THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2019

142 ADVANCED MATHEMATICS

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Published by
The National Examinations Council of Tanzania,
P.O. Box 2624,
Dar es salaam, Tanzania.

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FOREWORD

The National Examinations Council of Tanzania has prepared this report on the Candidates' Item Response Analysis (CIRA) for the Advanced Mathematics examination of Advanced Certificates for Secondary Education Examination (ACSEE) 2019. The aim of the report is to provide feedback to students, teachers and other education stakeholders on how the candidates responded to the items.

The analysis of the candidates' responses is done in order to identify the areas where the candidates faced challenges in attempting examination questions. Basically, the report highlights the candidates' strengths and weaknesses which determine whether the education system was successful or not.

The analysis shows that, the candidates performed well in the topics: Functions, Linear Programming, Sets, Logic, Statistics, Vectors, Calculating Devices, Coordinate Geometry II, Algebra, Hyperbolic Functions, Complex Numbers and Trigonometry. The average performance was observed in the topics such as Numerical Methods, Differentiation, Coordinate Geometry I and Differential Equations and the weak performance in the topics of Integration and Probability. The weak performance in these topics was due to the candidate's inability to derive and apply formulae, axioms, principles and special statistical distribution functions.

The comments given in this report will help students, teachers, parents, school managers and administrators to identify proper measures to be taken in order to improve the candidates' performance in future examinations administered by the Council.

Finally, the Council would like to thank everyone who participated in the preparation of this report.

Dr. Charles E. Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report analyses the candidates' performance in 142 Advanced Mathematics for the Advanced Certificate of Secondary Education Examination (ACSEE) 2019. The analysis highlights the strengths and weaknesses that were observed from the candidates' responses in order to provide a general overview of the candidates' performance.

The Advanced Mathematics Examination had two papers: 142/1 Advanced Mathematics 1 and 142/2 Advanced Mathematics 2. Advanced Mathematics 1 had ten (10) compulsory questions with 10 marks each. 142/2 Advanced Mathematics 2 consisted of sections A and B. Section A comprised four (4) compulsory questions with fifteen (15) marks each. Section B had four (4) optional questions, each carrying twenty (20) marks. The candidates were required to answer any two (2) questions from section B. The questions were set basing on the 2009 Advanced Level Mathematics syllabus.

A total of 10,649 candidates sat for the Advanced Mathematics Examination, out of which 9,237 (86.74%) candidates passed. Generally, the candidates' performance has increased by 3.0 percent as compared to the year 2018 whereby 83.74 percent of 11,991 candidates passed. The candidates who passed these examinations got different grades as indicated in Figure 1.

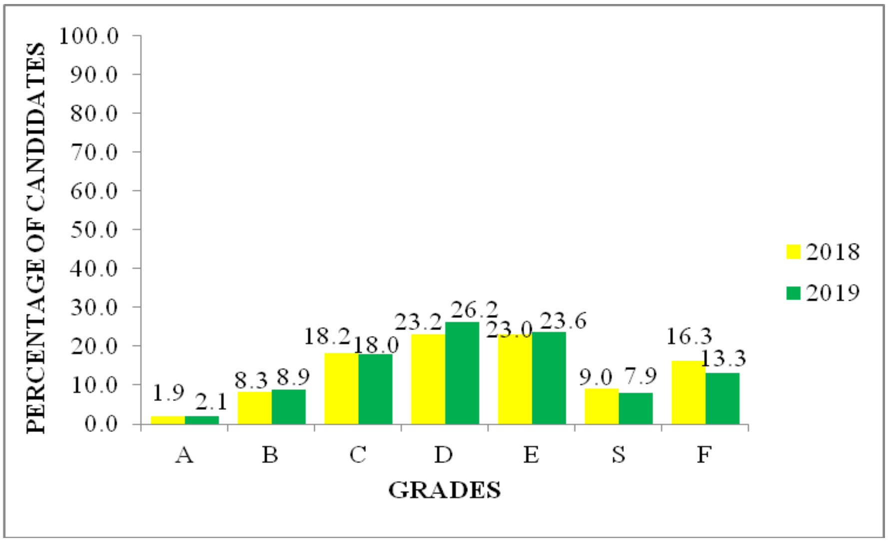


Figure 1: *Distribution of Grades for the 2018 and 2019 Advanced Mathematics Examinations*

This report therefore intends to provide a brief account of the requirements of the questions and the candidates' performance in each item. The factors that accounted for good and poor performance in each question have been indicated and illustrated using samples of candidates' responses.

The analysis of the candidates' performance in each topic is shown in the appendices in which the green colour stands for good performance, the yellow colour stands for average performance and the red colour stands for poor performance. The percentage boundaries 0-34, 35-59 and 60-100 are used to represent poor, average and good performance respectively. It is expected that the recommendations in this report will enhance the teaching and learning of Advanced Mathematics and therefore to improve the candidates' performance in the future.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 142/1 ADVANCED MATHEMATICS 1

2.1.1 Question 1: Calculating Devices

This question had parts (a) and (b). In part (a), the candidates were required to use a non-programmable calculator to (i) calculate $\log_e\left(e^4+2\ln 5\right)+\log 5$ correct to six decimal places, and (ii) obtain the value of $\sqrt{\dfrac{(4.03)^3\times(814765)^{0.5}}{\sqrt{5}}}$ correct to three significant figures. In part (b), the candidates were given the monthly salaries in Tanzania shillings for 20 employees of KNCU as 260,000.00, 170,000.00, 85,000.00, 505,000.00, 129,000.00, 89,000.00, 220,000.00, 157,000.00, 103,000.00, 480,000.00, 790,000.00, 600,000.00, 340,000.00, 144,000.00, 128,000.00, 90,000.00, 102,000.00, 185,000.00, 219,000.00 and 195,000.00. They were required to use the statistical functions of the scientific calculator to calculate (i) the mean $(\bar{x})$ and (ii) standard deviation $(x\delta n)$.

This question was attempted by 10,649 (99.4%) candidates. The percentages of candidates with weak, average and good scores are shown in Figure 2.

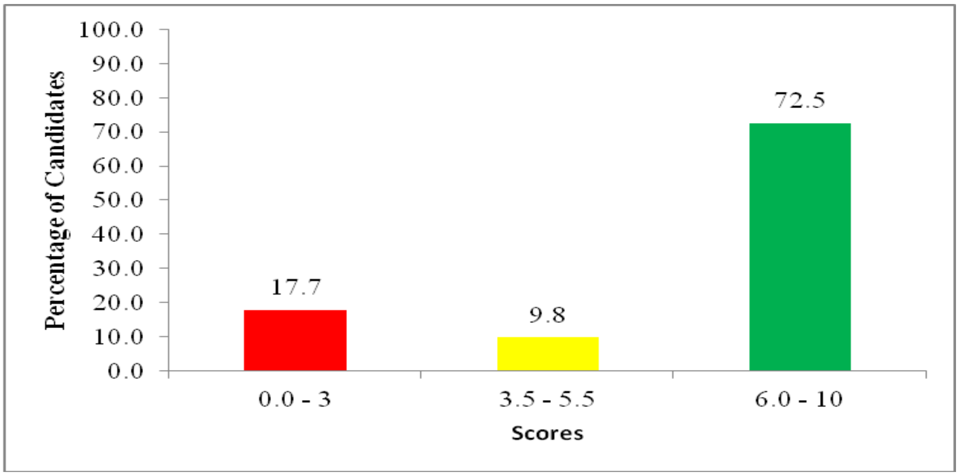


Figure 2: *Candidates' Performance in Question 1*

Figure 2 shows that 8760 (82.3%) candidates scored above 3 marks, therefore they had a good performance in this question. It also shows that 72.5 percent scored from 6 to 10 marks, amongst 33.6 percent scored all 10 marks. The candidates in this category were able to compute the given expressions and presented the answers according to the given instructions. In part (a) (i), they were able to use a non-programmable calculator to compute the value of $\log_e\left(e^4 + 2\ln 5\right) + \log 5$ correct to six decimal places as 4.756253. In part (a) (ii), they computed the square root of the expression $\frac{(4.03)^3 \times (814765)^{0.5}}{\sqrt{5}}$ correct to three significant figures as 163. In part (b), they correctly entered the monthly salaries for 20 employees into the scientific calculator to get the mean ($\bar{x}$) and standard deviation ($x\delta n$) as $249{,}550.00$ and $190{,}709.3272$ respectively. Extract 1.1 shows the responses of a candidate who answered the question correctly.

1	a) (i) 4.756253302 . ≃ 4.756253 (6dps).
	(ii) 162.5448295 ≃ 163 (3sf).
	b)(i) Mean ($\bar{x}$) = 249,550 Tshs. or Tsh 249,550.
	(ii) Standard Deviation = Tshs 190,709.3272.

Extract 1.1 *A correct response from one of the candidates.*

Figure 2 also shows that 17.7 percent of the candidates scored from 0 to 3 out of 10 marks and among them 4.7 percent scored zero. Such candidates could not fix non-programmable calculators to six decimal places in part (a) (i) and three significant figures in part (a) (ii). Thus, the majority of candidates obtained the inappropriate answers such as 4.7562533 in (i) and 162.54483, 162.545 etc. in (ii). In part (b), a significant number of candidates calculated the mean and standard deviation manually by using the frequency distribution table instead of using the statistical functions of a non programmable calculator. This shows that they did not adhere to the requirements of the question. A sample response from one of the candidates who did the question poorly is shown in Extract 1.2.

0) i)	= 4.7562533 Ans.
ii)	= 162 Ans

Extract 1.2: *An incorrect response from one of the candidates*

2.1.2 Question 2: Hyperbolic Functions

In this question, the candidates were required to: (a) solve the equation $\operatorname{cosech}^{-1}(x)+\ln x-\ln 3=0$, (b) prove that $x=\ln(\sec\theta+\tan\theta)$ given that $\sinh x=\tan\theta$ and (c) use the hyperbolic functions substitutions to find $\int \frac{1}{\sqrt{(x^2+8x+25)}}dx$.

A total of 9946 candidates equivalent to 92.9 percent attempted this question. Amongst, 7644 (76.9%) candidates scored marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was good. Figure 3 gives the percentage of candidates who had weak, average and good scores.

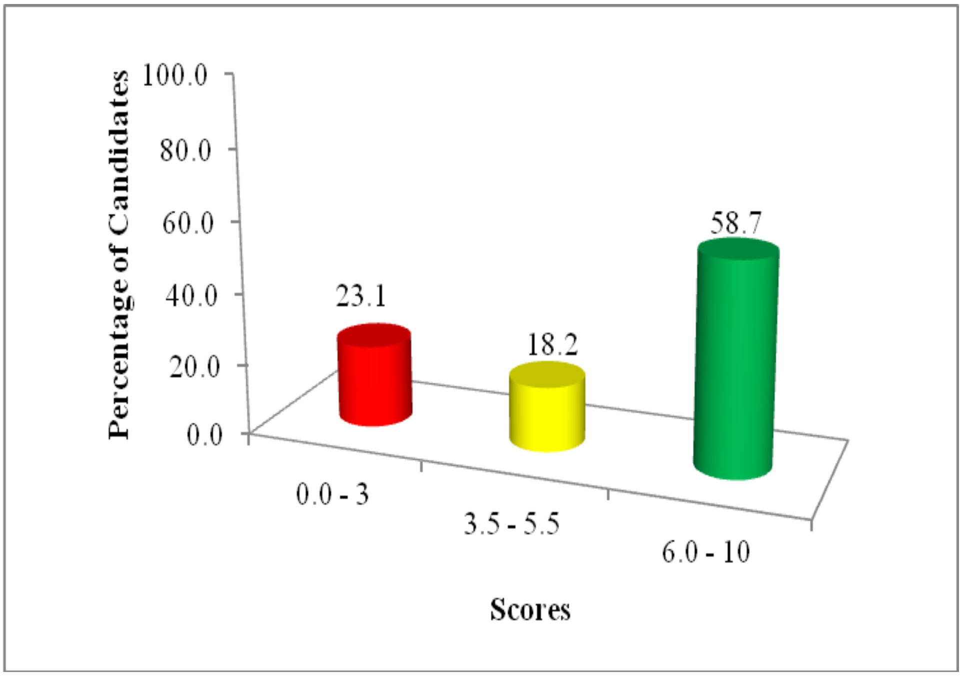


Figure 3: *Candidates' Performance in Question 2*

The candidates who performed well in part (a) applied the laws of logarithms and the definition of $\operatorname{cosech}^{-1}(x)$ to express $\operatorname{cosech}^{-1}(x)+\ln x-\ln 3=0$ in the form $x^3-3x=0$ and solved it to get $x=\sqrt{3}$. Likewise, in part (b), the candidates were able to use either the definition $\sinh x=\frac{e^x-e^{-x}}{2}$ or $\sinh^{-1}(x)=\ln\left(x+\sqrt{1+x^2}\right)$ to prove that $x=\ln(\sec\theta+\tan\theta)$. Moreover, in part (c), the candidates were able to write $x^2+8x+25$ as $(x+4)^2+9$ and substituted $x+4=3\sinh\theta$ and $dx=3\cosh\theta\, d\theta$ into $\int \frac{1}{\sqrt{(x+4)^2+9}}dx$ to get $\int \frac{1}{\sqrt{(x^2+8x+25)}}dx=\sinh^{-1}\left(\frac{x+4}{3}\right)+C$. Extract 2.1 is a solution presented by one of the candidates who answered the question correctly.

2 a) $= \ln\left[\left(\frac{1+\sqrt{1+x^2}}{x}\right)(x) \div 3\right] = 0$

$= \ln\left[\frac{1+\sqrt{1+x^2}}{3}\right] = 0$

$= e^{\ln\left(\frac{1+\sqrt{1+x^2}}{3}\right)} = e^0$

$= \frac{1+\sqrt{1+x^2}}{3} = 1$

$= 1+\sqrt{1+x^2} = 3$

$= \sqrt{1+x^2} = 3-1$

$\sqrt{1+x^2} = 2$

Squaring both sides

$1+x^2 = 4$

$x^2 = 4-1$

$x^2 = 3$

$x = \sqrt{3}$

$\therefore \underline{x = \sqrt{3}}.$

b) $\sinh x = \tan\theta$

Required to prove $x = \ln(\sec\theta + \tan\theta)$.

soln

$\sinh x = \tan\theta$.

$\left(\frac{e^x - e^{-x}}{2}\right) = \tan\theta$.

$e^x - e^{-x} = 2\tan\theta$.

$e^{2x} - 1 = 2e^x \tan\theta$.

$e^{2x} - 2\tan\theta\, e^x - 1 = 0$

By quadratic equation

$e^x = \frac{2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$= \frac{2\tan\theta \pm 2\sqrt{\tan^2+1}}{2}$

2 b) $e^x = \tan\theta \pm \sqrt{\tan^2\theta + 1}$

But from,

$\tan^2\theta + 1 = \sec^2\theta$

$\sec\theta = \sqrt{\tan^2\theta + 1}$

$\Rightarrow e^x = \tan\theta \pm \sec\theta.$

$e^x = \tan\theta + \sec\theta$

Applying ln both sides

$\ln e^x = \ln[\tan\theta + \sec\theta].$

$x = \ln[\sec\theta + \tan\theta],$

$\therefore \underline{x = \ln[\sec\theta + \tan\theta].}$ Hence Proved.

c) Given $\int \frac{1}{\sqrt{x^2+8x+25}}\,dx$

$= \int \frac{1}{\sqrt{(x+4)^2 - 16 + 25}}\,dx$

$= \int \frac{1}{\sqrt{(x+4)^2 + 9}}\,dx$

$= \int \frac{1}{\sqrt{9 + (x+4)^2}}\,dx$

Let

$(x+4) = 3\sinh\theta.$

$dx = 3\cosh\theta\, d\theta.$

$\theta = \sinh^{-1}\left(\frac{x+4}{3}\right)$

Now,

$\int \frac{3\cosh\theta\, d\theta}{\sqrt{9 + 9\sinh^2\theta}}$

$= \int \frac{3\cosh\theta\, d\theta}{3\sqrt{1+\sinh^2\theta}}.$

But $\sqrt{1+\sinh^2\theta} = \cosh\theta.$

2 c) $\Rightarrow \int \frac{\cosh\theta\, d\theta}{\cosh\theta}$

$= \int d\theta$

$= \theta + c$

But $\theta = \sinh^{-1}\left(\frac{x+4}{3}\right).$

$\therefore \int \frac{1}{\sqrt{x^2+8x+25}}\,dx = \sinh^{-1}\left(\frac{x+4}{3}\right) + c$

Extract 2.1: *A correct response*

In Extract 2.1 the candidate defined $\operatorname{cosech}^{-1}(x)$ and $\sinh x$ correctly and applied the appropriate techniques of integration.

On the other hand, 2302 (23.1%) candidates scored 3.0 marks or less. In part (a), some candidates defined $\operatorname{cosech}^{-1} x$ incorrectly whereby $\frac{1}{\sinh^{-1} x}$, $\frac{1}{\operatorname{cosech} x}$ and $\ln\left(1+\sqrt{1+x^2}\right)$ were frequently seen. Other candidates reached at $x = \frac{1}{\sinh\left(\ln\left(\frac{3}{x}\right)\right)}$ correctly but they confused the definition of the hyperbolic sine with that of hyperbolic cosine. Such candidates wrote $x = \frac{2}{e^{\ln\left(\frac{3}{x}\right)} + e^{-\ln\left(\frac{3}{x}\right)}}$ instead of $x = \frac{2}{e^{\ln\left(\frac{3}{x}\right)} - e^{-\ln\left(\frac{3}{x}\right)}}$. In part (b), the confusion in defining hyperbolic sine and hyperbolic cosine was also noted. Some candidates defined $\sinh x$ as $\frac{e^x + e^{-x}}{2}$ instead of $\frac{e^x - e^{-x}}{2}$. Such candidates got $x = \ln\left(\tan\theta + \sqrt{\tan^2\theta - 1}\right)$, which do not give a way to $x = \ln(\sec\theta + \tan\theta)$. Also, some candidates used the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to produce the equation $\sin\theta = \cos\theta \sinh x$. This equation does not give direct connection to the form of the required proof. They were supposed to define $\sinh x$ as $\frac{e^x - e^{-x}}{2}$ to produce an equation $\left(e^x\right)^2 - (2\tan\theta)e^x - 1 = 0$ that could be solved to get $x = \ln(\sec\theta + \tan\theta)$. In part (c), several candidates showed weaknesses in completing the square of $x^2 + 8x + 25$ so as to produce $(x+4)^2 + 9$. Majority of these candidates had a wrong solution $\int \frac{1}{\sqrt{x^2 + 8x + 25}}\,dx = \int \frac{1}{\sqrt{(x+4)^2 + 41}}\,dx$ which led to a wrong answer, $\int \frac{1}{\sqrt{x^2 + 8x + 25}}\,dx = \sqrt{41}\cosh^{-1}\left(\frac{x+4}{41}\right) + c$. Extract 2.2 is a sample response from a candidate who did the question badly.

2a) $\operatorname{cosech}^{-1}(x) + \ln x - \ln 3 = 0$

$\operatorname{cosech}^{-1} x = {}^{-}\ln x + \ln 3$

from $\operatorname{cosech}^{-1} x = \dfrac{1}{\sinh^{-1} x} =$

$\sinh^{-1} x = \ln (x + \sqrt{x^2 - 1})$

$= \operatorname{cosech}^{-1} x = \dfrac{1}{\ln (x + \sqrt{x^2 - 1})}$

$\dfrac{1}{\ln (x + \sqrt{x^2 - 1})} = {}^{-}\ln x + \ln 3$

$= (\ln (x + \sqrt{x^2 - 1}))({}^{-}\ln x + \ln 3)$

$(\ln x + \ln \sqrt{x^2 - 1}) \cdot ({}^{-}\ln x + \ln 3)$

$(\ln x)^2 + \ln^2 3x + \ln^2 (x(\sqrt{x^2 - 1}) + \ln^2 3(\sqrt{x^2 - 1} = 0$

$\ln^2 4x + \ln^2 \sqrt{x^2 - 1}\ (x + 3) = 0$

$\ln^2 \sqrt{x^2 - 1}\ (x + 3) = {}^{-}\ln^2 4x$

$(\ln \sqrt{x^2 - 1})^2 (x + 3) = (-\ln 4x)^2$

$\ln (x^2 - 1)(x + 3) = (-\ln 4x)^2$

$\ln x^3 + 3x^2 - x - 3 = (-\ln 4x)^2$

$\ln x^3 + 3x^2 - x - 3 + (\ln 4x)^2 = 0$

$\ln (x^3 + 3x^2 - x - 3 + 4x^2) = 0$

$\ln (x^3 + 7x^2 - x - 3) = 0$

$x = 0.693$ or -7.081 or -0.611

Extract 2.2: *An incorrect response*

In Extract 2.2, the candidate defined $cosech^{-1}x$ incorrectly.

2.1.3 Question 3: Linear Programming

In this question, it was given that "Mr Masumbuko has two traditional stores A and B for storing groundnuts. He stored 80 bags in A and 70 bags in B. Two customers C and D placed orders for 35 and 60 bags respectively. The transport costs per bag from A to C, A to D, B to C and B to D are 8/=, 12/=, 10/= and 13/= respectively." The candidates were required to (a), find the number of bags delivered to each customer in order to minimize the transport cost and (b) determine the minimum cost of transport.

The analysis shows that 6.3 percent of the candidates who attempted this question scored from 0 to 3 marks, 13.4 percent from 3.5 to 5.5 and 80.3 percent from 6 to

10 marks. Generally, the candidates' performance was very high, as 93.7 percent of them got more than 3 marks. Figure 4 illustrates the candidates' performance in this question.

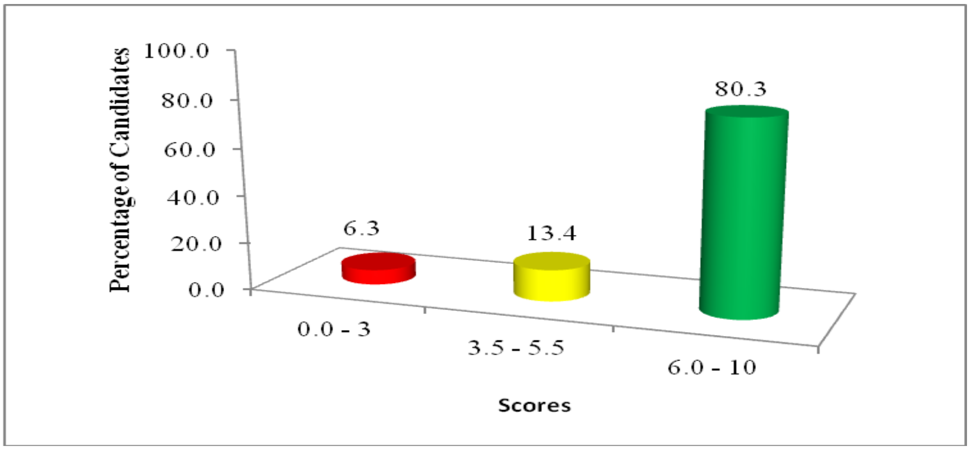


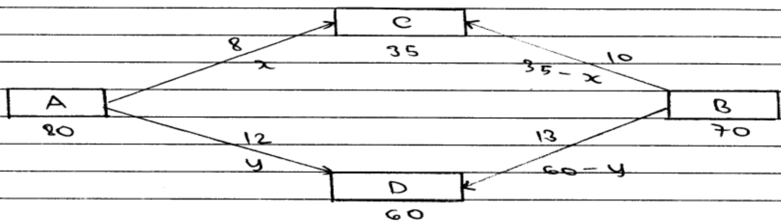
Figure 4: *Candidates' Performance in Question 3.*

The candidates who performed well in this question were able to present the given information on a drawing which helped them to formulate the constraints $x+y \le 80$, $x+y \ge 25$, $x \le 35$ and $y \le 60$ and the objective function $f(x, y)=1130-2x-y$. The candidates also represented the inequalities graphically and identified the feasible region as well as the corner points $(25, 0)$, $(35, 0)$, $(35, 45)$, $(20, 60)$, $(0, 60)$ and $(0, 25)$. Furthermore, they inserted the corner points into the objective function and recognized that the point $(35, 45)$ gives 35, 45 and 15 as the required number of bags to be transported from A to C, A to D and B to D respectively. Moreover, the candidates obtained 1015/= as the minimum cost of transport. Extract 3.1 is a sample answer obtained by one of the candidates who answered the question correctly.

3. (a) Let x be number of bags transported from A to C

y be the number of bags transported from A to D.

