In contrast, 2,222 (7.51%) candidates scored high marks, including 854 (2.88%) who obtained all 10 marks. All candidates who scored full marks were aware of the definition $\frac{d(x^n)}{dx} = nx^{n-1}$. In part (a) (i), these candidates used chain rule correctly. They let $x^2 + 1$ as u resulting in $y = \sqrt{u}$; hence $\frac{du}{dx} = 2x$ and $\frac{dy}{du} = \frac{1}{2\sqrt{u}}$. Then, they applied the chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ correctly and replaced u with $x^2 + 1$ to obtain $\frac{dy}{dx} = \frac{x}{\sqrt{x^2 + 1}}$. Similarly, in part (a) (ii), the candidates determined that $\frac{dx}{dt} = 4t(t^2 - 1)$ after applying chain rule in $x = (t^2 - 1)^2$ and getting $\frac{dy}{dt} = 3t^2$ from $y = t^3$. Finally, they substituted the expressions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$ into the formula $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ ending up with $\frac{dy}{dx} = \frac{3t}{4(t^2 - 1)}$.

In part (b), these candidates realized that the rectangular fence that could be made by bending a metal wire of 600 m is represented by the equation $l + w = 300$. They also developed the formula for calculating area of such fence as $A = w \times (300 - w)$. Therefore, the candidates applied the concept of differentiation to determine the dimensions for maximum area, as Extract 4.2 illustrates.


(b)

W

L

Area = WL —(1)

Total circumference = (W+L)2

	$2W+2L = 600m$
	$W+L = 300m$ — (ii) $\Rightarrow W = 300-L$
	fm (i)
	$A = WL$
	$A = (300-L)L$
	$A = 300L - L^2$
	Differentiate dA/dL
	$dA/dL = 300 - 2L$ — (iii)
	for maximum area
	$dA/dL = 0$
	$0 = 300 - 2L$
4(b)	$2L = 300m$
	$L = 150m$
	$W = 300 - L$
	$W = (300-150)m$
	$W = 150m$
	∴ The ~~Less~~ Dimension of the fence are
	length, $L = 150m$
	Width $= 150m$

Extract 4.2: A sample of correct response to part (b) of question 4

In Extract 4.2, the candidate applied the correct condition for maximizing area, $\frac{dA}{dl} = 0$ and worked appropriately to get the required dimensions.

2.5 Question 5: Integration

This question comprised parts (a) and (b). Part (a) measured the extent to which the candidates understood the relationship between derivatives and integrals. Its question stated as follows; *The first derivative of* $f(t)$ *is* $f'(t) = 6t + 1$. *Find* $f(t)$ *and the numerical value of* $f(10)$ *given that* $f(0) = 2$. In part (b), the question measured competence of the candidates to apply the knowledge of integration to find the area of a region. The candidates were asked to *find the area enclosed between the curve* $y = x(x-1)(x-2)$ *and the* x *axis.*

A total of 25,626 (73.27%) candidates responded to this question. Out of these, 3,965 (15.47%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was generally weak. Figure 5 shows the percentage of the candidates who scored low, average and high marks in this question.

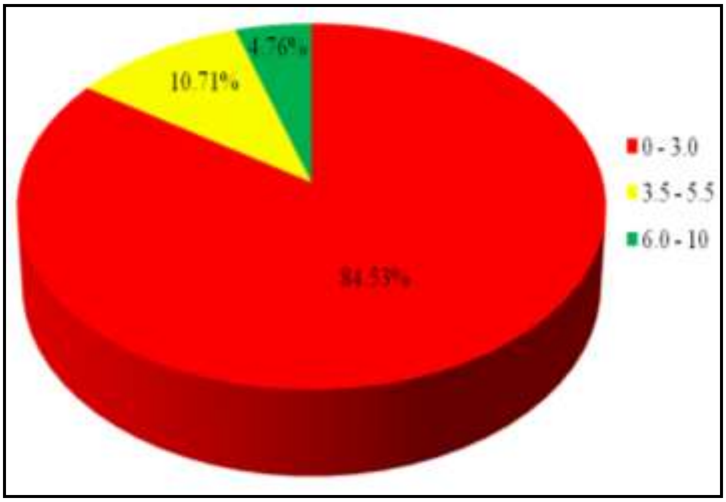


Figure 5: *The candidates' performance in question 5*

In this question, 21,661 (84.53%) candidates scored low marks. The analysis of candidates' responses revealed various weaknesses: In part (a), many candidates had the correct idea of evaluating $\int f'(t)dt$ so as to get $f(t)$, but most of them wrote $f(t) = \int (6t+1)dt = 3t^2 + t$ and therefore, $f(10) = 300$. These responses suggests that the candidates had insufficient knowledge of indefinite integrals as their solution lacks arbitrary constant. The solution should be $f(t) = \int (6t+1)dt = 3t^2 + t + c$. The statement $f(0) = 2$ implies that $f(t) = 2$ when $t = 0$. The solution $f(t) = 3t^2 + t + c$ becomes $2 = 3(0)^2 + (0) + c$, hence, $c = 2$. Therefore, a particular integral is $f(t) = 3t^2 + t + 2$, which gives $f(10) = 312$. Other candidates answered this part by calculating inverse of $f'(t)$. They interchanged y with t in $y = 6t + 1$ to get $t = 6y + 1$. Then, they expressed y in terms of t to get $y = \frac{t-1}{6}$ and therefore, ended up with the incorrect answers $f(t) = \frac{t-1}{6}$ and $f(10) = \frac{10-1}{6} = \frac{3}{2}$.

In part (b), many candidates understood that the question is set from applications of integration. However, most of them failed to identify the required regions and consequently used inappropriate limits of integration. They ignored the graphical representation of $y = x(x-1)(x-2)$, which seems to be one of the easiest means of identifying regions and limits of integration (since it is a visual method). Instead, the candidates opted to use algebraic technique. Most of them solved the equation $x(x-1)(x-2)=0$ correctly and got $x_1 = 0$, $x_2 = 1$ and $x_3 = 2$ but, they failed to use them appropriately. Most of these candidates used two of the three values of x. Work of the candidates who used $x_2 = 1$ and $x_3 = 2$ included the following steps: $A = \int_1^2 \left(x^3 - 3x^2 + 2x\right)dx = \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2 = -\frac{1}{4}$. Apart from excluding one region whose limits are $x = 0$ and $x = 1$, the candidates interchanged upper and lower function. In addition, a significant number of candidates used $x = 0$ and $x = 2$ as limits of integration and the sampled solution is presented in Extract 5.1. To respond correctly to this question, the area of each region is calculated using appropriate upper and lower functions as well as corresponding limits. The solution should include the following steps:

$$A = \int_0^1 \left(x^3 - 3x^2 + 2x\right)dx + \int_1^2 \left(-x^3 + 3x^2 - 2x\right)dx = \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left[-\frac{x^4}{4} + x^3 - x^2\right]_1^2 = \frac{1}{2} \text{ square unit.}$$

