THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2020

141 BASIC APPLIED MATHEMATICS

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141 BASIC APPLIED MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is delighted to issue this report on the Candidates' Item Response Analysis (CIRA) for the Basic Applied Mathematics paper of the Advanced Certificate of Secondary Education Examination (ACSEE) 2020. The report essentially provides feedback to education stakeholders on how the examined candidates responded to the items.

This report identifies the strengths and weaknesses of candidates in answering each item. In general, good performance of majority of the candidates was contributed by the ability to: use scientific calculators to compute and evaluate mathematical problems; present statistical data by graphs and estimating mode and median; apply knowledge of linear programming to solve real life problem; and sketch graphs of linear and quadratic functions.

On the other hand, the weak performance of other candidates was due to inability to: sketch graphs of logarithmic functions; apply the knowledge of exponential functions to solve real life problems; solve problems using trigonometric ratios and identities; apply integration to find area of enclosed region; and apply the knowledge of probability in real life settings.

Finally, the Council would like to thank everyone who participated in the preparations of this report.

Dr. Charles Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the candidates' responses to the items examined in 141 Basic Applied Mathematics paper for the Advanced Certificate of Secondary Education Examination (ACSEE) 2020. The paper was set according to the 2019 examination format and the 2010 Basic Applied Mathematics syllabus for Advanced Secondary Education. The paper consisted of 10 compulsory questions weighing 10 marks each.

In 2020, a total of 31310 candidates sat for the 141 Basic Applied Mathematics examination, of which 18483 (59.31%) candidates passed. This performance is better than that of 2019 whereby 19489 (56.21%) candidates passed. Therefore, the performance has increased by 3.10 percent. The candidates who passed these examinations got different grades ranging from grade A to S as shown in Figure 1.

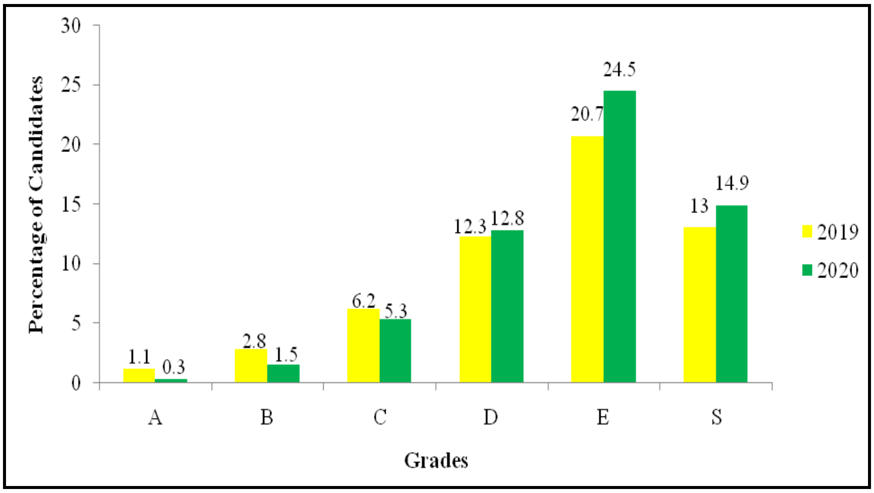


Figure 1: *Distribution of grades for 2019 and 2020.*

In Section 2.0, the report analyses the candidates' performance in all items of each question. The analysis in this section states the requirements of each item as well as explanations on how the candidates responded to the items. The candidates' performance in each question is judged using the percentage of candidates who scored 3.5 marks or more. The performance was categorized into three groups: 60 to 100 percent for good performance; 35 to 59 percent for average performance; and 0 to 34 percent for weak performance. In graphs or charts; green, yellow and red colours were used to denote good, average and weak performance respectively.

Section 3.0 of the report provides analysis of candidates' performance in each topic. The topics which attained good, average and weak performance are listed. Also, factors attributed to good and weak performance are outlined.

In Section 4.0, the report gives conclusion and recommendations for improving the performance in future Basic Applied Mathematics examinations. Finally, Appendices I and II are presented for further clarification of candidates' performance.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 Question 1: Calculating Devices

The candidates were required to use a non-programmable scientific calculator to compute:

(a) the value of $\frac{3+3(\sqrt[3]{0.65})}{3-3(\sqrt[3]{0.65})}$ correct to 4 significant figures.

(b) the mean and standard deviation of $33, 28, 26, 35$ and 38 correct to 2 decimal places.

(c) the value of $\frac{{}^5C_2 + {}^9P_6}{11!}$ correct to 4 decimal places.

A total of 31176 (99.57%) candidates attempted this question. Amongst, 24975 (80.11%) candidates scored marks ranging from 3.5 to 10. Therefore, candidates' performance was good. The summary of candidates' performance is presented in Figure 2.

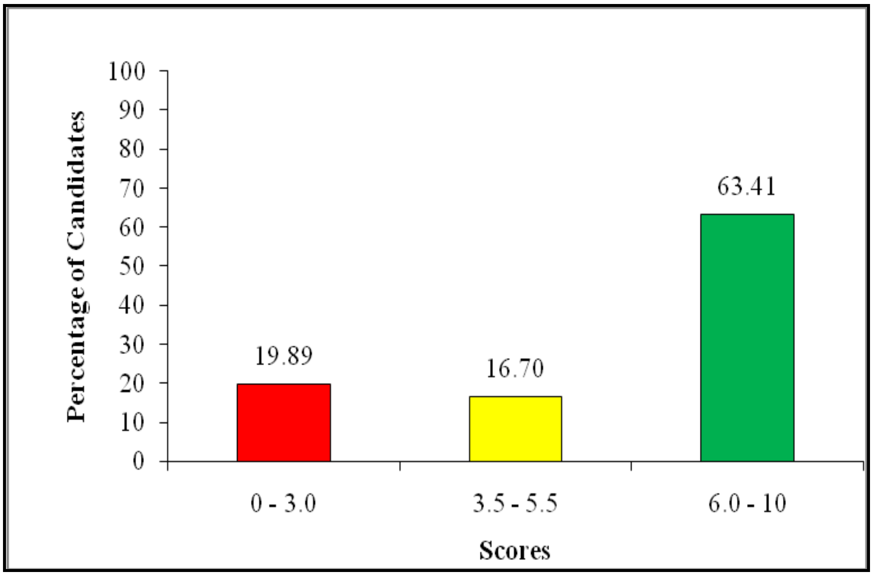


Figure 2: *The candidates' performance in question 1.*

Analysis of data revealed that 5216 (16.73%) candidates scored all 10 marks. In part (a), the candidates wrote $\frac{3+3(\sqrt[3]{0.65})}{3-3(\sqrt[3]{0.65})}=13.95$. This indicates that they correctly fixed calculator in four significant figures and correctly inserted the term $\frac{3+3(\sqrt[3]{0.65})}{3-3(\sqrt[3]{0.65})}$ into the calculator. In part (b), the candidates wrote $Mean = 32.00$ and $Standard\ deviation = 4.43$. For this case, it is clear that the candidates correctly navigated to statistical measures in calculators, inserted the given numbers and fixed the answer into 2 decimal places. The candidates who correctly answered part (c) entered $\frac{{}^5C_2+{}^9P_6}{11!}$ into calculator and fixed the answer to 4 decimal places, hence they got $\frac{{}^5C_2+{}^9P_6}{11!}=0.0015$.

Despite candidates' good performance, 1386 (4.45%) candidates scored zero. Majority of these candidates failed to fix calculator into required number of significant figures or decimal places. They wrote the answer with many significant figures or decimal places. For instance, in part (c) some candidates got $\frac{{}^5C_2+{}^9P_6}{11!}=0.001515402036$. This indicates that the candidates did not set calculator to give 4 decimal places. Also, some candidates used formulae to answer this question (instead of using calculators). For example, the formula $Mean = \frac{\sum x}{N}$ was notable to be used in answering part (b) of this question. With this approach, the candidates got $Mean = 32$ instead of 32.00. Furthermore, some candidates used calculators inappropriately (see Extract 1.1).

c.	$= {}^5C_2 + {}^9P_6$
	$11!$
	$= 10 + 60480$
	$39,916,800$
	$= 1.515402036 \times 10^{-3}$

Extract 1.1: *A sample of inappropriate procedures for part (c) of question 1.*

In Extract 1.1, the candidate set calculator to give the answer in standard notation instead of four (4) decimal places.

2.2 Question 2: Functions

The question measured candidates' ability to sketch graphs of functions, determining domain and range from the graph and developing composite function.

The question consisted of parts (a) and (b). In part (a), the candidates were required to (i) sketch the graph of $f(x) = \begin{cases} 2x-1 & \text{if } -2 < x \le 1 \\ x^2 & \text{if } 1 < x \le 2 \\ 10-3x & \text{if } 2 < x < 3 \end{cases}$ and (ii) state the domain and range of *f(x)*. In part (b), the candidates were given $f(x) = 3x+3$ and $g(x) = x+3$ and were required to find $(fog)(x)$ and $(fog)^{-1}(x)$.

A total of 30731 (98.14%) candidates attempted the question. Amongst, 10948 candidates scored 3.0 marks or less, 7160 scored marks ranging from 3.5 to 5.5 and 12623 scored marks ranging from 6.0 to 10. Figure 3 shows percentage of candidates who scored low, average and high marks.

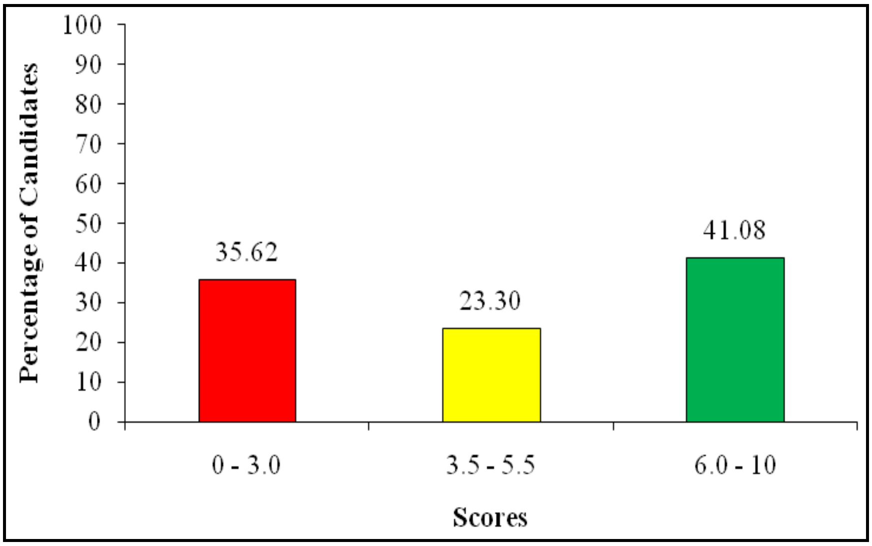


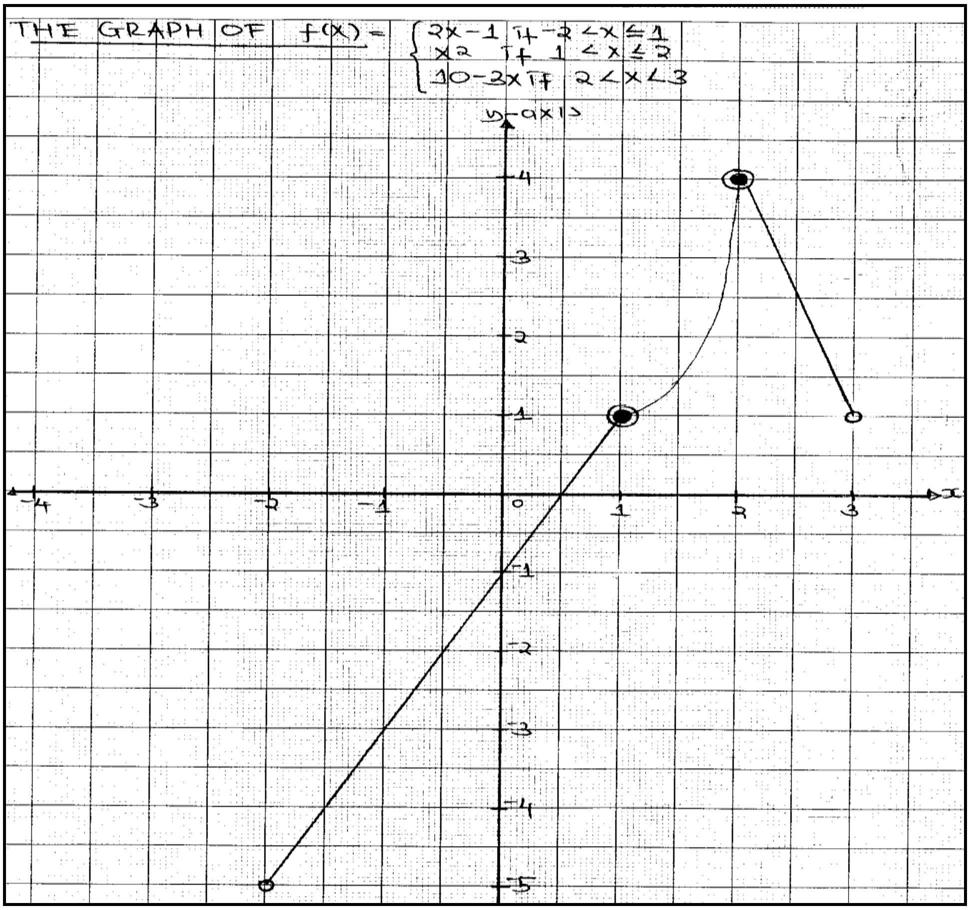
Figure 3: *The candidates' performance in question 2.*

As Figure 3 shows, 64.38 percent, equivalent to 19783 candidates scored marks ranging from 3.5 to 10. Therefore, the question was well performed.