Consider:



Constraints:

$35 - x \geq 0$

$x \leq 35$ - - - - - - (i)

$60 - y \geq 0$

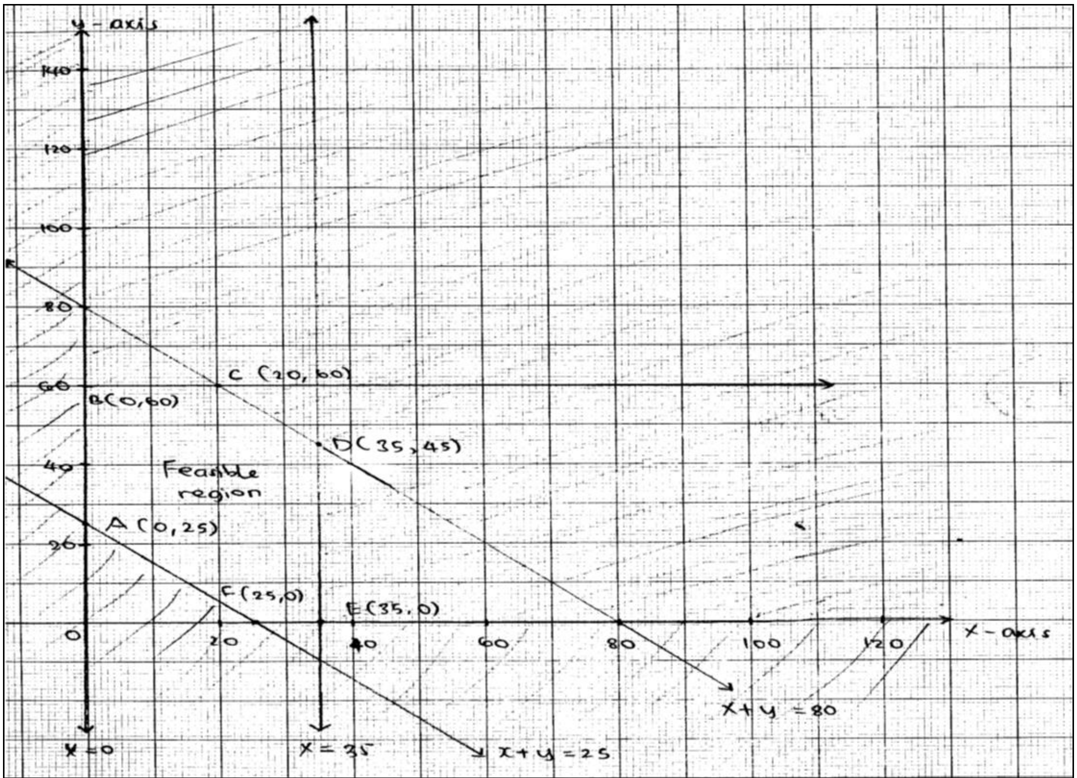
$y \leq 60$ - - - - - - (ii)

$x + y \leq 80$ - - - - - (iii)

$(35 - x) + (60 - y) \leq 70$

$x + y \geq 25$ - - - (iv)

$x, y \geq 0$ - - - - (v)



3 (a) Objective function:

$f(x, y) = 8x + 12y + 10(35 - x) + 13(60 - y)$

$= 8x + 12y + 350 - 10x + 780 - 13y$

$= 1130 - 2x - y$

Equations:

$x + y = 25$

x	0	25
y	25	0

$x + y = 80$

x	0	80
y	80	0

TABLE OF VALUES:

Corner points	$f(x,y) = 1130 - 2x - y$	Values
A(0,25)	$1130 - 2(0) - (25)$	1,105 /=
B(0,60)	$1130 - 2(0) - (60)$	1,070 /=
C(20,60)	$1130 - 2(20) - 60$	1,030 /=
D(35,45)	$1130 - 2(35) - 45$	1,015 /=
E(35,0)	$1130 - 2(35) - 0$	1,060 /=
F(25,0)	$1130 - 2(25) - 0$	1,080 /=

Optimal points are (35, 45).

∴ He should transport as follows;

From	To	
	C	D
A	35	45
B	0	15

3(b) ∴ The minimum cost of transport is 1,015 /=

Extract 3.1: *A correct response from one of the candidates*

On the other hand, the analysis of the candidates' responses shows that the candidates who got the question wrong failed to formulate the inequalities which satisfy the given conditions. For example, they presented the inequalities as $x+y \geq 80$, $x+y \leq 25$, $x \geq 35$ and $y \geq 60$. Further analysis shows that a significant number of candidates failed to present the given information on a transportation schedule. Some of them regarded the given question as a normal linear programming problem while it was a transportation problem. Consequently, they formulated the constraints as $8x+12y \geq 80$, $10x+13y \geq 70$ and objective function as $f(x, y) = 35x + 60y$. Others wrote a system of linear equations $8x+12y = 80$, $10x+13y = 70$ instead of inequalities. Extract 3.2 is a sample answer from the script of the candidate showing incorrect inequalities and objective function.

Solution:

3. from: Data:

let x be for ~~stores~~ Customer A C

y be for ~~stores~~ Customer B D

i) Inequalities. Line points.

$8x + 12y \geq 80$ $x = 10$ $y = 6.67 \approx 6.7$.

$10x + 13y \geq 70$ $x = 7$ $y = 5.4$.

$x \geq 0$ $x = 0$

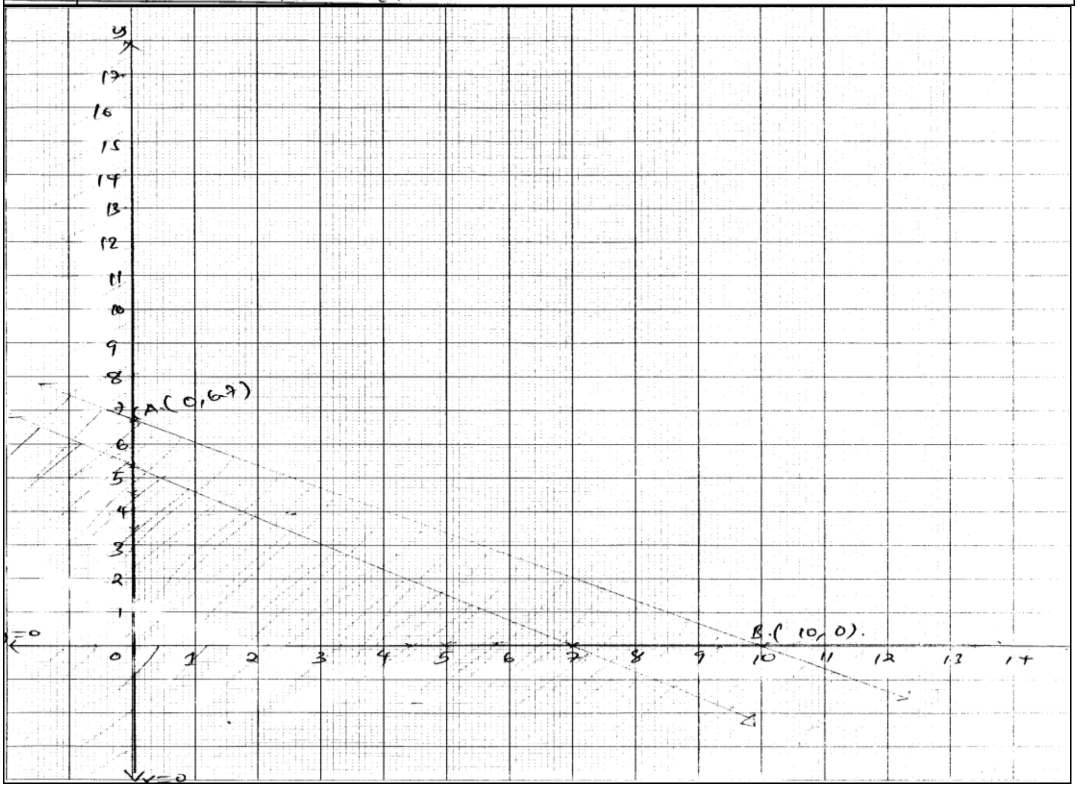
$y \geq 0$ $y = 0$.

objective function $f(x,y) = 35x + 60y$.

Corner points	Objective function. $35x + 60y$	Total.
A	$35(0) + 60(6.7)$	402.
B	$35(10) + 60(0)$.	350.

i) Therefore the farmer should deliver 400 to 402 bags of groundnuts for store A. and 350 bags of groundnuts for store B.

ii) The Minimum cost is 350



Extract 3.2: *An incorrect response from one of the candidates*

2.1.4 Question 4: Statistics

The question was;

(a) The sum of 20 numbers is 320 and the sum of the squares of these numbers is 5840.

 (i) Calculate the mean and standard deviation of 20 numbers.

(ii) If one number is added to the 20 numbers so that the mean is unchanged, find this number and show whether the standard deviation will change or not.

(b) A watchmen at Mlimani city shopping centre recorded the length of time, to the nearest minute, that a sample of 131 cars was parked in their car park. The results were:

Time (minutes)	5 – 10	11 – 16	17 – 22	23 – 28	29 – 34	35 - 40
frequency	15	28	37	26	18	7

(i) Calculate the median time correct to four decimal places.

(ii) By using the coding method and the assumed mean $A = 19.5$, calculate the mean in two decimal places.

The analysis of data shows that 10594 (98.9%) candidates attempted this question out of which 8926 (84.3%) candidates scored marks ranging from 3.5 to 10. This implies that the candidates' performance in this question was good as illustrated in Figure 5.

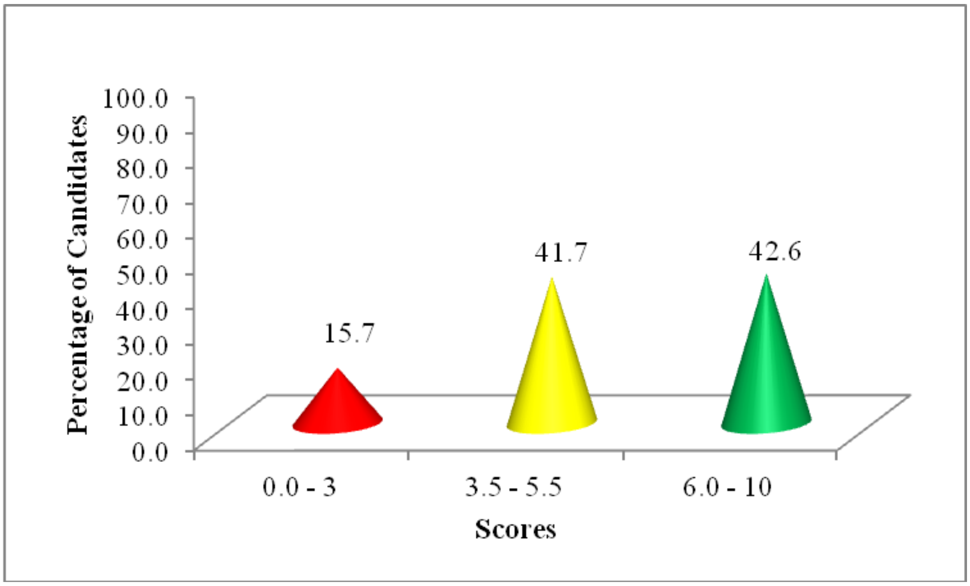


Figure 5: *Candidates' Performance in Question 4.*

A total of 904 candidates (8.4%) scored all 10 marks allotted to this question. In part (a) (i), they extracted the data $\sum_{i=1}^{20} x_i = 320$, $\sum_{i=1}^{20} x_i^2 = 5840$ and $N = 20$ from the given word problem. They applied the formulae $\bar{x} = \frac{\sum_{i=1}^{20} x_i}{N}$ and

$\delta = \sqrt{\dfrac{\sum_{i=1}^{n} x_i^{\,2}}{n} - \left(\dfrac{\sum_{i=1}^{n} x_i}{n}\right)^2}$ to get $\bar{x} = 16$ and $\delta = 6$. In part (a) (ii), the candidates realised that the value of the added number is the same as the computed mean in (i). Consequently, they computed $\sum_{i=1}^{21} x_i^{\,2}$ as $\sum_{i=1}^{21} x_i^{\,2} = \sum_{i=1}^{20} x_i^{\,2} + (16)^2 = 6096$ and applied the formula $\delta = \sqrt{\dfrac{\sum_{i=1}^{21} x_i^{\,2}}{21} - (\bar{x})^2}$ to get $\delta = 5.855$. Finally, they were able to comment that the standard deviation will change. In part (b) (i), the candidates extracted the data $L = 16.5$, $N = 131$, $n_b = 43$, $c = 6$ and $n_w = 37$ from the given frequency distribution table. Then, they used the formulae $Median = L + \left(\dfrac{\frac{N}{2} - n_b}{n_w}\right)c$ to obtain time equals 20.15 minutes. In part (b) (ii), the candidates used $A = 19.5$ and $c = 6$ to develop the columns of $y = \dfrac{x - A}{c}$ and fy, which enabled them to get $\sum fy = 25$. Moreover, they applied the formula $Mean = A + c\dfrac{\sum fy}{N}$ to obtain mean equals 20.65 minutes. Extract 4.1 shows a sample of a solution from one of the candidates who answered the question correctly.

4a) Solution

Sum of 20 number = 320, So n = 20,

$\Sigma x = 320, \quad \Sigma x^2 = 5840$

(i) Mean,

$\bar{x} = \dfrac{\Sigma x}{N} = \dfrac{320}{20} = 16$

Standard deviation,

Using the Formula, $\delta = \sqrt{\dfrac{\Sigma x^2}{N} - \left(\dfrac{\Sigma x}{N}\right)^2}$

$\delta = \sqrt{\dfrac{5840}{20} - \left(\dfrac{320}{20}\right)^2}$

$\delta = 6$

∴ Mean is 16 and Standard deviation is 6

(ii) Let y be the number added to 20 numbers

then, $N = 21$, $\Sigma x = 320 + y$

Since the mean is unchanged, the $\bar{x} = 16$

So, From Mean $\bar{x} = \frac{\sum x}{N}$

$$16 = \frac{320 + y}{21}$$

$$320 + y = 336$$

$$y = 336 - 320 = 16$$

The number that is added is 16

+ Standard deviation is calculate using the relation,

$$\delta = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

Here, $\sum x^2 = 5840 + 16^2 = 6096$

$\sum x = 336$, $N = 21$

then $\delta = \sqrt{\frac{6096}{21} - \left(\frac{336}{21}\right)^2}$

4a) (ii) Gives, $\delta = \sqrt{34.285}$

$\delta = 5.8554$

$\therefore$ The standard deviation will change from 6 to 5.8554

(b) Solution

The Frequency Distribution table.

Time (min)	f	x	Cf	$y = \frac{x - 19.5}{6}$	fy
5 - 10	15	7.5	15	-2	-30
11 - 16	28	13.5	43	-1	-28
17 - 22	37	19.5	80	0	0
23 - 28	26	25.5	106	1	26
29 - 34	18	31.5	124	2	36
35 - 40	7	37.5	131	3	21

then, $N = 131$ $\sum fy = 25$

(i) Median,

Median is given by, Median $= L + \left(\frac{N/2 - nb}{nw}\right) i$

Here, $L =$ Is obtained from class interval 17 - 22

then, $\frac{N}{2} = 65.5$, $L = 16.5$, $nb = 43$, $nw = 37$

and i - interval $= 6$

then, Median $= 16.5 + \frac{(65.5 - 43) \times 6}{37}$

$= 20.14864865 \simeq 2.015 \times 10^1$

$\therefore$ Median is 20.15 to four significant figure

(ii) Using Coding method, $A = 19.5$

Mean by Coding method is $\bar{x} = A + c\left(\frac{\sum fy}{N}\right)$

Here $A = 19.5$, $C = 6$, $\sum fy = 25$, $N = 131$

	then.
4b)	(ii) $\bar{x} = 19.5 + 6 \times \left(\frac{25}{131}\right)$
	$\bar{x} = 20.64503817 \simeq 20.65$
	$\therefore$ Mean $\bar{x} = 20.65$ to two decimal places.

Extract 4.1: *A* ***correct response***

In Extract 4.1, the candidate extracted data from the word problem and frequency distribution table correctly and applied the appropriate formulae.

04b. $A = 19.5$. $C = 6$.

Time (min).	f.	c.f.	x.	d. x − A.	u = d/c.	fu.
5 − 10	15	15	7.5	−12	−2	−30.
11 − 16	28	43	13.5	−6	−1.	−28
17 − 22	37	80	19.5	0	0	0
23 − 28	26	106	25.5	6	1	26
29 − 34	18	124	31.5	12	2.	36.
35 − 40	7.	131.	37.5	18	3.	21
	Σ = 131.					Σ = 25.

(i) Required: Median time correct to four significant figures.

Median = $L + \left(\frac{t_1}{t_1 + t_2}\right) c$.

where L = lower class boundary of median class.

t_1 − frequency of median class − the one before it.

C = class size.

t_2 − frequency of median class − the one after it.

4b (i) Thus

$L = 16.5$.

$C = 6$.

$t_1 = 37 - 28$

$= 9$.

$t_2 = 37 - 26$.

$= 11$.

Thus

Median $= 16.5 + \left(\frac{9}{9 + 11}\right) 6$.

Median $= 16.5 + 2.7$.

Median time $= 19.2$ minutes.

Extract 4.2: *A correct response*

In Extract 4.2 the candidate calculated the mode instead of median.

2.1.5 Question 5: Sets

This question had parts (a) and (b). In part (a), the candidates were required to prove that $(A\cap B')\cup(B\cap A')=(A\cup B)-(A\cap B)'$ using the properties of sets. In part (b), it was given that a student at Sokoine University of Agriculture made a study about the types of livestock's in a nearby village. The student came up with the following findings: 82 villagers kept cattle, 110 villagers kept goats, 73 villagers kept pigs, 59 villagers kept cattle and goats, 53 kept goats and pigs, 32 kept cattle and pigs, 20 kept all three types of livestock's. If the village has 200 occupants, the candidates were instructed to use Venn diagram to find the number of villagers who kept (i) only one type of livestock, (ii) only two types of livestock and (iii) none of the livestock.

The question was attempted by 10,629 (99.3%) candidates, out of which 80.4 percent scored from 6 to 10 marks and 5.0 percent scored all 10 marks. Further analysis shows that 11.5 percent of the candidates had average performance; their scores ranging from 3.5 to 5.5 and 8.1 percent scored from 0 to 3.0 marks. Generally, the candidates' performance was good as illustrated in Figure 6.

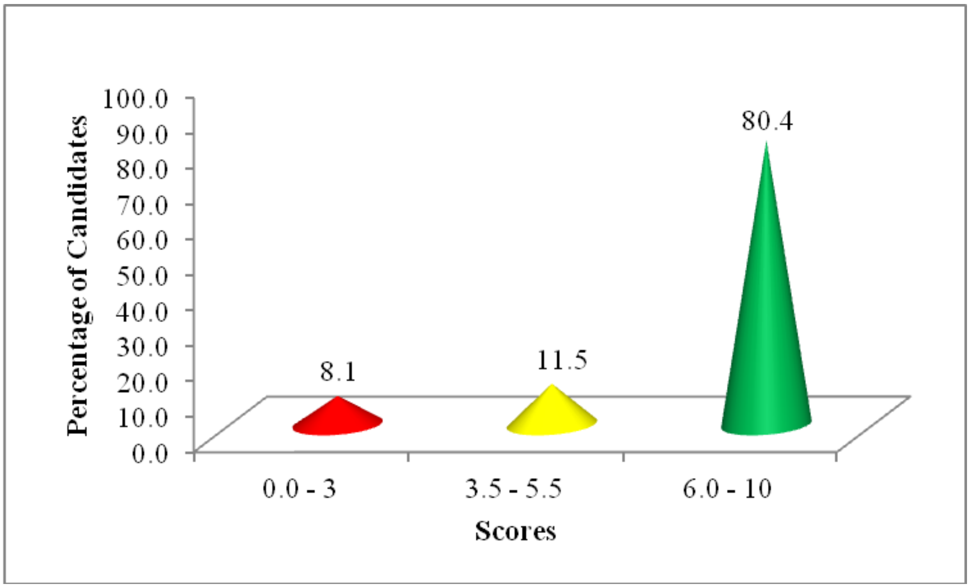


Figure 6: *Candidates' Performance in Question 5*

The analysis of the candidate's responses shows that the candidates who did well in part (a) used correctly the distributive law, complement law, identity law and the de Morgan's law in showing that $(A\cap B')\cup(B\cap A')\neq(A\cup B)-(A\cap B)'$. In part (b), they transformed the given word problem mathematically and presented it on a

Venn diagram to obtain the correct answer. Extract 5.1 is a sample response taken from the script of a candidate who answered the question correctly.

5a soln.

$(A \cap B') \cup (B \cap A') = (A \cup B) - (A \cap B)'$ --- given.

consider L.H.S -

$(A \cap B') \cup (B \cap A')$ ---- given

$((A \cap B') \cup B) \cap ((A \cap B') \cup A')$ --- distributive law.

$[(A \cup B) \cap (B' \cup B)] \cap [(A \cup A') \cap (B' \cup A')]$ -- distributive law

$[(A \cup B) \cap U] \cap [U \cap (B' \cup A')]$ --- complement law.

$(A \cup B) \cap (B' \cup A')$ --- Identity law.

$(A \cup B) \cap (A' \cup B')$ --- commutative law.

$(A \cup B) \cap (A \cap B)'$ --- De-Morgan's law.

$\underline{(A \cup B) - (A \cap B)'}$ --- from definition $A - B = A \cap B'$.

$(A \cap B') \cup (B \cap A') \neq (A \cup B) - (A \cap B)'$ and ~~not~~ is equal to $(A \cup B) - (A \cap B)$

5b soln.

let

B - people keeping goats.

A - villagers keeping cattles

C - villagers keeping pigs.

Data:

$n(A) = 82$ villagers.

$n(B) = 110$ villagers

$n(C) = 73$ villagers

$n(A \cap B) = 59$ villagers

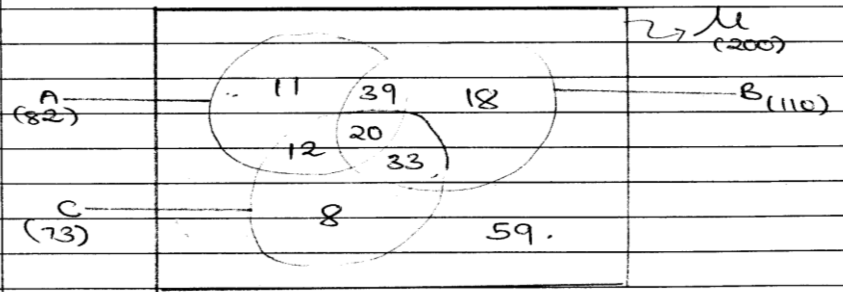
$n(B \cap C) = 53$ villagers

$n(A \cap C) = 32$ villagers

$n(A \cap B \cap C) = 20$ villagers

$U = 200$ villagers.

By venn-diagram.

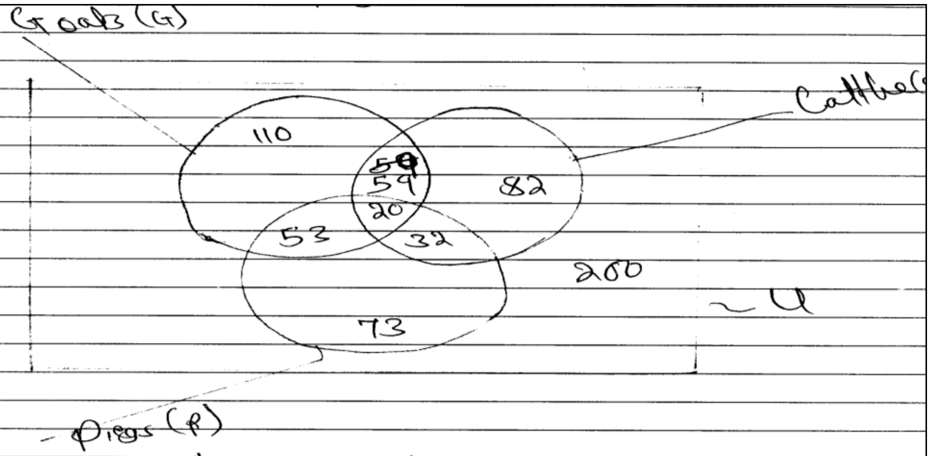


5b i/	soln.
	only one type of livestock
	$= n(A)\text{only} + n(B)\text{only} + n(C)\text{only}$.
	$= 11 + 18 + 8$
	$= 37$ villagers.
	$\therefore$ 37 villagers kept only one type of livestock.
5b ii/	soln.
	only two types of livestock.
	$= 12 + 39 + 33$.
	$= 84$ villagers
	$\therefore$ 84 villagers kept only two types of livestock.
5b iii/	soln.
	none of the livestock; $n(A\cap B\cap C)$
	$n(A\cup B\cup C)' = 200 - [11+12+20+39+33+8+18]$
	$= 200 - 141$
	$= 59$ villagers.
	$\therefore$ 59 villagers kept none of the livestock.

Extract 5.2: *A correct response from one of the candidates*

Despite the good performance, 110 (1.0%) candidates got the question wrong. In part (a), some candidates used Venn diagram in proving the given expression contrary to the requirements of the question. Others confused the concept of set with that of Logic as they wrote the disjunction symbol $(\vee)$ frequently instead of the union symbol $(\cup)$ and the conjunction $(\wedge)$ instead of the intersection $(\cap)$. Another common mistake committed by a number of candidates was failure to state the names of each law applied. In part (b), some candidates were able to draw Venn diagram but could not position correctly the number of villagers on each region of the diagram. For instance, they placed 79 instead of 18 in the region for goats only, 29 instead of 11 in the region for cattle only, 73 instead of 8 in the region for pigs etc. Thus, it was difficult for them to find the number of villagers who kept one type of livestock, only two types of livestock and none of the livestock. In addition, several candidates used the formula to solve the problem contrary to the requirements of the question. Extract 5.2 illustrate this case.

5	(b) let For cattle be "C"
	For Goat be G
	For Pigs be P



(i) The number of villagers who kept only one type of livestock

$110 + 82 + 73 + x = 200$

$265 + x = 200$

$265 - 200 = x$

$x = 65.$

Therefore 65 villagers kept only one type of livestock

(ii) Only two types of livestock

$59 + 53 + 32 + x = 200$

$144 + x = 200$

$x = 200 - 144 = 56$

5 (b)(ii) 56 villagers kept only two types of live stock.

(iii) None of the livestock

$200 - 20 = 180$

Therefore 180 villagers kept non of the livestock

Extract 5.2: *An incorrect response from one of the candidates*

2.1.6 Question 6: Functions

The question had parts (a) and (b). In part (a), the candidates were required to (i) mention any two properties of $f(x) = b^x$ and (ii) draw the graph of $f(x) = \left(\frac{1}{2}\right)^x$ for $-3 \le x \le 3$. In part (b), the candidates were given $y = \frac{x^2 - 2x - 3}{x^2 - 4}$ and asked to; (i) find the vertical and horizontal asymptotes, and (ii) sketch the graph of *y*.

A total of 10,658 (99.5%) candidates did this question. As Figure 7 shows, 94.5 percent of those candidates scored the marks ranging from 3.5 to 10. Therefore, the question was best performed in this examination.