5b) $y = x^3 - 2x^2 - x^2 + 2x.$

$y = x^3 - 3x^2 + 2x.$, $x_1 = 2$ and $x_2 = 0$

Then,

Area $= \int_0^2 \left(x^3 + -3x^2 + 2x\right).$

$= \int_0^2 \left(x^3 - 3x^2 + 2x\right).$

$= \left[\frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}\right]_0^2 \cancel{+c}.$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^2$$

$$\therefore \text{ Area } = \left(\frac{(2)^4}{4} - (2)^3 + (2)^2\right) - \left(\frac{(0)^4}{4} - (0)^3 + (0)^2\right).$$

$$= (4 - 8 + 4) - (0).$$

$\therefore$ Area = 0 unit square

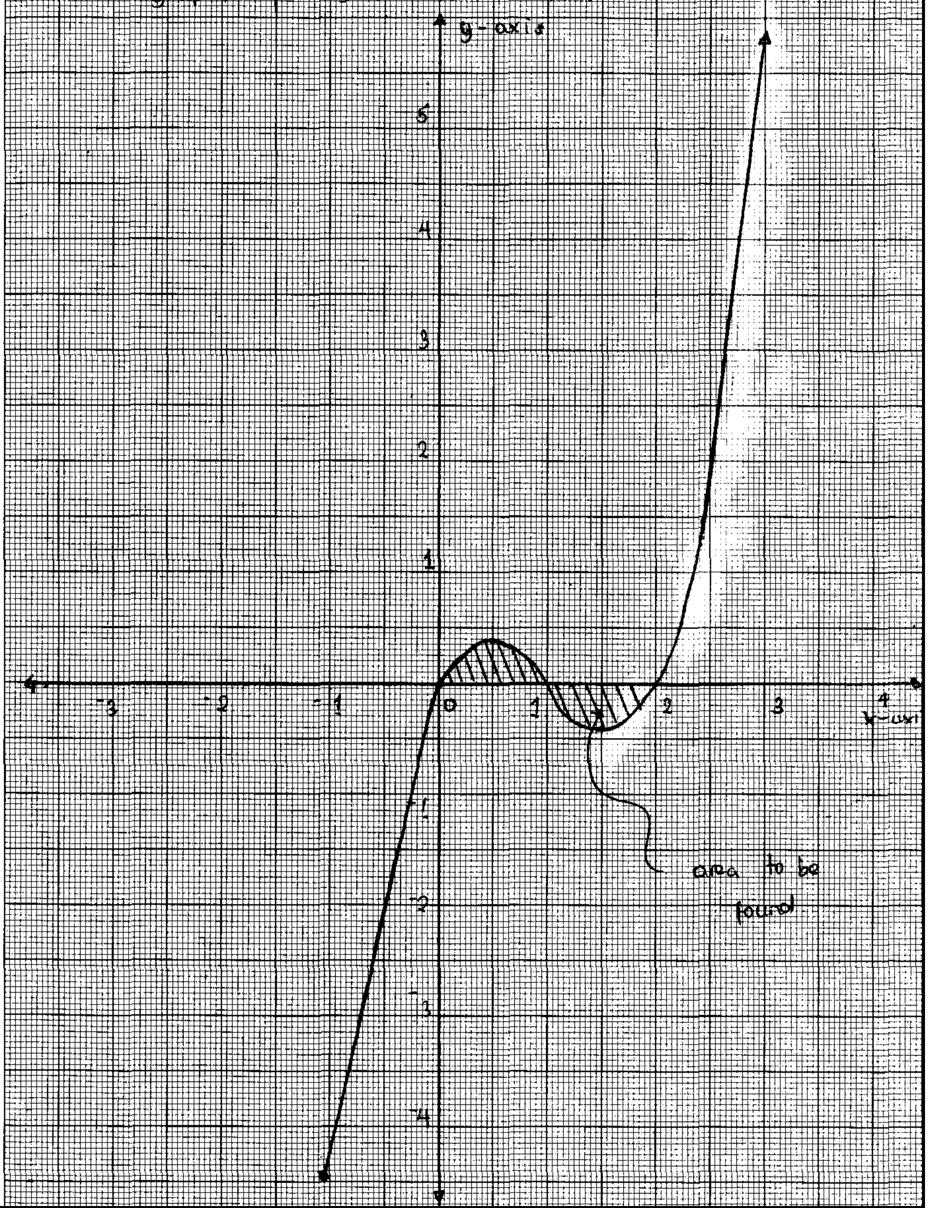
Extract 5.1: A sample of incorrect response to part (b) of question 5

In Extract 5.1, the candidate thought there is only one region from $x = 0$ to $x = 2$.

On the other hand, 179 (0.70%) candidates responded to this question correctly. In part (a) the candidates realized that the function $f(t)$ is obtained by evaluating the integral of the given derivative, that is $f'(t)$. Therefore, they evaluated $\int(6t+1)dt$ correctly to get $f(t) = 3t^2 + t + c$. The condition $f(0) = 2$ helped them to develop the equation $3t^2 + t + c = 2$ and solved it by taking $t = 0$ to obtain $c = 2$; and consequently $f(t) = 3t^2 + t + 2$. The candidates also correctly calculated $3t^2 + t + 2$ where $t = 10$ to get $f(10) = 312$.

In part (b), the candidates identified the required region correctly by drawing the graphs of $y = x(x-1)(x-2)$ (or otherwise). Then, they correctly determined the two regions and applied the formula for calculating area of the regions correctly, as shown in Extract 5.2.

05 (b) The graph of $y=(x-1)(x-2)x$.



0.5. (b) From

$$\text{Area} = \int_a^b f(x)\,dx.$$

$$A = \int_0^1 x(x-1)(x-2)\,dx + \int_1^2 x(x-1)(x-2)\,dx$$

$$= \int_0^1 (x^3 - 3x^2 + 2x)\,dx + \int_1^2 (x^3 - 3x^2 + 2x)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}\right]_0^1 + \left[\frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}\right]_1^2$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$$

$$0.25 + |-0.25|.$$

$$= 0.5 \text{ square units.}$$

Extract 5.2: A sample of correct response to part (c) of question 5

In Extract 5.2, the candidate introduced absolute brackets to avoid negative value that could distort the answer.

2.6 Question 6: Statistics

The question was framed as follows; *A biology teacher asked each of her 20 students to bring a grasshopper as a specimen for practical and the length of each grasshopper was recorded in centimeters as follows:*

1	*3*	*5*	*4*	*5*	*2*	*4*	*2*	*4*	*2*
4	*2*	*5*	*3*	*2*	*3*	*2*	*3*	*1*	*3*

(a) Prepare frequency distribution table (do not group the data).

(b) Calculate median of the data.

(c) Use assumed mean $A = 3$ *and coding method to calculate mean and standard deviation correct to 2 decimal places.*

The question was attempted by 33,456 (95.65%) candidates. A total of 17,191 (51.38%) candidates scored marks ranging from 3.5 to 10. Therefore, overall performance of the candidates in this question was average. The percentage of candidates who obtained low, average and high marks are shown in Figure 7.

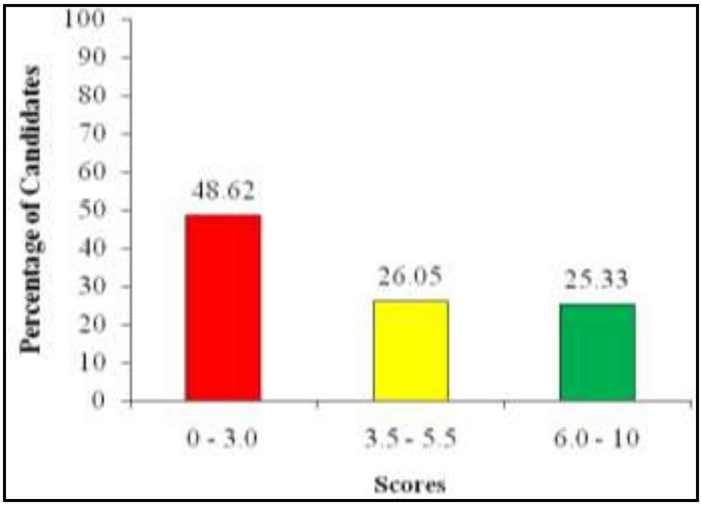


Figure 6: *The candidates' performance in question 6*

Data also reveals that 472 (1.41%) candidates scored all 10 marks allotted to this question. In part (a), the candidates prepared a table with columns/rows of lengths and frequency as follows:

Lengths (cm)	1	2	3	4	5
Frequency	2	6	5	4	3

The candidates who answered part (b) correctly knew the meaning of median (Extract 6.1). Also, some candidates identified that median class is the class whose value is 3. Therefore, class size is $c = 1$, lower boundary is $L = 2.5$, frequency of median class is $f_w = 5$ and total frequency of the classes with lesser values than that of median class is $f_b = 8$. Then, they substituted the values into the formula $Median = L + \left(\frac{\frac{N}{2} - f_b}{f_w} \right) c$ and computed correctly to obtain $Median = 2.9$. Few candidates answered this part by representing the data using ogive and estimating median from it. Through this approach, the acceptable value for median ranged from 2.5 to 3.4. In part (c), the candidates presented an extended frequency distribution table that includes all columns necessary for calculating mean and standard deviation using coding formulae (Extract 6.2).

b/ median of the data.