The data also reveals that 919 (3.0%) candidates answered the question correctly scoring all 10 marks. In part (a), many candidates constructed correctly tables of values for each definition of $f(x)$ and used them to sketch graphs (see Extract 2.1). Also, they observed the graphs and correctly stated the domain and range of $f(x)$ as $Domain = \{x : -2 < x < 3\}$ and $Range = \{y : -5 < y \leq 4\}$.

Majority of the candidates who answered part (b)(i) correctly replaced variable x in definition of $f(x)$ with the definition of $g(x)$ and worked out to get $(fog)(x) = 3x + 12$. That is; $(fog)(x) = 3(g(x)) + 3 = 3(x+3) + 3 = 3x + 12$. For part (b)(ii), the candidates let $(fog)(x) = y$ and got $y = 3x + 12$. Thereafter, they interchanged variables in $y = 3x + 12$ leading to $x = 3y + 12$. Finally, they correctly made y the subject of $x = 3y + 12$ and ended up with $(fog)^{-1}(x) = \frac{x}{3} - 4$.

Also, there were few candidates who used the definition $(fog)^{-1}(x) = g^{-1} o f^{-1}(x)$ correctly after obtaining the inverses of $f(x)$ and $g(x)$ as $f^{-1}(x) = \frac{x-3}{3}$ and $g^{-1}(x) = x - 3$ respectively.

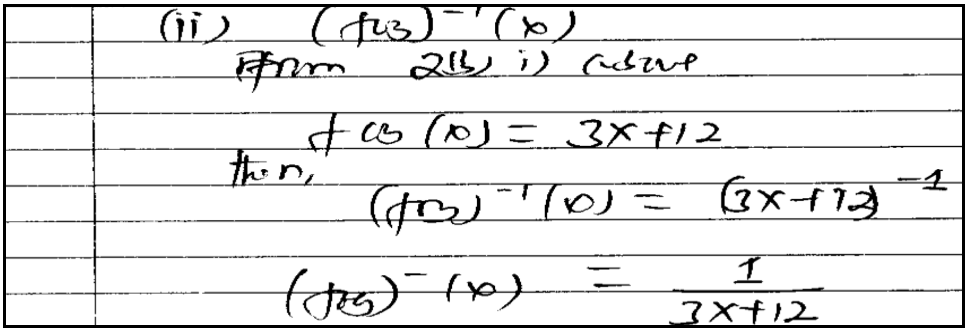


Extract 2.1: *A sample of correct solution for part (a) of question 2.*

In Extract 2.1, the candidate correctly joined the points for $f(x)=2x-1$ and $f(x)=10-3x$ using ruler to produce straight lines (as they are linear functions). The points of $f(x)=x^2$ were joined using free hand and produced a curve (as it is a quadratic function).

On the other hand, 29812 (97%) candidates failed to score all 10 marks. In part (a)(i), many candidates presented incorrect graph of $f(x)=x^2$ for $1<x\le 2$. They joined the points in this range incorrectly producing straight line instead of a curve. Also, a notable number of candidates ignored the symbol for excluding a value (open circle) in their graphs. In part (a) (ii), analysis of candidates' responses revealed that many candidates were unable to determine the domain and range from the graph. Instead, they crammed and wrote the statement "all real numbers" for both domain and range. Other candidates also excluded some values in domain and range due to lack of understanding on inclusive and exclusive symbols in graphs.

In part (b)(i), many candidates presented incorrect approach in finding $(fog)(x)$. For instance, some candidates wrote $(fog)(x)=(3x+3)(x+3)=3x^2+12x+9$. These candidates wrongly interpreted $(fog)(x)$ as the product of $f(x)$ and $g(x)$. Other candidates equated expressions of $f(x)$ and $g(x)$ hence, they got an equation and solved it. Instead, these candidates were supposed to replace x in the definition of $f(x)$ with the definition of $g(x)$. Incorrect answers obtained in part (b)(i) led to incorrect solutions in part (b)(ii). In particular, the incorrect answer $(fog)(x)=3x^2+12x+9$ led to $(fog)^{-1}(x)=\dfrac{-12\pm\sqrt{36+12x}}{6}$ instead of $(fog)^{-1}(x)=\dfrac{x}{3}-4$. Also, some candidates defined $(fog)^{-1}(x)$ wrongly as $(fog)^{-1}(x)=f^{-1}og^{-1}(x)$ instead of $(fog)^{-1}(x)=g^{-1}of^{-1}(x)$. For this reason, even the candidates who computed $f^{-1}(x)$ and $g^{-1}(x)$ correctly ended up with the wrong answer $(fog)^{-1}(x)=\dfrac{x-6}{3}$. Furthermore, some candidates regarded -1 in $(fog)^{-1}(x)$ wrongly as exponent (see Extract 2.2).


(ii) $(fog)^{-1}(x)$
From 2(b) i) above
$fog(x) = 3x+12$
then,
$(fog)^{-1}(x) = (3x+12)^{-1}$
$(fog)^{-}(x) = \frac{1}{3x+12}$

Extract 2.2: *A sample of incorrect solution for part (b)(ii) of question 2.*

In Extract 2.2, the candidate applied the concept of negative exponent in finding inverse of a function. This indicates that he/she is not aware of the mathematical notation particularly in functions.

2.3 Question 3: Algebra

The question measured the ability of candidates to solve system of two equations, exponential equation as well as finding the sum of the first n terms of Arithmetic Progression.

The question was as follows:

(a) Use the substitution method to solve the following system of equations: $\begin{cases} 3x - y = 9 \\ x^2 + xy + 2 = 0 \end{cases}$

(b) Find the value(s) of x satisfying the equation $4^x - 6(2^x) - 16 = 0$.

(c) Find the sum of the first n terms of the series $1 + 3 + 5 + ...$

This question was attempted by 30286 (96.72%) candidates, of whom 15537 (51.3%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was average. The performance summary in this question is presented in Figure 4.

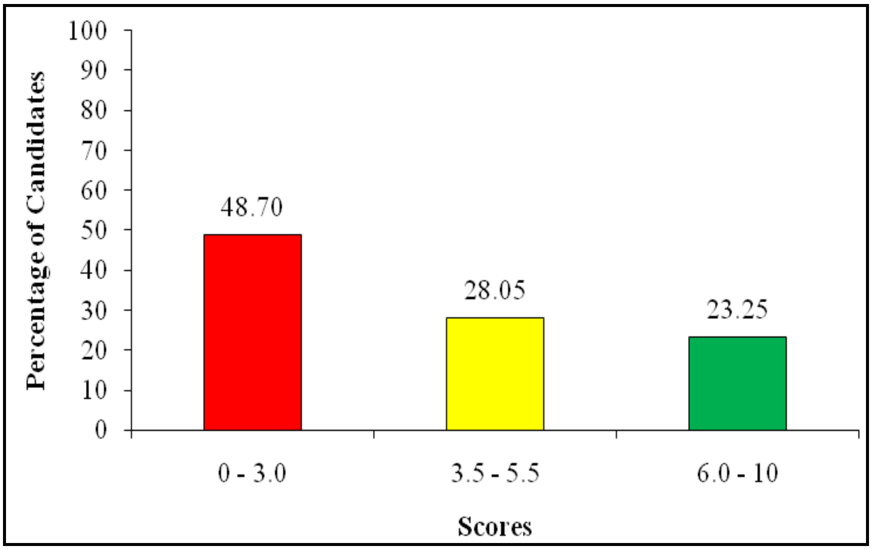


Figure 4: *The candidates' performance in question 3.*

The candidates who responded to part (a) correctly were able to find the arbitrary value of either x or y from $3x - y = 9$. In particular, some candidates rewrote $3x - y = 9$ in the form $y = 3x - 9$, replaced y in $x^2 + xy + 2 = 0$ with $3x - 9$ and simplified to get $4x^2 - 9x + 2 = 0$ (or other equivalent equation). The candidates also managed to solve this equation and got $x = \frac{1}{4}$ and $x = 2$. To solve for y, the candidates substituted $x = \frac{1}{4}$ and $x = 2$ simultaneously into $y = 3x - 9$ and worked out to get $y = -\frac{33}{4}$ and $y = -3$ respectively.

The candidates who did part (b) correctly realized that the term 4^x can be written in the form $(2^x)^2$. Therefore, they changed $4^x - 6(2^x) - 6 = 0$ into a quadratic equation in 2^x as $(2^x)^2 - 6(2^x) - 16 = 0$. Then, the candidates solved the quadratic equation to get $2^x = 8$ and $2^x = -2$. Applying the concepts of exponents, the candidates solved $2^x = 8$ to get $x = 3$ and realized that there is no real value of x satisfying the equation $2^x = -2$.

In part (c), the candidates who scored all marks realized that the series $1 + 3 + 5 + ...$ is an Arithmetic Progression and worked out correctly to get the sum of first *n* terms as illustrated in Extract 3.1.

3c.	Soln.
	1 + 3 + 5 + ...
	d = 3 - 1 = 5 - 3
	= 2.
	A_1 = 1.
	Sn = [2A_1 + d(n-1)] n/2.
	= [(2 x 1) + 2(n-1)]n/2.
	= (2 + 2n - 2)n / 2
	= n^2

Extract 3.1: *A sample of correct solution for part (c) of question 3.*

Extract 3.1 is a response of the candidate who correctly determined common difference $(d = 2)$ and first term $(A_1 = 1)$. Also, he/she employed correct formula $S_n = \frac{n}{2}[2A_1 + (n-1)d]$ and worked out correctly to get $S_n = n^2$.

In contrast, 14749 (48.7%) candidates had weak performance of 3.0 marks or less. In part (a), common weakness was failure of some candidates to perform binomial expansion. Most of the candidates crammed the identity $(a+b)^2 = a^2 + 2ab + b^2$ incorrectly as $(a+b)^2 = a^2 + b^2$. For example, some candidates correctly reached at $\left(\frac{9+y}{3}\right)^2 + \left(\frac{9+y}{3}\right)y + 2 = 0$, but they obtained incorrect equation, such as $4y^2 + 27y + 87 = 0$. Also, some candidates failed to solve the resulted quadratic equation.

In part (b), many candidates did not strive to write 4^x in the form $\left(2^x\right)^2$. In addition, most of them wrongly changed $6\left(2^x\right)$ into 12^x and hence came up with an incorrect equation $4^x - 12^x - 16 = 0$. The candidates also applied the laws of exponents inappropriately indicating lack of knowledge and skills on manipulating exponents (see Extract 3.2). Some candidates introduced logarithms to solve the equation. Apart from complicating the question, the candidates also failed to apply the laws of logarithms correctly. Furthermore, there were few candidates who forced to get real value of x from $2^x = -2$ while it does not exist. This indicates that they were not competent on the concept of exponents.

In part (c), a notable number of candidates worked out to find n^{th} term of Arithmetic Progression, contrary to the requirement of the question. The question instructed them to find the sum of the first n terms. These candidates used the formula $A_n = A_1 + (n-1)d$ instead of $S_n = \frac{n}{2}\left[2A_1 + (n-1)d\right]$. Also, some candidates failed to recognize that the given series is an Arithmetic Progression. Instead, they treated the series as Geometric Progression. For instance, they calculated common ratio, $r = \frac{G_2}{G_1} = \frac{3}{1} = 3$, instead of common difference.