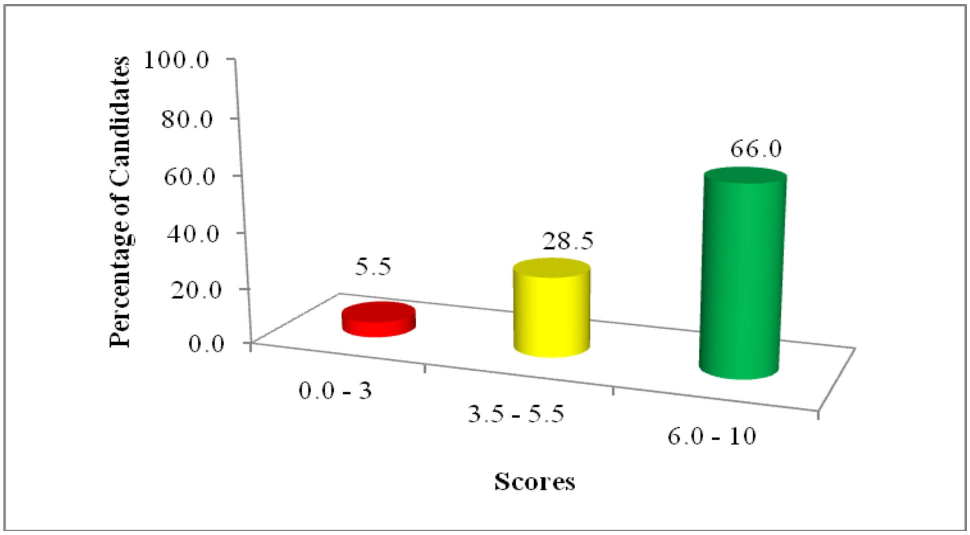


Figure 7: *Candidates' Performance in Question 6*

In part (a) (i), the candidates who performed well mentioned two properties of $f(x)=b^x$ out of the following: $f(x)$ is a one-to-one function; domain of *f(x)* is $(-\infty,\infty)$ and its range is $(0,\infty)$; the graph of $f(x)$ is a curve passing through $(1,b)$ and $(0,1)$; $f(x)$ is an increasing function that approaches zero as x approaches negative infinity for $b>1$; and $f(x)$ is a decreasing function that approaches zero as x approaches positive infinity for $0<b<1$. The properties which were frequently observed in candidates' responses are domain is a set of all real numbers and range is a set of all positive numbers. In part (a) (ii), they used the properties mentioned in part (a) (i) or otherwise to draw the graph of $f(x)=\left(\frac{1}{2}\right)^x$ correctly. In part (b) (i), the candidates solved an equation $x^2-4=0$ to get vertical asymptotes $x=2$ and $x=-2$ and evaluated $y=\lim_{x\to\infty}\left(\frac{x^2-2x-3}{x^2-4}\right)$ to get horizontal asymptote $y=1$. In part (b) (ii), they solved an equation $\frac{x^2-2x-3}{x^2-4}=0$ to get x - intercepts, $x=-1$ and $x=3$. Also they evaluated the value of $y=\frac{x^2-2x-3}{x^2-4}$ when $x=0$ to get $y-\text{intercept}=\frac{3}{4}$. Moreover, they used the information obtained in both parts (i) and (ii) as well as a table of values to sketch the graph. Extract 6.1 is a sample response taken from a candidate who did the question correctly.

6(a)i Properties of $f(x) = 6^x$

- The domain of f(x) satisfies all real numbers for values of x.
  ie Domain = $\{x : x \in \mathbb{R}\}$

- The Range of f(x) does not satisfy all real numbers for values of y.
  ie

$$y = 6^x$$
$$\log y = \log 6^x$$
$$\log y = x \log 6$$
$$x = \frac{\log y}{\log 6}$$

$$\therefore y > 0$$

$$\therefore \text{Range} = \{y : y \in \mathbb{R} \text{ and } y > 0\}$$

→ If y = 0 or negative value, the function is not satisfied and it is undefined

6(a) ii Solu
Required ;- Graph of $f(x) = \left(\frac{1}{2}\right)^x$ for $-3 \le x \le 3$

Table of values ; $y = \left(\frac{1}{2}\right)^x$

x	-3	-2	-1	0	1	2	3
y	8	4	2	1	1/2	1/4	1/8

Intercepts;
X - intercept , y = 0

6(a) i.

$0 = \left(\frac{1}{2}\right)^x$

$\log 0 = x \log\left(\frac{1}{2}\right)$

$x =$ undefined

$\therefore$ No X-intercept

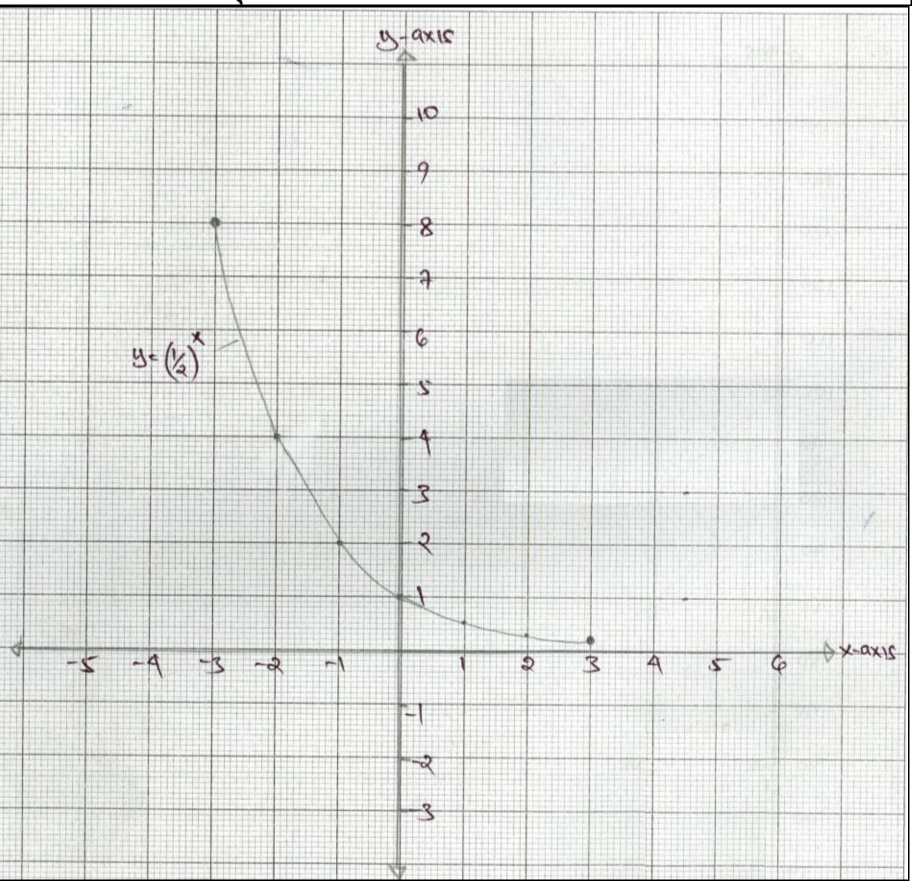
y-intercept, $x = 0$

$y = \left(\frac{1}{2}\right)^0$

$y = 1$

$\therefore$ y-intercept ; $(0, 1)$

6(a) ii.



6(b) Soln

Given $y = \dfrac{x^2 - 2x - 3}{x^2 - 4}$

i/. Vertical and Horizontal Assymptotes.

Vertical assymptotes :-

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

$\therefore$ Vertical assymptotes :- $x = 2$ and $x = -2$

Horizontal assymptotes

$y = \dfrac{x^2 - 2x - 3}{x^2 - 4}$

6(b)i $y = \dfrac{x^2/x^2 - 2x/x^2 - 3/x^2}{x^2/x^2 - 4/x^2}$

$y = \dfrac{1 - 2/x - 3/x^2}{1 - 4/x^2}$

As $x \to \infty$

$$y = \frac{1 - \frac{2}{\infty} - \frac{3}{\infty}}{1 - \frac{4}{\infty}}$$

$$y = \frac{1 - 0}{1 - 0}$$

$$y = 1$$

$\therefore$ Horizontal asymptote; $y = 1$

Q6(b)ii To sketch graph of y

Intercepts

X-intercept, $y = 0$

$$0 = \frac{x^2 - 2x - 3}{x^2 - 4}$$

$$x^2 - 2x - 3 = 0$$

$x = 3$ and $x = -1$

$\therefore$ X-intercepts; $(3,0)$ and $(-1,0)$

Y-intercept, $x = 0$

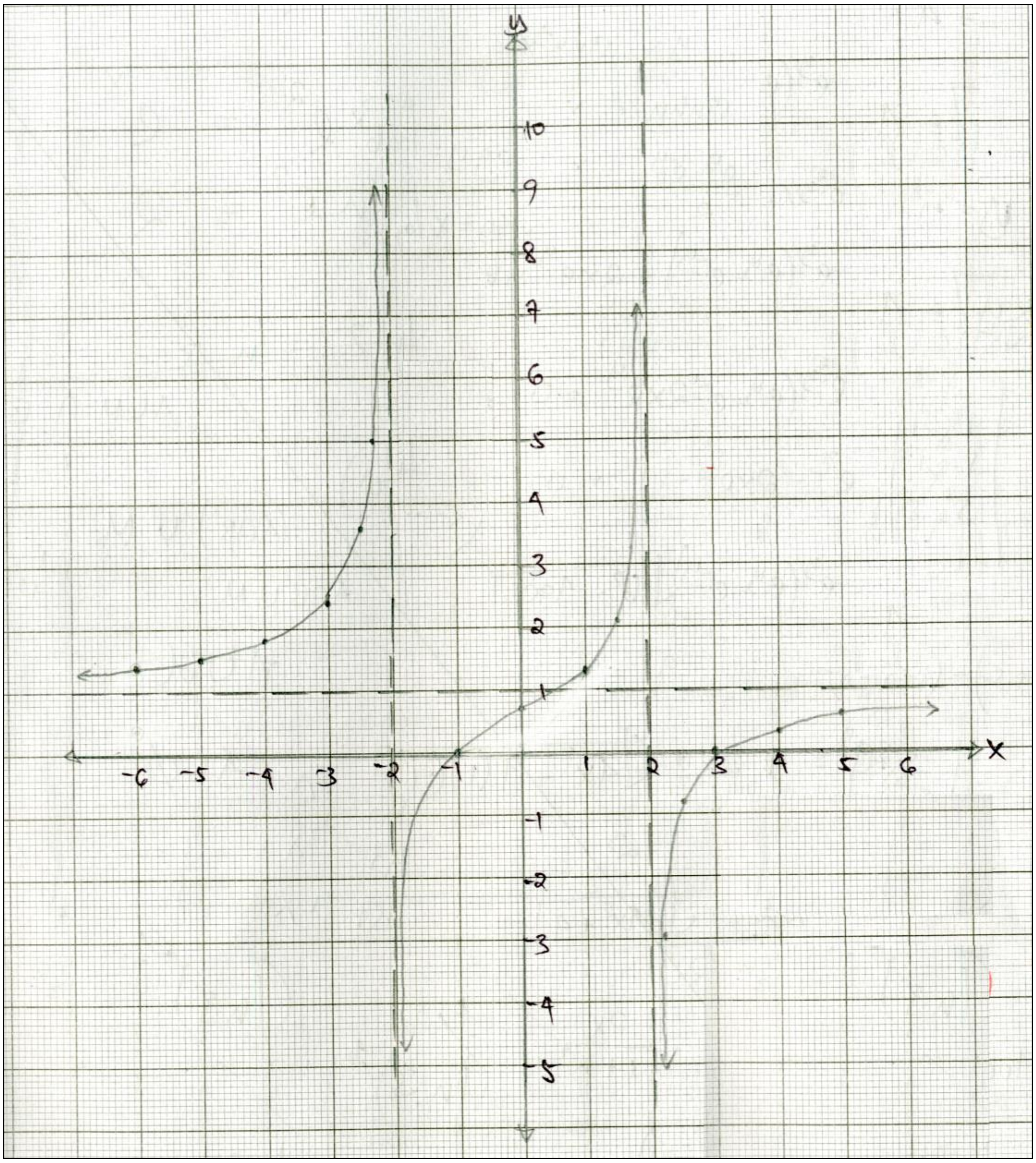
$$y = \frac{0^2 - 2(0) - 3}{0^2 - 4}$$

$$y = \frac{3}{4}$$

$\therefore$ Y-intercept; $(0, \frac{3}{4})$

Q6(b)ii Table of values

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	1.4	1.5	1.8	2.4	∞	0	0.75	1.3	∞	0	0.4	0.6



Extract 6.1: *A correct response*

In Extract 6.1, the candidate had sufficient understanding of the properties and graphs of exponential and rational functions.

On the other hand, 591 (5.5%) candidates scored low marks ranging from 0 to 3.0. In part (a) (i), majority of them could not state the properties of $f(x) = b^x$ correctly. For instance, some candidates wrote "the graph is not complete curve and not complete straight line" while others stated that "the function is commutative or distributive or associative". These responses indicate that they had insufficient knowledge on the properties of exponential functions. In part (a) (ii), several candidates prepared a table of values correctly but did not adhere to the properties of exponential functions based on the given interval $-3 \le x \le 3$. Some candidates drew the graph that resembles step function while others presented graph with arrows. In part (b) (i), the analysis of candidates' responses shows that majority of the candidates made confusions on asymptotes as they exchanged the values of the horizontal asymptote with that of vertical asymptotes. Also, some candidates could not obtain the correct values of asymptotes due to computational errors. Moreover, in part (b) (ii), several candidates did not find intercepts of the function. Lack of intercepts and use of incorrect values of asymptotes led them to sketch incorrect graph. Extract 6.2 shows a sample response from a candidate who did the question poorly.

(i) Horizontal asymptotes

$$x^2 - 4 = 0$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = 2$$

$\therefore$ Horizontal asymptotes $= 2$

(ii) Vertical asymptotes

From $y = \dfrac{x^2 - 2x - 3}{x^2 - 4}$

$$y = \frac{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}}$$

$$y = \frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 - \frac{4}{x^2}} \qquad x \to \infty$$

$$y = 1$$

$\therefore$ Vertical asymptotes $= 1$

Extract 6.2: *An incorrect response*

In Extract 6.2, the candidate computed did not understand the requirements of the question as he computed the required asymptotes interchangeably.

2.1.7 Question 7: Numerical Methods

This question had parts (a), (b) and (c). In part (a), it was given that the value $A = \int_a^b f(x)\,dx$ represents the area under the graph of $y = f(x)$ between $x = a$ and $x = b$. The candidates were required to derive the trapezium rule with six ordinates in finding the approximation of $A = \int_a^b f(x)\,dx$. In part(b), the candidates were required to approximate $\int_1^7 \frac{x^3}{1+x^4}dx$ correct to three decimal places by using trapezium rule obtained in (a). In part (c), the candidates were required to use the approximation obtained in part (b) and the actual integral of $\int_1^7 \frac{x^3}{1+x^4}dx$ to calculate the relative error correct to three decimal places.

A total of 10,253 candidates corresponding to 95.8 percent attempted the question. Out of such candidates, 48.9 percent scored 0 to 3 marks, 28.6 percent 3.5 to 5.5 marks and 22.5 percent scored 6 to 10 marks. The candidates' performance in this question was average, since 51.1 percent scored above 3 marks. Figure 8 shows the candidates' performance in this question.

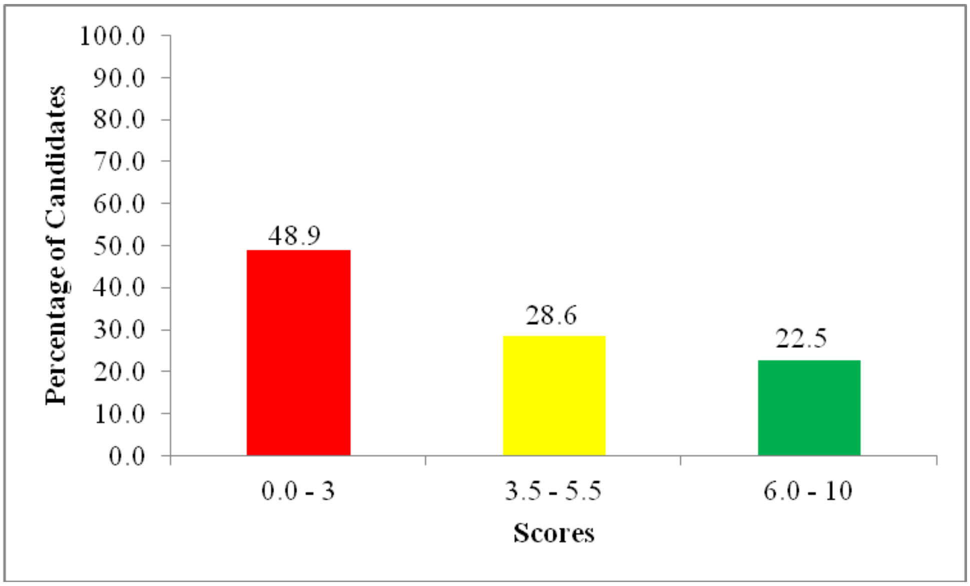
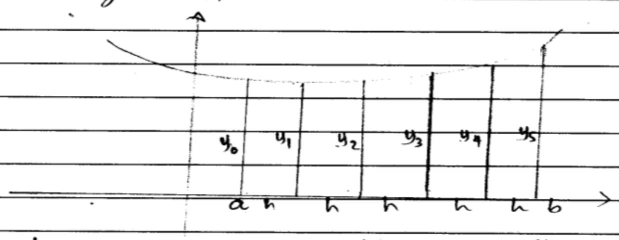


Figure 8: *Candidates' Performance in Question 7*

The analysis shows that the candidates who had a good performance demonstrated the following strengths: In part (a), they sketched the graph of $f(x)$, divided the area under the graph of $f(x)$ into five strips whereby the shape of each strip was nearly a trapezium, calculated the approximated area of each strip and added them to get the formula $\int_a^b f(x)dx = \frac{b-a}{10}\left(y_1 + y_6 + 2(y_2 + y_3 + y_4 + y_5)\right)$. In part (b), they constructed a table of values with correct entries for the ordinates y_1, y_2, y_3, y_4, y_5 and y_6 as 0.500, 0.436, 0.292, 0.217, 0.172 and 0.143 respectively. They then substituted the upper and lower limits of the given definite integral and the ordinates into the formula obtained in (a) to get $\int_1^7 \frac{x^3}{1+x^4}dx$ equal to 1.726. In part (c), most candidates opted to use the substitution $u = x^4$ to solve for dx to get $dx = \frac{du}{4x^3}$ and to change the boundaries of integration into $u = 1$ and $u = 2401$. They then expressed the integral $\int_1^7 \frac{x^3}{1+x^4}dx$ into $\frac{1}{4}\int_1^{2401}\frac{du}{1+u}$ and solved it to get 1.773. Finally they used the formula $\frac{\varepsilon_a}{a_0} = \frac{|a_0 - a|}{a_0}$ and the results obtained in part (a) and (b) to get 0.027 as the required relative error. Extract 7.1 is a sample answer from one of the candidates who performed well.

deriving trapezium rule.

7 (a).



6 ordinates are $y_0 + y_1 + y_2 + y_3 + y_4 + y_5$

to find area of trapezium

$A_1 = \frac{1}{2}(y_0 + y_1)h.$

$A_2 = \frac{1}{2}(y_1 + y_2)h.$

$A_3 = \frac{1}{2}(y_2 + y_3)h.$

$A_4 = \frac{1}{2}(y_3 + y_4)h.$

$A_5 = \frac{1}{2}(y_4 + y_5)h.$

$A = A_1 + A_2 + A_3 + A_4 + A_5.$

$A = \frac{h}{2}(y_0 + y_1 + y_1 + y_2 + y_2 + y_3 + y_3 + y_4 + y_4 + y_5).$

$A = \frac{h}{2}(y_0 + y_5 + 2\sum \text{remaining ordinates}).$

7 (b). $\int_1^7 \frac{x^3}{1+x^4}dx.$

$A = \int_1^7 \frac{x^3}{1+x^4}dx = \frac{h}{2}(y_0 + y_5 + 2\sum \text{remaining ordinates})$

$$h = \frac{b-a}{n} = \frac{7-1}{5} = \frac{6}{5}$$

x	1	2.2	3.4	4.6	5.8	7.
y	0.5	0.435936	0.291933	0.216907	0.172262	0.142798
	y_0	y_1	y_2	y_3	y_4	y_5

$$\int_1^7 \frac{x^3}{1+x^4}\,dx = \frac{1.2}{2}\left(0.5 + 0.142798 + 2\left(\begin{matrix}0.435936 + 0.291933 + \\ 0.216907 + 0.172262\end{matrix}\right)\right)$$

$$\int_1^7 \frac{x^3}{1+x^4}\,dx = \underline{1.726}$$

7c

$$\int_1^7 \frac{x^3}{1+x^4}\,dx$$

consider.

$$\int \frac{x^3}{1+x^4}\,dx.$$

Let $u = 1 + x^4.$

$du = 4x^3\,dx$

$$\int \frac{x^3}{u} \cdot \frac{du}{4x^3}.$$

$$\frac{1}{4}\int \frac{du}{u}$$

$$\frac{1}{4}\ln u. \quad + c.$$

$$\frac{1}{4}\ln(1+x^4)\Big|_1^7$$

$$\frac{\ln(2402)}{4} - \frac{\ln 2}{4}$$

$$= 1.77273$$

actual value is $1.773.$

$$\text{relative error} = \frac{\text{Absolute error}}{\text{True value.}}$$

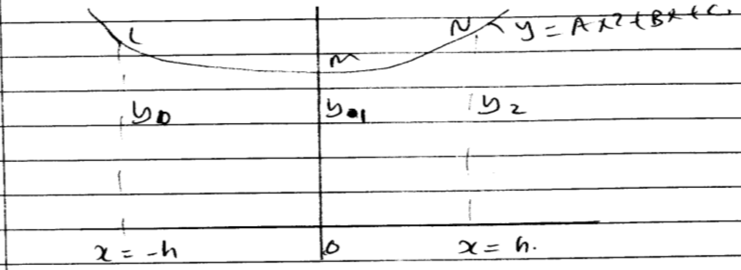
$$\text{relative error} = \frac{|1.7737 - 1.726|}{1.773}$$

$\therefore$ Relative error is $\underline{0.027}$.

Extract 7.2: *A correct response*

Despite the average performance in this question, a total of 2915 (28.4%) candidates did not answer the question correctly. In part (a), many candidates derived the Simpsons rule instead of trapezium rule as shown in Extract 7.2. Some candidates derived the Trapezium with n ordinates instead of 6 ordinates. Such candidates ended with the formula $A = \frac{h}{2}\left[y_1 + y_n + 2\left(y_2 + y_3 + ... + y_{n-1}\right)\right]$. Other candidates regarded ordinates as strips as a result they ended up with the formula with seven ordinates ie. $\int_a^b f(x)dx = \frac{b-a}{10}\left(y_1 + y_7 + 2\left(y_2 + y_3 + y_4 + y_5 + y_6\right)\right)$ instead of six ordinates. In part (b), a number of candidates failed to realize that h is an interval between two consecutive ordinates as they used formulae such as $h = \frac{a+b}{n}$ and $h = \frac{b-a}{n}$ instead of $h = \frac{b-a}{n-1}$ which let to wrong approximation of the given integral. In part (c), the candidates encountered the following challenges: Some candidates did not express the limits of integration in terms of u. For example, they wrote $\int_1^7 \frac{du}{u} = \ln u\Big|_1^7 = \ln 7 - \ln 1 = 1.81$ instead of $\ln\left(1+x^4\right)\Big|_1^7$. This shows that they could not remember that if you are working with a definite integral, the limits of integration will change as well. Other candidates failed to identify that the ant derivatives of the integrand $\frac{1}{u}$ is $\ln u$. Also, several candidates evaluated the actual integral of $\int_1^7 \frac{x^3}{1+x^4}$ dx correctly but they calculated the percentage error instead of relative error.

7a.



$\cancel{\int_a^b y\,dx}$ $A = \int_a^b f(x)\,dx.$

$A = \int_a^b (Ax^2+Bx+C)\,dx.$

$A = \int_{-h}^{h} (Ax^2+Bx+C)\,dx$

$$A = \frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx \Big]_{-h}^{h}$$

$$A = \frac{Ah^3}{3} + \frac{Bh^2}{2} + Ch - \left[\frac{-Ah^3}{3} + \frac{Bh^2}{2} - Ch\right]$$

$$A = \frac{Ah^3}{3} + \frac{Bh^2}{2} + Ch + \frac{Ah^3}{3} - \frac{Bh^2}{2} + Ch.$$

$$A = \frac{2Ah^3}{3} + 2Ch.$$

From the Curve LMN

The equation $y = Ax^2 + Bx + C$ when $y = y_0$ for $x = -h$, $y = y_1$ for $x = 0$ and $y = y_2$ for $x = h$.

$y_0 = Ah^2 - Bh + C.$

$y_1 = C$

$y_2 = Ah^2 + Bh + C$

$y_1 + y_2.$

$= Ah^2 + Bh + C + Ah^2 - Bh + C$

$= 2Ah^2 + 2C.$

$2Ah^2 = y_1 + y_2 - 2C.$

$$A = \frac{2Ah^3 + 6Ch}{3}$$

$$A = \frac{h(2Ah^2 + 6C)}{3}.$$

$$A = \frac{h}{3}(y_1 + y_2 - 2C + 6C)$$

$$A = \frac{h}{3}(y_1 + y_2 + 4y_1)$$ For simpson's rule

For Trapezium rule

$$A = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + y_{n-1})]$$

Extract 7.2: *An incorrect response*

In Extract 7.2, the candidates derived the Simpson's rule instead of Trapezium rule.

2.1.8 Question 8: Coordinate Geometry I

This question had parts (a), (b) and (c). In part (a), the candidates were required to derive the formula for calculating the area of a rectangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$. In part (b), the candidates were instructed to use the formula obtained in part (a) to find the area of the rectangle whose vertices are $(1, 1)$, $(3, 5)$, $(-2, 4)$ and $(-1, -5)$. In part (c), they were asked to show that the line $3x-4y+14=0$ is a tangent to the circle $x^2+y^2+4x+6y-3=0$.

The analysis of data shows that 9023 (84.3%) candidates attempted this question, of whom 3663 (40.6%) candidates scored the marks ranging from 3.5 to 10 and among them 0.8 percent scored all 10 marks. Therefore, the candidates' performance in this question was average. Figure 9 gives a summary of the candidates' performance.