By using the ungrouped data formula.

b.

arranged order

1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5

$$\text{Median} = \frac{\left(\frac{1}{2}N\right)^{th} + \left(\frac{1}{2}N + 1\right)^{th}}{2}$$

$$= \frac{10^{th} + 11^{th}}{2} = \frac{3+3}{2} = 3$$

Hence the median is 3

Extract 6.1: A sample of correct response to part (b) of question 6

In Extract 6.1, the candidate arranged data in ascending order, identified that the tenth and eleventh data are the appropriate middle values and calculated their average, ending up with $Median = 3$.

06. (c). Asumed mean $\bar{x} = 3$.

x	f_i	$d_i (x - \bar{x})$	$u = d_i/c$	$f_i u^2$	$f_i u_i$	$f_i u_i^2$
1	2	~~2~~ -2	-2	4	-4	8
2	6	~~6~~ -1	-1	1	-6	6
3	5	~~5~~ 0	0	0	0	0
4	4	~~4~~ 1	1	1	4	4
5	3	~~3~~ 2	2	4	6	12
summations	20				0	30

$$\text{mean} = A + c\,\frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f}$$

$$= 3 + \frac{1}{20}(0) = 3.$$

Hence mean $= 3.00$

$$Var(x) = c^2\left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right]$$

$$= 1^2\left(\frac{1}{20} \times 30. - \frac{1}{20}(0)\right)$$

$$= 1.5$$

standard deviation $= \sqrt{var(x)}$

$$= \sqrt{1.5}$$

$$\sigma(x) = 1.22.$$

Extract 6.2: A sample of correct response to part (c) of question 6

In Extract 6.2, the candidate filled up the table correctly and applied appropriate formulae to compute mean and standard deviation.

Moreover, 16,256 (48.62%) candidates got low marks due to various weaknesses. Most candidates answered part (a) wrongly by naming the upper and lower rows of the table as data values (x) and frequency (f) respectively. These candidates did not understand the explanations given before the table. The explanations clarify that each number in the table represents length of one grasshopper. Some candidates grouped the data, which is contrary to the instructions of the question (*do not group the data*). In part (b), many candidates applied correct formula however; they substituted incorrect data (Extract 6.3), and consequently got a wrong answer. Also, some candidates applied incorrect formula for calculating median, for example $Median = L_1 + \left(\frac{\frac{N}{2} + fb}{fw}\right)i$ instead of

$Median = L_1 + \left(\dfrac{\frac{N}{2} - fb}{fw} \right) i$. Furthermore, few candidates wrongly defined median as *the most occurring value in the set;* which is the definition of mode, not median. As a result they got $Median = 2$. In part (c), many candidates failed to recall correct coding formulae for calculating mean and standard deviation. Instead, they used incorrect formulae such as $Mean = \dfrac{\sum fu}{\sum f}$ and $\text{Standard deviation} = \dfrac{\sum fu}{\sum f} - \dfrac{\sum fu^2}{\sum f}$. Some of the candidates used correct formulae, but substituted incorrect values such as $Mean = 3 + \left(\dfrac{30}{20} \right) 1 = 4.5$.

from median $= L + \left(\dfrac{N/2 - nb}{nw} \right) i$

where $L = 4.5$

$N/2 = 40/2$

$N = 20$

$nb = 6$, $nw = 5$

Median $= L + \left(\dfrac{N/2 - Nb}{nw} \right) i$

$= 4.5 + \left(\dfrac{20 - 6}{5} \right) i$

$= 4.5 + \left(14/5 \right) 1$

Median $= 4.5 + 2.8$

$= 7.3$

$\therefore$ Median $= 7.3$

Extract 6.3: A sample of incorrect response to part (a) of question 6

In Extract 6.3, the candidate substituted incorrect values for lower boundary (L) and total frequency of the classes with lesser values than that of median class (nb).

2.7 Question 7: Probability

This question tested the candidates' competence in finding probability of simple and combined events. It comprised the parts (a) and (b) as follows:

(a) A six sided die is thrown. Find the probability that an odd number will show up.

(b) Three coins are tossed at once.

(i) Use tree diagram to illustrate all possible outcomes.

(ii) Find the probability of getting at least two heads.

This question was attempted by 30,549 (87.34%) candidates, of which 23,986 (78.52%) got from 3.5 to 10 marks. Therefore, the candidates had good performance in this question. Figure 7 shows percentages of candidates who got low, average and high marks.

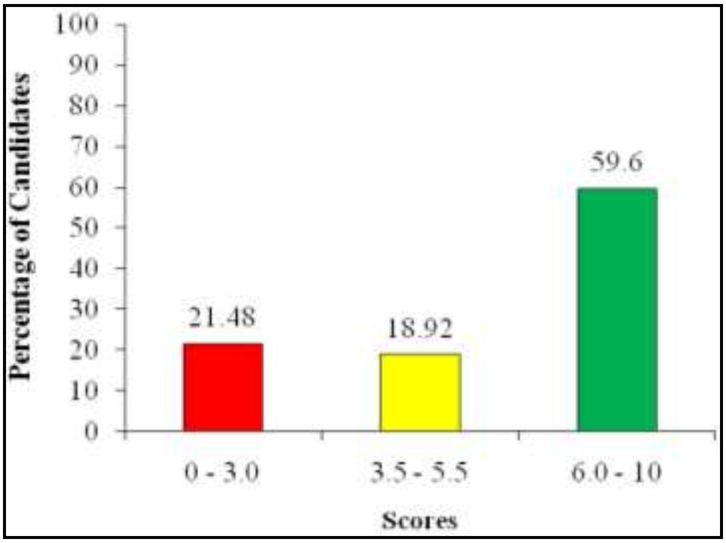


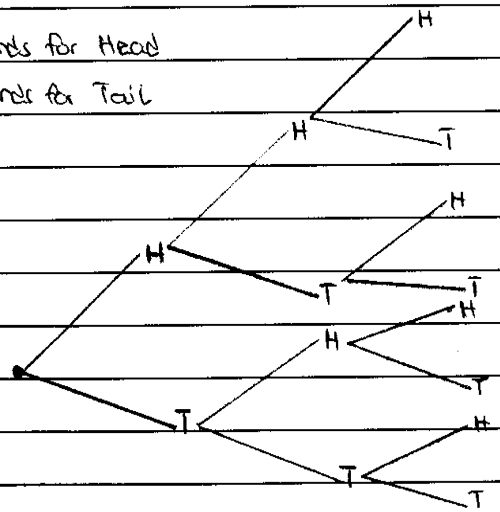
Figure 7: *The candidates' performance in question 7*

A total of 7,071 (23.15%) candidates did the question correctly, scoring all 10 marks. In part (a), the candidates identified the sample space as 1, 2, 3, 4, 5 and 6, and got the number of sample space $(n(S))$ as 6. They also identified odd numbers from a sample space as 1, 3 and 5, hence the number of event $(n(E))$ is 3. Then, they applied the formula for calculating probability of an event, $P(E)=\frac{n(E)}{n(S)}$ to get $P(E)=\frac{1}{2}$. As Extract 7.1 shows, these candidates also adhered to the instructions of part (b) and computed probability of getting at least two heads correctly, $P(E)=\frac{1}{2}$.

b) Note

H stands for Head

T stands for Tail



i) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) : 8$

ii) E = (atleast two heads) = $\{HHH, HHT, HTH, THH\}$

$n(E) : 4$

$P(E) : \frac{n(E)}{n(S)}$

$: 4/8 = 1/2$ $\therefore$ Probability for at least two heads is $1/2$

Extract 7.1: A sample of correct response to part (a) of question 7

In Extract 7.1, the candidate drew a tree diagram and determined the number of sample space and that of event correctly. This candidate had sufficient knowledge and skills related to combined events.