Qn 3 (b)	$4^x - 6(2^x) - 16 = 0$
	$4^x - 12^x - 16 = 0$
	$-8^x = 16$
	$-2^{3x} = 2^4$
	$-3x = 4$
	$x = -4/3$

Extract 3.2: *A sample of incorrect solution for part (b) of question 3.*

In Extract 3.2, apart from developing incorrect equation $4^x - 12^x - 16 = 0$, the candidate subtracted unlike terms $\left(\text{i.e } 4^x - 12^x = -8^x\right)$. This operation is incorrect since unlike terms cannot be added or subtracted (only like terms can be added or subtracted).

2.4 Question 4: Differentiation

This question measured the competence of the candidates to use the First Principles of differentiation as well as apply differentiation in finding slope of a tangent to the curve and stationary points of the curve.

The question consisted of the following parts:

(a) Find the first derivative of $f(x) = x^2$ from First Principles.

(b) Find the slope of the tangent to the curve $8x^3 + xy^3 - 5y^2 = 0$ at $(1,-1)$.

(c) Use second derivative test to classify the stationary point(s) of the curve $f(x) = 2x^3 + 3x^2 - 12x - 5$.

This question was attempted by 27672 (88.38%) candidates. The percentages of candidates who scored low, average and high marks are shown in Figure 5.

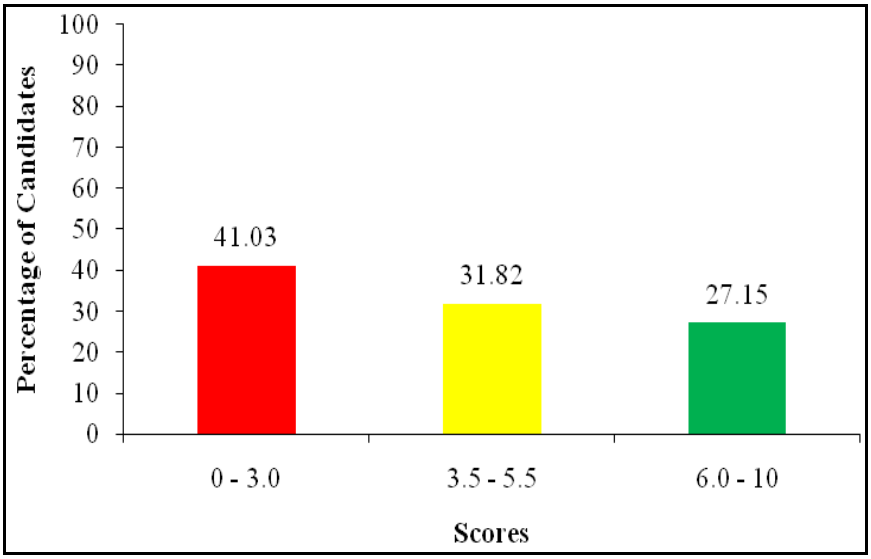


Figure 5: *The candidates' performance in question 4.*

About 7512 (58.97%) candidates obtained 3.5 marks or more. Therefore, the question was averagely performed.

The data reveals further that 1179 (4.26%) candidates scored all 10 marks allotted to this question. In part (a), these candidates correctly developed $f(x+h) = (x+h)^2$ from $f(x) = x^2$. Then, they substituted the expressions for $f(x+h)$ and $f(x)$ into the formula $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ and worked out accurately to get $f'(x)=2x$ (see Extract 4.1).

In part (b), the candidates appreciated the concept of implicit differentiation and correctly determined first derivative of $8x^3+xy^3-5y^2=0$ as $\frac{dy}{dx}=-\frac{24x^2+y^3}{3xy^2-10y}$. Thereafter, they substituted the point $(1,-1)$ into $-\frac{24x^2+y^3}{3xy^2-10y}$ and evaluated to get $slope=-\frac{23}{13}$. In part (c), the candidates calculated the derivative of $f(x)=2x^3+3x^2-12x-5$ and got $f'(x)=6x^2+6x-12$. They used the fact that "at stationary point $f'(x)=0$" to determine x- coordinates of the stationary points, that is $x=1$ and $x=-2$. Further, the candidates substituted $x=1$ and $x=-2$ into $f(x)=2x^3+3x^2-12x-5$ so as to determine y- coordinates of the stationary points, that is $y=-12$ and $y=15$. Therefore, the stationary points of the curve are $(1,-12)$ and $(-2,15)$. Then, the candidates correctly classified the stationary points as shown in Extract 4.2.

4. (a) $f(x) = x^2$

$f(x+h) = (x+h)^2$

from the first principle;

$\frac{dy}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$

$\frac{dy}{dx} = \lim_{h\to 0}\left(\frac{(x+h)^2 - x^2}{h}\right)$

$\frac{dy}{dx} = \lim_{h\to 0}\left(\frac{x^2+2hx+h^2-x^2}{h}\right)$

$dy/dx = \lim_{h\to 0}\left(\frac{2hx+h^2}{h}\right)$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{h(2x+h)}{h}$

$\frac{dy}{dx} = \lim_{h\to 0} 2x+h$

As $h\to 0$, $\frac{dy}{dx} = 2x$

Extract 4.1: *A sample of correct solution for part (a) of question 4.*

Extract 4.1 shows workings of the candidate who correctly evaluated the limit of $\frac{(x+h)^2-x^2}{h}$ as h approaches to zero.

	from $\frac{dy}{dx} = 6x^2 + 6x - 12$
	$\frac{d^2y}{dx^2} = 12x + 6.$
	for $(x, y) = (1, -12)$.
	$\frac{d^2y}{dx^2} = 12(1) + 6$
	$= 18.$
	as $\frac{d^2y}{dx^2} > 0.$
	Point $(1, -12)$ is a minimum point
4c	$\frac{d^2y}{dx^2}$ for $(x_2, y_2) = -2(12) + 6$
	$= -18.$
	as $\frac{d^2y}{dx^2} < 0.$
	∴ Point $(-2, 15)$ is a maximum point.

Extract 4.2: *A sample of correct solution for part (c) of question 4.*

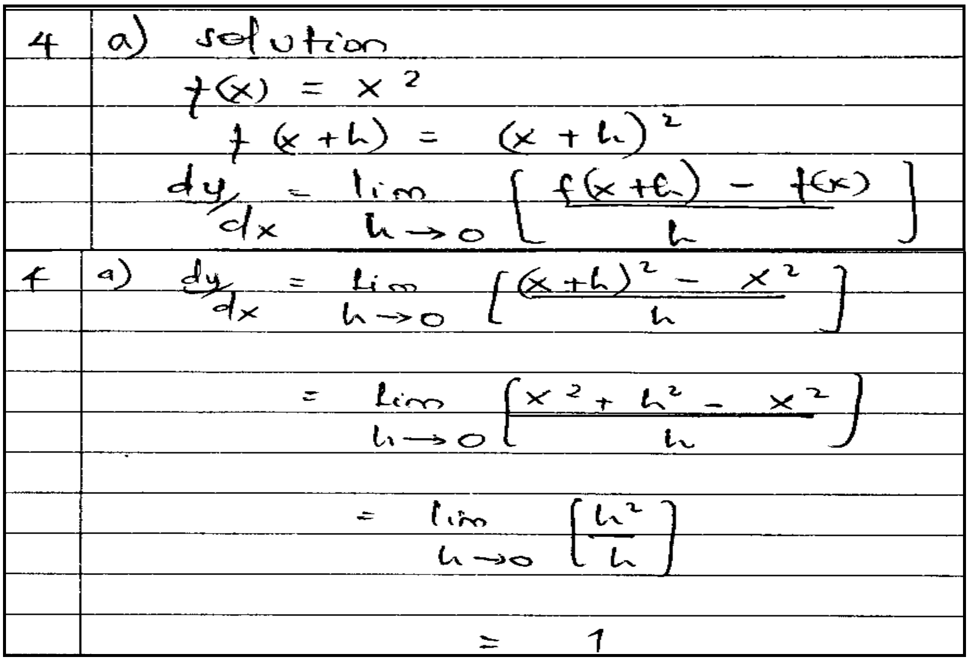
In Extract 4.2, the candidate used $f''(x) = 12x + 6$ to determine the nature of each stationary point.

Conversely, 11354 (41.03%) candidates scored 3.0 marks or less, of whom 1839 (6.65%) candidates scored zero. In part (a), many candidates failed to get the correct expression for $f(x+h)$ using $f(x) = x^2$. For instance, some candidates wrote $f(x+h) = x^2 + h$ indicating lack of knowledge and skills related to substitution. For this case, the candidates got incorrect derivative (see Extract 4.3). Furthermore, few candidates employed the formula for finding slope of a straight line, $f'(x) = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ instead of $f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$.

In part (b), many candidates were unable to find the derivative of the given implicit function. For example, some candidates got $24x^2 + 3xy^2\frac{dy}{dx} - 10y\frac{dy}{dx} = 0$ from $8x^3 + xy^3 - 5y^2 = 0$. These candidates did not apply the concept of implicit differentiation, specifically to the term xy^3. Other candidates answered this part wrongly by substituting $x = 1$ and $y = -1$ simultaneously into $8x^3 + xy^3 - 5y^2 = 0$ and solved for y and x

respectively. Furthermore, there were candidates who substituted point $(1,-1)$ into $8x^3 + xy^3 - 5y^2 = 0$. These candidates got incorrect answer, 2 in particular.

In part (c), many candidates confused the conditions for maximum and minimum point. They wrote $\frac{d^2y}{dx^2} < 0$ and commented minimum point (instead of maximum point) and when $\frac{d^2y}{dx^2} > 0$, they commented maximum point (instead of minimum point). Therefore, these candidates classified the points $(1,-12)$ and $(-2,15)$ incorrectly as maximum and minimum point respectively. Also, some candidates correctly got $\frac{dy}{dx} = 6x^2 + 6x - 12$. But, they did not employ the fact that at stationary points $\frac{dy}{dx} = 0$. Instead, some of them wrote $y = 6x^2 + 6x - 12$ and solved for x (when $y = 0$) and y (when $x = 0$). This shows that the candidates confused the concepts of intercepts and stationary points.



4 a) solution

$f(x) = x^2$

$f(x+h) = (x+h)^2$

$\frac{dy}{dx} = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$

4 a) $\frac{dy}{dx} = \lim_{h \to 0} \left[\frac{(x+h)^2 - x^2}{h} \right]$

$= \lim_{h \to 0} \left[\frac{x^2 + h^2 - x^2}{h} \right]$

$= \lim_{h \to 0} \left[\frac{h^2}{h} \right]$

$= 1$

Extract 4.3: *A sample of incorrect solution for part (a) of question 4.*

In Extract 4.3, the candidate incorrectly expanded $f(x+h) = (x+h)^2$ as $f(x+h) = x^2 + h^2$. This indicates that he/she lacked knowledge and skills on finding square of binomial expression.

2.5 Question 5: Integration

The question measured the ability of the candidates to use substitution technique in integrating the functions and to apply the concept of integration in calculating area of a region bounded by two curves.

In this question, the candidates were instructed to: (a) integrate $\sin^2 2x\cos 2x$ with respect to x; (b) evaluate $\int_0^{\sqrt{a}} \frac{x}{x^2+a}dx$; and (c) find the area enclosed between the curves $y = x^2 + 2$ and $y = 10 - x^2$.

A total of 19713 (62.96%) candidates responded to this question. Out of these, 4729 (23.99%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was weak. Figure 6 is a summary of candidates' performance in this question.