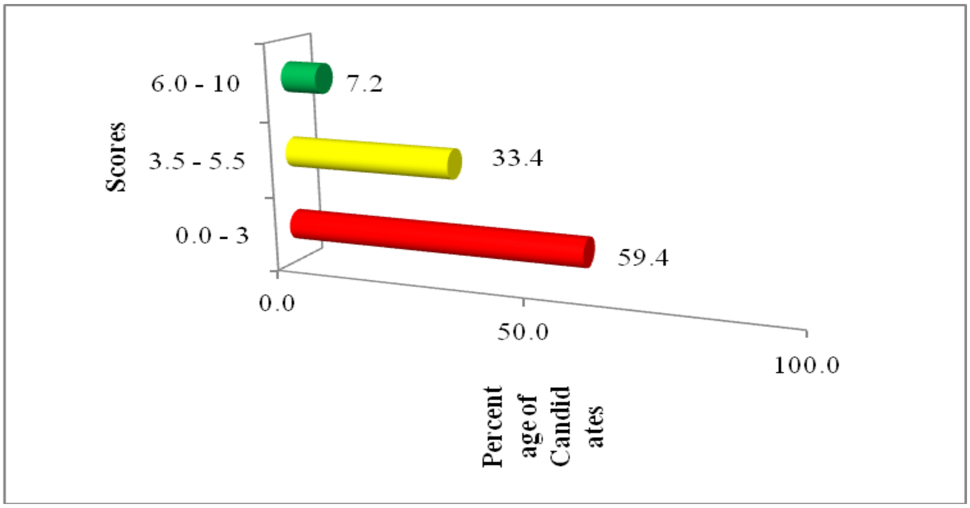
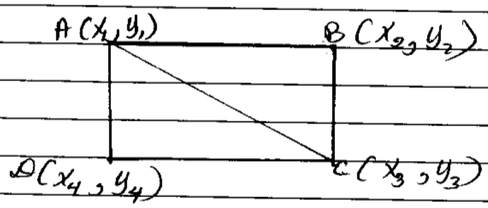


Figure 9: *Candidates' Performance in Question 8*

The candidates with good performance were able to sketch the given rectangle on *xy* plane and construct vertical lines that connect each vertex to the *x* - axis to develop four trapeziums as required in part (a). They also derived the formula for calculating area of the rectangle by finding sums and differences of the areas of trapeziums. Others used determinant approach of finding area of triangle to derive the required formula as shown in Extract 8.1. The candidates who attempted part (b) correctly substituted the values of $(x_1, y_1)=(1, 1)$, $(x_2, y_2)=(3, 5)$, $(x_3, y_3)=(-2, 4)$ and $(x_4, y_4)=(-1, -5)$ into the formula derived in part (a) to get $Area=21$ square units. In part (c), the candidates managed to express $3x-4y+14=0$ as $y=\dfrac{3x+14}{4}$ and substituted it into the equation $x^2+y^2+4x+6y-3=0$ to obtain the quadratic equation $25x^2+220x+448=0$. Finally, they substituted $a=25$, $b=220$ and $c=448$ into the condition $b^2=4ac$ to get $48400=48400$, which fulfil the requirements of the question. Also, a few candidates attempted the question by showing that the radius of the circle equals to the perpendicular distance of a centre from the line. They calculated the radius (r = 4) and centre C(-2, -3) from $x^2 + y^2 + 4x + 6y - 3 = 0$. Then, they computed the perpendicular distance of the centre from the line $3x - 4y + 14 = 0$ to get d = 4 units verifying that the given line is a tangent to the circle. Extract 8.1 is a sample response taken from the script of one of the candidates.

08 (a)



Consider triangle ACD

$$A_1 = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$$

$$A_1 = \frac{1}{2}\left[(x_1y_3 + x_3y_4 + x_4y_1) - (x_3y_1 + x_4y_3 + x_1y_4)\right]$$

~~A_1~~ Consider triangle ABC

$$A_2 = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

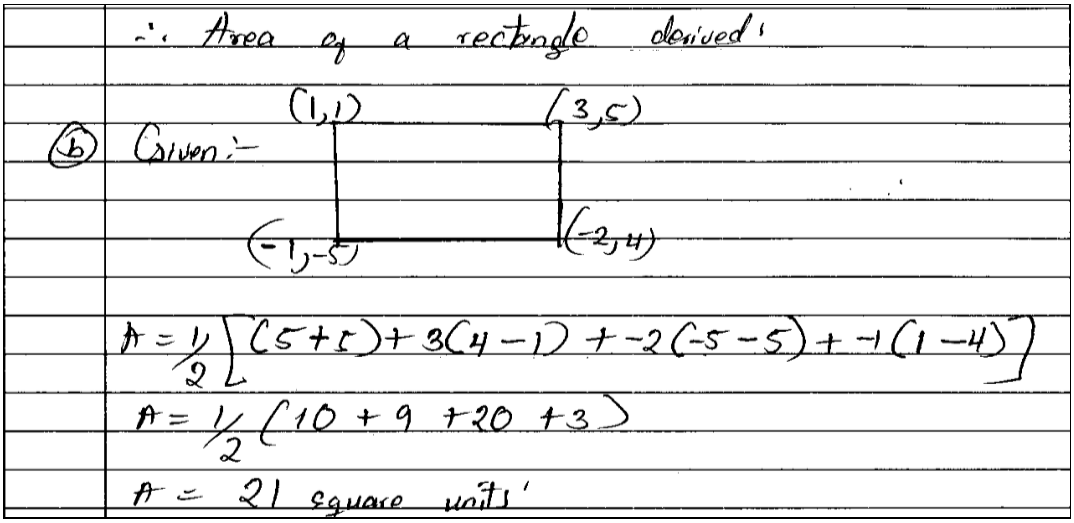
$$A_2 = \frac{1}{2}\left[(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right]$$

Add $A_1 + A_2 = A$

$$A = \frac{1}{2}(x_1y_3 + x_3y_4 + x_4y_1 - x_3y_1 - x_4y_3 - x_1y_4) +$$
$$\frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)$$

$$A = \frac{1}{2}\left(x_1(y_3 - y_4 + y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_4 - y_2) + x_4(y_1 - y_2)\right)$$

08 (a) $A = \frac{1}{2}\left[x_1(y_2 - y_4) + x_2(y_3 - y_1) + x_3(y_4 - y_2) + x_4(y_1 - y_3)\right]$



∴ Area of a rectangle derived.

(b) Given:-

$A = \frac{1}{2}\left[(5+5) + 3(4-1) + -2(-5-5) + -1(1-4)\right]$

$A = \frac{1}{2}(10 + 9 + 20 + 3)$

$A = 21$ square units.

Extract 8.1: *A correct response*

In Extract 8.1, the candidate correctly derived and applied the formula for calculating area of rectangle using coordinates of vertices.

Conversely, 1738 (19.3%) candidates scored zero. In part (a), it was noted that a large number of candidates did not apply the concepts of area of trapeziums to derive the formula for calculating area of the rectangle. Some candidates used the formula for finding the distance between two points and $Area = length \times width$. This approach excludes one vertex as it involves only three vertices. Also, a greater proportion of candidates decided to use the determinant method, which seems to be an alternative means of writing the required formula. But the incorrect formulae $A = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$ and $A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{vmatrix}$ dominated in candidates' responses. The mistakes in part (a) and computational errors in part (b) caused many candidates to have incorrect responses for part (b). For example, the candidates who applied the formula $A = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$ got 42 square units instead of 21 square units. In part (c), majority of the candidates used the condition $b^2 = 4ac$ rather than the concept of radius of a circle. Such candidates produced an incorrect equation as shown in Extract 8.2. This indicates that they had insufficient knowledge and skills in expanding squares of linear expressions. A few candidates who used the concept of radius did computational errors particularly in calculating the perpendicular distance of a centre from the line $3x - 4y + 14 = 0$. Several candidates differentiated $x^2 + y^2 + 4x + 6y - 3 = 0$, the procedure which does not lead to the correct answer. Other candidates developed a wrong equation $3x - 4y + 17 = 0$ from $x^2 + y^2 + 4x + 6y - 3 = 0$ with intention of applying the concept of perpendicular lines, $m_1 m_2 = -1$. This concept could not work for the asked case. Extract 8.2 shows a sample response from one of the candidates who attempted the question incorrectly.

c) $3x - 4y + 14 = 0$ and $x^2 + y^2 + 4x + 6y - 3 = 0$

Soln

from $y = mx + c$

$\frac{3x + 14}{4} = \frac{4y}{4}$

$y = \frac{3x}{4} + \frac{7}{2}$

Substitute y in the equation of circle

$x^2 + \left(\frac{3x}{4} + \frac{7}{2}\right)^2 + 4x + 6\left(\frac{3x}{4} + \frac{7}{2}\right) - 3 = 0$

$x^2 + \frac{9x^2}{16} + \frac{42x}{8} + \frac{49}{4} + 4x + \frac{18x}{4} + \frac{42}{2} - 3 = 0$

$\frac{x^2}{1} + \frac{9x^2}{16} + \frac{42x}{8} + \frac{4x}{1} + \frac{18x}{4} + \frac{49}{4} + 21 - 3 = 0$

$\frac{16x^2 + 9x^2}{16} + \frac{42x + 32x + 18x}{8} + \frac{49}{4} + \frac{18}{1} = 0$

$\frac{25x^2}{16} + \frac{92x}{8} + \frac{121}{4}$

Solve quadratic

$x_1 = -3.68$ and $x_2 = -3.68$

Extract 8.2: *An incorrect response*

In Extract 8.2, the candidate developed a wrong equation.

2.1.9 Question 9: Integration

This question comprised parts (a) and (b). In part (a), the candidates were required to obtain a reduction formula of I_n in terms of I_{n-2} from $I_n = \int \sec^n x dx$ and use it to integrate $\int \sec^5 x dx$. Part (b) required the candidates to find the length of an arc given by $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ between $\theta = 0$ *to* $\theta = 2\pi$

A total of 6,820 candidates (63.7%) attempted this question. This implies that more than one third of the candidates did not answer the question. The analysis of data shows that 70.7 percent scored 0 to 3 marks, 25.0 percent scored 3.5 to 5.5 marks and 4.3 percent from 6 to 10 marks. Further analysis shows that 41.7 percent scored zero. Therefore, the question was poorly performed as indicated in Figure 10.

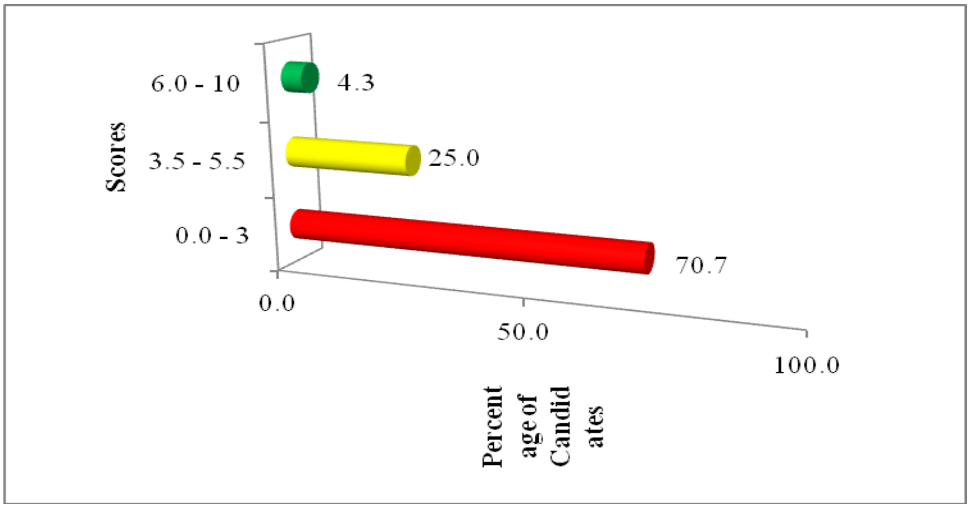


Figure 10: *Candidates' Performance in Question 9*

The factors that contributed to the poor performance are as follows: In part (a), some candidates were unable to write $\sec^n x$ as the product of $\sec^{n-2} x$ and $\sec^2 x$ so that they can integrate $\int \sec^{n-2} x \sec^2 x dx$ by parts to obtain the formula $I_n = \left(\frac{n-2}{n-1}\right) I_{n-2} + \frac{1}{n-1}\tan x \sec^{n-2} x$. Such candidates failed to find $\int \sec^5 x\, dx$ as well because it required them to apply the reduction formula obtained previously.

Most of them worked out $\int \sec^5 x\, dx$ using the method of integration by parts as illustrated in Extract 9.1. These candidates lost some marks because they did not adhere to the instructions. In part (b), some candidates could not find the arc length because they had a problem in differentiating the parametric equations $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$. Others substituted 360^{o} as the upper limit into $\frac{a\theta^2}{2}$ instead of 2π. Such candidates came up with 64,800*a* units instead of $2a\pi^2$ units. It was also found that a number of candidates used the incorrect formula $\int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, dx$ instead of $\int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$. These candidates got the incorrect length of the arc as they integrated $a\theta$ with respect *x* to get $2a\pi$ instead of $2a\pi^2$.

9 a. $\int \sec^5 x\, dx$

$\int \sec^2 x\, \sec^3 x\, dx$

let $u = \sec^3 x$ ~~dx~~

$\frac{du}{dx} = 3\sec x \tan x \sec^2 x$.

$du = 3\sec x \tan x \sec^2 x\, dx$

$dv = \sec^2 x\, dx$

$v = \tan x$.

$\int \sec^5 x\, dx = \tan x \sec^3 x - \int \tan x . 3\sec x \tan x \sec^2 x\, dx$

$= \tan x \sec^3 x - \int 3\sec x \tan^2 x \sec^2 x\, dx$

$\int \sec^5 x\, dx = \tan x \sec^3 x - \int 3\sec^3 x(\sec^2 x - 1)\, dx$

$\int \sec^5 x\, dx = \tan x \sec^3 x - 3\int(\sec^5 x - \sec^3 x)\, dx$.

$4\int \sec^5 x\, dx = \tan x \sec^3 x + 3\int \sec^3 x\, dx$.

$\int \sec x \sec^2 x\, dx$.

let $u = \sec x$

$du/dx = \sec x \tan x$

$du = \sec x \tan x\, dx$.

$dv = \sec^2 x\, dx$

Extract 9.1: *An incorrect response*

In Extract 9.1, the candidates did not use the reduction formula to integrate $\int \sec^5 x dx$

On the other hand, there were 80 (1.2%) candidates who answered this question correctly. They derived the reduction formula $I_n = \left(\frac{n-2}{n-1}\right) I_{n-2} + \frac{1}{n-1} \tan x \sec^{n-2} x$ from $I_n = \int \sec^n x dx$ and used it to find $\int \sec^5 x dx$ in part (a). In part (b), the candidates were able to find the length of an arc from the given parametric equations. Extract 9.2 represents a sample solution from one of the candidates who had adequate skills on the concepts of Integration.

9. a/ Given

$I_n = \int \sec^n x dx.$

$I_n = \int \sec^{n-2} x \cdot \sec^2 x \cdot dx.$

let

$u = (\sec x)^{n-2}.$

$\frac{du}{dx} = n-2 (\sec x)^{n-3} \cdot \sec x \tan x.$

$\frac{du}{dx} = n-2 \sec^{n-2} x \tan x.$

$du = (n-2) \sec^{n-2} x \tan x \, dx.$

$dv = \sec^2 x \, dx.$

$\int dv = \int \sec^2 x \, dx.$

$v = \tan x$

from

$\int u dv = uv - \int v du.$

$\int \sec^n x \, dx = \sec^{n-2} x \cdot \tan x - (n-2) \int \tan x \cdot \tan x \sec^{n}$

$I_n = \sec^{n-2} x \tan x - (n-2) \int \tan^2 x \sec^{n-2} x$

but $\tan^2 x = \sec^2 x - 1.$

9. a/ $I_n = \sec^{n-2} x \tan x - (n-2) \int (\sec^2 x - 1) \sec^{n-2} x \, dx.$

$I_n = \sec^{n-2} x \tan x - (n-2) \int \sec^n x - \sec^{n-2} x \, dx.$

$I_n = \sec^{n-2} x \tan x - (n-2) \int \sec^n x dx + (n-2) \int \sec^{n-2} x$

$I_n = \sec^{n-2} x \tan x - (n-2) I_n + (n-2) I_{n-2}$

$(1 + n - 2) I_n = \sec^{n-2} x \tan x + (n-2) I_{n-2}$

$I_n = \frac{1}{n-1} \left(\sec^{n-2} x \tan x + (n-2) I_{n-2} \right)$

Given $n = 5$.

$$\int \sec^5 x \cdot dx = \frac{1}{5-1}\left(\sec^3 x \tan x + 3\int \sec^3 x\, dx\right)$$

$$\int \sec^5 x\, dx = \frac{1}{4}\left(\sec^3 x \tan x + 3\int \sec^3 x\, dx\right)$$

consider $\int \sec^3 x\, dx$

$$\int \sec^3 x\, dx = \int \sec^2 x \cdot \sec x \cdot dx$$

let $u = \sec x$. and $dv = \sec^2 x\, dx$

$$\frac{du}{dx} = \sec x \tan x. \qquad \int dv = \int \sec^2 x \cdot dx$$

$$v = \tan x$$

but

$$\int u\,dv = uv - \int v\,du.$$

$$\int \sec^3 x\, dx = \sec x \tan x - \int \tan^2 x \sec x\, dx.$$

$$\int \sec^3 x\, dx = \sec x \tan x - \int (\sec^2 x - 1)\sec x\, dx.$$

$$\int \sec^3 x\, dx = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx.$$

$$2\int \sec^3 x\, dx = \sec x \tan x + \ln|\sec x + \tan x|.$$

$$\cancel{\int \sec^3 x\, dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|)}.$$

Now.

$$\int \sec^5 x\, dx = \frac{1}{4}\left(\sec^3 x \tan x + \frac{3}{2}\sec x \tan x + \frac{3}{2}\ln|\sec x + \tan x|\right.$$

9. b/ $x = a\cos\theta + a\theta\sin\theta$.

$$\frac{dx}{d\theta} = -a\sin\theta + a\theta\cos\theta + a\sin\theta.$$

$$\frac{dx}{d\theta} = a\theta\cos\theta.$$

$$y = a\sin\theta - a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a\cos\theta - a(-\theta\sin\theta - \cos\theta).$$

$$\frac{dy}{d\theta} = a\cos\theta + a\theta\sin\theta - a\cos\theta.$$

$$\frac{dy}{d\theta} = a\theta\sin\theta.$$

from

$$l = \int_{\theta_1}^{\theta_2} \sqrt{\left(\frac{dy}{d\theta}\right)^2 + \left(\frac{dx}{d\theta}\right)^2}\, d\theta.$$

$$l = \int_0^{2\pi} \sqrt{(a\theta)^2(\sin^2\theta + \cos^2\theta)} \cdot d\theta.$$

$$l = \int_0^{2\pi} \sqrt{a^2\theta^2}\, d\theta \quad \text{since } \cos^2\theta + \sin^2\theta = 1$$

$$l = \int_0^{2\pi} a\theta\, d\theta$$

$$l = a\int_0^{2\pi} \theta\, d\theta$$

$$l = a\left[\frac{\theta^2}{2}\right]_0^{2\pi}$$

$$l = \frac{a}{2}\left[4\pi^2 - 0\right]$$

$$l = 2a\pi^2 \text{ units}$$

$\therefore$ length of an arc is $2a\pi^2$ units

Extract 9.2: *A correct response*

In Extract 9.2, the candidates derived correctly the reduction formula and was able to calculate the length of an arc.

2.1.10 Question 10: Differentiation

This question had parts (a), (b) and (c) which asked as follow: (a) if $y = \left(\frac{1-x^2}{1+x^2}\right)^n$ show that $(1-x^4)\frac{dy}{dx} + 4nxy = 0$; (b) if the minimum value of $f(x) = 2x^3 + 3x^2 - 12x + k$ is one –tenth of its maximum value, find the value of k; and (c) (i) If $f(x, y) = x^3 y + e^{xy^2}$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ and (ii) If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, find $\frac{\partial^2 z}{\partial x \partial y}$ at (1,1).

The data analysis reveals that out of 8263 (77.2%) candidates who answered this question, 3551 (43.0%) scored the marks ranging from 3.5 to 10. Therefore, the candidates' performance in this question was average. Figure 11 shows the percentage of candidates with weak, average and good scores.

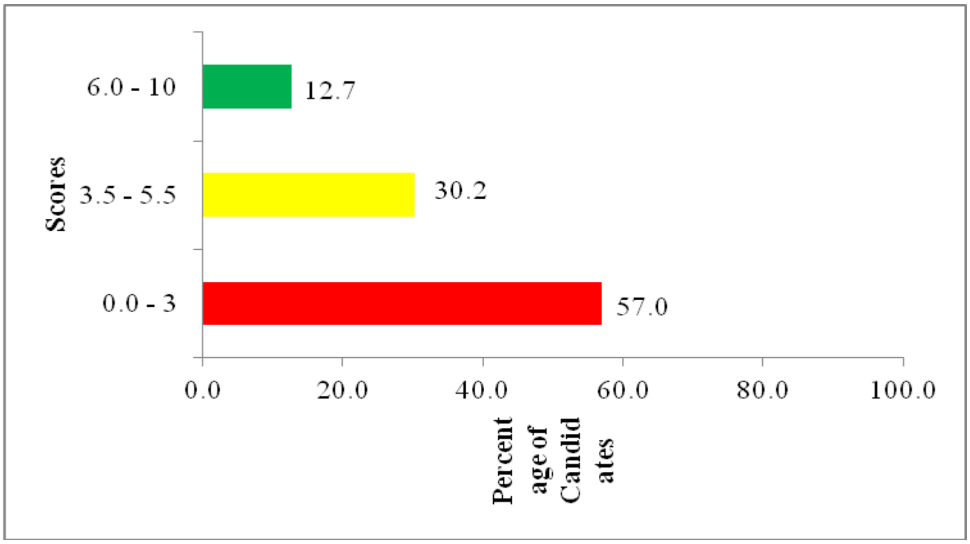


Figure 11: *Candidates' Performance in Question 10*

Further analysis shows that 93 (1.1%) candidates scored all 10 marks. The candidates who answered part (a) correctly, differentiated $y = \left(\frac{1-x^2}{1+x^2}\right)^n$ by introducing natural logarithm or otherwise and arranged the resulting equation into $(1-x^4)\frac{dy}{dx} + 4nxy = 0$ to meet the requirements of the question. In part (b), they differentiated $f(x) = 2x^3 + 3x^2 - 12x + k$ at once to get $f'(x) = 6x^2 + 6x - 12$. Also, the candidates realized that at maximum or minimum point $f'(x) = 0$, hence they solved $6x^2 + 6x - 12 = 0$ to get $x = -2$ or $x = 1$. Thereafter, they realized that $f(x)$ has maximum value at $x = -2$ and minimum value at $x = 1$ using the second derivative test. Finally, they got $f(-2) = 20 + k$ and $f(1) = -7 + k$, which were used to formulate the equation $-7 + k = \frac{1}{10}(20 + k)$ that gives $k = 10$. The candidates who answered part (c) (i) correctly obtained $\frac{\partial f}{\partial x} = 3x^2 y + y^2 e^{xy^2}$ and $\frac{\partial f}{\partial y} = x^3 + 2xye^{xy^2}$ from $f(x, y) = x^3 y + e^{xy^2}$. Likewise, in part (c) (ii) they obtained $\frac{\partial^2 z}{\partial x \partial y} = \frac{x^4 + 3x^2 y^2}{(x^2 + y^2)^2} = 1$ from $z = x^2 \tan^{-1}\left(\frac{y}{x}\right)$ and $(x, y) = (1, 1)$. These candidates had sufficient knowledge and skills on partial derivatives. Extract 10.1

is a sample answer of one of the candidates who performed well parts (a) and (c) of the question.

10 a) $y = \frac{(1-x^2)^n}{(1+x^2)^n}$

$\ln y = n \ln \left[\frac{1-x^2}{1+x^2}\right]$

$\ln y = n \ln (1-x^2) - n \ln (1+x^2)$

$\frac{1}{y} dy/dx = \frac{-2xn}{1-x^2} - \frac{2xn}{1+x^2}$

$\frac{1}{y} \frac{dy}{dx} = \frac{-2xn(1+x^2) - 2xn(1-x^2)}{1-x^4}$

$\frac{1}{y} \frac{dy}{dx} = \frac{-2xn - 2x^3n - 2xn + 2x^3n}{1-x^4}$

$\frac{1}{y} dy/dx = \frac{-4xn}{1-x^4}$

$dy/dx = \frac{-4nxy}{1-x^4}$

$(1-x^4) dy/dx + 4nxy = 0.$

10 c) $z = x^2 \tan^{-1}(y/x)$

$\partial z/\partial y$ = required. where x is constant

let $u = x^2$

$\partial u/\partial y = 0$

$v = \tan^{-1}(y/x)$

$\tan v = y/x$

$\sec^2 v \, \partial v/\partial y = 1/x$

$\partial v/\partial y = \frac{1}{x(1+y^2/x^2)}$

$\partial v/\partial y = \frac{x}{x^2+y^2}$

$\partial z/\partial y = u \partial v/\partial y + v \partial u/\partial y$

$\partial z/\delta y = x^2 \cdot \left[\frac{x}{x^2+y^2}\right]$

10

$$\frac{\partial z}{\partial y} = \frac{x^3}{x^2+y^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{x^3}{x^2+y^2}\right)$$

$$= \frac{(x^2+y^2)\frac{\partial (x^3)}{\partial x} - (x^3)\frac{\partial (x^2+y^2)}{\partial x}}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{(x^2+y^2)\cdot 3x^2 - (x^3)\cdot(2x)}{(x^2+y^2)^2}$$

at $(1,1)$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{(1+1)3 - (1\times 2)}{(1+1)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 1.$$

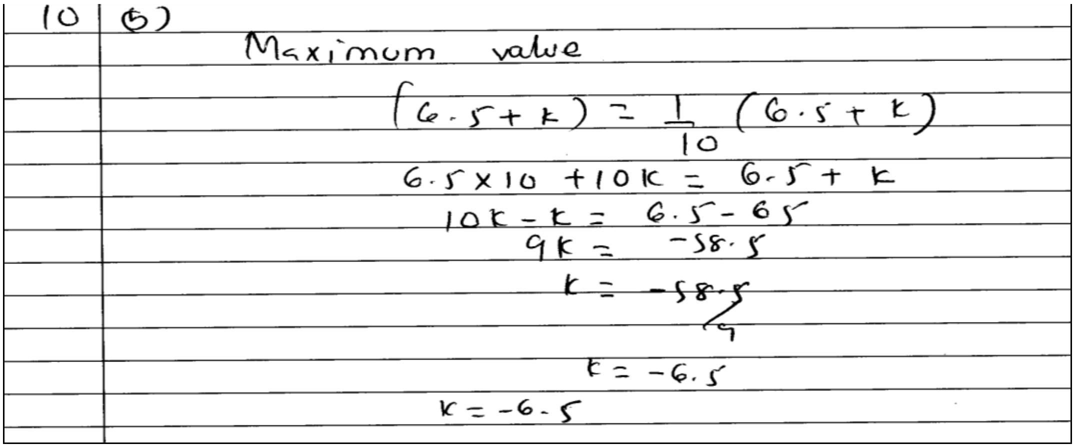
Extract 10.1: *A correct response*

In Extract 10.1, the candidate had correct understanding on both total and partial differentiation.