In contrast, 23,478 (76.85%) candidates scored low marks including 1,936 (6.34%) candidates who scored zero. In part (a), some of these candidates misinterpreted the problem. They perceived a die was tossed twice, and consequently got incorrect number of sample space and event, which were 36 and 18 respectively. Other candidates applied inappropriate technique of permutation by calculating ${}^{6}P_{3}$ (see Extract 7.2) without considering the fact that permutations do not give probability of an event. They failed to recall that permutation is the technique for finding the number of arrangements of r objects from a set of n objects.

In part (b), many candidates drew the tree diagram incorrectly. As a result, they came up with a wrong number of sample space and event, and thus incorrect probability. Extract 7.3 is a sample of a solution depicting an incorrect tree diagram. Also, some candidates presented a wrong sample space by listing outcomes of all three coins in the same braces, $S = \{H, T, H, T, H, T\}$. Thereafter, they wrote $n(s) = 6$ and $n(E) = 2$ which led to incorrect probability of an event, $P(E) = \frac{2}{6} = \frac{1}{3}$.

7. a) Soln:

Odd number can be 1, 3, 5

$nPr = \frac{n!}{(n-r)!r!}$

$n = 6$

$r = 3$

$6P_3 = \frac{6!}{(6-3)!\,3!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3!\,3!}$

$= \frac{\cancel{6}^{2} \times 5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{3} \times \cancel{2} \times \cancel{1}}$

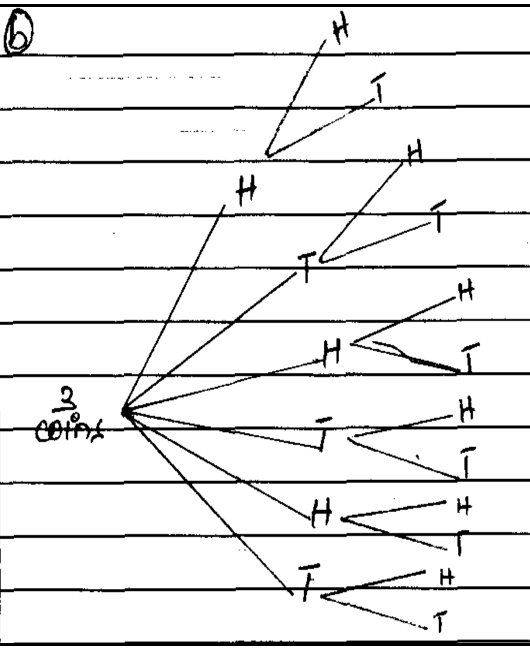
$= 20$

$\therefore$ The probability is 20

Extract 7.2: A sample of incorrect response to part (a) of question 7

In Extract 7.2, the answer *probability is 20* indicates that the candidate lacked knowledge of probability axiom, which states that probability of an event ranges from 0 to 1.

07 (b)



$n(S) = \{HH, HT, TH, TT, HH, HT, TH, TT, HH, HT, TH, TT\}$

probability of getting at least two heads

$n(E) = 3$

$n(S) = 12$

$P = \frac{n(E)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

$\therefore$ The probability of getting at least two heads is $\frac{1}{4}$.

Extract 7.3: A sample of incorrect response to part (b) of question 7

The tree diagram shown in Extract 7.3 leads to incorrect outcomes, each containing two sub-elements instead of three sub-elements as seen in Extract 7.1.

2.8 Question 8: Trigonometry

The question tested competence of the candidates to: (a) prove trigonometric identities; (b) solve trigonometric equations; and (c) apply cosine rule to calculate angles and sides of triangle. The question read as follows;

(a) If $\cos(x+\beta)=2\sin(x-\beta)$ *, show that* $\tan x=\dfrac{1+2\tan\beta}{2+\tan\beta}$.

(b) Solve the equation $2\sin\theta+\cos 2\theta=1$ *for* $0^\circ\le\theta\le 180^\circ$.

(c) In a triangle XYZ, $\overline{XY}=30\,\text{m}$, $\overline{YZ}=40\,\text{m}$ *and* $\overline{ZX}=60\,\text{m}$. *Calculate the angle formed by the sides* $\overline{XY}$ *and* $\overline{YZ}$.

This question, 26.71 percent, equivalent to 6,633 candidates got 3.5 marks or more. This indicates that overall performance of the candidates in this question was weak. A summary of candidates' performance in this question is presented in Figure 8.

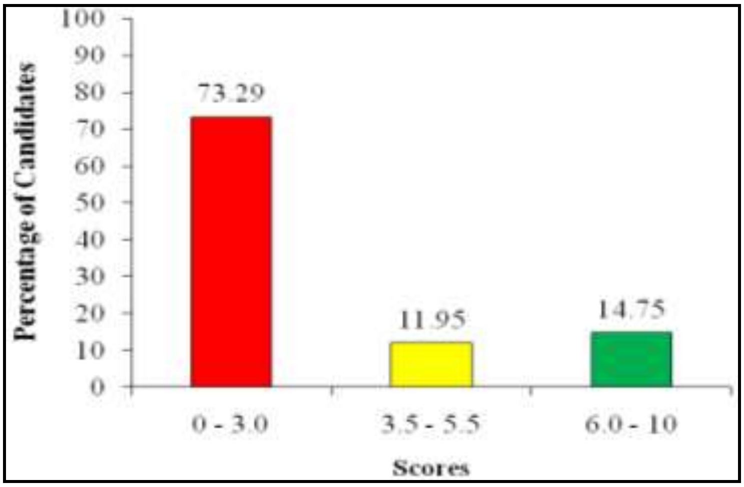


Figure 8: *The candidates' performance in question 8*

The data reveals further that 18,205 (73.29%) candidates scored 3.0 marks or less. In part (a), most of these candidates failed to demonstrate knowledge and skills of compound angles. They expanded $\cos(x+\beta)$ and $\sin(x-\beta)$ incorrectly. For instance, some candidates expanded $\cos(x+\beta)$ as $\cos x\cos\beta+\sin x\sin\beta$ and $\sin(x-\beta)$ as $\sin x\cos\beta+\cos x\sin\beta$, and thus ended up with $\tan x=\dfrac{2\sin\beta-\cos\beta}{\sin\beta+2\cos\beta}$, which cannot be maneuvered to get $\tan x=\dfrac{1+2\tan\beta}{2+\tan\beta}$. The correct expansions are $\cos(x+\beta)=\cos x\cos\beta-\sin x\sin\beta$ and $\sin(x+\beta)=\sin x\cos\beta+\cos x\sin\beta$. Also, some candidates were not conversant with trigonometric terms as demonstrated in Extract 8.1. It should be known that the words *sin* and *cos* are mathematically meaningless unless they are written together with an angle, for example $\sin A$ or $\cos x$.

In part (b), some candidates did not realize the necessity of writing the $\cos 2\theta$ in terms of $\sin\theta$. They worked on equation containing both $\cos 2\theta$ and $\sin\theta$ which do not give required factors and answers. Other candidates struggled to formulate the equivalent equation, $2\sin^2\theta - 2\sin\theta = 0$ but, they applied inappropriate double angle identity. For example, some candidates defined both $\cos 2\theta$ and $\cos^2\theta$ wrongly as $\cos^2\theta + \sin^2\theta$ and $1 + \sin^2\theta$ respectively. The correct identities are $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$. A sample of an incorrect answer to part (b) of this question is presented in Extract 8.2.