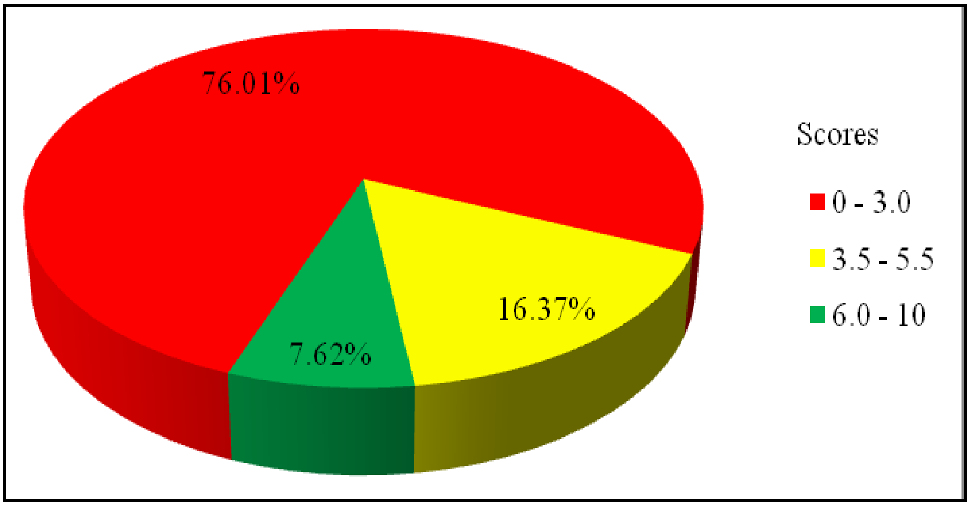
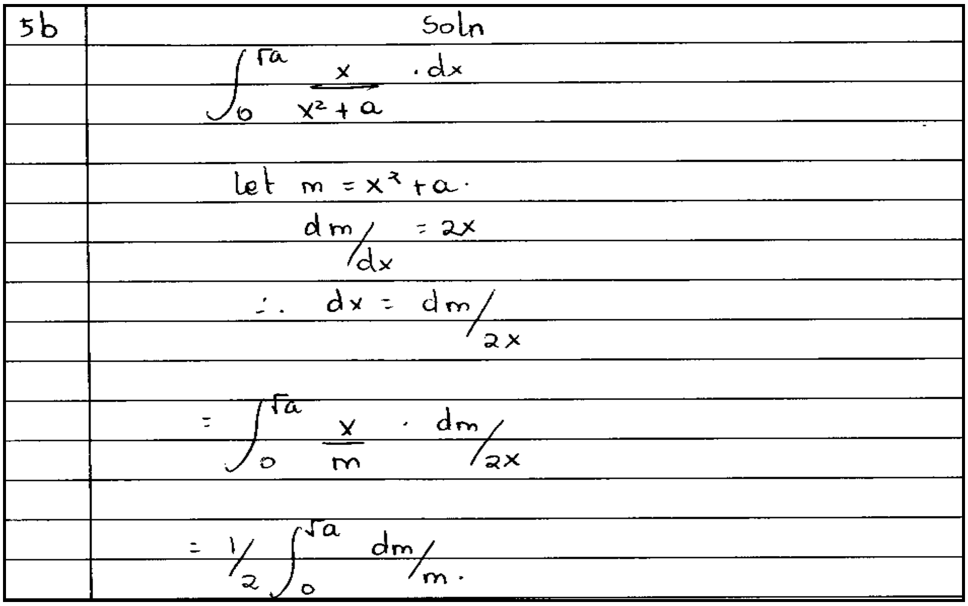


Figure 6: *The candidates' performance in question 5.*

Furthermore, data reveals that 6787 (34.43%) candidates scored zero. In part (a), one of the noted weaknesses was the failure to choose suitable function that could transform the integrand into simple one. Some candidates let $u = \cos 2x$ which results to $du = -2u\sin 2xdx$ and $-\frac{1}{2}\int u\sin 2xdu$. This approach involves tedious work such that the candidates failed to arrive at the correct answer. Also, some candidates did not apply the chain rule when differentiating $\sin 2x$ or $\cos 2x$. For instance, some candidates wrote $\frac{du}{dx} = \cos 2x$ from $u = \sin 2x$ instead of $\frac{du}{dx} = 2\cos 2x$.

In part (b), many candidates realized that the integrand involves function and its derivative. But, most of them changed the variable of integrand without changing the values of limits. Extract 5.1 illustrates this case. Other candidates incorrectly simplified $\frac{x}{x^2+a}$ into $\frac{1}{x}+\frac{x}{a}$. Therefore, they changed $\int_0^{\sqrt{a}} \frac{x}{x^2+a}dx$ into $\int_0^{\sqrt{a}} \frac{1}{x}dx + \frac{1}{a}\int_0^{\sqrt{a}} xdx$ which led them to incorrect answer. Furthermore, few responses had the following wrong statement; $\int_0^{\sqrt{a}} \frac{x}{x^2+a}dx = \left[\frac{x^{1+1}}{x^{2+1}+a^{1+1}}\right]_0^{\sqrt{a}}$. This indicates that the candidates intended to use the standard integral $\int_a^b x^n dx = \frac{x^{n+1}}{n+1}+C$ in each term of $\int_0^{\sqrt{a}} \frac{x}{x^2+a}dx$. This technique is helpful in integrating polynomial functions and not rational functions, including $\frac{x}{x^2+a}$.

In part (c), many candidates wrote the correct formula $Area = \int_a^b ((upper\ function) - (lower\ function))dx$. However, the common weaknesses were failure to determine the integrand and limits of integration for the required region. In finding integrand, many candidates interchanged upper and lower functions. They wrote $Area = \int_a^b ((x^2+2)-(10-x^2))dx$ instead of $Area = \int_a^b ((10-x^2)-(x^2+2))dx$. As a result, they got $Area = -\frac{64}{3}$ and gave extra explanations to justify that the correct answer is $Area = \frac{64}{3}\ units\ squares$. In finding limits, many candidates correctly equated x^2+2 and $10-x^2$; then simplified the resulted equation as $8-2x^2=0$ (or other equivalent equation). However, they obtained incorrect limits $x=0$ and $x=2$ due to failure to adhere the techniques of solving quadratic equations. Some candidates incorrectly computed the limits by solving for *x* (taking $y=0$) and *y* (taking $x=0$). This indicates that the candidates worked out to get *x*-intercept and *y*-intercept instead of finding limits of integration.



5b Soln

$\int_0^{\sqrt{a}} \frac{x}{x^2+a} \cdot dx$

let $m = x^2 + a$.

$dm/dx = 2x$

$\therefore\ dx = dm/2x$

$= \int_0^{\sqrt{a}} \frac{x}{m} \cdot dm/2x$

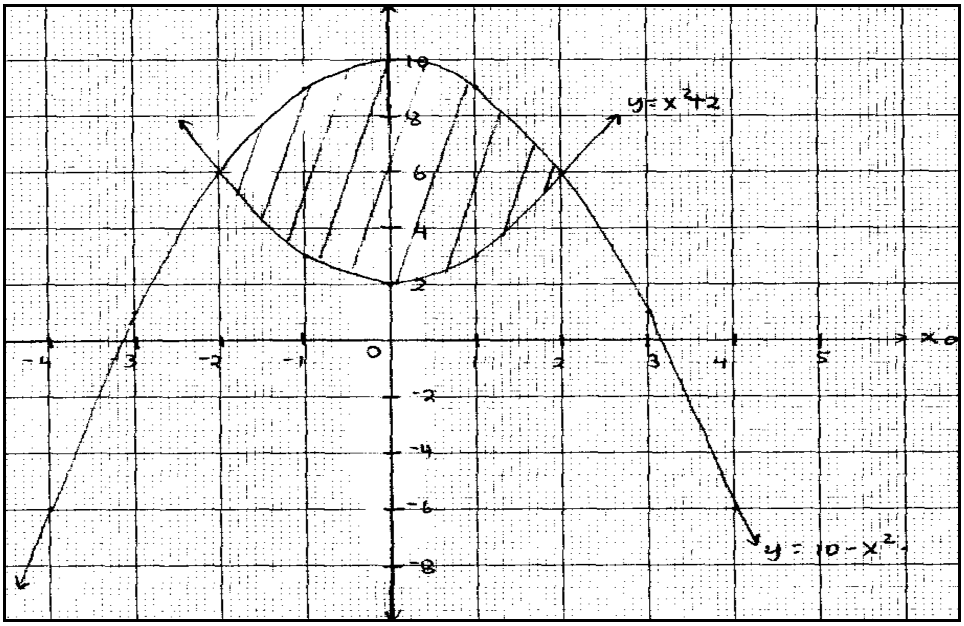
$= 1/2 \int_0^{\sqrt{a}} dm/m$.

Extract 5.1: *A sample of incorrect solution for part (b) of question 5.*

In Extract 5.1, the candidate used incorrect limits, $m=0$ and $m=\sqrt{a}$. He/she was supposed to substitute $x=0$ and $x=\sqrt{a}$ into $m=x^2+a$ so as to obtain correct limits, $m=a$ and $m=2a$ respectively.

Despite the weak performance in this question, 64 (0.32%) candidates attempted the question correctly scoring all 10 marks. In parts (a) and (b), the candidates recognized that the integrands are formed by the function and its derivative. Therefore, in part (a), they assumed $u=\sin 2x$ which led to $du = 2\cos 2x dx$ and consequently $\int \sin^2 2x \cos 2x dx = \frac{1}{2}\int u^2 du = \frac{1}{6}u^3 + C$. Finally, they replaced u in the expression with $\sin 2x$ to get the required answer, that is, $\int \sin^2 2x \cos 2x dx = \frac{1}{6}\sin^3 2x + C$. In part (b), some candidates assumed $u = x^2 + a$ and performed other procedures in a similar way as in part (a). Their workings included the following steps: $\int_0^{\sqrt{a}} \frac{x}{x^2+a} dx = \frac{1}{2}\int_a^{2a} \frac{du}{u} = \frac{1}{2}\left|\ln|u|\right|_a^{2a} = \ln\sqrt{2}$. In part (c), the candidates correctly identified the required region by drawing the graphs of $y = x^2 + 2$ and $y = 10 - x^2$ (or otherwise). From the graph, the candidates correctly determined the upper and lower functions as well as limits of integration (see Extract 5.2). Then, they applied the correct formula and obtained

$Area = \int_{-2}^{2} \left(\left(10 - x^2\right) - \left(x^2 + 2\right) \right) dx$. Finally, the candidates correctly evaluated the integral and got $Area = \frac{64}{3}\ square\ units$.



Extract 5.2: *A sample of correct solution for part (c) of question 5.*

In Extract 5.2, the candidate drew the graphs correctly, hence easily identified that the upper function is $y = 10 - x^2$, lower function is $y = x^2 + 2$, lower limit is $x = -2$ and upper limit is $x = 2$.

2.6 Question 6: Statistics

The question measured candidates' ability to compute mode, draw the cumulative frequency curve and estimate median from the cumulative frequency curve.

In this question, the candidates were given the following table showing litres of milk produced by 131 cows each day.

Litres of milk	5 - 10	11 - 16	17 - 22	23 - 28	29 - 34	35 - 40
Number of cows	15	28	37	26	18	7

From the tabulated data, the candidates were required to: (a) estimate the mode; and (b) draw the cumulative frequency curve and use it to estimate median.

The question was attempted by 30896 (98.67%) candidates. Out of these, 7022 candidates scored from 0 to 3.0 marks, 9559 candidates from 3.5 to 5.5 and 14315 from 6.0 to 10. The percentage of candidates who obtained low, average and high marks are shown in Figure 7.