Analysis of the candidates' performance also reveals that 4712 (57.0%) candidates scored 3.0 marks or less. Their responses showed a number of weaknesses. In part (a), a considerable number of the candidates could not recall the quotient rule correctly. For instance, some used $\frac{dy}{dx} = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$ instead of $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$. Apart from that, some of the candidates who decided to introduce natural logarithm did some errors in applying the laws of logarithms particularly the quotient rule.

In part (b), a significant number of candidates could not interpret the sentence "the minimum value of $f(x) = 2x^3 + 3x^2 - 12x + k$ is one –tenth of its maximum value" correctly. As a result, they wrote $\frac{1}{10}(k-7) = 20 + k$ instead of $k - 7 = \frac{1}{10}(20+k)$. Also, few candidates could not solve the equation $-7 + k = \frac{1}{10}(20+k)$ correctly. They ended up with incorrect answer such as $k = 1$ and $k = 9$. In addition, some candidates evaluated maximum or minimum point incorrectly by solving $f''(x) = 0$ instead of $f'(x) = 0$. Extract 10.2 illustrate such case. In part (c) (i), a notable number of the candidates were unable to get correct partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x, y) = x^3 y + e^{xy^2}$. The incorrect answers $\frac{\partial f}{\partial x} = 3x^2 y + e^{xy^2}$ and $\frac{\partial f}{\partial y} = x^3 + 2ye^{xy}$ or $\frac{\partial f}{\partial x} = 3x^2 y + e^{y^2}$ and $\frac{\partial f}{\partial y} = x^3 + 2xe^{xy^2}$ were frequently seen in the candidates' responses. In part(c) (ii), many candidates misinterpreted $\frac{\partial^2 z}{\partial x \partial y}$ as $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial x} . \frac{\partial z}{\partial y}$ instead of $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$. This misinterpretation led to incorrect answer such as $\frac{\partial^2 z}{\partial x \partial y} = 0.61072$. Moreover, majority of the candidates who attempted this question denoted partial differentiation as total differentiation. These candidates wrote $\frac{df}{dx}$, $\frac{df}{dy}$ and $\frac{d^2 z}{dxdy}$ instead of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial^2 z}{\partial x \partial y}$ indicating insufficient knowledge in mathematical language.

10	(b) $f(x) = 2x^3 + 3x^2 - 12x + k$
	Soln
	$\frac{dy}{dx} = 6x^2 + 6x - 12$
	$\frac{d^2y}{dx^2} = 12x + 6$
	$0 = 12x + 6$
	$12x = -6$
	$x = -1/2$
	$y = 2 \times (-1/2)^3 + 3 \times (-1/2)^2 - 12(-1/2) + k$
	$y = \frac{-1}{4} + 3/4 + 6 + k$
	$y = 1/2 + 6 + k$
	$y = 6.5 + k$ Maximum value



10 (b)

Maximum value

$f(6.5+k) = \frac{1}{10}(6.5+k)$

$6.5 \times 10 + 10k = 6.5 + k$

$10k - k = 6.5 - 65$

$9k = -58.5$

$k = \frac{-58.5}{9}$

$k = -6.5$

$k = -6.5$

Extract 10.2: *An incorrect response*

In Extract 10.2, the candidate solved $f''(x) = 0$ instead of $f'(x) = 0$

2.2 142/2 ADVANCED MATHEMATICS 2

2.2.1 Question 1: Complex Numbers

This question comprised parts (a), (b) and (c). In part (a), the candidates were required to express the complex number $\left(\frac{1+i}{1-i}\right)^8 + \left(\frac{\sqrt{3}}{1-i}\right)^4$ in the form $a+ib$. In part (b), they were required to show that $[(r\cos\theta + i\sin\theta)]^n = r^n e^{in\theta}$. In part (c), it was given that the point P represents the complex number $z = x+iy$ on the Argand diagram and the candidates were required to describe the locus of P if $|z-1| = 3|z+i|$.

The analysis shows that the question was attempted by 10344 (96.6%) candidates, of whom 36.9 percent scored from 9 to 15 marks, 30.8 percent from 5.5 to 8.5 marks and 32.2 percent from 0 to 5.0 marks. Further analysis indicates that 327 candidates (3.2%) did the question well and scored all 15 marks while 425 candidates (4.1%) scored 0. The general candidates' performance in this question was good because 67.7 percent scored marks ranging from 5.5 to 15. Figure 12 illustrates this case.

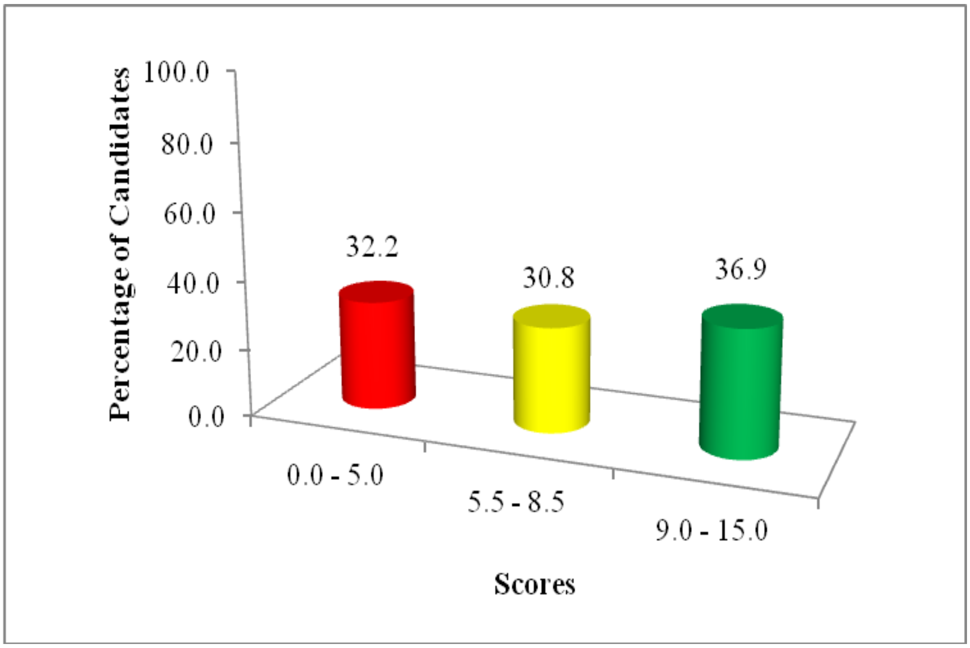


Figure 12 *Candidates' Performance in Question 1*

The analysis of the candidates' responses shows that the candidates who got the question right displayed the following strengths. In part (a), they multiplied the term $\frac{1+i}{1-i}$ and $\frac{\sqrt{3}}{1-i}$ in the expression $\left(\frac{1+i}{1-i}\right)^8+\left(\frac{\sqrt{3}}{1-i}\right)^4$ by $1+i$ which is the conjugate of the denominator $1-i$, to get $\left(\frac{2i}{2}\right)^8+\left(\frac{\sqrt{3}(1+i)}{2}\right)^4$ and simplified it to $-\frac{5}{4}+0i$. In part (b), many candidates realized that $\cos\theta+i\sin\theta$ equal to $e^{i\theta}$ and hence were able to show that $[(r\cos\theta+i\sin\theta)]^n=r^n e^{in\theta}$. A few candidates employed the principle of mathematical induction by showing the left hand side of the given equation equals to $re^{i\theta}$ when $n=1$, $r^k e^{ik\theta}$ when $n=k$ and $r^{k+1}e^{i(k+1)\theta}\times re^{i\theta}$ when $n=k+1$. In part (c), they were able to insert $z=x+iy$ into $|z-i|=3|z+i|$ and manoeuvred it to get $x^2+\left(y+\frac{5}{4}\right)^2=\left(\frac{3}{4}\right)^2$. Moreover, they were able to conclude that the locus of point P represents the circle whose centre is

$\left(0, \frac{-5}{4}\right)$ and radius $r = \frac{3}{4}$ units. Extract 11.1 shows a response of a candidate who did well in this question.

1. a)

$$\left(\frac{1+i}{1-i}\right)^8 + \left[\frac{\sqrt{3}}{1-i}\right]^4$$

$$\left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^8 + \left[\frac{\sqrt{3}(1+i)}{(1-i)(1+i)}\right]^4$$

$$\left[\left(\frac{1+i+i+i^2}{1-i^2}\right)\right]^8 + \left[\frac{\sqrt{3}+\sqrt{3}i}{1-i^2}\right]^4$$

$$\left[\frac{1+2i-1}{1+1}\right]^8 + \left[\frac{\sqrt{3}+\sqrt{3}i}{1+1}\right]$$

$$\left[\frac{2i}{2}\right]^8 + \left[\frac{\sqrt{3}+\sqrt{3}i}{2}\right]^4$$

$$i^8 + \left[\sqrt{3}/2 + \sqrt{3}/2\, i\right]^4$$

$$(i^2)^4 + \left[(\sqrt{3}/2)^4 + 4(\sqrt{3}/2)^3[\sqrt{3}/2\, i] + 6(\sqrt{3}/2)^2[\sqrt{3}/2\, i]^2 + 4(\sqrt{3}/2)(\sqrt{3}/2\, i)^3 + (\sqrt{3}/2\, i)^4\right]$$

$$(-1)^4 + 9/16 + 9/4\, i - 27/8 - 9/4\, i + 9/16$$

$$= -5/4$$

So

$$\left[\frac{1+i}{1-i}\right]^8 + \left[\frac{\sqrt{3}}{1-i}\right]^4 = -5/4 + (0)i$$

b) Soln

$$[r(\cos\theta + i\sin\theta)]^n = r^n e^{in\theta}$$

From expansion series of e^x by maclaurin's theorem.

$$e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + \cdots$$

so

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \cdots$$

$$= 1 + i\theta - \theta^2/2 - \theta^3/6\, i + \theta^4/24 + \cdots$$

$$e^{i\theta} = (1 - \theta^2/2 + \theta^4/24 + \cdots] + i[\theta - \theta^3/6 + \cdots$$

but Expansion of $\cos\theta$ and $\sin\theta$ are

$$\cos\theta = 1 - \theta^2/2 + \theta^4/24 + \cdots$$
$$\sin\theta = \theta - \theta^3/6 + \cdots$$

so

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

Times by r both sides

$$re^{i\theta} = r[\cos\theta + i\sin\theta]$$

raise to power n both sides

$$(re^{i\theta})^n = \left[r[\cos\theta + i\sin\theta]\right]^n.$$

$$r^n e^{in\theta} = \left[r[\cos\theta + i\sin\theta]\right]^n.$$

Hence Shown

(c). Soln

$$z = x + iy.$$

then

$$|z - i| = 3|z + i|$$

$$|x + iy - i| = 3|x + iy + i|$$

$$|x + i(y-1)| = 3|x + i(y+1)|.$$

1. (c)

$$\sqrt{x^2 + (y-1)^2} = 3\sqrt{x^2 + (y+1)^2}$$

Square both sides.

$$x^2 + (y-1)^2 = 9(x^2 + (y+1)^2)$$

$$x^2 + y^2 - 2y + 1 = 9(x^2 + y^2 + 2y + 1)$$

$$x^2 + y^2 - 2y + 1 = 9x^2 + 9y^2 + 18y + 9.$$

$$8x^2 + 8y^2 + 20y + 8 = 0$$

$$2x^2 + 2y^2 + 5y + 2 = 0 \quad \text{--- Locus.}$$

on comparison with

$$x^2 + y^2 + \tfrac{5}{2}y + 1 = 0$$

and $x^2 + y^2 + 2gx + 2fy + c = 0$

then

$2g = 0$ $\quad 2f = \tfrac{5}{2}.$ $\quad c = +1$

$g = 0$ $\quad f = \tfrac{5}{4}$

and

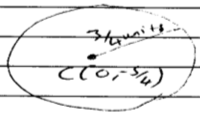
$$\text{radius } (r) = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{0^2 + (\tfrac{5}{4})^2 - (+1)}$$

$$r = 3/4.$$

$$\text{centre}(-g, -f) = (-0, -\tfrac{5}{4})$$

So the locus is circle of equation $x^2 + y^2 + \tfrac{5}{2}y + 1 = 0$ whose centre is $(0, -\tfrac{5}{4})$ and radius of $3/4$ units circle.



Extract 11.1: *A correct response from one of the candidates*

On the other hand, the candidates who scored low marks faced the following challenges: In part (a), they failed to identify that the conjugate of $1-i$ is $1+i$ and hence unable to express the given expression in the form $a+ib$ as shown in Extract 11.2. In part (b), they forgot to use the Euler's formula to prove that $r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta}$ instead they tried to derive it. In part (c), some candidates did not know how to find the magnitude of the resulting expression after putting $z = x+iy$ into $|z-i| = 3|z+i|$ while others could not complete the square of $x^2 + y^2 + \frac{5}{2}y = -1$ to get $x^2 + \left(y+\frac{5}{4}\right)^2 = \left(\frac{3}{4}\right)^2$ due to weaknesses in algebra.

(1.) $\left(\frac{1+i}{1-i}\right)^8 + \left(\frac{\sqrt{3}}{1-i}\right)^4$

Soln.

$= \left(\frac{8+8i}{1-i}\right) + \left(\frac{4\sqrt{3}}{1-i}\right)$

$= \frac{8+8i+4\sqrt{3}}{1-i}$

$= \frac{8+8i+6.93}{1-i}$

$= \frac{\cancel{8+}\ 8i+14.93}{1-i}$

$= 8i+14.93 \times \frac{1}{1-i}$

Extract 11.2: *An incorrect response*

In Extract 11.2, the candidates was unable to use the concept of conjugate.

2.2.2 Question 2: Logic

The question consisted of parts (a), (b) and (c). In part (a), it was given that P stands for "She is tall" and Q for "She is beautiful". The candidates were required to write the verbal representation of the statements (i) $P \wedge Q$, (ii) $P \wedge \sim Q$, (iii) $\sim P \wedge \sim Q$ and (iv) $\sim(P \vee \sim Q)$. In part (b) the candidates were instructed to use the laws of algebra of propositions to simplify $[P \wedge (P \vee Q)] \vee [Q \wedge (P \vee Q)]$. In part (c), the candidates were required to (i) find a simplified sentence having the following truth table and (ii) draw the corresponding electric network.

p	q	r	
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

The analysis of data shows that 9467 (88.7%) candidates out of 10,675 candidates scored the marks ranging from 5.5 to 15 and among them 2252 (21.1%) scored all 15 marks. Generally, the candidates' performance was good as shown in Figure 13.

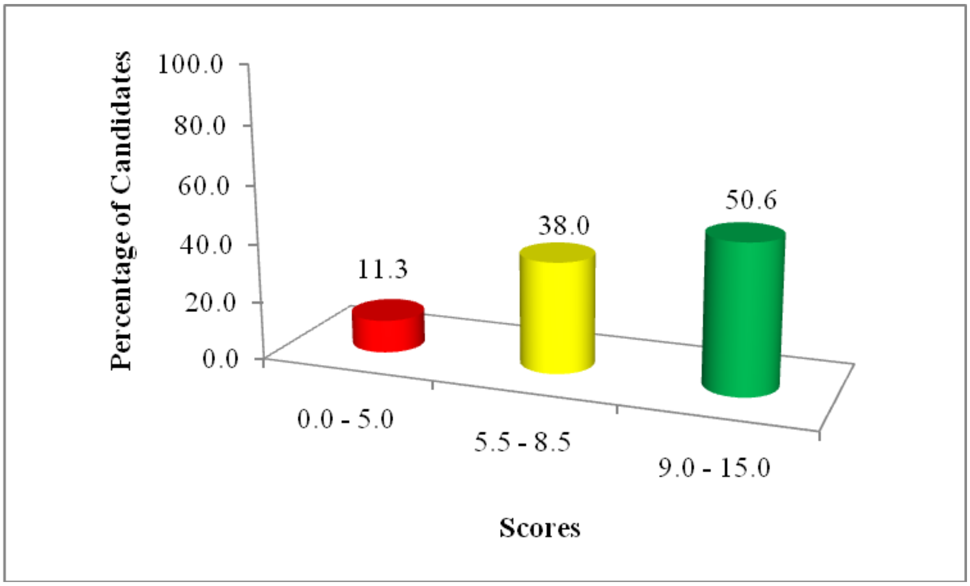


Figure 13: *Candidates' Performance in Question 2*

In part (a), the candidates were able to relate the logical connectives with English language words; $\wedge$ (and/but), $\vee$ (or) and ~ (not). Hence, they managed to write the given symbolic statements by using words. In part (b), the candidates managed to simplify $[P \wedge (P \vee Q)] \vee [Q \wedge (P \vee Q)]$ by applying identity and distributive laws to get $P \vee Q$. In part (c), most candidates focused on either the truth value T or F for the asked sentence in the given truth table to formulate the non - simplified compound statement. The candidates who focused on T had the statement $(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r)$ while those who focused on F had the statement $(p \vee \sim q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee q \vee r)$.

The statements were simplified to $p \vee (q \wedge r)$. Therefore, they all came up with the similar electrical network. Extract 12.1 shows a response of one of the candidates who performed well in this question.

2a. (i). $P \wedge Q$

$\equiv$ She is tall and beautiful.

(ii). $P \wedge \sim Q$

$\equiv$ She is tall but not beautiful

(iii). $\sim P \wedge \sim Q$

$\equiv$ She is neither tall nor beautiful.

(iv). $\sim(P \vee \sim Q)$

$\equiv$ It is not true that she is tall or not beautiful.

2b. Soln

$(P \wedge (P \vee Q)] \vee [Q \wedge [P \vee Q]]$

$\equiv [(P \vee F) \wedge (P \vee Q)] \vee [(F \vee Q) \wedge (P \vee Q)]$ Identity law

$\equiv [P \vee (F \wedge Q)] \vee [(F \wedge P) \vee Q]$ Distributive law

$\equiv [P \vee F] \vee [F \vee Q]$ Identity law

$\equiv P \vee Q$ Identity law

$[P \wedge (P \vee Q)] \vee [Q \wedge [P \vee Q]] \equiv P \vee Q$

2c. Soln:

let the sentence be M.

Considering F

$M \equiv (p \vee \sim q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee q \vee r)$

$M \equiv (p \vee \sim q \vee r) \wedge ((p \vee q) \vee \sim r) \wedge ((p \vee q) \vee r))$ Associative law

$\equiv (p \vee \sim q \vee r) \wedge ((p \vee q) \vee [\sim r \wedge r]]$ Distributive law

$\equiv (p \vee \sim q \vee r) \wedge ((p \vee q) \vee F)$ Compliment law

$\equiv (p \vee \sim q \vee r) \wedge (p \vee q)$ Identity law

$\equiv (p \vee (\sim q \vee r)) \wedge (p \vee q)$ Associative law

$\equiv p \vee ((\sim q \vee r) \wedge q)$ Distributive law

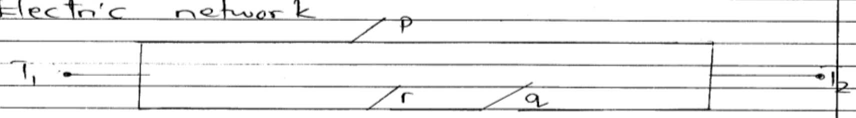
$\equiv p \vee ((\sim q \wedge q) \vee (r \wedge q))$ Distributive law

$\equiv p \vee (F \vee (r \wedge q))$ Compliment law

$\equiv p \vee (r \wedge q)$ Identity law

Statement, $M \equiv p \vee (r \wedge q)$

2e(i) Electric network



Extract 12.1: *A correct response*

In Extract 12.1 the candidate had clear understanding on verbal representation of logical sentences, laws of algebra of proposition, truth table and electrical network.

In spite of good performance, 1208 (11.3%) candidates scored 5.0 marks or less. In part (a), a number of candidates used "full stop" to replace the logical connective $\wedge$. This indicates that they lacked knowledge of logic language in relation to English language. Some candidates repeated the words while others introduced the articles or new auxiliary verbs. For instance, they wrote "she is tall and she is beautiful" instead of "she is tall and beautiful" while others wrote "she doesn't tall and she doesn't beautiful”. Furthermore, a few number of them constructed the truth tables for each sentence. This indicates that they did not understand the requirements of the question. In part (b), most candidates used the laws of algebra of proposition incorrectly to simplify $[P \wedge (P \vee Q)] \vee [Q \wedge (P \vee Q)]$. As a result, they ended up with incorrect simplified statements such as $p \wedge q$ and T. Analysis of the candidates’ responses also shows that some candidates applied the correct laws but did not state the appropriate names of laws or did not write the names at all.

In part (c) (i), several candidates failed to extract a correct logical statement from the truth table. For instance, some candidates obtained $(p \vee q \vee r) \wedge (p \vee q \vee \sim r)$ which indicates that they do not focus on either truth value T or false value F. Also, some candidates extracted a correct statement but could not simplify it to $p \vee (q \wedge r)$. Consequently, the candidates drew a wrong electrical network in part (c) (ii). In addition, some candidates drew an electrical network without simplifying the statement derived from the table. Extract 12.2 is a sample response of a candidate who did the question badly.

2: a: soln.

i) $P \wedge Q$.

= She is tall if she is beautfull.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

ii) $P \wedge \sim Q$.

= she is tall if she not beutfull.

P	Q	$\sim Q$	$P \wedge \sim Q$.
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F.

iii/ $\sim P \wedge \sim Q$.
= She is not a tall if she is not a butfull.

P	Q	~P	~Q	~P∧~Q.
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

iv/ $\sim(P \vee \sim Q)$
= Not only a tall and she is not a beutiful.

P	Q	~Q	P∨~Q	~(P∨~Q)
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

2.b) soln.

Given:

$= [P\wedge(P\vee q) \vee (Q\wedge(P\vee Q)]$

$= ((P\vee q)\wedge(P\vee P))\vee((Q\wedge Q)\vee(P\wedge Q)))$

$= (P\vee q)\wedge(T) \vee (T)\vee(P\wedge Q)$

$= \sim(P\vee q)\vee T$

$\sim(P\vee q)\vee T$.

$= T$.

$\therefore T$.

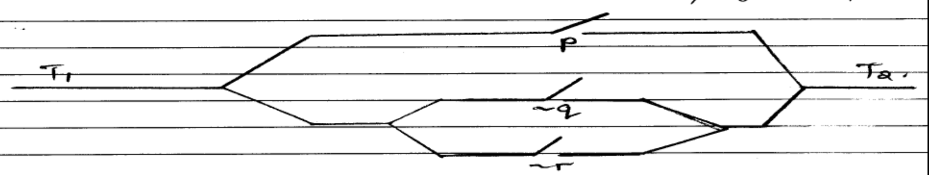
C: i/ soln.

From the truth table.

$= P\vee(\sim q\wedge\sim r)$

$\therefore P\vee(\sim q\wedge\sim r)$

ii/ $P\vee(\sim q\wedge\sim r)$ draw the electricity flow:



Extract 12.2: *An incorrect response*

In Extract 12.2, the candidate constructed the truth table of the given statements instead of writing in words, applied the laws incorrectly and failed to extract a correct statement from the table.

2.2.3 Question 3: Vectors

This question comprised parts (a), (b) and (c). In part (a), it was given that $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$. The candidates were required to find (i) the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$, (ii) the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ and (iii) the unit vector of $\underset{\sim}{a} \times \underset{\sim}{b}$. In part (b), it was given that the position vectors of point K and L are $3\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$ and $\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$ respectively. The candidates were required to find the position vector of the point M which divides $\overline{KL}$ in the ratio of $4:3$. In part (c), the candidates were required to find the velocity and acceleration when $t=0$ from the displacement vector $\underline{r} = a\,\underline{i}\cos nt + b\,\underline{j}\sin wt$ where a and b are arbitrary constants.

The analysis of data shows that 17.3 percent of 10,562 candidates who attempted this question scored from 0 to 5.0 marks, 24.3 percent from 5.5 to 8.5 marks and 58.4 percent from 9.0 to 15 marks. Further analysis shows that 82.6 percent of the candidates scored more than 5.0 marks, of whom 383 (3.6%) candidates scored all 15 marks. On the basis of this analysis, the candidates' performance in this question was good. Figure 14 illustrates the information given above.

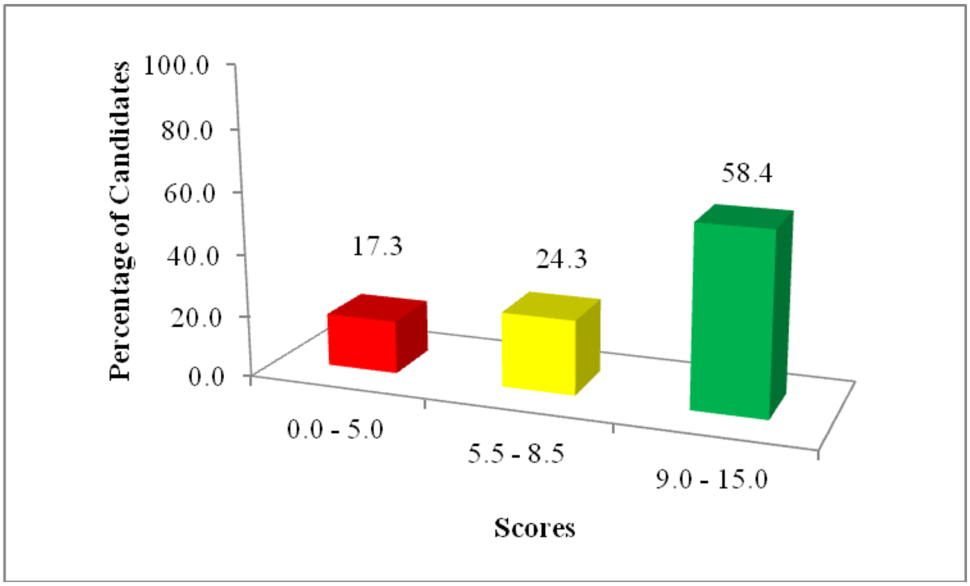


Figure 14: *Candidates' Performance in Question 3*

The good performance was due to the following reasons: In part (a) (i), the candidates used the formula $\frac{\underline{a}\,.\underline{b}}{|\underline{b}|}$ and obtained 5 as the projection of $\underline{a}$ onto $\underline{b}$. In part (a) (ii), most candidates computed $\underline{a}\cdot\underline{b}$, $|\underline{a}|$ and $|\underline{b}|$ as 15, $\sqrt{29}$ and $\sqrt{9}$ respectively, then inserted in either the formulae $\underline{a}\,.\underline{b} = |\underline{a}||\underline{b}|\cos\theta$ or $|\underline{a}\times\underline{b}| = |\underline{a}||\underline{b}|\sin\theta|$, to get an angle whose magnitude is 21.8^{o}. In part (a) (iii), most candidates used the formula $\frac{\underline{a}\times\underline{b}}{|\underline{a}\times\underline{b}|}$ to obtain $\frac{1}{6}(2\underline{i}+4\underline{j}-4\underline{k})$ as the unit vector of

$\underset{\sim}{a} \times \underset{\sim}{b}$. In part (b), a number of candidates employed the formulae $OM = \dfrac{nOK + mOL}{n+m}$ and $OM = \dfrac{nOK - mOL}{n-m}$ to obtain $\dfrac{1}{7}\left(13\underline{i} + 18\underline{j} - 7\underline{k}\right)$ and $-5\underline{i} + 6\underline{j} + 23\underline{k}$ as the position vectors which divide $\overrightarrow{KL}$ internally and externally respectively in the ratio $4:3$. In part (c), the candidates were able to differentiate $\underline{r} = a\underline{i}\cos nt + b\underline{j}\sin wt$ with respect to t to get $\underline{v} = -na\underline{i}\sin nt + wb\underline{j}\cos wt$. On replacing t with 0, they got $\underline{v} = wb\underline{j}$. Finally, they differentiated $\underline{v} = -na\underline{i}\sin nt + wb\underline{j}\cos wt$ to get $\underline{a} = -n^2 a\underline{i}$ which is the acceleration at $t = 0$. Extract 13.1 is a sample response from one of the candidates who answered question 3 correctly.