In part (c), many candidates failed to realize that the precise approach for a particular question is cosine rule (Extract 8.3). Instead, they applied knowledge of other concepts that do not give the correct answer. For instance, some candidates computed $\tan^{-1}\left(\frac{40}{30}\right)$ which applies to right angled triangles (not otherwise). This approach is irrelevant to the given question because the triangle is not right angled as its sides do not obey the Pythagoras theorem, that is $(30)^2 + (40)^2 \neq (60)^2$. Other candidates applied sine rule, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{C}$. The data given in this question do not fulfill the requirements of sine rule. Sine rule requires two sides and one angle which is opposite to one of the two sides or two angles and one side which is opposite to one of the two angles.

(a) Solution

$\tan x = \frac{1 + 2\cdot\tan \beta}{2 + \tan \beta}$

from

$\cos(x + \beta) = 2\sin(x - \beta)$

$\cos x + \cos \beta = 2\sin x - 2\sin \beta$

Extract 8.1: A sample of incorrect response to part (a) of question 8

In Extract 8.1, the candidate considered each of the words *sin* and *cos* as a complete mathematical term.

8. b) $2\sin\theta + \cos 2\theta = 1$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$.
but $\cos^2\theta = 1 - \sin^2\theta$
$\cos 2\theta = (1 - \sin^2\theta) - \sin^2\theta$
$\cos 2\theta = -\sin^2\theta + \sin^3\theta$.
$2\sin\theta - \sin^2\theta + \sin^3\theta = 1$
$\sin^3\theta - \sin^2\theta + 2\sin\theta = 1$ 167.6
let $\sin\theta = a$
$a^3 - a^2 + 2a = 1$
$a = 0.5698$, $a = 0.215$

$a = \sin\theta$ $\cancel{a=}$
$0.5698 = \sin\theta$. OR.
$\theta = \sin^{-1}(0.56984)$
$\theta = 34.7°$.
OR
$a = \sin\theta$
$0.215 = \sin\theta$
$\theta = \sin^{-1}(0.215)$
$\theta = 12.4°$

for $0° \leq \theta \leq 180°$.
1st Quadrat.
$\theta = 34.7°$ or $\theta = 12.4°$.

2nd quadrat
$\theta = 180 - \alpha$
$\theta = 180 - 34.7°$
$\theta = 145.3°$

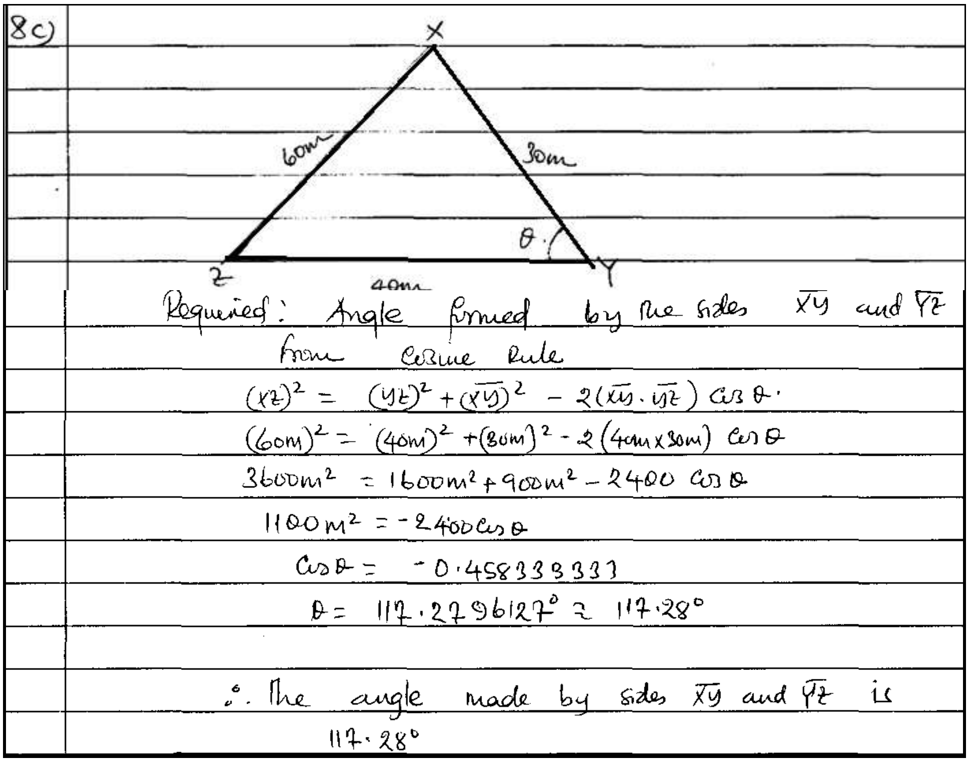
Extract 8.2: A sample of incorrect response to part (b) of question 8

In Extract 8.2, the candidate opened the brackets in $(1-\sin^2\theta)-\sin^2\theta$ wrongly by performing multiplications instead of subtractions.

In spite of weak performance, 450 (1.81%) candidates scored all 10 marks. In part (a) the candidates recalled the expansion of $\cos(x+\beta)$ and $\sin(x-\beta)$. Therefore, they wrote $\cos(x+\beta) = 2\sin(x-\beta)$ as

$\cos x\cos\beta - \sin x\sin\beta = 2\sin x\cos\beta - 2\sin\beta\cos x$ and rearranged it to get $\frac{\sin x}{\cos x} = \frac{\cos\beta + 2\sin\beta}{2\cos\beta + \sin\beta}$. Then, they applied the identity $\tan x = \frac{\sin x}{\cos x}$ and divided both numerator and denominator of $\frac{\cos\beta + 2\sin\beta}{2\cos\beta + \sin\beta}$ by $\cos\beta$ to obtain $\tan x = \frac{1 + 2\tan\beta}{2 + \tan\beta}$.

In part (b), the candidates applied appropriate double angle identity, that is $\cos 2\theta = 1 - 2\sin^2\theta$. Therefore, they replaced $\cos 2\theta$ in $2\sin\theta + \cos 2\theta = 1$ with $1 - 2\sin^2\theta$ and rearranged the resulting equation into $2\sin^2\theta - 2\sin\theta = 0$. Then, they solved it using either factorization, completing the square or general quadratic formula to get $\sin\theta = 0$ or $\sin\theta = 1$ and consequently $\theta = \sin^{-1}(0)$ or $\theta = \sin^{-1}(1)$ which give $\theta = 0^\circ, 90^\circ, 180^\circ$. In part (c), the candidates stated and applied the cosine rule correctly as shown in Extract 8.3.


8c)

Required: Angle formed by the sides $\overline{XY}$ and $\overline{YZ}$

from Cosine Rule

$(XZ)^2 = (YZ)^2 + (XY)^2 - 2(\overline{XY} \cdot \overline{YZ})\cos\theta$

$(60m)^2 = (40m)^2 + (30m)^2 - 2(40m \times 30m)\cos\theta$

$3600m^2 = 1600m^2 + 900m^2 - 2400\cos\theta$

$1100m^2 = -2400\cos\theta$

$\cos\theta = -0.458333333$

$\theta = 117.2796127^\circ \approx 117.28^\circ$

$\therefore$ The angle made by sides $\overline{XY}$ and $\overline{YZ}$ is 117.28°

Extract 8.3: A sample of correct response to part (b) of question 8

Extract 8.3 shows that the candidate sketched a well labeled triangle and correctly computed to get the required angle, $\theta = 117.28^\circ$.