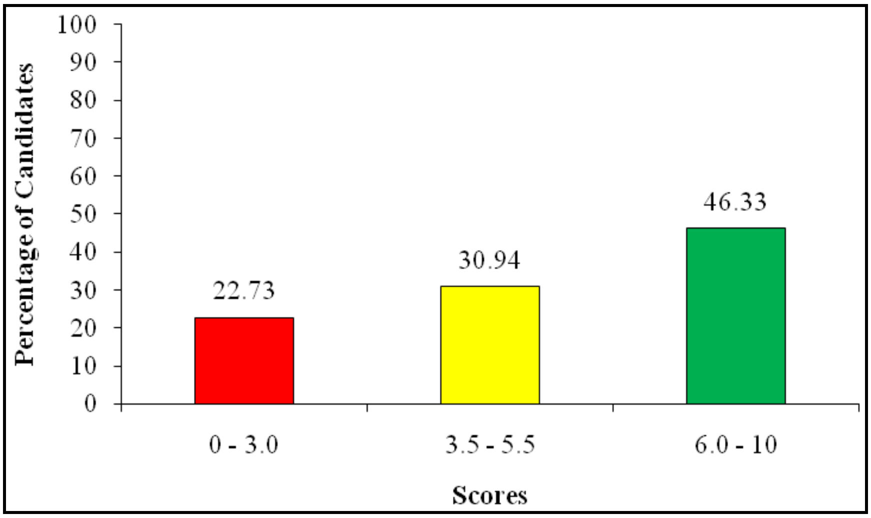
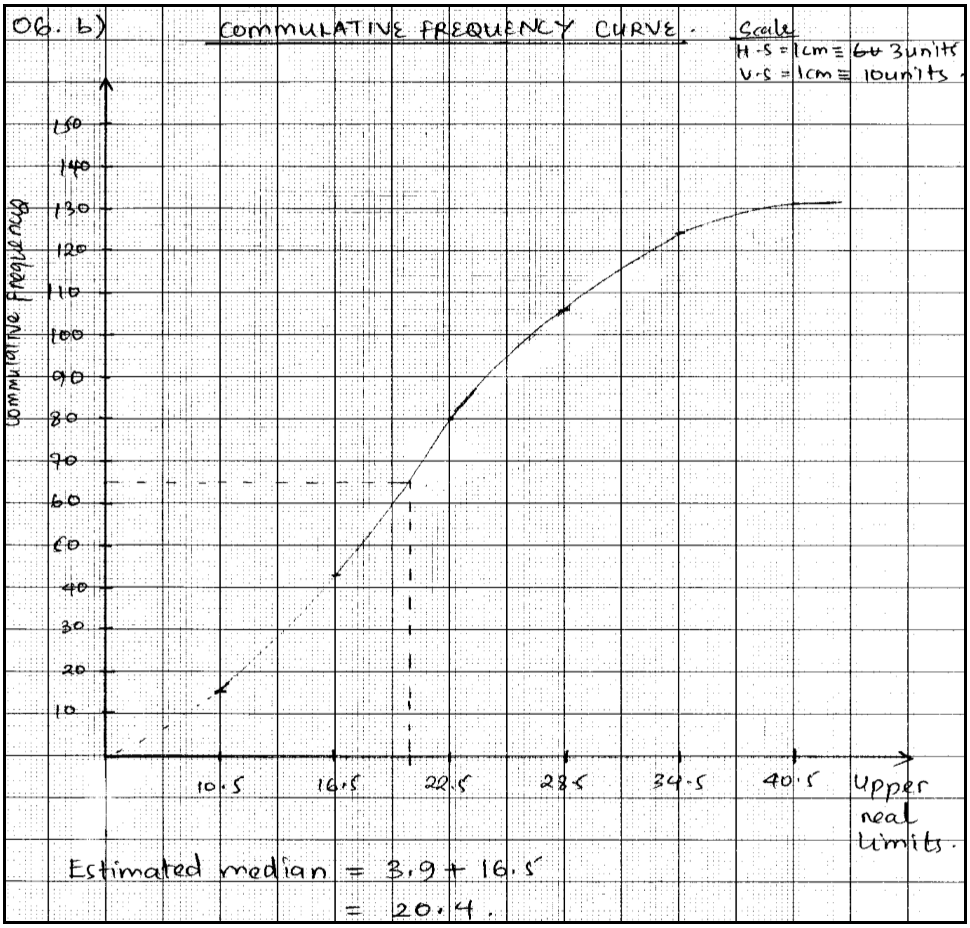


Figure 7: *The candidates' performance in question 6.*

As Figure 7 shows, 77.27 percent, equivalent to 23874 candidates scored marks ranging from 3.5 to 10. Therefore, performance of the candidates in this question was good.

Majority of the candidates who answered part (a) correctly were able to identify a correct modal class (a class with highest frequency) as 17 - 22. As a result, they correctly determined class size $(c = 6)$, lower boundary $(L = 16.5)$, difference in frequency of modal class and the frequency of the next class of lesser values $(t_1 = 9)$ as well as the difference in frequency of modal class and next class of higher values $(t_2 = 11)$. Then, they correctly substituted the values into the formula $Mode = L + \left(\frac{t_1}{t_1 + t_2}\right)c$ and computed to obtain $Mode = 19.2$. Apart from this approach, a considerable number of candidates answered this part by representing the data using histogram and used it to estimate the mode. In this approach, the tolerable value for mode ranged from 19.1 to 19.4.

Competent candidates in part (b) constructed cumulative frequency curve after correctly obtaining upper boundaries and cumulative frequency. Thereafter, they used it to estimate the median as shown in Extract 6.1. The accepted value for the median ranged from 20.0 to 20.5.

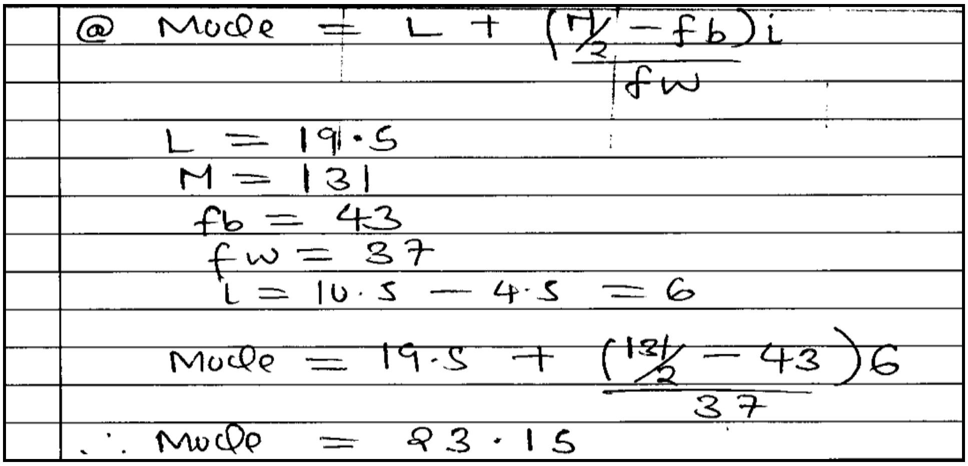


Extract 6.1: *A sample of correct solution for part (b) of question 6.*

In Extract 6.1, the candidate correctly drew horizontal dotted line from 65.5th position along cumulative frequency axis to the point where it meets the curve. From this point of contact, he/she drew a vertical line downwards to the horizontal axis (axis indicating the upper boundaries). The estimated value of upper boundary at the point was taken as median.

Though the performance was good, a total of 30333 (98.18%) candidates lost some marks in this question whereby 790 (2.56%) candidates scored zero. In part (a), many candidates interchanged the values of t_1 and t_2. They incorrectly substituted into the formula $t_1 = 11$ and $t_2 = 9$ instead of $t_1 = 9$ and $t_2 = 11$. In addition, some candidates also failed to determine the correct modal class and hence computed the lower boundary incorrectly. For instance, some candidates wrote $L = 5 - 0.5 = 4.5$. Other candidates computed median instead of mode (see Extract 6.2). This indicates that the candidates confused between mode and median.

In part (b), some candidates used class marks (instead of upper boundaries) or frequency (instead of cumulative frequency) to draw the curve. Also, a significant number of candidates joined the plotted points using a ruler, hence, they presented cumulative frequency polygon (not cumulative frequency curve as required).



(a) Mode = $L + \frac{(N/2 - fb)i}{fw}$

L = 19.5
N = 131
fb = 43
fw = 37
i = 10.5 − 4.5 = 6

Mode = $19.5 + \frac{(131/2 - 43)6}{37}$

∴ Mode = 23.15

Extract 6.2: *A sample of incorrect solution for part (a) of question 6.*

In Extract 6.2, the candidate deployed inappropriate formula for calculating mode. Also, he/she wrongly computed lower boundary (L=19.5 instead of L=16.5).

2.7 Question 7: Probability

The question examined the candidates' ability to establish sample space, identify event from sample space and use the concept of combination in solving mathematical problems.

The question had parts (a), (b) and (c). In part (a), the candidates were informed that "A family has three children and the chances for boys and girls are equal". They were required to (i) use tree diagram to show all possible outcomes; and (ii) find the probability that two are girls and one is a boy.

Part (b) read "A bag contains 6 red and 4 blue beads. A bead is picked out and retained and then a second bead is picked out". The candidates were required to find the probability that; (i) both beads are red (ii) the beads are of different colours.

In part (c), the candidates were required to find the number of odd numbers greater than 2000 but less than 3000 that can be formed using the digits 1, 2, 3, 4, 5 and 6 if each digit is not repeated.

This question was attempted by 27078 (86.48%) candidates. The analysis of data shows that 12818 (47.34%) candidates got marks ranging from 3.5 to 10. Therefore, the candidates had average performance in this question. Figure 8 shows percentage of candidates who got low, average and high marks in this question.

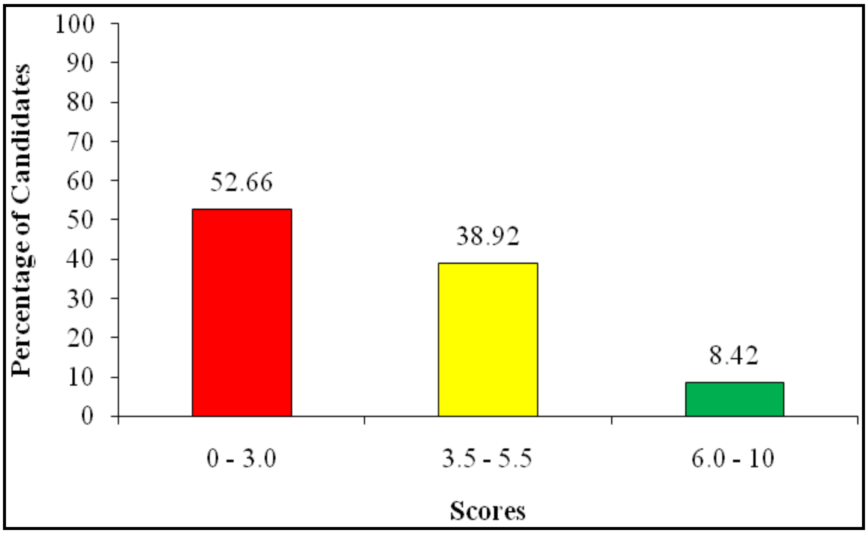
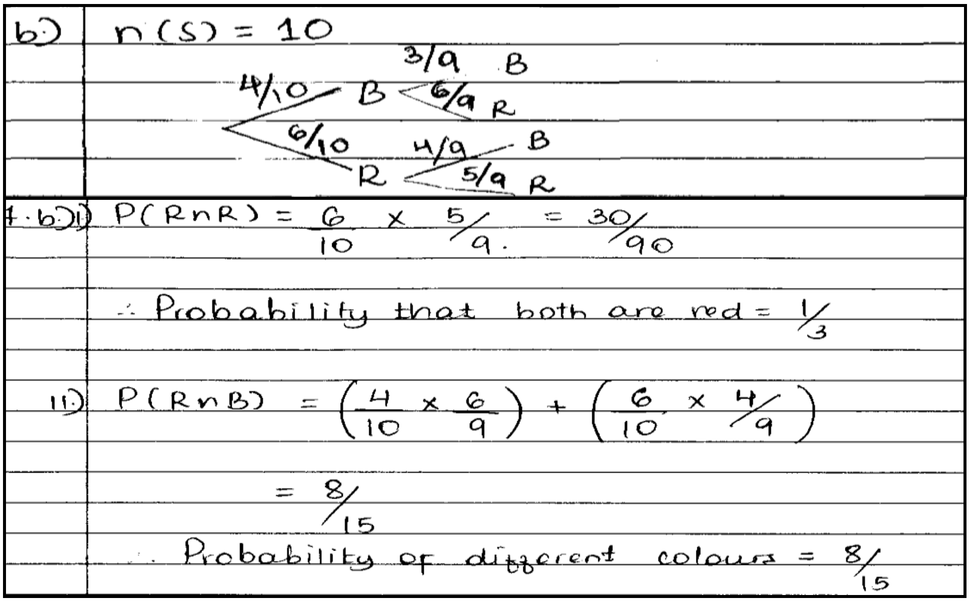


Figure 8: *The candidates' performance in question 7.*

A total of 563 (1.18%) did the question correctly scoring all 10 marks. In part (a) (i), the candidates drew tree diagram correctly and listed all possible outcomes, $S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$. In answering subpart (ii), the candidates correctly identified the elements of the event "two are girls and one is a boy" from the sample space as $E = \{BGG, GBG, GGB\}$. The two sets enabled them to have $n(E) = 3$, $n(S) = 8$ and consequently $P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$.

In part (b), the candidates analyzed the probabilities of drawing each bead without replacement in a tree diagram. From the tree diagram, the candidates were able to compute probability of picking two red beads and two beads of different colours by applying the principle of multiplication and addition (see in Extract 7.1).