3 (a)(i)

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{pmatrix}2\\3\\4\end{pmatrix} \cdot \begin{pmatrix}2\\1\\2\end{pmatrix}$$

$$= 4 + 3 + 8$$

$$= 15$$

$$|\underset{\sim}{b}| = \sqrt{2^2 + 1^2 + 2^2}$$

$$= 3$$

$$\text{Proj}_b\, a = \frac{15}{3}$$

$$= 5$$

3 (a)(ii) Solution:

from:

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$$

where

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = 15$$

$$|\underset{\sim}{b}| = 3$$

$$|\underset{\sim}{a}| = \sqrt{2^2 + 3^2 + 4^2}$$

$$= \sqrt{29}$$

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \frac{15}{3\sqrt{29}}$$

3(a)(ii) $\cos\theta = 0.92847$

$\theta = \cos^{-1}(0.92847)$

$\theta = 21.80^\circ$

3 (a) (iii) Solution:

Unit vector $= \dfrac{\underset{\sim}{a} \times \underset{\sim}{b}}{|\underset{\sim}{a} \times \underset{\sim}{b}|}$

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{pmatrix} i & j & k \\ 2 & 3 & 4 \\ 2 & 1 & 2 \end{pmatrix}$$

$\underset{\sim}{a} \times \underset{\sim}{b} = 2\underset{\sim}{i} + 4\underset{\sim}{j} - 4\underset{\sim}{k}$

$|\underset{\sim}{a} \times \underset{\sim}{b}| = \sqrt{2^2 + 4^2 + (-4)^2}$

$= 6$

Unit vector $= \dfrac{2\underset{\sim}{i} + 4\underset{\sim}{j} - 4\underset{\sim}{k}}{6}$

$= \frac{1}{3}\underset{\sim}{i} + \frac{2}{3}\underset{\sim}{j} - \frac{2}{3}\underset{\sim}{k}$

3 (b) Solution:

By internal division method,

$\underset{\sim}{P} = \cancel{AA}\ \dfrac{n\underset{\sim}{a} + m\underset{\sim}{b}}{m+n}$ $\quad m:n = 4:3$

3 (b) $\underset{\sim}{a} = 3i + 2j - 5k$

$\underset{\sim}{b} = i + 3j + 2k$

let

Vector $\underset{\sim}{M}$ be $\underset{\sim}{P}$

$\underset{\sim}{P} = \dfrac{3(3\underset{\sim}{i} + 2\underset{\sim}{j} - 5k) + 4(\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k})}{3+4}$

$= \dfrac{9\underset{\sim}{i} + 6\underset{\sim}{j} - 15\underset{\sim}{k} + 4\underset{\sim}{i} + 12\underset{\sim}{j} + 8\underset{\sim}{k}}{7}$

$\underset{\sim}{M} = \dfrac{13\underset{\sim}{i} + 18\underset{\sim}{j} - 7\underset{\sim}{k}}{7}$

By external division,

$$\underset{\sim}{r} = \frac{n\underset{\sim}{a} - m\underset{\sim}{b}}{m-n}$$

$$\underset{\sim}{r} = \frac{3(3\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}) + 4(\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k})}{4-3}$$

$$\underset{\sim}{r} = 5\underset{\sim}{i} - 6\underset{\sim}{j} - 23\underset{\sim}{k}$$

3 (c) Solution:

$$r = ai\cos nt + bj\sin wt$$

$$\frac{dr}{dt} = -ani\sin nt + bwj\cos wt$$

$$\frac{d^2r}{dt^2} = -an^2i\cos nt - bw^2j\sin wt$$

When $t = 0$,

For r,

$$\frac{dr}{dt} = -ani\sin 0 + bwj\cos 0$$

$$\frac{dr}{dt} = 0 + bw\underset{\sim}{j}$$

$$\text{Velocity} = (bw\underset{\sim}{j})\ m/s$$

For Acceleration,

$$\frac{d^2r}{dt^2} = -an^2i\cos n(0) - bw^2\underset{\sim}{j}\sin 0$$

$$= -an^2\underset{\sim}{i}\ m/s^2$$

Extract 13.1: *A correct response from one of the candidates*

On the other hand, the weak performance of candidates was due to the following reasons: In part (a), it was noted that a significant number of candidates used the incorrect formulae. Examples of the more usual mistakes were to write $P_b^a = \frac{\underline{a}.\underline{b}}{|\underline{a}|}$, $P_b^a = \frac{\underline{a}.\underline{b}}{|\underline{a}|}|\underline{b}|$ and $P_b^a = \frac{\underline{a} \times \underline{b}}{|\underline{b}|}$. Such candidates got the incorrect projections. For example, the candidates who applied the formula $P_b^a = \frac{\underline{a}.\underline{b}}{|\underline{a}|}$ got $\frac{15}{29}\sqrt{29}$ instead of

5. It was also noted that most candidates had some success in quoting the formula $P_b^a = \frac{\underline{a}.\underline{b}}{|\underline{b}|}$ but several candidates found the arithmetic difficult and did not obtain a correct answer of the expression $= \frac{\left(2\underset{\sim}{i}+3\underset{\sim}{j}+4\underset{\sim}{k}\right)\bullet\left(2\underset{\sim}{i}+\underset{\sim}{j}+2\underset{\sim}{k}\right)}{\sqrt{4+1+4}}$. Such case is shown in Extract 13.2. In part (a) (ii), it was surprising to see how a considerable number of candidates used the incorrect formulae such as $\underline{a}\bullet\underline{b}=|\underline{a}||\underline{a}|\sin\theta$, $\underline{a}\times\underline{b}=|\underline{a}||\underline{a}|\sin\theta$ etc. These candidates got the incorrect angles between $\underline{a}$ and $\underline{b}$. For instance, the candidates who used the formula $\underline{a}\bullet\underline{b}=|\underline{a}||\underline{a}|\sin\theta$ got $\theta = 68.2°$ instead 21.8^0. In part (a) (iii), most candidates recalled the formula $\frac{\underline{a}\times\underline{b}}{|\underset{\sim}{a}\times\underset{\sim}{b}|}$ correctly but did not get the required cross product of $\underline{a}$ and $\underline{b}$ as they calculated the determinant of $\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 3 & 4 \\ 2 & 1 & 2 \end{pmatrix}$ incorrectly. For instance, some candidates wrote $2\underline{i}-4\underline{j}-4\underline{k}$ as the vector representing $\underline{a}\times\underline{b}$ instead of $2\underline{i}+4\underline{j}-4\underline{k}$. In part (b), many candidates exchanged the value of n with that of m ie. 4:3 corresponds to m:n and not n:m. For instance the position vector of the point which divides $\overrightarrow{KL}$ internally was found using the formula $OM = \frac{mOK+nOL}{n+m}$ instead of $OM = \frac{nOK+mOL}{n+m}$. This case is illustrated in Extract 13.3. In part (c), some candidates treated vector $\underline{r} = a\underline{i}\cos nt + b\underline{j}\sin wt$ as a normal function and therefore presented the corresponding velocity and acceleration as $v = bw$ and $a =$ - an^2 instead of $\underline{v} = wb\underline{j}$ and $\underline{a} = -n^2 a\underline{i}$ respectively. Other candidates went through the trouble of integrating $\underline{r} = a\underline{i}\cos nt + b\underline{j}\sin wt$, this was a wastage of time as the question did not require them to do so. Such candidates wrote - $\frac{b}{w}\underline{j}$ and - $\frac{a}{n^2}\underline{j}$ as the corresponding velocity and acceleration at $t = 0$ contrally to the demand of the question.

8 a) is Projection:

$$\text{proj}_b\, a = \frac{(a \cdot b)}{|b|}$$

$$(a \cdot b) = a.a + b.b + c.c$$
$$= (2)(2) + (3)(1) + (4)(2)$$
$$= 4 + 4 + 8$$
$$= 16$$

$$|b| = \sqrt{2^2 + 1^2 + 2^2}$$
$$= \sqrt{9}$$
$$|b| = 3$$

$$\text{proj}_b\, a = \frac{16}{3}$$

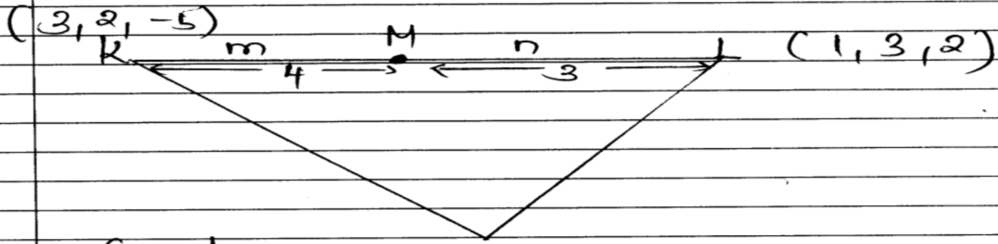
$\therefore$ The projection of a onto b = 5.33

Extract 13.2: *An incorrect response*

In Extract 13.2, the candidates computed the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ incorrectly.

(b). $3i + 2j - 5k = K$

$L = i + 3j + 2k$



Consider

$$\frac{KM}{ML} = \frac{m}{n}$$

$$\frac{\underset{\sim}{M} - \underset{\sim}{K}}{\underset{\sim}{L} - \underset{\sim}{M}} = \frac{4}{3}$$

$$\frac{m - K}{L - m} = \frac{4}{3}$$

$$n(\underset{\sim}{M} - K) = m(L - m)$$

$$n\underset{\sim}{M} - n\underset{\sim}{K} = m\underset{\sim}{L} - m\underset{\sim}{M}$$

$$nm + mM = m\underset{\sim}{L} + n\underset{\sim}{K}$$

$$m(n + M) = mL + nk$$

$$M = \frac{mL + nk}{m + n}.$$

3 Vector $m = \frac{mL + nk}{m+n}$

But $M = 4$, $n = 3$

$\underset{\sim}{M} = \frac{4x(3, 2, -5) + 3(1, 3, 2)}{4+3}$

$\underset{\sim}{M} = \frac{(12, 8, -20) + (3, 9, 6)}{7}$

$\underset{\sim}{M} = \frac{15i + 17j - 14k}{7}$

$\therefore$ position vector $\underset{\sim}{M} = \frac{15\underset{\sim}{i}}{7} + \frac{17\underset{\sim}{j}}{7} - \frac{14k}{7}$

$\therefore \underset{\sim}{M} = \frac{15\underset{\sim}{i}}{7} + \frac{17\underset{\sim}{j}}{7} - \frac{14\underset{\sim}{k}}{7}$

Extract 13.3 *An incorrect response*

2.2.4 Question 4: Algebra

This question comprised parts (a), (b) and (c). In part (a), the candidates were required to (i) express $\frac{1}{r(r+1)}$ in partial fractions and (ii) deduce the formula for $\sum_{r=1}^{n} \frac{1}{r(r+1)}$. Part (b) had the descriptions "A teacher bought pens, pencils and note books for her students. She bought 3 pens, 6 pencils and 3 notebooks in the first week.1 pen, 2 pencils and 2 notebooks in the second week as well as 4 pens, 1 pencil and 4 notebooks in the third week. If she spent 3000, 1100 and 2600 shillings in the first, second and third week respectively". The candidates were required to find the price of each item using the inverse matrix method. In part (c), the candidates were required to use synthetic division in finding the quotient and the remainder when $2x^4 + 3x^3 - 2x + 5$ is divided by $x + 5$.

The analysis of data indicates that 10480 (97.9%) candidates attempted the question, out of whom 8233 (78.6%) candidates scored the marks ranging from 5.5 to 15 marks. Therefore, the general performance of the candidates in this question was good. Figure 15 shows the percentage distribution of candidates with weak, average and good scores.

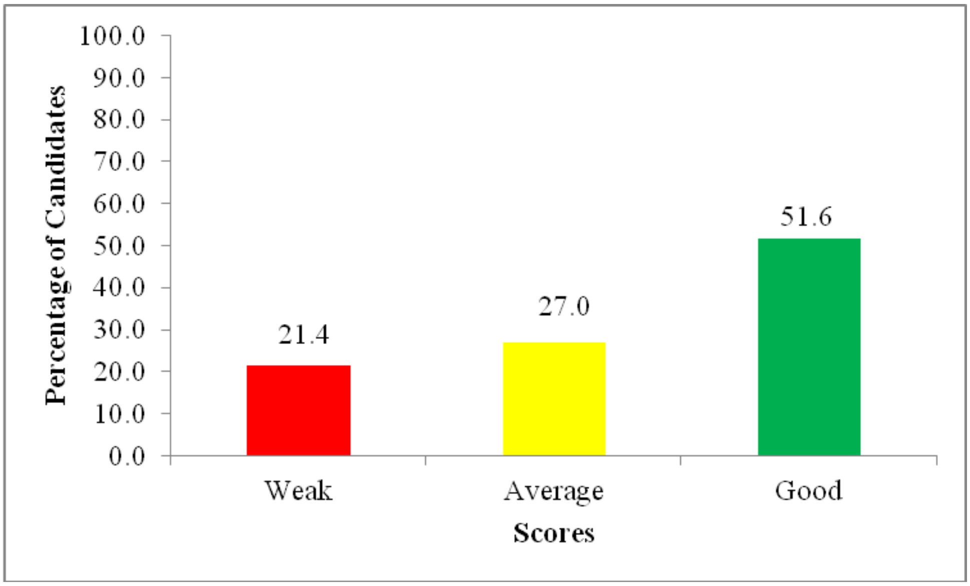


Figure 15: *Candidates' Performance in Question 4*

Further analysis shows that 400 (3.8%) candidates scored all marks allotted to the question. Their responses showed various strengths as follows: In part (a) (i), the candidates used the method of undetermined coefficients or otherwise to write $\frac{1}{r(r+1)}$ as $\frac{1}{r}-\frac{1}{r+1}$. With this form, the candidates managed to get $\sum_{r=1}^{n}\frac{1}{r(r+1)}=\frac{n}{n+1}$. In part (b), they represented the price of each pen, pencil and notebook as *x*, *y* and *z* respectively. This enabled them to translate the given word problem in matrix form as $\begin{pmatrix}3 & 6 & 3\\ 1 & 2 & 2\\ 4 & 1 & 4\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}3000\\ 1100\\ 2600\end{pmatrix}$. Thereafter, they computed the determinant of $\begin{pmatrix}3 & 6 & 3\\ 1 & 2 & 2\\ 4 & 1 & 4\end{pmatrix}$ as 21, its cofactors as $\begin{pmatrix}6 & 4 & -7\\ 21 & 0 & 21\\ 6 & -3 & 0\end{pmatrix}$ and its inverse as $\frac{1}{21}\begin{pmatrix}6 & -21 & 6\\ 4 & 0 & -3\\ -7 & 21 & 0\end{pmatrix}$. Finally, the candidates pre-multiplied the inverse on both sides of the matrix equation to get $\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}500\\ 200\\ 100\end{pmatrix}$. In part (c), such candidates used synthetic division correctly to find the quotient and remainder as $2x^3 - 7x^2 + 35x - 177$ and 890 respectively. Extract 14.1 is a sample solution of a candidates who attempted part (a) and (c) correctly.

4. (a) (i) let: $\frac{1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$

$1 = A(r+1) + Br.$

when $r = 0.$

$1 = A$

when $r = -1$

$1 = -B \Rightarrow B = -1.$

$\therefore \frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$

4. (a) (ii) From: $\sum_{r=1}^{n} \frac{1}{r(r+1)}$

but $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$

$\therefore \sum_{r=1}^{n} \frac{1}{r(r+1)} = \sum_{r=1}^{n} \frac{1}{r} - \frac{1}{r+1}$

expanding the terms from 1 to n.

for $r = 1$: $1 - \cancel{\frac{1}{2}}$

$r = 2$: $+ \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$

$r = 3$: $+ \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}.$

$r = 4$: $+ \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}$

⋮

$r = n-1$: $+ \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}$

$r = n$: $+ \cancel{\frac{1}{n}} - \frac{1}{n+1}$

$$= 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1}$$

$$= \frac{n}{n+1}$$

4\. (c) from: $x + 5 = 0$

$x = -5$

-5	2	3	0	-2	5
$+$	0	-10	35	-175	885
	2	-7	35	-177	890

∴ when $2x^4 + 3x^3 - 2x + 5$ is divided by $x + 5$, the quotontient is $2x^3 - 7x^2 + 35x - 177$ and remaind or is 890

Extract 14.1: *A correct response*

In Extract 14.1, the candidate displayed proper knowledge and skills on sigma notation, telescopic series and synthetic division.

Despite good performance, 202 (1.9%) candidates scored zero. In part (a) (i), majority of these candidates could not express the given expression into partial fraction. For instance, some of them wrote $1 = \frac{A}{r} + \frac{B}{r+1}$, hence ended up with incorrect values of *A* and *B*. In part (a) (ii), some candidates seemed to apply the principle of mathematical induction which do not complement the requirements of the question. Others substituted $r = 0$ into $\frac{1}{r} - \frac{1}{r+1}$ resulting to the term which cannot be defined. This indicates that the candidates had insufficient knowledge on sigma notation because the question required them to start with $r = 1$ and not $r = 0$. In part (b), there were few candidates who did not realize that the word problem can be summarized into matrix equation. Majority of the candidates who realized, did not assign variable to the price of each item or assigned the variables incorrectly. For example some candidates assigned the variables to weeks while others formulated a correct matrix equation but could not solve it correctly. Errors in finding determinant or cofactors resulted to a wrong inverse and consequently a wrong final answer. In addition, several candidates skipped key steps of finding the determinant and the inverse of 3×3 matrix, probably they used scientific calculator. Such candidates were supposed to show clearly all algebraic steps in finding the inverse. Further analysis shows that some candidates solved the equation using crammers rule instead of inverse method while others had an understanding of the concept but they post-multiplied the inverse instead of pre-multiplying it. They wrote $\begin{pmatrix}3000\\1100\\2600\end{pmatrix}\times\begin{pmatrix}6 & -21 & 6\\4 & 0 & -3\\-7 & 21 & 0\end{pmatrix}\frac{1}{21}$, which cannot be operated. These candidates were supposed to write $\frac{1}{21}\begin{pmatrix}6 & -21 & 6\\4 & 0 & -3\\-7 & 21 & 0\end{pmatrix}\times\begin{pmatrix}3000\\1100\\2600\end{pmatrix}$.

In part (c), the common weakness was the use of long division method instead of synthetic division. Also, some candidates did computational errors in either synthetic or long division method. Extract 14.2 is a response showing some mistakes done by one of the candidates.

4. (a) (ii) test for $n = 1$

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$$

$$= \frac{1}{1} - \frac{1}{2}.$$

$$= \frac{1}{2}.$$

which is true for the value of $n = 1$.

Induction step.

$$\sum_{r=1}^{k} \frac{1}{r(r+1)} = \frac{1}{k} - \frac{1}{k+1} \quad \ldots\ \text{Hypothesis.}$$

for $n = k+1$

$$\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \sum_{r=1}^{k} \frac{1}{r(r+1)} + \sum_{r=1}^{1} \frac{1}{r(r+1)}$$

$$= \frac{1}{k} - \frac{1}{k+1} + \frac{1}{2}$$

$$= \frac{2(k+1) \mp 2k + k(k+1)}{2k(k+1)}.$$

$$= \frac{2k+2 \mp 2k + k^2 + k}{2k^2 + 2k}.$$

$= \dfrac{k^2+k+2}{2k^2+2k}$

$= \dfrac{k^2+k+2}{2k(k+1)}$ Hence shown

4 c/

$$
\begin{array}{r}
2x^3 - 7x^2 + 35x - 177 \\
x+5 \;\big)\; 2x^4 + 3x^3 + 0x^2 - 2x + 5 \\
-(2x^4 + 10x^3) \\
-7x^3 + 0x^2 \\
-(-7x^3 - 35x^2) \\
35x^2 - 2x \\
-(35x^2 + 175x) \\
-177x + 5 \\
-(-177x - 885) \\
(890)
\end{array}
$$

$\therefore\ 2x^3 - 7x^2 + 35x - 177 + \dfrac{890}{x+5}$

Extract 14.2: *An incorrect response*

In Extract 14.2, the candidate applied the principle of mathematical induction and long division inappropriately.

2.2.5 Question 5: Trigonometry

This question consisted of parts (a), (b), (c) and (d). The candidates were required to: (a) show that $\sin 5\alpha + \sin 2\alpha - \sin\alpha = \sin 2\alpha(2\text{co}3\alpha + 1)$ by using the factor formula; (b) simplify the expression $\dfrac{1+\sin\phi}{5+3\tan\phi - 4\cos\phi}$ using small angles approximation up to the term containing ϕ^2; (c) prove that $\cos\beta(\tan\beta+3)(3\tan\beta+1) = 3\sec\beta = 10\sin\beta$, and (d) find the greatest and least values of the expression $\dfrac{1}{4\sin x - 3\cos x + 6}$.

This question was opted by 5798 (54.2%) candidates, of which 34.7 percent scored below 7 marks, 36.0 percent from 7 to 11.5 marks and 29.3 percent from 12 to 20 marks. Further analysis shows that 20 candidates scored all 20 marks while 276 candidates (4.8%) scored zero. These results imply that the performance of candidates in question 5 was good as illustrated in Figure 16.

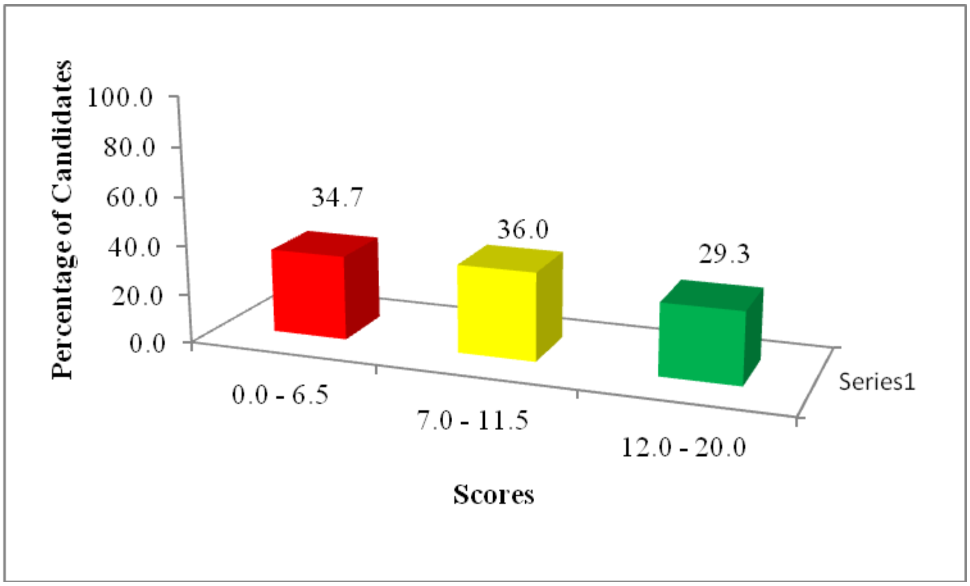


Figure 16*: Candidates' Performance in Question 5*

The candidates who did part (a) correctly expressed $\sin 5\alpha + \sin 2\alpha - \sin\alpha$ as $2\sin\frac{1}{2}(5\alpha-\alpha)\cos\frac{1}{2}(5\alpha+\alpha)+\sin 2\alpha$ which was then simplified to $\sin 2\alpha(2\cos 3\alpha+1)$. The candidates who got part (b) right realised the need to substitute ϕ for $\sin\phi$, $\tan\phi$ and $1-\frac{\phi^2}{2}$ for $\cos\phi$ in the expression $\frac{1+\sin\phi}{5+3\tan\phi-4\cos\phi}$ to get $\frac{1+\phi}{2\phi^2+3\phi+1}$; Factorised the expression $\frac{1+\phi}{2\phi^2+3\phi+1}$ into $\frac{1}{2\phi+1}$ and finally expanded $\frac{1}{2\phi+1}$ binomially to get $1-2\phi+4\phi^2$. The candidates who scored all marks in part (c) were able to use the identity $\tan\beta=\frac{\sin\beta}{\cos\beta}$ and $\sin^2\beta+\cos^2\beta=1$ in showing that $\cos\beta(\tan\beta+3)(3\tan\beta+1)=3\sec\beta+10\sin\beta$. The candidates who scored all marks in part (d) expressed $4\sin x - 3\cos x$ correctly in R-form as $5\sin(x-36.87^\circ)$, managed to identify that $-5\le 4\sin x-3\cos x\le 5$ and therefore $1\le 4\sin x-3\cos x+6\le 11$ and finally concluded correctly that the greatest and lowest value of the function $\frac{1}{4\sin x-3\cos x+6}$ are 1 and $\frac{1}{11}$ respectively. Extract 15.1 is a sample response from a candidate who had adequate skills on the question.