2.9 Question 9: Exponential and Logarithmic Functions

The question comprised the following parts:

(a) If $f(x) = a^x$ *where* $x \in R$*;*

(i) Write down the condition(s) on $"a"$ *such that* $f(x)$ *is an exponential function.*

(ii) Show that $\dfrac{f(x+1)}{f(x)} = a$.

(b) Evaluate $\int_3^5 \frac{5}{(3x-8)}dx$ *correct to 2 decimal places.*

(c) Suppose Tsh 2000 is invested to an account which offers 7.125% compounded monthly.

(i) Express the amount A in the account as a function of the term of the investment t *years.*

(ii) How long will it take for the initial investment to double?

Out of 28,131 candidates who attempted this question, 4,284 (15.23%) scored from 3.5 to 10 marks. Therefore, the candidates' performance in this question was weak. This performance is presented in Figure 9.

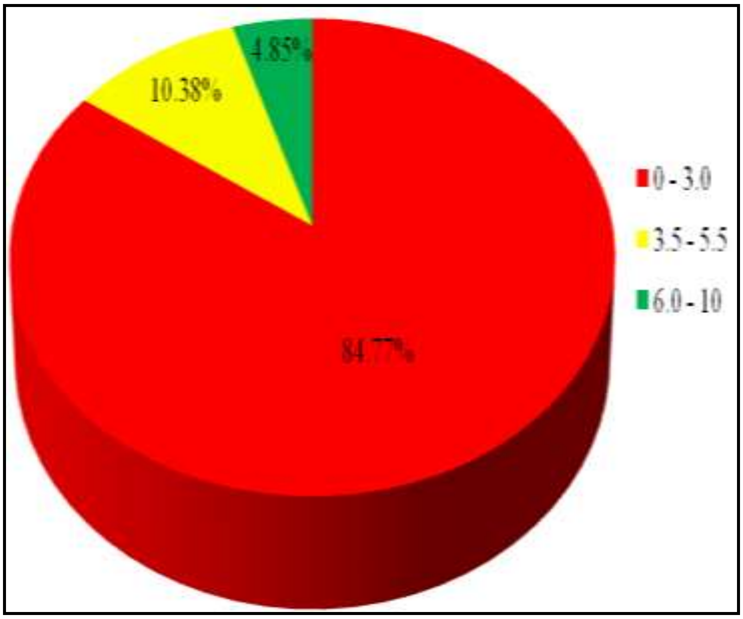


Figure 9: *The candidates' performance in question 9*

A total of 23,847 (84.77%) candidates scored low marks. Most of these candidates were not conversant with the definition of exponential function, and thus provided incorrect answers for part (a) (i). For instance, some candidates wrote $a \neq 0$ *and* a *is an integer*. This sentence includes 1 and all negative integers, which distorts the meaning of the given function. These candidates did not realize that if $a = 1$, then $f(x) = 1^x = 1$, which is a constant function. Also, if a is less than zero, a^x could not give the real value for some real numbers, particularly $x = \frac{1}{2}$. These candidates were supposed to write any statements that include all positive real numbers except 1, such as $a > 0$ and $a \neq 1$. In part (a) (ii), many candidates failed to develop $f(x+1)$ from $f(x) = a^x$. They wrote $f(x+1) = a^x + 1$ instead of $f(x+1) = a^{x+1}$. Although most of them applied the law of quotient of exponents correctly, they failed to verify the given statement. These candidates obtained $\frac{f(x+1)}{f(x)} = 1 + a^{-x}$, which is not equivalent to $\frac{f(x+1)}{f(x)} = a$.

In part (b) many candidates started well as they let $u = 3x + 8$ and developed $du = 3dx$ in order to change the variable from x to u. However, these candidates did not change the limits so as to reflect the variable u. They wrote $\int_3^5 \frac{5}{(3x-8)} dx = \frac{5}{3}\int_3^5 \frac{du}{u}$ instead of substituting $x = 3$ and $x = 5$ into $u = 3x - 8$ so as to obtain $u = 1$ and $u = 7$ respectively that would result in $\int_3^5 \frac{5}{(3x-8)} dx = \frac{5}{3}\int_1^7 \frac{du}{u}$. Moreover, some of these candidates failed to evaluate $\int \frac{du}{u}$ (study Extract 9.1). It is clear that $\frac{1}{u}$ is the standard integrand whose integral is $\ln u$ and therefore, $\int_a^b \frac{du}{u} = [\ln u]_a^b$.

In part (c) (i), many candidates applied inappropriate formula as shown in Extract 9.2 while others simply wrote $A(t) = 2000(1.07125)^{nt}$. These candidates misinterpreted the problem. They thought the investment is compounded annually instead of monthly. The correct formula for calculating compound interest for this particular case is $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ which gives $A(t) = 2000(1.0059375)^{12t}$ after substituting $P = 2000$, $r = 7.125\%$ and $n = 12$ into it and performing appropriate operations. The incorrect answers obtained in part (a) (i) led to incorrect solutions for part (c) (ii). Other candidates presented the correct equation $A(t) = 2000(1.0059375)^{12t}$, however, they failed to get the correct answer for part (ii) as they had difficulties in working with logarithms.

9 (b) $\int_3^5 \frac{5}{(3x-8)} dx$

let $u = 3x - 8$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

Also ~~from $u = 3x-8$~~

Then $= \int_3^5 \frac{5}{u} \cdot \frac{du}{3}$

$= 5/3 \int_3^5 \frac{du}{u}$

$= 5/3 \int_3^5 u^{-1} du$

9(b) $= 5/3 \left[\frac{u^{-1+1}}{-1+1} \right]_3^5$

$= \frac{5}{3} \left[\frac{u^0}{0} \right]_3^5$

$= 5/3 \times 1$

$= 5/3$

$\therefore \int_3^5 \frac{5}{3x-8} dx = 5/3$

Extract 9.1: A sample of incorrect response to part (b) of question 9

Extract 9.1 shows that apart from failing to change the limits, the candidate applied the definition $\int u^n dx = \frac{u^{n+1}}{n+1} + c$, which led to undefined expression

$\frac{u^0}{0}$.

9\. (c) from $I = PRT$

then

(i) $I = 2000 \times \frac{7.125}{100} \times \frac{t}{12}$

(ii) To double $\cancel{= 2(2000)}$ $I' = 2I$

$\cancel{= 4000}$

then

$I = 4000 \times \frac{7.125}{100} \times t$

. but for one year I is

$I = 2000 \times \frac{7.125}{100} \times 1$

$I = 142.5$ /=

then

for 4000

$2I' = 2000 \times \frac{7.125}{100} \times t$

but $I = 142.5$

9 (c) (ii) $285 = 2000 \times \frac{7.125}{100} \times t$

$t = 285 / 142.5$

$t = 2$

Time would be 2 years

Extract 9.2: A sample of incorrect response to part (c) of question 8

In Extract 9.2, the candidate applied the formula for calculating simple interest instead of compound interest.

Although the general performance in this question was weak, there were 1,364 (4.85%) candidates who obtained high marks, including 10 (0.04%) candidates who scored all 10 marks. In part (a) (i), these candidates presented the statements which excludes real numbers that make $f(x)$ undefined. Although they wrote different statements, all of them explained the required conditions. Such statements include: a is not less than 0, $a \neq 0$ and $a \neq 1$; $a > 0$ and $a \neq 1$; and a *is all positive real numbers except 1*. In part (a) (ii) the candidates realized that $f(x+1) = a^{x+1}$ and they applied the law of quotient of exponents as follows: $\frac{f(x+1)}{f(x)} = \frac{a^{x+1}}{a^x} = a^{x+1-x} = a$. In part (b), the candidates applied the substitution technique correctly. They let $u = 3x - 8$ that gives $u = 1$ when $x = 3$ and $u = 7$ when $x = 5$. They further differentiated $u = 3x - 8$ to get $du = 3dx$ and rearranged into $dx = \frac{1}{3}du$. After substituting, they changed $\int_3^5 \frac{5}{(3x-8)}dx$ into $\frac{5}{3}\int_1^7 \frac{du}{u}$, yielding $\frac{5}{3}\left[\ln|u|\right]_1^7$ and consequently $\frac{5}{3}\ln 7$.