In part (c), the candidates realized that the problem can be solved by employing the principle of combinations. Firstly, they computed the number of options that will give the fourth digit which is odd by selecting one digit out of the digits 1, 3 and 5 $\left(\text{i.e } {}^{3}C_{1}\right)$. Secondly, they computed the number of options that will make the number which is greater than 2000 but less than 3000 by selecting the first digit being 2 $\left(\text{i.e } ^1C_1\right)$. Thirdly, they computed the number of options for second digit from the other four digits $\left(^4C_1\right)$ and third digit from the other 3 digits $\left(^3C_1\right)$. Finally, the candidates computed the number of odd numbers that can be formed as $^1C_1 \times ^4C_1 \times ^3C_1 \times ^3C_1 = 36$.


b) $n(S) = 10$

4.b)(i) $P(R \cap R) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$

∴ Probability that both are red $= \frac{1}{3}$

(ii) $P(R \cap B) = \left(\frac{4}{10} \times \frac{6}{9}\right) + \left(\frac{6}{10} \times \frac{4}{9}\right)$

$= \frac{8}{15}$

∴ Probability of different colours $= \frac{8}{15}$

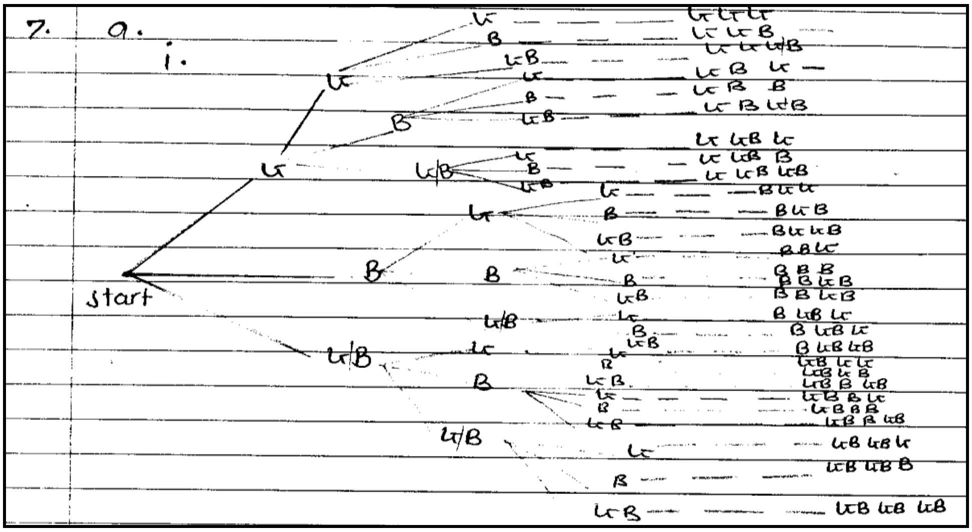
Extract 7.1: *A sample of correct solution for part (a) of question 7.*

Extract 7.1 shows that in second picking the number of sample space (denominator) is 1 less than that of the first picking. This indicates that the first bead was not replaced before the second bead being picked.

Moreover, the data shows that 14260 (52.66%) scored low marks and among them, 7982 (29.48%) candidates scored zero. In part (a) (i), many candidates presented incorrect tree diagram indicating that the candidates interpreted the problem wrongly (see Extract 7.2). Consequently, they got incorrect answer for part (a) (ii).

In part (b), a greater number of candidates incorrectly perceived that drawings of beads were done with replacement. The number of sample space in second picking remained 10 instead of changing to 9. For instance, in answering part (b)(i) some candidates wrote $P(R \cap R) = \frac{6}{10} \times \frac{6}{10} = \frac{9}{25}$ instead of $P(R \cap R) = \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$.

In part (c), majority of the candidates used tabular method incorrectly. Most of them failed to meet the conditions of making a number being odd and greater than 2000 but less than 3000. For example, some candidates solved two cases (number being *odd* and *greater than 2000 but less than 3000*) separately and added the answers. Also, there were candidates who used permutations technique instead of the concept of combination. For instance, they wrote ${}^{6}P_{4} = 360$ odd numbers. Moreover, few candidates incorrectly used the given digits as sample space whereby they wrote $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$, $n(E) = 2$ and consequently $P(E) = \frac{n(E)}{n(s)} = \frac{2}{6} = \frac{1}{3}$.



Extract 7.2: *A sample of incorrect solution for part (a) of question 7.*

In Extract 7.2, the tree diagram shows an impractical chance (G/B) for each child. Therefore, he/she had three possibilities of G, B and G/B for a particular child instead of two possibilities of G and B.

2.8 Question 8: Trigonometry

The question tested ability of the candidates to use trigonometric expansion of compound angles in evaluating the value of trigonometric ratio of the given angles and trigonometric identities to prove the given trigonometric statement(s).

In part (a), the candidates were required to: (i) evaluate the value of $\tan 15^0$ from sine and cosine of 45^0 and 30^0; and (ii) find the exact value of

$\sin(A+B)$ given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{15}{17}$ where A and B are angles in first and second quadrants respectively. In part (b), the candidates were required to prove that $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$.

Data reveals that 19594 (62.58%) candidates attempted this question. Amongst, 4266 (21.78%) scored marks ranging from 3.5 to 10 indicating weak performance. Figure 8 shows the performance of candidates in this question.

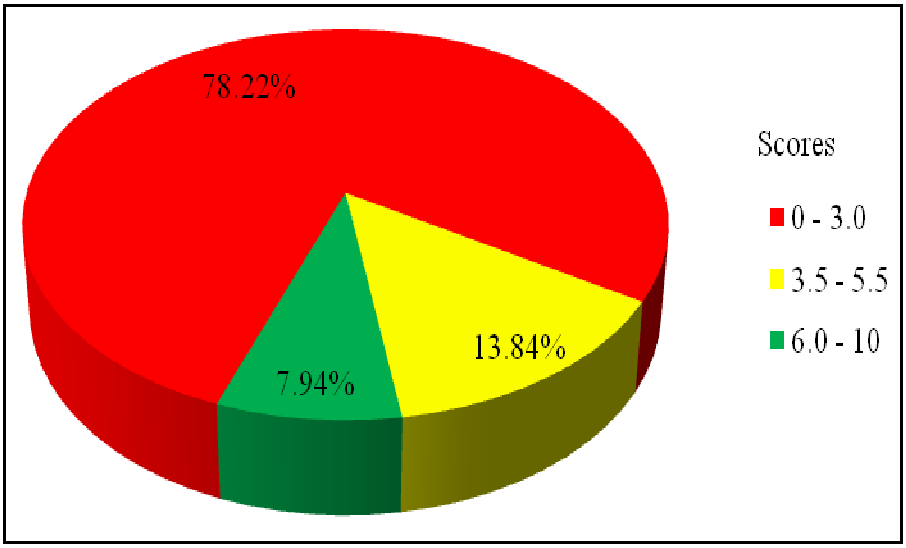
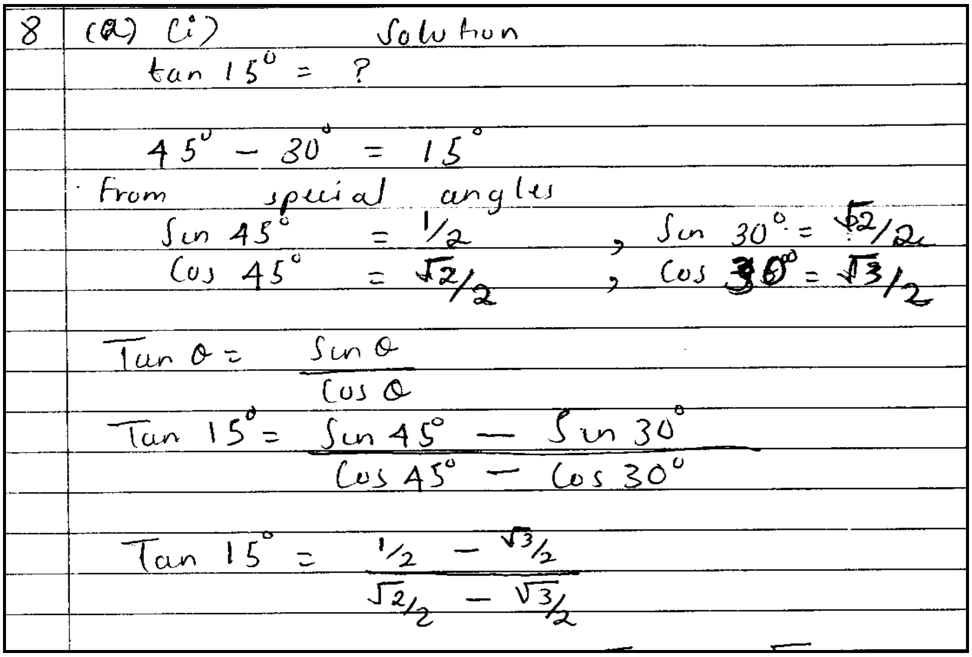


Figure 9: *The candidates' performance in question 8.*

Data shows that 15328 (78.22 %) candidates scored 3.0 marks or less. The common weakness in part (a) was the failure of candidates to apply compound angles formulae. In part (a) (i), many candidates expanded $\sin(45^o - 30^o)$ and $\cos(45^o - 30^o)$ incorrectly (see Extract 8.1). Similarly, in part (a) (ii) the candidates wrote $\sin(A+B) = \sin A + \sin B$ instead of $\sin(A+B) = \sin A \cos B + \sin B \cos A$. Also, some candidates read $\tan 15^o$ from table of natural tangents in mathematical table and wrote $\tan 15^o = 0.267949$. This approach was contrary to the instructions of the question. They were supposed to substitute the values of $\sin 15^o$ and $\cos 15^o$ obtained from $\sin 45^o \cos 30^o - \sin 30^o \cos 45^o$ and $\sin 45^o \cos 30^o + \sin 30^o \cos 45^o$ respectively into $\frac{\sin 15^o}{\cos 15^o}$ so as to evaluate the value of $\tan 15^o$.

A significant number of candidates skipped part (b). The minority who attempted this part were able to write $\sin 2A = 2\cos A \sin A$ however, did not realize the necessity of writing $\sin 2A = \frac{2\cos A \sin A}{1}$. Therefore, they failed to proceed with the proof.


8 (a) (i) Solution
tan 15° = ?

45° − 30° = 15°
From special angles
Sin 45° = 1/2 , Sin 30° = √2/2
Cos 45° = √2/2 , Cos 30° = √3/2

Tan θ = Sin θ / Cos θ
Tan 15° = (Sin 45° − Sin 30°) / (Cos 45° − Cos 30°)

Tan 15° = (1/2 − √3/2) / (√2/2 − √3/2)

Extract 8.1: *A sample of incorrect solution for part (a) of question 8.*

In Extract 8.1, the candidate wrote incorrect expansions of $\sin(45^o - 30^o)$ and $\cos(45^o - 30^o)$.

In spite of weak performance, 21 (0.13%) candidates answered the question perfectly scoring all 10 marks. In part (a) (i), the candidates realized that 15° is the difference between 45° and 30° (i.e $15^\circ = 45^\circ - 30^\circ$), therefore, they wrote $\sin 15^\circ = \sin(45^\circ - 30^\circ)$. Since 30° and 45° are special angles, the candidates realized that $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 45^\circ = \frac{\sqrt{2}}{2}$ and $\cos 45^\circ = \frac{\sqrt{2}}{2}$. The candidates then applied the compound angles formula, $\sin(A - B) = \sin A \cos B - \sin B \cos A$ where $A = 45^\circ$ and $B = 30^\circ$. Finally, they substituted the ratios of special angles into the expanded form of

$\sin(45^\circ - 30^\circ)$ and correctly worked on the expression and ended up with $\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$.

In part (a)(ii), the candidates applied the identity $\cos^2 x + \sin^2 x = 1$ which facilitated to obtain correct ratios for $\sin A$, $\cos B$, $\sin B$ and $\cos A$. Then, they substituted the values into the compound angle formula, $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and got $\sin(A+B) = -\dfrac{13}{85}$ (see Extract 8.1).

In part (b), the candidates correctly applied double angle formulae as they wrote $\sin 2A = \dfrac{2\sin A \cos A}{1}$. Further, they replaced 1 with $\sin^2 A + \cos^2 A$ and wrote $\sin 2A = \dfrac{2\sin A \cos A}{\cos^2 A + \sin^2 A}$. This indicates that the candidates were aware of the trigonometric identity $\sin^2 A + \cos^2 A = 1$. Finally, they divided both numerator and denominator of $\dfrac{2\sin A \cos A}{\cos^2 A + \sin^2 A}$ by $\cos^2 A$, the strategy which enabled them to reach at $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$.