5 a) $\sin 5\alpha + \sin 2\alpha - \sin\alpha$

$$= \sin 5\alpha - \sin\alpha + \sin 2\alpha$$

$$= 2\sin\left(\frac{5\alpha-\alpha}{2}\right)\cos\left(\frac{5\alpha+\alpha}{2}\right) + \sin 2\alpha$$

$$= 2\sin 2\alpha\cos 3\alpha + \sin 2\alpha$$

$$= \sin 2\alpha\,(2\cos 3\alpha + 1)$$

$\therefore \sin 5\alpha + \sin 2\alpha - \sin\alpha = \sin 2\alpha(2\cos 3\alpha + 1)$

hence shown

b) $\dfrac{1+\sin\phi}{5+3\tan\phi-4\cos\phi}$

as $\phi \to 0$, $\sin\phi \approx \phi$

$\tan\phi \approx \phi$

$\cos\phi \approx 1 - \frac{1}{2}\phi^2$

5 b) then

$$\frac{1+\sin\phi}{5+3\tan\phi-4\cos\phi} \approx \frac{1+\phi}{5+3\phi-4(1-\frac{1}{2}\phi^2)}$$

$$\approx \frac{1+\phi}{5+3\phi-4+2\phi^2}$$

$$\approx \frac{1+\phi}{2\phi^2+3\phi+1}$$

$$\approx \frac{1+\phi}{(\phi+1)(2\phi+1)}$$

$$\approx \frac{1}{1+2\phi}$$

$$\approx (1+2\phi)^{-1}$$

by using binomial theorem

$$\frac{1+\sin\phi}{5+3\tan\phi-4\cos\phi} \approx 1 - 2\phi + \frac{2(2\phi)^2}{2} + \dots$$

$$\approx 1 - 2\phi + 4\phi^2 + \dots$$

we neglect the higher powers

then

$$\frac{1+\sin\phi}{5+3\tan\phi-4\cos\phi} \approx 1-2\phi+4\phi^2$$

$$\therefore \quad \frac{1+\sin\phi}{5+3\tan\phi-4\cos\phi} \approx 1-2\phi+4\phi^2$$

5 c) $\cos\beta(\tan\beta+3)(3\tan\beta+1)$

$$= \cos\beta(3\tan^2\beta+\tan\beta+9\tan\beta+3)$$

$$= \cos\beta(3(\tan^2\beta+1)+10\tan\beta)$$

but $1+\tan^2\beta = \sec^2\beta$

$$= \cos\beta(3\sec^2\beta+10\tan\beta)$$

$$= 3\sec\beta + \frac{10\sin\beta}{\cos\beta}(\cos\beta)$$

$$= 3\sec\beta + 10\sin\beta$$

$\therefore \cos\beta(\tan\beta+3)(3\tan\beta+1) = 3\sec\beta + 10\sin\beta$

But the question was incorrect in typing since it was typed as prove that

$\cos\beta(\tan\beta+3)(3\tan\beta+1) = 3\sec\beta = 10\tan\beta$

instead of

$\cos\beta(\tan\beta+3)(3\tan\beta+1) = 3\sec\beta + 10\tan\beta$

hence proved

d) Given the function

$$f(x) = \frac{1}{4\sin x - 3\cos x + 6}$$

let

$$4\sin x - 3\cos x = R\sin(x+\alpha)$$

$$= R\sin x\cos\alpha - R\cos x\sin\alpha$$

by comparison

$$\begin{cases} R\cos\alpha = 4 \\ R\sin\alpha = 3 \end{cases}$$

$$R^2\cos^2\alpha + R^2\sin^2\alpha = 16+9$$

5 d) $R^2(\cos^2\alpha + \sin^2\alpha) = 25$

$R^2 = 25$

$R = 5$

$\frac{R\sin\alpha}{R\cos\alpha} = 3/4$

$\alpha = \tan^{-1}(3/4)$

$\alpha = 36.87^\circ$

Then

$4\sin x - 3\cos x = 5\sin(x - 36.87^\circ)$

then

$f(x) = \frac{1}{5\sin(x - 36.87^\circ) + 6}$

the greatest value of $f(x)$ is obtained when $\sin(x - 36.87^\circ) = -1$

then

$f(x) = \frac{1}{-5+6} = 1$

the least value is obtained when

$\sin(x - 36.87^\circ) = 1$

$f(x) = \frac{1}{5+6} = 1/11$

$\therefore$ The greatest value of the function is 1 and the least value is $1/11$

Extract 15.1: *A correct response from one of the candidates*

Despite the good performance, there were candidates who scored low marks. In part (a), they failed to recall the factor formula $\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$ to show the validity of the given expression. Most of them expressed the left hand side of the given equation as $2\cos\frac{1}{2}(5\alpha - 2\alpha)\sin\frac{1}{2}(5\alpha + 2\alpha) - \sin\alpha$ and hence unable to perform the proofs as illustrated in Extract 15.2. In part (b), some candidates failed to recall and apply the double angle identity and small angle approximation for trigonometric sine, cosine and tangent. Other candidates got stuck after they have reached at $\frac{1+\phi}{2\phi^2 + 3\phi + 1}$. Such candidates did not continue with the next stage of factorization. A few candidates managed to effect the factorization of $\frac{1+\phi}{2\phi^2 + 3\phi + 1}$ into $\frac{1}{2\phi + 1}$

but could not use the binomial theorem to expand it into an expression containing ϕ^2. In part (c), a number of candidates had no clue of using the trigonometric identities $\tan\beta = \frac{\sin\beta}{\cos\beta}$ and $\sin^2\beta + \cos^2\beta = 1$. Thus, they were unable to prove that $\cos\beta(\tan\beta+3)(3\tan\beta+1) = 3\sec\beta + 10\sin\beta$. In part (d), most candidates failed to transform the denominator $(4\sin x - 3\cos x + 6)$ of the expression $\frac{1}{4\sin x - 3\cos x + 6}$ in the form $R\sin(x-\alpha)$ which could enhance them to find the greatest and least values. In addition to that other candidates managed to express $4\sin x - 3\cos x$ as $5\sin(x - 36.87°)$ but failed to identify that $-5 \le 4\sin x - 3\cos x \le 5$ and hence unable to find the required values.

$$\sin 5\alpha - \sin\alpha = 2\sin\left(\frac{5\alpha+\alpha}{2}\right)\cos\left(\frac{5\alpha-\alpha}{2}\right)$$
$$= 2\sin\frac{6\alpha}{2}\cos\frac{4\alpha}{2}.$$
$$= 2\sin 3\alpha\cos 2\alpha.$$

Extract 15.2: *An incorrect response*

In Extract 15.2, the candidates failed to apply factor formula.

2.2.6 Question 6: Probability

The question aimed at determining the candidates' knowledge on the concept of Probability as follows:

(a) (i) Show that ${}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}$.

(ii) A machine produces a total of 10,000 nails a day which on average 5% are defective. Find the probability that out of 500 nails chosen at random 10 will be defective.

(b) (i) Find the probability that in four tosses of a fair die a 2 appears at most once.

(ii) The mean weight of 400 female pupils at a certain school is 65 kg and the standard deviation is 5 kg. Assuming that the weights were normally distributed, find how many pupils weigh between 50 kg and 60 kg.

(c) A random variable X has the probability density function

$$f(x) = \begin{cases} px, \text{ for} & 0 \le x \le 2 \\ p(4-x), \text{ for} & 2 \le x \le 4 \\ 0, & \textit{else where} \end{cases}.$$

(i) Find the value of the constant p.

(ii) Sketch the graph of $f(x)$.

(iii) Evaluate $P\left(\frac{1}{2} \le X \le \frac{5}{2}\right)$.

The analysis of data shows that 1543 (14.4%) candidates opted this question. This shows that most candidates did not attempt it. Also, it was the poorest performed question whereby 1182 (76.6%) candidates scored 6.5 marks or less. Figure 17 shows the percentage of candidates with low, average and good scores.

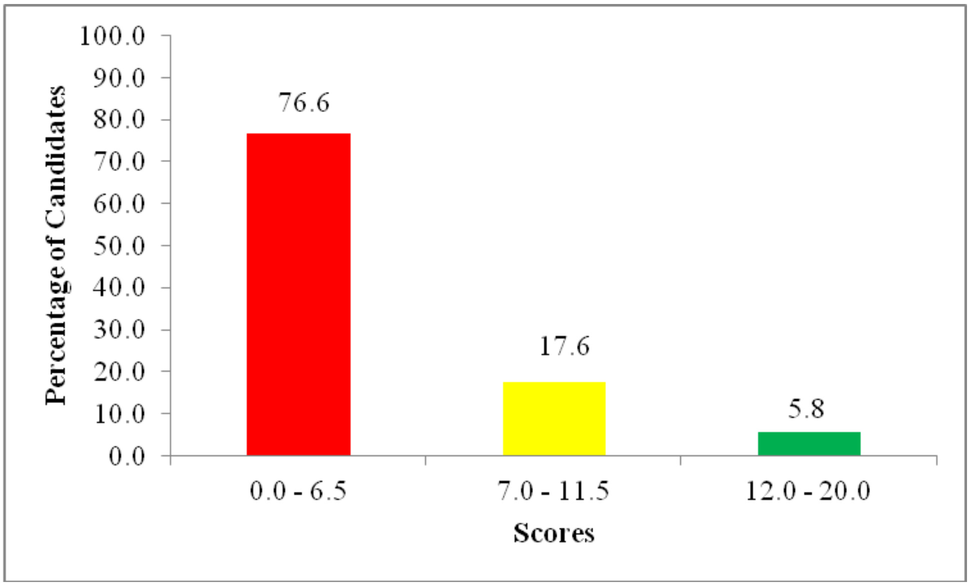


Figure 17: *Candidates' Performance in Question 6*

The analysis of the candidates' responses revealed a number of weaknesses. In part (a) (i), some candidates were unable to distinguish the formulae for combination and permutation. They used the incorrect formula ${}^{n}c_{r} = \frac{n!}{(n-r)!}$, hence failed to show that ${}^{n}C_{r+1} + {}^{n}C_{r} = {}^{n+1}C_{r+1}$. Such candidates were supposed to recall that ${}^{n}c_{r} = \frac{n!}{(n-r)!r!}$ and deduce that ${}^{n}C_{r+1} = \frac{n(n-1)(n-2)...(n-r)}{(r+1)r!}$ and

${}^{n+1}c_{r+1} = \frac{(n+1)!}{(r+1)!((n+1)-(r+1))!}$ which could help them to produce ${}^{n+1}C_{r+1}$ by manipulating ${}^{n}C_{r+1} + {}^{n}C_{r}$. In part (a) (ii), a significant number of candidates did not realize that the problem could be solved by applying binomial distribution formula. Some candidates used the formula $P(E) = \frac{n(E)}{n(S)}$ while the question required them to use the formula $P(X = x) = {}^{n}c_{x} p^{n} q^{n-x}$ where $n = 500$, $p = 0.05$, $q = 0.95$ and $x = 10$ to get $P(X = 10) = 0.000297$. Also, they could use the concept of combination such that $P(10\,\text{defective}) = \frac{{}^{500}C_{10} \times {}^{9500}C_{490}}{{}^{10000}C_{500}}$. Other candidates managed to recall the binomial distribution formula $P(X = x) = {}^{n}c_{x} p^{n} q^{n-x}$ but could not extract correctly the parameters x, n, p and q from the problem.

Likewise, in part (b) (i), the majority of the candidates did not apply the binomial distribution formula. Again, they used the formula $P(E) = \frac{n(E)}{n(S)}$ which led to incorrect answer. Also, some candidates used the binomial distribution formula but interpreted $P(X \le 1)$ incorrectly by writing $P(X \le 1) = 1 - [P(x = 0) + P(x = 1)]$. The candidates were supposed to define $P(X \le 1) = P(x = 0) + P(x = 1)$ and applying the formula $P(X = x) = {}^{n}c_{x} p^{n} q^{n-x}$ to get $P(X \le 1) = \frac{125}{144}$. In part (b) (ii), several candidates applied the correct formula $\textit{standard unit} = \frac{\textit{boundary} - \textit{mean}}{\textit{standard deviation}}$ but did not find boundaries for lower and upper limits. They calculated the standard units directly from 50 kg and 67 kg, hence ended up with incorrect answer $-3 \le z \le 0.4$. The candidates were supposed to realize that the weight recorded as being between 50 kg and 67 kg can actually have any value from 49.5 kg to 67.5 kg. Therefore, the appropriate range of standard units is $-3.1 \le z \le 0.5$ which gives $P(-3.1 \le z \le 0.5) = 0.69049$ and 276 $(i.e\ 400 \times 0.69049)$ as the number of pupils whose weight lie between 50 and 67. Also, a considerable number of candidates were unable to read the probability of the calculated range of standard units $(P(-3.1 \le z \le 0.5))$ using scientific calculators or mathematical tables. Moreover, a few candidates multiplied the mean (65) and standard deviation (5) to get 325 as the number of pupils weighing between 50 and 67. In part (c) (i), many candidates did not apply the property $\int_{-\infty}^{\infty} f(x)\,dx = 1$, thus failed to get the correct value of p from $f(x) = \begin{cases} px & 0 \le x < 2 \\ p(4-x) & 2 \le x \le 4 \\ 0 & \text{else where} \end{cases}$. Some candidates formulated an incorrect equation $px = 4p - px$ and solved it to get $p = 0$. Consequently, such candidates sketched the incorrect graph of *f(x)* in (ii) and obtained the incorrect value of $P\left(\frac{1}{2} \le x \le \frac{5}{2}\right)$ in (iii). The question could be solved by formulating an equation $\int_0^2 px\,dx + \int_2^4 p(4-x)\,dx = 1 \Rightarrow 2p + 2p = 1$ and solving it to get $p = \frac{1}{4}$ which leads to $f(x) = \begin{cases} \frac{1}{4}x & 0 \le x < 2 \\ \frac{1}{4}(4-x) & 2 \le x \le 4 \\ 0 & \text{else where} \end{cases}$ and $P\left(\frac{1}{2} \le x \le \frac{5}{2}\right) = \frac{11}{16}$. Extract 16.1 is a sample response of a candidate who performed this question poorly.

6 (a) ii/ $n(E) = 10$ nails.
$n(S) = 500$ nails.

$P = n(E)/n(S)$.

$P = 10/500$

Probability $= 0.02$ or $1/50$.

6 (b) ii/ Number of pupils weigh between 50 and 67 Kg
= Mean X Standard deviation
= 65 Kg X 5 Kg.
= 325 pupils.

∴ 325 pupils weight between 50 and 67 Kg.

6 (c) i/ $f(x) = PX$ ——— (i).
$f(x) = P(4 - x)$ ——— (ii)
Equate eqn i & ii
$PX = 4P\cancel{x} - PX$.

$PX = 4P - PX$.
$2PX = 4P$.
$2PX - 4P = 0$.

$$\frac{2P\cancel{(x-2)}}{\cancel{(x-2)}} = \frac{0}{x-2}$$

$$2P = \frac{0}{x-2}.$$

$$\frac{\cancel{2}P}{\cancel{2}} = \frac{0}{2}$$

$$P = 0.$$

Extract 16.1: *An incorrect response*

In Extract 16.1, the candidate applied inappropriate formulae and ignored the properties of probability density function.

Although many candidates performed poorly in this question, one candidate attempted the question correctly and scored all 20 marks. Extract 16.2 shows a solution of such candidate.

ii) Solution

total 10,000. Per days.

$P = 5/100 = 0.05$

$n = 500$

$x = 10$

from.

$P + Q = 1.$

$Q = 1 - P.$

$Q = 1 - 0.05$

$Q = 0.95.$

Then from the formula.

$P_{(x)} = {}^nC_x P^x Q^{n-x}.$

$P_{(x=10)} = {}^{500}C_{10} (0.05)^{10} (0.95)^{500-10}.$

$= {}^{500}C_{10} (0.05)^{10} (0.95)^{490}$

$P_{(x=10)} = 2.9 \times 10^{-4}$

6 b) i) four tosses of a fair die.

2 appears at most once.

This means that either not occur or occur once.

$n(s) = 6.$ $n = 4.$

$P(E)$ of 2 to occur.

$P(E) = \frac{1}{6}.$

$P = 1/6.$

$P + Q = 1.$

$Q = 1 - P.$

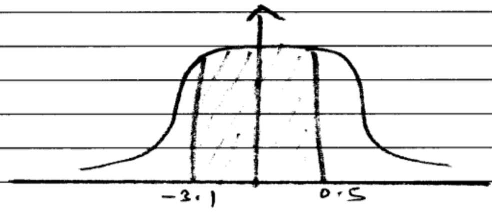
$Q = 1 - 1/6$

$Q = 5/6.$

Then.

$P(2) = P_{(x=0)} + P_{(x=1)}$

$= {}^nC_x P^x Q^{n-x} + {}^nC_x P^x Q^{n-x}$

	$= {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{4-0} + {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{4-1}$
	$= \left(\frac{5}{6}\right)^4 + 4 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^3$
	$= \frac{625}{1296} + \frac{125}{324}$
	$= \frac{125}{144}$
6	b) iii) Given.
	μ (mean) = 65.
	δ (s.d) = 5.
	50 and 67.
	from.
	$Z = \frac{X-\mu}{\delta}$
	Since is normal distributed.
	$\frac{(50-0.5)-65}{5} \leq x \leq \frac{(67+0.5)-65}{5}$.
	$-3.1 \leq x \leq 0.5$.
	
	$Q(-3.1) + Q(0.5)$.
	$= 0.49903 + 0.19146$
	$= 0.69049$.
	$P(Z) = 0.69049$.
	$n. = 0.69049 \times 400$
	$= 276$
	∴ Number of pupils weigh between 50 and 67 kg are 276
6 a)	$f(x) = \begin{cases} Px & 0 \leq x < 2. \\ P(4-x) & 2 \leq x \leq 4 \\ 0 & \text{else wher.} \end{cases}$
	i) required Value of P.
	Since $\int_a^b P(x) = 1$.
	$\int_0^2 Px\,dx + \int_2^4 P(4-x) = 1$
	$P\int_0^2 x\,dx + P\int_2^4 (4-x) = 1$.

$$P\int_0^2 x\,dx + P\int_2^4 (4-x) = 1.$$

$$P\left[\frac{x^2}{2}\right]_0^2 + P\left[4x - \frac{x^2}{2}\right]_2^4 = 1.$$

$$P(2-0) + P((8-6)) = 1.$$

$$2P + 2P = 1.$$

$$4P = 1.$$

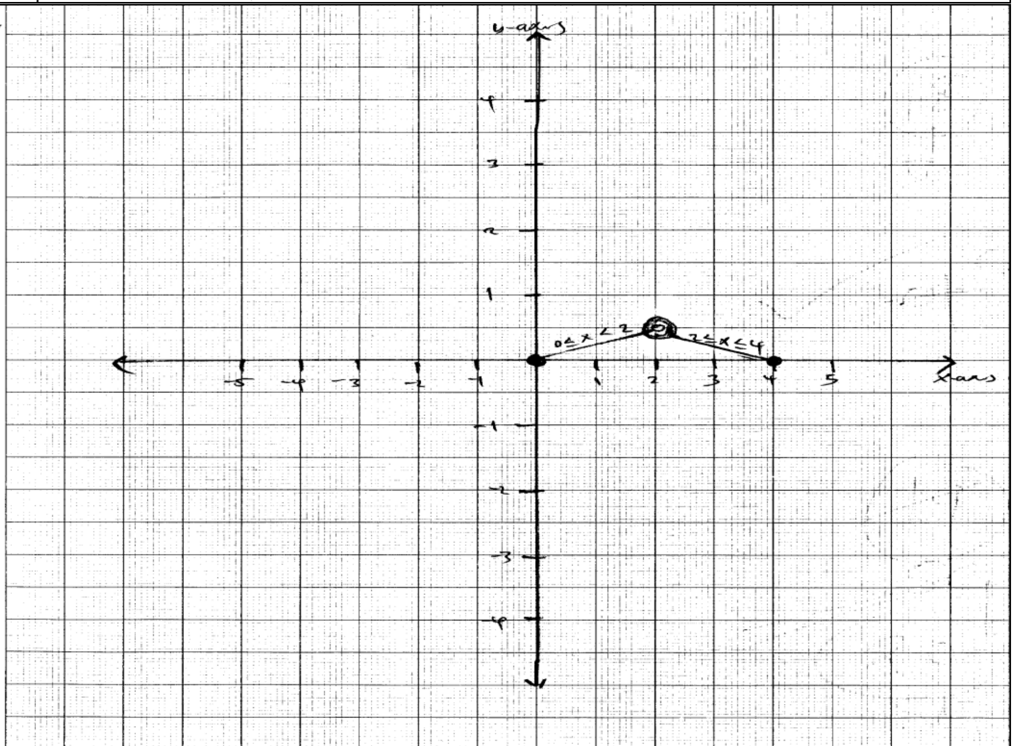
$$P = \frac{1}{4}$$

$\therefore$ The value of constant P is $\frac{1}{4}$

ii) graph of the function.

$$f(x) = \begin{cases} x/4 & 0 \le x \le 2 \\ 1/4\,(4-x) & 2 \le x \le 4 \\ 0 & \text{else where}). \end{cases}$$

6Q2.



iii) $P\left(\frac{1}{2} \le x \le \frac{5}{2}\right)$

$$P\left(\tfrac{1}{2} \le x \le \tfrac{5}{2}\right) = P\left(\tfrac{1}{2} \le x \le 2\right) + P\left(2 \le x \le \tfrac{5}{2}\right)$$

$$= \int_{1/2}^{2} \frac{x}{4}\,dx + \int_2^{5/2} \frac{1}{4}(4-x)$$

$$= \frac{1}{4}\left[\frac{x^2}{2}\right]_{1/2}^{2} + \frac{1}{4}\left[4x - \frac{x^2}{2}\right]_{2}^{5/2}.$$

$$= \frac{1}{4}\left(2 - \frac{1}{8}\right) + \frac{1}{4}\left(\frac{55}{8} - 6\right)$$

$$= \frac{1}{4}\left(\frac{15}{8}\right) + \frac{1}{4}\left(\frac{7}{8}\right)$$

$$= \frac{15}{32} + \frac{7}{32}.$$

$$= \frac{15+7}{32}.$$

$$= \frac{11}{16}$$

Extract 16.2: *An incorrect response*

In Extract 16.2, the candidate demonstrated a correct understanding on binomial distribution, normal distribution and the general concept of probability density function.

2.2.7 Question 7: Differential Equations

This question consisted of parts (a), (b) and (c). In part (a), the candidates were required to form a differential equation whose general solution is $x = e^{2t}(A + Bt)$ where *A* and *B* are constants. In part (b), the candidates were required to (i) show that $y = 2 - \cos x$ is a particular integral of the differential equation $\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$ and find the general solution and (ii) find the particular solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$, such that when $x = 0$, $y = 1.5$ and $\frac{dy}{dx} = 0$. In part (c), it was given that "A rumour is spreading through a large city at a rate which is proportional to the product of the fractions of those who heard it and of those who have not heard it, so that *x* is the fraction of those who heard it after time *t*. The candidates were required to (i) show that $x = \frac{c}{c + (1-c)e^{-kt}}$ provided that, initially a fraction *c* had heard the rumour and (ii) find *x* as function of *t* and the fraction of the population expected to have heard it by 6:00 pm if 10% have heard the rumour at noon and another 10% by 3:00 pm,.

This question was opted by 5106 (47.7%) candidates, of which 36.6 percent scored from 7 to 20 marks and among them only 11 (0.2%) candidates scored all 20 marks. Therefore, the performance in this question was satisfactory as illustrated in Figure 18.

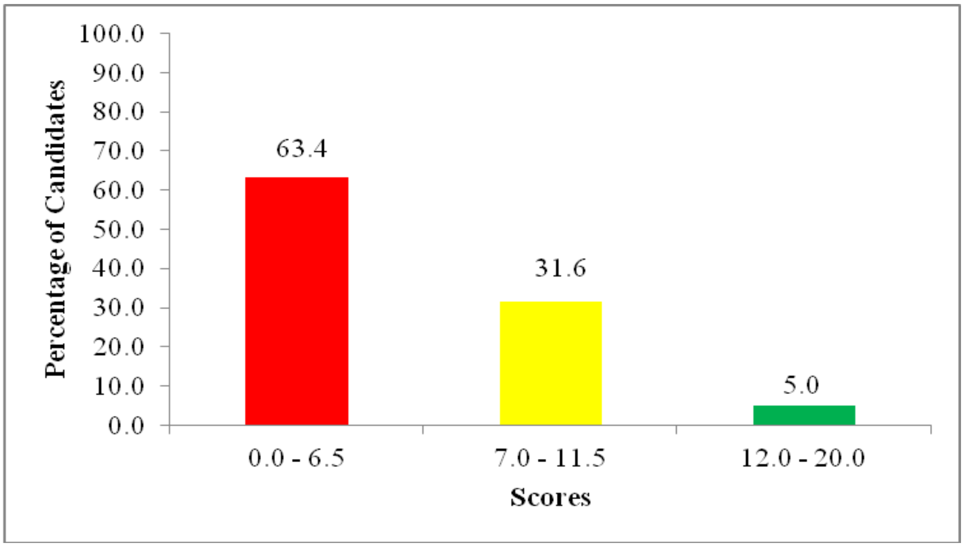


Figure 18: *Candidates' Performance in Question 7*

As illustrated in Extract 17.1, the candidates who answered part (a) correctly differentiated the equation $x = e^{2t}(A + Bt)$ to get $\frac{dx}{dt} = 2e^{2t}\left(A + Bt\right) + Be^{2t}$ and $\frac{d^2x}{dt^2} = 4e^{2t}\left(A + Bt\right) + 4Be^{2t}$ as the first and second derivatives respectively. Finally, they correctly eliminated the constants A and B to get $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = 0$ which was the required differential equation. In part (b) (i), majority of the candidates realized that they were required to substitute the second derivative of $y = 2 - \cos x$ into $\frac{d^2y}{dx^2} + 4y$ to get $8 - 3\cos x$ and conclude that $y = 2 - \cos x$ is a particular integral of $\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$. Also, they managed to find the complementary solution $y_c = A\cos 2x + B\sin 2x$ and combined it with the particular integral $y_p = 2 - \cos x$ to produce the general solution $y = A\cos 2x + B\sin 2x + 2 - \cos x$. In part (b) (ii), a considerable number of the candidates obtained correctly $y = \frac{1}{2}\cos 2x + 2 - \cos x$ as the particular solution for the differential equation $\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$. In part (c), most candidates were able to formulate the differential equation $\frac{dx}{x(1-x)} = kdt$. They were also able to break $\frac{1}{x(1-x)}$ into the sum of $\frac{1}{x}$ and $\frac{1}{1-x}$ so that they can integrate both sides of the equation $\frac{1}{x} + \frac{1}{1-x} = kt$ to show that $x = \frac{c}{c+(1-c)e^{-kt}}$. Furthermore, they expressed x as a function of t and got $x = \frac{1}{1+9e^{-kt}}$ which enabled them to find the fraction of the population who heard the rumour by 6:00 pm as $\frac{9}{25}$.