In part (c) (i) the candidates' responses demonstrated good understanding of using the formula for calculating compound interest. The commonly observed formulae were $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ (taking rate, $r = 0.07125$) and $A(t) = P\left(1 + \frac{r}{100n}\right)^{nt}$ (taking rate, $r = 7.125$). Other parameters were P that stands for principal and n that stands for the number of periods of calculating compound interest in one year. Therefore, they substituted $P = 2000$ and $n = 12$ into the formula, and they computed correctly to get $A(t) = 2000(1.0059375)^{12t}$. In part (c) (ii), the candidates realized that if initial investment doubles, then $A(t) = 4000$. Therefore, they replaced $A(t)$ in $A(t) = 2000(1.0059375)^{12t}$ with 4000 and computed correctly to get $t = 9.75$ years, as Extract 9.3 shows.

iv/ Solution

For double of investment

$$2P = 2000\,(1.0059375)^{12t}$$

$$2 \times 2000 = 2000\,(1.0059375)^{12t}$$

$$2 = (1.0059375)^{12t}$$

Applying log both sides.

9c

$$\log 2 = 12t \log 1.0059375$$

$$\frac{\log 2}{\log 1.0059375} = 12t$$

$$\frac{117.086}{12} = \frac{12t}{12}$$

$$t = 9.76 \text{ years.}$$

$\therefore$ Time taken will be 9 years and 9 months

Extract 9.3: A sample of correct response to part (c) of question 9

In Extract 9.3, the candidate applied the laws of logarithms and correctly evaluated the time for initial investment to double.

2.10 Question 10: Linear Programming

In this question, the candidates were given the following word problem: *An engineer wants to make at least 6 steel tables and 9 wooden tables every day. He does not want to make more than 30 tables per day. A steel table requires 3 units and a wooden table 2 unit of workshop space and there are at least 54 units of workshop space available. The profit of making a steel table is Tsh 5800 and wooden table is Tsh 3600.*

(a) How many steel and wooden tables should be manufactured per day to realize the maximum profit?

(b) What is maximum profit?

A total of 32,794 (93.76%) candidates attempted this question and their performance is summarized in Figure 10.

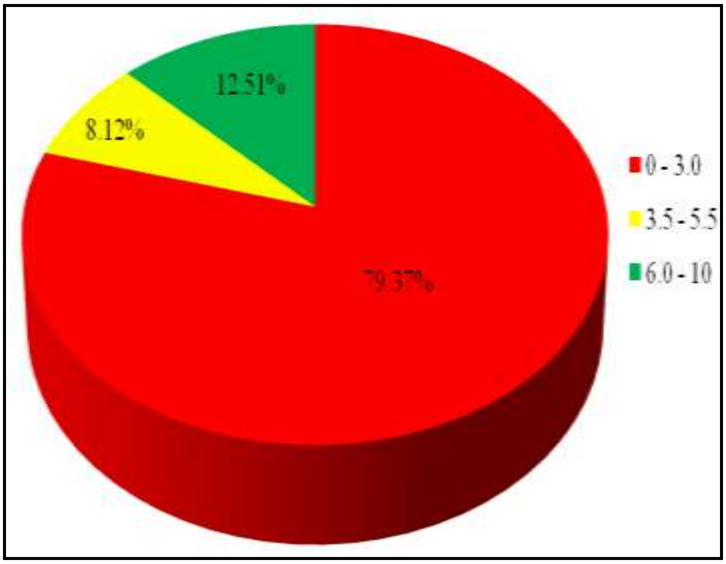


Figure 10: *The candidates' performance in question 10*

As Figure 10 shows, 26,029 (79.37%) candidates scored marks ranging from 0 to 3.5, suggesting that the performance in this question was weak. Although most of them started well by letting x for number of steel tables and y for number of wooden tables, they failed to formulate correct constraints. Most of the candidates misinterpreted the statements *An engineer wants to make at least 6 steel tables and 9 wooden tables every day*. *He does not want to make more than 30 tables per day*. They translated the statements into one constraint as $6x+9y\leq 30$, instead of three as $x\geq 6$ (for making at least 6 steel tables), $y\geq 9$ (for making at least 9 wooden tables), and $x+y\leq 30$ (for making not more than 30 tables). Other candidates misinterpreted the conditional words. For example, some candidates misinterpreted the word *at least* as less than or equal instead of greater than or equal. They thus translated the statement *A steel table requires 3 units and wooden table 2 units of workshop space and there are at least 54 units of workshop space available* as $3x+2y\leq 54$. Consequently, these candidates presented an incorrect graph, as illustrated in Extract 10.1. A few candidates obtained correct constraints but drew incorrect graphs.

10 Solution

let x represent the number of units of steel
let y represent the number of units of wood

	Days	Workshop	Profit
Steel	6	3	5800
Wood	9	2	3600
Total	30	54	

The objective function.
$f(x,y) = 5800x + 3600y$

The constraints
$6x + 9y \leq 30$
$3x + 2y \leq 54$
$y \geq 0, x \geq 0$

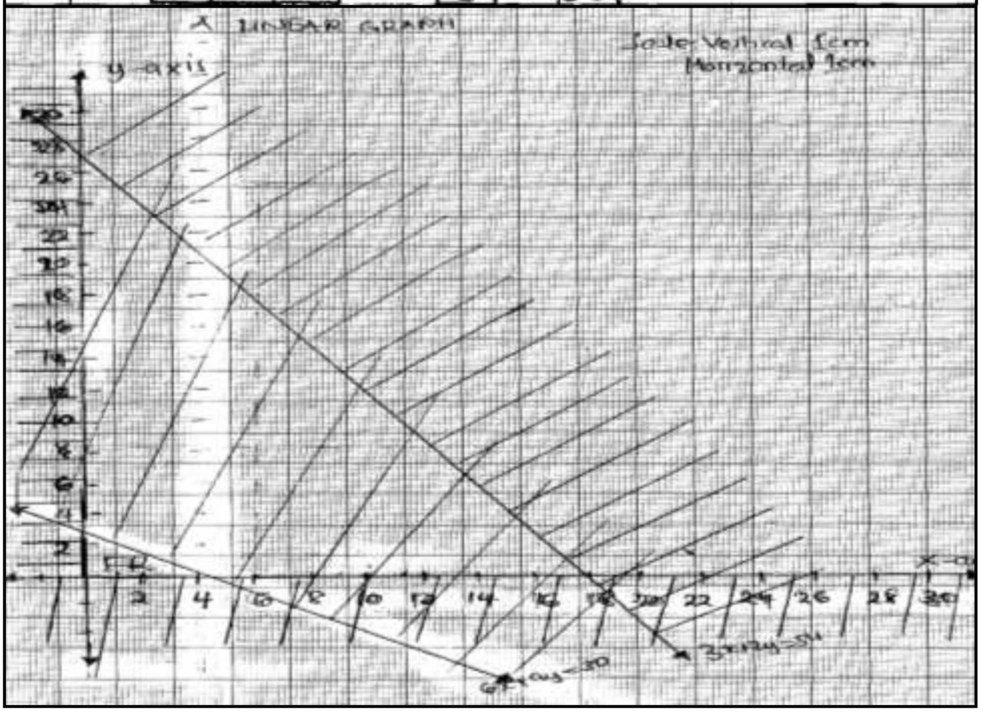
Table values
$6x + 9y = 30$
$3x + 2y = 54$