8(a) $\cos A = 4/5$

and also

$\cos^2 B + \sin^2 B = 1$

$\sin^2 B = 1 - \cos^2 B$

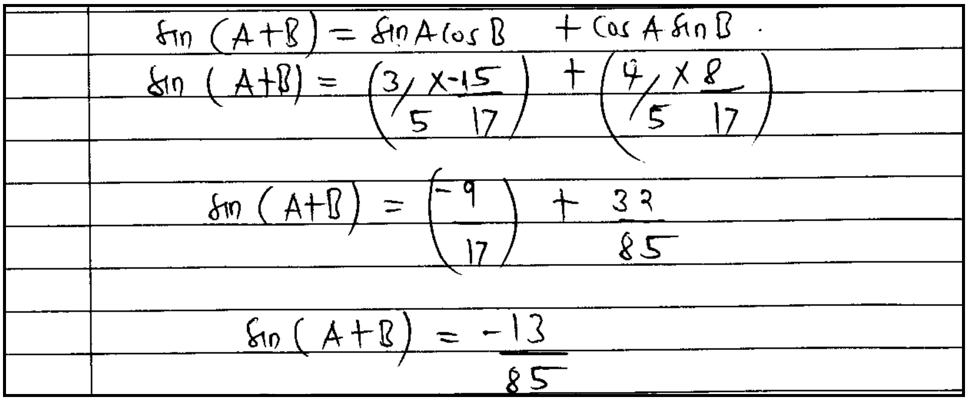
$\sin B = \sqrt{1 - \cos^2 B}$

$\sin B = \sqrt{1 - \left(\frac{15}{17}\right)^2}$

$\sin B = 8/17$

In the 2nd quandrants the sine of angle have positive values

Hence; cosine of B will be negative



Extract 8.2: *A sample of correct solution for part (a)(ii) of question 8.*

In Extract 8.2, the candidate carefully observed the respective quadrant of the angles A and B. Therefore, he/she correctly got $\sin A = \frac{3}{5}$, $\cos B = -\frac{15}{17}$, $\sin B = \frac{8}{17}$ and $\cos A = -\frac{15}{17}$.

2.9 Question 9: Exponential and Logarithmic Functions

The question had parts (a) and (b). Part (a) measured candidates' ability to sketch graph of natural logarithmic function, particularly $f(x) = \ln|x|$ for $x \in R$, and state its domain and range. Part (b) measured candidates' ability to use the concept of exponential function to solve the problem related to half-life of objects. The problem read "The amount (A) of radioactive isotope carbon-14 at any time t is given by $A(t) = A_0 e^{kt}$ and its half-life was 5730 years". From the problem, the candidates were required to: (i) express the amount of carbon-14 left from an initial amount N milligrams as a function of time t in years; and (ii) find the percentage of the original amount of carbon-14 left after 20000 years.

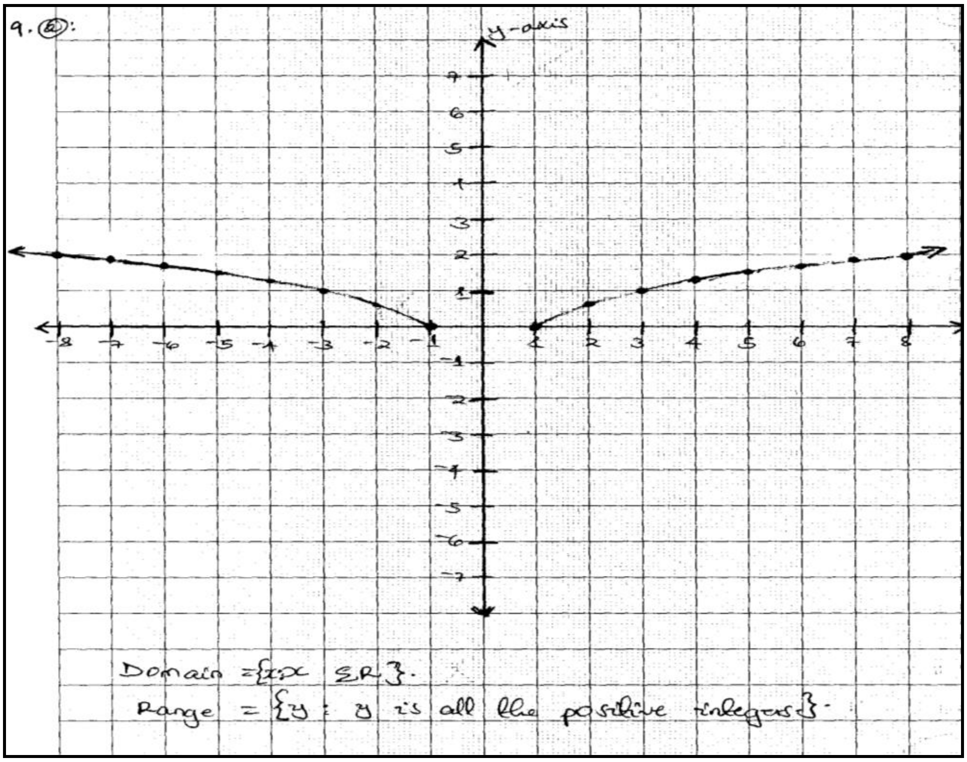
The question was attempted by 15646 (49.97%) candidates. This was the mostly skipped question in this examination. Also, it is the question whose performance was the weakest whereby only 1396 (8.92%) candidates scored from 3.5 to 10 marks (see Table 1).

Table 1: The summary of candidates' performance in question 9.

Scores	0 - 3.0	3.5 - 5.5	6.0 - 10
Number of Candidates	14250	1075	321
Percentage of Candidates	91.08	6.87	2.05

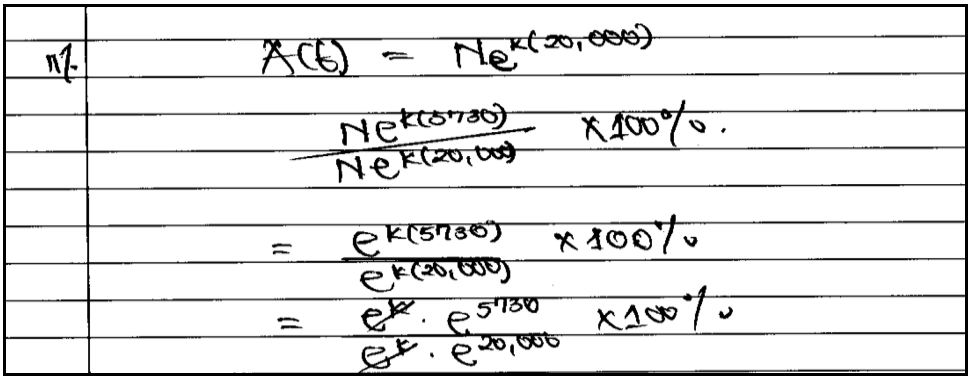
Data shows that 14250 (91.08%) candidates scored low marks ranging from 0 to 3.0. In part (a), some candidates presented table of values showing either only positive or negative values of *x*. Therefore, they drew graph showing only one portion of the curve, either on the left side or right side of the *y* - axis. The candidates were supposed to draw table of values for both positive and negative values of *x* including the values approaching to zero. For this case, their graphs did not show all features of the function. Also, some candidates joined the plotted points using straight line. This proves that the candidates were not aware of the nature of graphs of logarithmic functions which is always a curve. Failure to have correct graph led to incorrect domain and range (see Extract 9.1). However, further analysis on the responses indicated that the candidates crammed the statement $domain = \{x : x \in R\}$ without observing nature of the graph. The statement is incorrect for this particular question. The correct answer is $domain = \{x : x \in R, x \neq 0\}$ and $range = \{y : y \in R\}$.

In part (b), many candidates failed to model the problem using the given data. For instance, some candidates wrote $t = \frac{1}{2}$ when $A = 5730$ instead of $t = 5730\ years$ when $A = \frac{N}{2}$. As a result, they ended up with incorrect equation, $5730 = A_0 e^{\frac{k}{2}}$ in particular. Another notable weakness was inability of the candidates to solve exponential equations. Some candidates correctly got $\frac{N}{2} = Ne^{k(5730)}$ but failed to get $k = -\left(\frac{\ln 2}{5730}\right)$. Most of them applied laws of exponents or logarithms incorrectly (see Extract 9.2). For those two cases, the candidates got incorrect answer for both subparts (i) and (ii).



Extract 9.1: *A sample of incorrect solution for part (b)(ii) of question 9.*

In Extract 9.1, the graph excludes the intervals $0 < x < 1$ and $-1 < x < 0$, which resulted to incorrect range of the function.

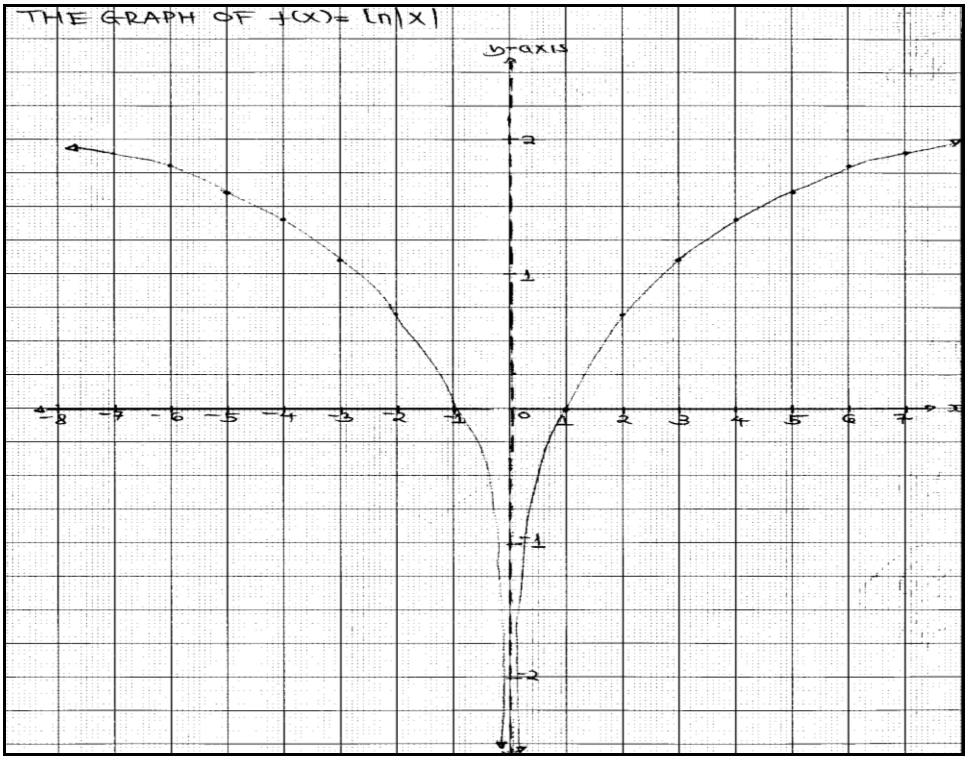


Extract 9.2: *A sample of incorrect solution for part (b)(ii) of question 9.*

In Extract 9.2, the candidate performed the terms of the form $e^{a(b)}$ incorrectly as $e^{a(b)} = e^a e^b$ instead of $e^{a(b)} = e^{a \times b}$.

Nevertheless, 21(0.13%) attempted the question correctly scoring all 10 marks. The candidates who attempted part (a) correctly substituted the positive and negative values of *x* into $y = \ln|x|$ to get corresponding values of *y*. The points were plotted on *xy* - plane and joined to produce a curve (see Extract 9.3). From the graph the candidates identified that the domain includes all real numbers except zero while the range includes all real numbers.

The candidates who attempted part (b) correctly responded to (i) by substituting $t=0$ and $A_0 = N$ into $A(t) = A_0 e^{kt}$ to get the model equation as $A(t) = Ne^{kt}$. Thereafter, they substituted $t = 5730$ years to have $A(5730) = \frac{N}{2}$ and evaluated to get $k = -\left(\frac{\ln(2)}{5730}\right)$ and consequently $A(t) = Ne^{-\left(\frac{\ln(2)}{5730}\right)t}$. This equation relates the amount $A(t)$ of carbon-14 left from initial amount N milligrams and time (t) in years. Using this equation, the candidates responded to (ii) by computing amount left after 20000 years, $A(20000) = 0.0889978N$ and finally its percentage, $\frac{A(20000)}{N}\% = \frac{0.0889978N}{N} \times 100\% = 8.9\%$.



Extract 9.3: *A sample of correct solution for part (a) of question 9.*

In Extract 9.3, the candidates drew two curves extending downwards as value of x - coordinate approaches to zero and extending upwards right and left along x - axis.

2.10 Question 10: Linear Programming

The question intended to measure candidates' ability to formulate constraints and objective function of a Linear Programming problem as well as making optimal decisions using graphical method.

The problem read: "An aircraft has 600 m^2 of cabin space and can carry 5,000 kg of luggage. An economy class passenger gets 3 m^2 of space and is allowed to travel with 20 kg of luggage. The first class passenger gets 4 m^2 of space and is allowed to have 50 kg of luggage in the aircraft. In the aircraft, there is space for at least 50 economy class passengers. The profit per flight for the economy and first class passengers are 40,000/- and 100,000/- respectively". From the information of the problem, the candidates were required to: (a) write down all the constraints; and (b) use graphical method to find the number of economy passengers and first class passengers which will give the maximum profit per flight.

The question was attempted by 28928 (92.39%) candidates, of whom 9082 candidates scored 3.0 or less, 10532 from 3.5 to 5.5 and 9314 from 6.0 to 10. Thus, the performance for this question was good. Figure 10 shows percentage of the candidates who got low, average and high marks.

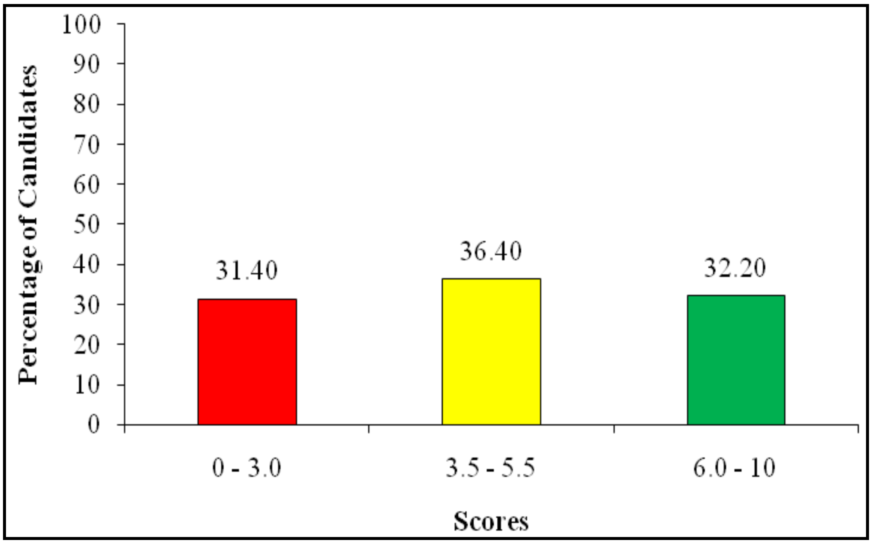
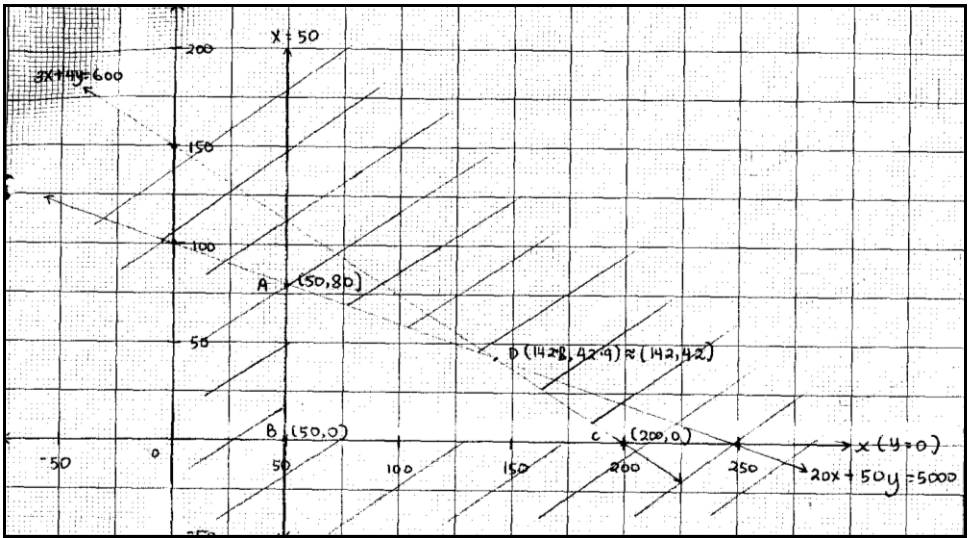


Figure 10: *The candidates' performance in question 10.*

The candidates realized that the number of economy and first class passengers are the decision variables to be controlled so as to meet the limited space and weight. The candidates used *x* as the number of economy passengers and *y* as the number of first class passengers. Using these variables they wrote correct constraints; $x \geq 50$, $y \geq 0$, $3x + 4y \leq 600$ and

$20x + 50y \leq 5000$ and represented them graphically and showing correct feasible region. Also, they formulated the correct objective function, $Maximize: f(x, y) = 40000x + 100000y$ and used it to determine the optimal solution using the corner points of the feasible region (see Extract 10.1).



Since number of people can never be in decimal D can be assumed as (142,42)

So:

RESULT TABLE

CORNER POINTS	f(x) = 40,000x + 100,000y	Values
A (50, 80)	40,000×50 + 100,000×80	10,000,000
B (50, 0)	40,000×50 + 100,000×0	2,000,000
C (200, 0)	40,000×200 + 100,000×0	8,000,000
D (142, 42)	40,000×142 + 100,000×42	9,880,000

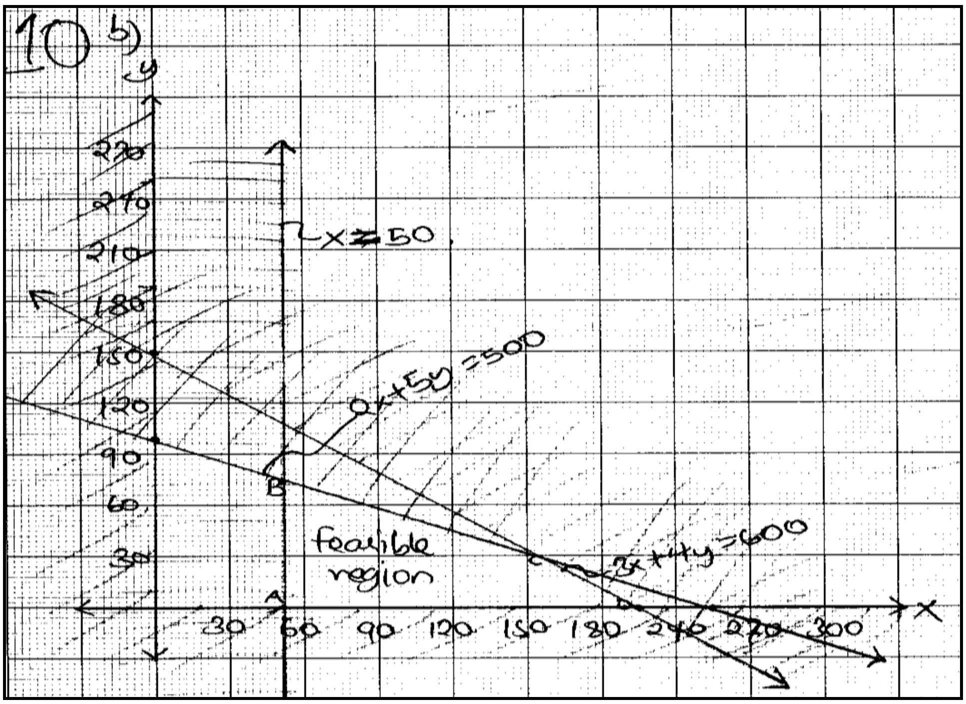
∴ For maximum profit, 50 economy passengers and 80 first class passengers should board the flight.

Extract 10.1: *A sample of correct solution for question 10.*

Extract 10.1 shows the response of the candidate who rounded off 142.8 to 142 and 42.9 to 42. He/she was aware that number of people cannot be expressed in fractions/decimals and the values should satisfy the constraints.

On the other hand, 28791 (99.53%) candidates lost some marks in this question. For part (a), there were notable number of candidates who wrote incorrect constraints. Most of them wrote $3x+4y \geq 600$ and $20x+50y \geq 5000$ instead of $3x+4y \leq 600$ and $20x+50y \leq 5000$ respectively. Other candidates did not write the constraint $x \geq 50$. Furthermore, few candidates got constraints which indicate that they lack knowledge of relating the resources and the respective quantities. For instance, some candidates wrote $x+y \leq 80$, $3x+20y \geq 40000$ and $4x+50y \geq 100000$.

In part (b), the common weakness of the candidates was the failure to round off the values of the corner point $(142.8, 42.9)$. Many candidates rounded it to $(143, 43)$, this does not satisfy the constraints $3x+4y \leq 600$ and $20x+50y \leq 5000$. Other candidates wrote the point $(142.8, 42.9)$ while the values represent number of people (passengers) which cannot be expressed in fraction/decimals. This weakness led to incorrect optimum point, for example, the point $(143, 43)$ gives profit of 10,020,000/- which seems as the maximum. But, when point $(142, 42)$ is used gives profit of 9880000/- making the maximum profit being 10000000/- and consequently optimum point is $(50, 80)$. Moreover, some candidates drew graph of $x \geq 50$ incorrectly, hence they got incorrect feasible region and optimum point.



10 b) A = (50, 0)
B = (50, 100)
C = (142.8, 42.9)
D = (250, 0)

50(40000) + 100,000(0) = 2000,000
40000(50) + 100,000(100) = 12,000,000
40000(142.8) + 100,000(42.9) = 10,002,000
40000(250) + 100,000(0) = 10,000,000.
∴ 50 economy passengers and 100 first class passenger will maximize profit of about 12,000,000/=

Extract 10.2: *A sample of incorrect solution for question 10.*

In Extract 10.2, the candidate incorrectly read point B as (50, 100) instead of (50, 80).

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Basic Applied Mathematics Examination consisted of one (1) paper namely 141 Basic Applied Mathematics. The examination tested 10 topics namely *Calculating Devices, Functions, Algebra, Differentiation, Integration, Statistics, Probability, Trigonometry, Exponential and Logarithmic Functions* and *Linear Programming.*

Analysis of data revealed that the candidates' performance in four (4) topics was good. These topics are *Calculating Devices (80.11%), Statistics (77.27%), Linear Programming (68.60%)* and *Functions (64.38%).* Also, the analysis showed that the candidates performed averagely in the topics of *Differentiation (58.97%), Algebra (51.30%)* and *Probability (47.34%).* Furthermore, the candidates' performance in *Exponential and Logarithmic Functions*, *Trigonometry* and *Integration* was weak whereby the percentage of candidates who scored 3.5 marks or more were 8.92, 21.78 and 23.99 respectively (see **Appendix I**).

4.0 CONCLUSION AND RECOMMENDATIONS

4.1. Conclusion

Data reveals that 18483 (59.31%) candidates passed the examination. Therefore, the overall performance of candidates in this examination was average. Good performance was highly contributed by the ability of candidates to: use scientific calculators; draw and use statistical graphs; and apply knowledge of mathematics to solve real life problem. However, the weak performance was attributed to: inadequate knowledge and skills in sketching graphs of logarithmic functions; applying the knowledge of exponential functions to solve real life problems; solving problems using trigonometric ratios and identities; and applying integration to find area of enclosed region.

4.2. Recommendations

In order to improve the candidates' performance in Basic Applied Mathematics future examinations, the following are recommended:

(a) Teachers should lead students to investigate and discuss real life processes that can be described by exponential functions.

(b) Teachers should lead students to prepare table of values for logarithmic functions and use it to draw graph.

(c) Teachers should guide students to deduce double angle formulae for sine, cosine and tangent.

(d) Teachers should guide students to define sine, cosine and tangent of angles in a particular quadrant and their reciprocals.

(e) Teachers should guide students on how to find area between two curves using integrals.

(f) Teachers should lead students to investigate the applications of probability in real life situations and solve the related problems.

(g) Teacher should guide students to solve simultaneous equations involving linear and quadratic equations.

(h) Teacher should introduce exponential function to students and lead them to discuss the condition(s) for the function to be defined.

Appendix I

Analysis of Candidates' Performance per Topic in 141 Basic Applied Mathematics 2020

S/N	Topic	Questions Number	Percentage of Candidates who Scored an Average of 35% or Above	Remarks
1	Calculating Devices	1	80.11	Good
2	Statistics	6	77.27	Good
3	Linear Programming	10	68.6	Good
4	Functions	2	64.38	Good
5	Differentiation	4	58.97	Average
6	Algebra	3	51.3	Average
7	Probability	7	47.34	Average
8	Integration	5	23.99	Weak
9	Trigonometry	8	21.78	Weak
10	Exponential and Logarithmic Functions	9	8.92	Weak

Appendix II

Candidates' Performance in each Topic for 2019 &2020