$3x+2y=54$

x	0	18
y	27	0

$6x+9y=30$

x	5	0
y	0	3.3



Point.	Objective function	Profit
A([illegible])	5800(0) + 3600(27)	92,200
B([illegible])	5800(0)+3600(3.[illegible]	10,800
C(5,0)	5800(5)+3600(0)	29,000
D(18,0)	5800(18)+3600(0)	104,400

ii/ Steel tables to be made is 18 and wooden table is none

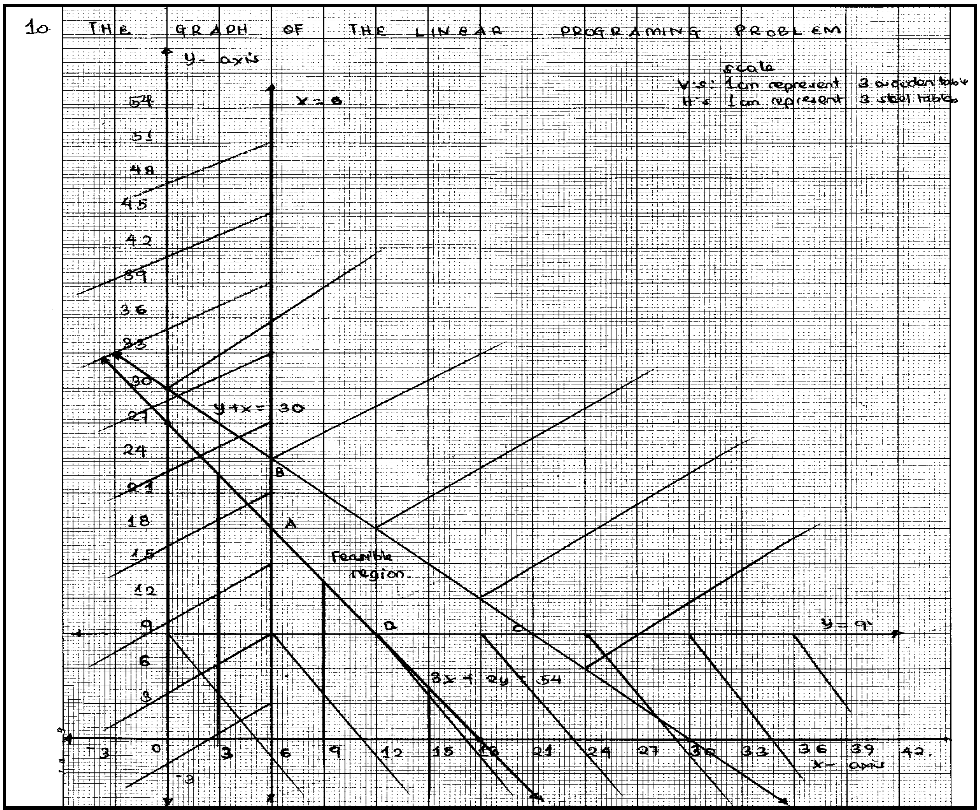
iv/ The maximum profit is 104,400

Extract 10.1: A sample of incorrect response to question 10

In Extract 10.1, the candidate obtained inappropriate feasible region and consequently incorrect maximum profit as well as number of steel and wooden tables.

On the other hand, 1,546 (4.71%) candidates answered the question correctly scoring all 10 marks. They used x and y to represent number of steel and wooden tables respectively. This enabled them to rewrite the given word problem into mathematical model, whereby objective function is $f(x, y) = 5800x + 3600y$ and the constraints are $x + y \leq 30$, $3x + 2y \geq 54$, $x \geq 6$, $y \geq 9$, $x \geq 0$ and $y \geq 0$. Then, they used graphical method to determine corner points of feasible region, and they substituted the points into objective function to optimize the problem (Extract 10.2).

10. THE GRAPH OF THE LINEAR PROGRAMING PROBLEM



10. From the graph

Corner points	objective function (5800x + 3600y)	net profit
A (6, 18).	6 x 5800 + 3600 x 18.	99 600.
B (6, 24).	6x 5800 + 3600 x 24.	121 200.
C (21, 9).	21 x 5800 + 3600 x 9.	154, 200.
D (12, 9)	12x 5800 + 3600x 9.	102, 000.

Hence the optimum point is C (21, 9).

(i). Therefore the engineer should make 21 steel tables and 9 wooden table per day to realize maximum profit.

(ii). The maximum profit is 154, 200/-

Extract 10.2: A sample of correct response for question 10

As Extract 10.2 shows, the candidate identified maximum profit correctly and therefore, got correct number of steel and wooden tables.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The 141 Basic Applied Mathematics examination of 2021 comprised ten (10) questions constructed from 10 topics. The data reveal that three topics had good performance. These topics included *Probability* with performance by 78.52 percent, *Calculating Devices* by 72.31 percent and *Functions* with performance by 60.40. Good performance in these topics proves that the candidates were competent in determining sample space and identifying event of experiments; using a scientific calculator to solve mathematical problems; and finding range of linear function as well as turning point and inverse of quadratic function. The data further indicates that *Statistics* had an average performance of 51.38 percent.

The rest of the topics had weak performance, including *Trigonometry* (26.71%), *Differentiation* (25.93%), *Linear Programming* (20.63%), *Integration* (15.47%), *Exponential and Logarithmic Functions* (15.23%) and *Algebra* (12.39%). The analysis of candidates' performance in different topics is shown in **Appendix I** and the comparison of performance between 2020 and 2021 is shown in **Appendix II**.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The data shows that 60.85 percent of the candidates passed the 141 Basic Applied Mathematics examination in 2021, whereas 59.31 percent of the candidates passed in 2020. Therefore, the performance has increased by 1.54 percent. This report has also presented in detail the strengths and weaknesses observed in candidates' responses in each question.

The weak performance was due to failure of many candidates to find n^{th} term and sum of the first n terms of the geometrical progression; apply the knowledge of perfect square to solve quadratic equation; relate derivatives and integrals; apply knowledge of integration to find area of the region; define exponential function; evaluate integral of the form $\int \frac{a}{bx+c}dx$; and find compound interest. Furthermore, the weak performance on some topics was due to inability of the candidates to write the mathematical model of the word problem; find derivative of Cartesian and parametric expressions; apply concepts of differentiation to solve real life problems; prove trigonometric identities; solve trigonometric equations; and apply cosine rule.

4.2 Recommendations

Based on the analysis presented in this report, the following measures are recommended so as to improve candidates' performance in future examinations:

(a) All cases which give different formulae for calculating sum of first *n* terms of Geometric Progression should be well understood by the students. Also, the knowledge of perfect square must be acquired especially condition for quadratic equation to have equal roots as well as its general solution.

(b) Students should be guided to identify distinctive features of exponential function and its conditions. In addition, the students must be knowledgeable on standard integral of $\frac{1}{ax+b}$ and general equation for compound interest.

(c) Students must be knowledgeable on relation of derivatives and anti-derivatives. Also, they must clearly acquire knowledge of identifying regions enclosed by curves and limitations of the formula for calculating area between two curves.

(d) English and mathematical languages should be simultaneously taught to the students. This will enhance ability of the students to write various word problems into mathematical model.

(e) Students must be guided to integrate mathematics with real life situations so as to improve their ability of solving real-life problems.

Appendix I

Analysis of Candidates' Performance per Topic in 141 Basic Applied Mathematics 2021

S/N	Topic	Questions Number	Percentage of Candidates who Scored an Average of 35% or Above	Remarks
1	Probability	7	78.52	Good
2	Calculating Devices	1	72.31	Good
3	Functions	2	60.4	Good
4	Statistics	6	51.38	Average
5	Trigonometry	8	26.71	Weak
6	Differentiation	4	25.93	Weak
7	Linear Programming	10	20.63	Weak
8	Integration	5	15.47	Weak
9	Exponential and Logarithmic Functions	9	15.23	Weak
10	Algebra	3	12.39	Weak

Appendix II

Candidates' Performance in each Topic for 2020 and 2021