7. (a) $X = e^{2t}(A + Bt)$

$(e^{-2t})X = e^{2t}(A + Bt)(e^{-2t})$

$Xe^{-2t} = A + Bt$

$e^{-2t}\,dx/dt + -2Xe^{-2t} = 0 + B$

$e^{-2t}\,dx/dt + -2xe^{-2t} = B$

$0 = -2e^{-2t}\,dx/dt + e^{-2t}\,d^2x/dt^2 + 4xe^{-2t} - 2e^{-2t}\frac{dx}{dt}$

$0 = e^{-2t}\frac{d^2x}{dt^2} - 4e^{-2t}\frac{dx}{dt} + 4xe^{-2t}$

$0 = e^{-2t}\left(\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4\right)$

$0 = \frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4$

7. (b) (i) $y = 2 - \cos x$

$\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$

$y = 2 - \cos x$ ——— (i)

$\frac{dy}{dx} = \sin x$

and $\frac{d^2y}{dx^2} = \cos x$ ——— (ii)

Substituting (i) and (ii) into

$$\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$$

$$(\cos x) + 4(2 - \cos x) = 8 - 3\cos x$$

$$\cos x + (4 \times 2) - 4\cos x = 8 - 3\cos x$$

$$8 + \cos x - 4\cos x = 8 - 3\cos x$$

$$8 - 3\cos x = 8 - 3\cos x$$

LHS = RHS.

$\therefore$ & $y_p = 2 - \cos x$ is a particular solution.

7.

$$\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$$

Complementary solution;

Consider $\frac{d^2y}{dx^2} + 4y = 0$

7. (b) (i) let $y = e^{px}$

$dy/dx = pe^{px}$

$d^2y/dx^2 = p^2e^{px}$

Substituting in $\frac{d^2y}{dx^2} + 4y = 0$

$$p^2e^{px} + 4e^{px} = 0$$

$$(p^2 + 4)e^{px} = 0$$

$$p^2 + 4 = 0$$

$$p^2 = -4$$

$$p = \pm 2i \quad , \quad p = 0 \pm 2i$$

now $y_c = Ae^{\alpha}(\cos\beta\theta + \sin\beta\theta)$

where α - real part $= 0$

β - Imag. part $= 2$.

$$y_c = Ae^0(\cos 2\cancel{x} + \sin 2\cancel{x})$$

$$y_c = A(\cos 2\cancel{x} + \sin 2\cancel{x}).$$

General solution, $y = y_c + y_p$

$$y = A(\cos 2x + \sin 2x) + 2 - \cos x$$

$$y = A(\cos 2x + \sin 2x) + 2 - \cos x$$

(b) (ii) $\frac{d^2y}{dx^2} + 4y = 8 - 3\cos x$

particular solution.

$y = A(\cos 2x + \sin 2x) + 2 - \cos x$

$y = 1.5$, $x = 0$ and $dy/dx = 0$

$1.5 = A(\cos 2(0) + \sin 2(0)) + 2 - \cos(0)$

$1.5 = A(\cos 0 + \sin 0) + 2 - \cos 0$

$1.5 = A(1 + 0) + 2 - 1$

$1.5 = A + 1$

$1.5 - 1 = A$

$A = 0.5$

Particular solution

$y = \frac{1}{2}(\cos 2x + \sin 2x) + 2 - \cos x$

7. (c) Given x - fraction of those who heard the rumour

now $(1-x)$ is the fraction of those who didn't hear.

7. (c) but rate, $r \propto x(1-x)$

(i) $r = kx(1-x)$

$r = k(x - x^2)$

but $r = dx/dt$

$dx/dt = k(x - x^2)$

$$\int_{x_0}^{x_f} \frac{dx}{x-x^2} = \int_0^t k\,dt$$

$$\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$$

$$\frac{1}{x(1-x)} = \frac{A(1-x) + B(x)}{x(1-x)}$$

$$\therefore \quad 1 = A(1-x) + Bx$$

$$1 + 0x = A + x(B-A)$$

On comparing the two sides

$$A = 1 \quad , \quad B = A = 1$$

now $$\frac{1}{x(1-x)} = \frac{1}{x} + \frac{1}{1-x}$$

7. (c) $$\int_{x_0}^{x_f} \frac{dx}{x} + \int_{x_0}^{x_f} \frac{dx}{1-x} = \int_0^t k\,dt$$

$$\ln x \Big|_{x_0}^{x_f} + \Big| -\ln(1-x) \Big|_{x_0}^{x_f} = kt \Big|_0^t$$

$$\ln\left(\frac{x_f}{x_0}\right) - \ln\left(\frac{1-x_f}{1-x_0}\right) = kt$$

$$\ln\left(\frac{x_f}{x_0}\right) + \ln\left(\frac{1-x_0}{1-x_f}\right) = kt$$

$$\ln\left[\frac{x_f}{x_0}\left(\frac{1-x_0}{1-x_f}\right)\right] = kt$$

$$\frac{x_f}{x_0}\left(\frac{1-x_0}{1-x_f}\right) = e^{kt}$$

but $x_f = x$

and $x_0 = c$

$$\frac{x}{c}\left(\frac{1-c}{1-x}\right) = e^{kt}$$

$\div x \quad \frac{c}{x}\left(\frac{1-x}{1-c}\right) = e^{-kt}$

$$\frac{c}{x}(1-x) = (1-c)e^{-kt}$$

$$c - cx = x(1-c)e^{-kt}$$

$$c = x\left(c + (1-c)e^{-kt}\right)$$

$$x = \frac{c}{c + (1-c)e^{-kt}}$$

shown!

7. (c) (ii)

Given $c = 10\% = 0.1$

$t = 3\text{ hrs}$

~~x = 7.~~ $x = 20\% = 0.2$

from

$$x = \frac{c}{c + (1-c)e^{-kt}}$$

$$0.2 \; \cancel{x} = \frac{0.1}{0.1 + (1-0.1)e^{-k(3)}}$$

$$0.2\left(0.1 + 0.9e^{-3k}\right) = 0.1$$

$$0.1 + 0.9e^{-3k} = \frac{0.1}{0.2}$$

$$e^{-3k} = \frac{\left(\frac{0.1}{0.2} - 0.1\right)}{0.9}$$

$$e^{-3k} = \frac{4}{9}$$

$$e^{3k} = \frac{9}{4}$$

$$3k = \ln(9/4)$$

$$k = \frac{1}{3}\ln 9/4$$

x ~~at~~ at 6:00 pm

$$x = \frac{c}{c + (1-c)e^{-kt}}$$

7.	(c) (ii) where $t = 3$
	$c = 20\% = 0.2$
	$x = \dfrac{0.2}{0.2 + (1-0.2)e^{-\frac{3}{3}\ln 9/4}}$
	$x = 0.36$
	36% will have hear the rumour by 6:00 pm

Extract 17.1: *A correct response from one of the candidates*

In this question 63.4 percent scored below 7 marks and among them 47.7 percent scored zero. The candidates who scored zero lacked knowledge and skills to deal with Differential Equations problems. In part (a), most of them differentiated correctly the equation $x = e^{2t}(A + Bt)$ but they could not eliminate the constants A and B to formulate a second order differential equation. In part (b), a very large number of candidates attempted to find the particular solution instead of the general solution. This was a wastage of time because the particular integral was one of the given information. Part (c) (i) was also poorly done due to failure of candidates to comprehend the given information into the first order differential equation $\frac{dx}{dt} = kx(1-x)$ as a prerequisite step in showing that $x = \frac{c}{c+(1-c)e^{-kt}}$.

In part (c) (ii), a number of candidates failed to proceed much further through not recognizing that the given equation in (i) provided a clue on finding the fraction expected to have heard the rumour by 6:00 pm. Extract 17.2 shows a sample solution of a candidate who answered part (a) incorrectly.

7 a	$x = Ae^{2t} + Bte^{2t}$ —— (i)
	$\frac{dx}{dt} = 2Ae^{2t} + Be^{2t} + 2Bte^{2t}$ —— (ii)
	$\frac{d^2x}{dt^2} = 4Ae^{2t} + 2Be^{2t} + 2Be^{2t} + 4Bte^{2t}$ —— (iii).
	$\frac{d^2x}{dt^2} = 4Ae^{2t} + 4Be^{2t} + 4Bte^{2t}$ —— (iii).
	Force eqn (ii) x 2 − (iii)
	$2\frac{dx}{dt} - \frac{d^2x}{dt^2} = 2Be^{2t}$ —— (iv)

Take eqn (i) × 2 − (iv)

$2x - \frac{dx}{dt} = Be^{4t}$ ——— (v)

put eqn (v) into (iv)

$2\frac{dx}{dt} - \frac{d^2x}{dt^2} = -2\left(2x - \frac{dx}{dt}\right)$

$\cancel{2\frac{dx}{dt}} - \frac{d^2x}{dt^2} = -4x + \cancel{2\frac{dx}{dx}}$

$\frac{d^2x}{dt^2} - 4x = 0$ is the required d.e.

Extract 17.2: *An in correct response from one of the candidates*

2.2.8 Question 8: Coordinate Geometry II

The question consisted of parts (a), (b), (c), (d) and (e). The candidates were required to:

(a) show that the equation of a tangent to the parabola $y^2 = 4ax$ at point (x_1, y_1) is $yy_1 = 2a(x + x_1)$,

(b) find the perpendicular distance of the point $(10,10)$ from a tangent to the curve $4x^2 + 9y^2 = 25$ at $(-18, 1)$,

(c) show that $16x^2 + 25y^2 - 64x + 150y - 111 = 0$ is an equation of ellipse,

(d) (i) show that $y = mx + c$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ when $c^2 = a^2m^2 - b^2$,

(ii) determine the equation of tangent line to the hyperbola $5x^2 - 4y^2 = 1$ given the slope of the tangent line is -2,

(e) (i) transform the equation $x^2 + y^2 + 4x = 2\sqrt{x^2 + y^2}$ into a polar equation,

(ii) draw the graph of the polar equation obtained in (i) above in the interval $0 \le \theta \le 2\pi$.

About three quarters of the candidates (75.4%) attempted this question out of whom 79.6 percent scored the marks ranging from 7.0 to 20. Therefore, the candidates' performance was good. Figure 19 shows the percentage distribution of candidates for weak, average and good scores.

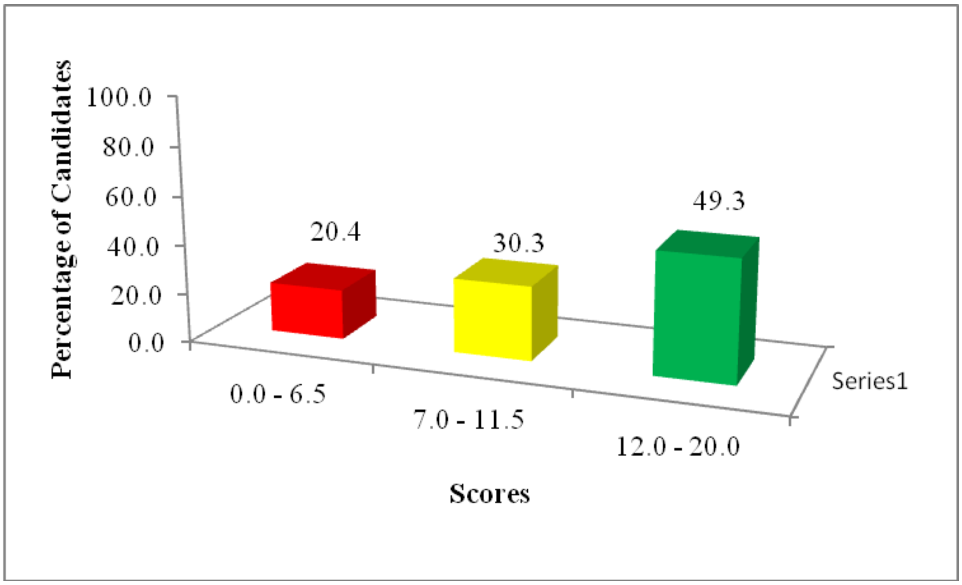


Figure 19: *Candidates' Performance in Question 8*

Further analysis shows that 160 (1.5%) candidates scored all 20 marks. Their responses described various strengths as follows. In part (a), such candidates obtained $\frac{dy}{dx}=\frac{2a}{y}$ from $y^2=4ax$ which produced the slope of a tangent at (x_1, y_1) as $\frac{2a}{y_1}$. Finally, the candidates substituted the slope $\frac{2a}{y_1}$ into the formula $slope=\frac{y-y_1}{x-x_1}$ or otherwise to arrive at $yy_1=2a(x+x_1)$. In part (b), some candidates managed to obtain $\frac{dy}{dx}=-\frac{4x}{9y}$ from $4x^2+9y^2=25$ which was used to compute the slope of a tangent at $(-18,1)$ as 8. Then, they formulated an equation of the tangent, $8x-y+145=0$ which was used together with the formula $d=\left|\frac{ax+by+c}{\sqrt{a^2+b^2}}\right|$ and the point $(10,10)$ to get 26.67 units as the required perpendicular distance. In part (c), the candidates were able to complete the square on $16x^2+25y^2-64x+150y-111=0$ to get $\frac{(x-2)^2}{25}+\frac{(y+3)^2}{16}=1$ which is a standard form of the equation for an ellipse. In part (d) (i), they substituted $y=mx+c$ into $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and simplified the resulting equation to $\left(b^2-a^2m^2\right)x^2-2a^2mcx-a^2c^2-a^2b^2=0$. Thereafter, they substituted $A=b^2-a^2m^2$, $B=-2a^2cm$ and $C=-a^2c^2-a^2b^2$ into the condition $B^2=4AC$ to obtain $c^2=a^2m^2-b^2$. In part (d) (ii), some candidates identified that $a^2=\frac{1}{5}$ and $b^2=\frac{1}{4}$ by comparing $5x^2-4y^2=1$ with the standard equation of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Then, they used the condition $c^2=a^2m^2-b^2$ and the slope of the tangent $m=-2$ to get $c=\pm\frac{1}{10}\sqrt{55}$ and consequently the equation of the tangent $y=-2x\pm\frac{1}{10}\sqrt{55}$. Also, other candidates substituted the equation of the tangent $y=-2x+c$ into $5x^2-4y^2=1$ to formulate an equation $11x^2-16cx+4c^2+1=0$. By applying the condition $B^2=4AC$, the candidates were able to get $c=\pm\frac{1}{10}\sqrt{55}$ and $y=-2x\pm\frac{1}{10}\sqrt{55}$. In part (e) (i), the majority of the candidates replaced x and x^2+y^2 in the equation $x^2+y^2+4x=2\sqrt{x^2+y^2}$ with $r\cos\theta$ and r^2 respectively and simplified the resulting polar equation $r^2+4r\cos\theta=2r$ to get $r=2-4\cos\theta$. In part (e) (ii), the candidates constructed a table of values for r and θ in the interval $0\le\theta\le 2\pi$ using the polar equation obtained in (i). The table enabled them to draw the graph of the polar equation. Extract 18.1 shows a solution of a candidate who performed well in this question.

E (a)

$$y^2 = 4ax$$

$$2y\cdot \frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

Slope of tangent $= \frac{2a}{y}$

at point (x_1, y_1)

slope of tangent $= \frac{2a}{y_1}$

Eqn of tangent.

~~20~~ slope $= \frac{\Delta y}{\Delta x}$

~~do~~ $\frac{2a}{y_1} \cancel{\times} \frac{y - y_1}{x - x_1}$

$2a(x - x_1) = y_1 y - y_1^2$

$2ax - 2ax_1 = y_1 y - y_1^2$

but $y_1^2 = 4ax_1$

$2ax - 2ax_1 = y_1 y - 4ax_1$

$2ax - 2ax_1 + 4ax_1 = y_1 y$

$y_1 y = 2a(x + x_1)$ Hence shown.

(b) $4x^2 + 9y^2 = 25$

$8x + 18y\frac{dy}{dx} = 0$

$18y\frac{dy}{dx} = -8x$

$$\frac{dy}{dx} = \frac{-8x}{18y}$$

$$\frac{dy}{dx} = \frac{-4x}{9y}$$

slope of tangent at point $(-18, 1)$

$M_T = \frac{-4(-18)}{9(1)}$

$M_T = 72/9.$

$$M_T = 8$$

Eqn of tangent.

$$\text{slope} = \Delta y / \Delta x$$

$$\frac{8}{1} = \frac{y-1}{x+18}$$

$$y - 1 = 8x + 144$$

$$8x - y + 144 + 1 = 0$$

$$8x - y + 145 = 0$$

Eqn of tangent =;

$$8x - y + 145 = 0$$

$$\text{Perpendicular distance} = \left| \frac{ax + by + c}{\sqrt{a^2+b^2}} \right|$$

$$= \frac{|8x - y + 145|}{\sqrt{64+1}}$$

$$= \frac{|8x - y + 145|}{\sqrt{65}}$$

at point $(10, 10)$

$$= \frac{|8(10) - (10) + 145|}{\sqrt{65}}$$

$$= 215 / \sqrt{65}$$

$\therefore$ Perpendicular distance $= 215/\sqrt{65}$ units

$$16x^2 - 64x + 25y^2 + 150y = 111$$

Completing the square

$$16(x^2 - 4x) + 25(y^2 + 6y) = 111$$

$$111 = 16\left(x^2 - 4x + (4/2)^2 - (4/2)^2\right) + 25\left(y^2 + 6y + (6/2)^2 - (6/2)^2\right)$$

$$16\left((x-2)^2 - 4\right) + 25\left((y+3)^2 - 9\right) = 111$$

$$16(x-2)^2 - 64 + 25(y+3)^2 - 225 = 111$$

$$16(x-2)^2 + 25(y+3)^2 = 111 + 64 + 225$$

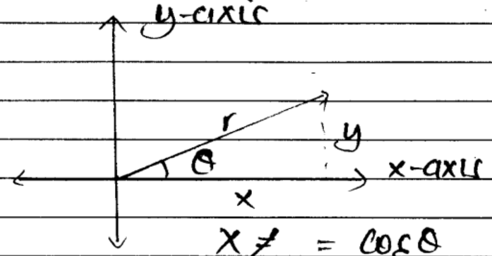
$$16(x-2)^2 + 25(y+3)^2 = 400$$

Divide by 400

$$\frac{16(x-2)^2}{400} + \frac{25(y+3)^2}{400} = 1$$

$$\frac{(x-2)^2}{25} + \frac{(y+3)^2}{16} = 1$$ shown

8 (c) i) $x^2 + y^2 + 4x = 2\sqrt{x^2+y^2}$



$$\frac{x}{r} = \cos\theta$$

$$x = r\cos\theta$$

but also; $r = \sqrt{x^2+y^2}$

$$r^2 = x^2 + y^2$$

$$r^2 + 4r\cos\theta = 2r$$

$$r^2 + 4r\cos\theta - 2r = 0$$

$$r^2 + r(4\cos\theta - 2) = 0$$

$$r(r + (4\cos\theta - 2)) = 0$$

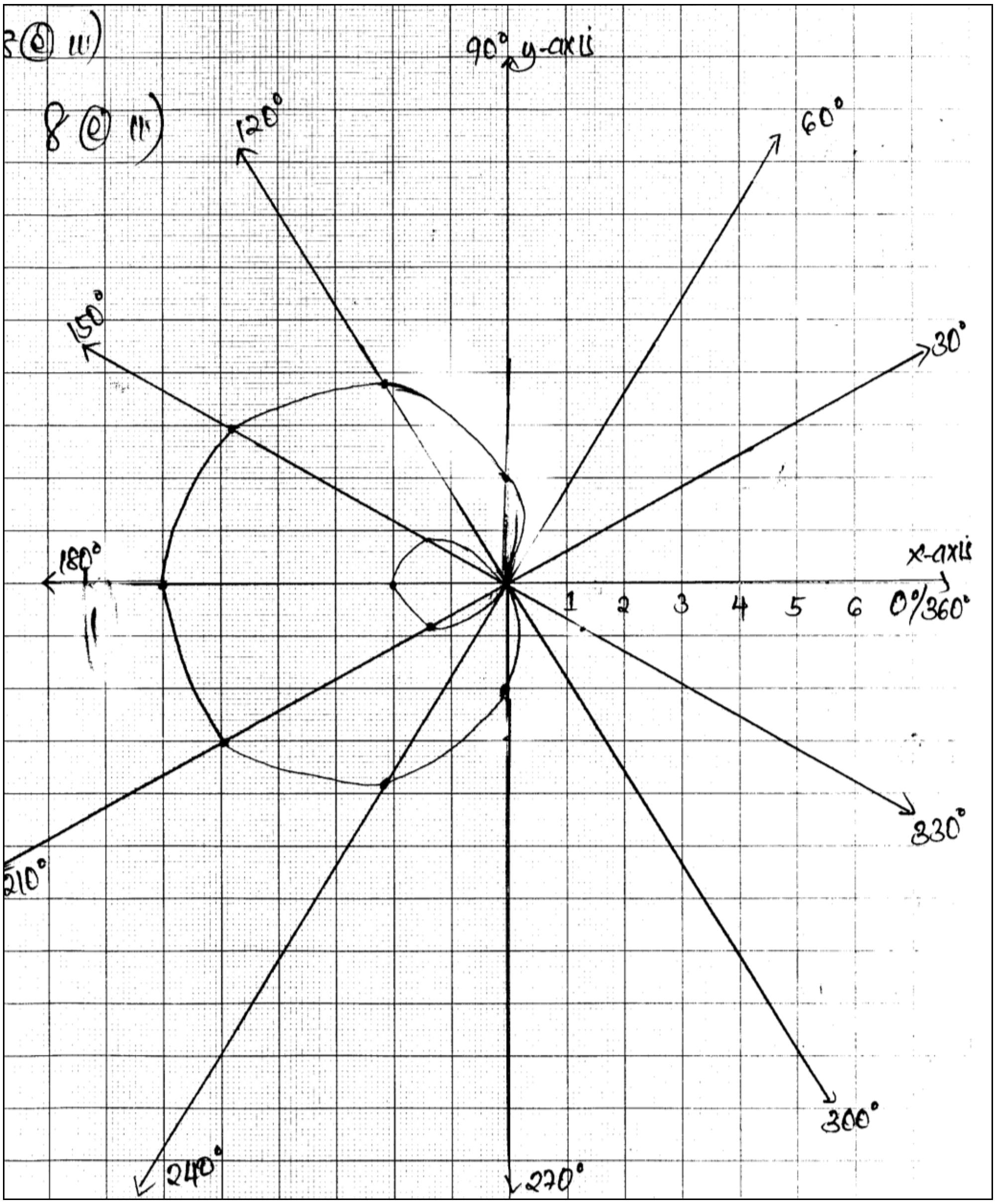
$\therefore$ $r = 0$

and $r = 2 - 4\cos\theta$

$\therefore$ $x^2 + y^2 + 4x = 2\sqrt{x^2+y^2}$ in polar form it is

$$r = 2 - 4\cos\theta$$

	θ	0°	30°	60°	90°	120°	150°	180°	210°
	r=2-4cosθ	-2	-1·46	0	2	4	5·46	6	5·46
		240°	270°	300°	330°	360°			
		4	2	0	-1·46	-2			



Extract 18.1: *A correct response*

In Extract 18.1, the candidate had clear understanding on tangency to parabola and hyperbola, equation of ellipse and polar equations and graphs.

On the other hand, there were 1648 (20.4%) candidates who scored 5.0 marks or less. In part (a), several candidates reached at $yy_1 = 2ax + 2x_1 + y_1^2$ correctly but could not replace y_1^2 with $4ax_1$ so as to arrive at $yy_1 = 2a(x + x_1)$. In part (b), most candidates got $\frac{dy}{dx} = -8$ instead of $\frac{dy}{dx} = 8$ which led to a wrong equation of the tangent particularly $8x + y + 143 = 0$. Also, other candidates substituted $(-18, 1)$ into $d = \left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$ instead of $(10, 10)$. In part (c), a number of candidates had a correct struggle in rearranging the equation $16x^2 + 25y^2 - 64x + 150y - 111 = 0$ into the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ but most of them were unable to use the concept of completing the square and therefore, they failed to express the given equation into $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{16} = 1$. In part (d) (i), several candidates failed to produce the equation $(b^2 - a^2m^2)x^2 - 2a^2mcx - a^2c^2 - a^2b^2 = 0$ from $y = mx + c$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Also, other candidates did not realize that they were required to use the condition of tangency $B^2 = 4AC$ to produce a relation that could be manoeuvred to obtain $c^2 = a^2m^2 - b^2$. In part (d) (ii), many candidates formulated a wrong equation of the tangent by differentiating $5x^2 - 4y^2 = 1$ to get $\frac{dy}{dx} = \frac{5x}{4y}$, equating $\frac{5x}{4y}$ to -2 and simplifying it to $8y + 5x = 0$. Also, some candidates used the answer obtained in (i). They correctly determined that $m = -2$, $a^2 = \frac{1}{5}$, $b^2 = \frac{1}{4}$ and substituted the numerals to $c^2 = a^2m^2 - b^2$ but they did computational errors which led to inappropriate values of *c*, particularly $c = \sqrt{\frac{11}{20}}$ instead of $c = \pm\sqrt{\frac{11}{20}}$. In part (e), several candidates had a correct solution in (i). Majority of the candidates had a correct table of values but drew incorrect graph of $r = 2 - 4\cos\theta$ in (ii). Also, there were some candidates who presented a table with incorrect values due to failure to set the scientific calculator to read radians instead of degrees. Extract 18.2 shows a sample solution of a candidate who attempted part (d) (ii) incorrectly.

8 (d) ii	$5x^2 - 4y^2 = 1$.
	slope of tangent is -2.
	From $5x^2 - 4y^2 = 1$.
	on differentiation.
	$2(5x) - 2(4y)\frac{dy}{dx} = 0$.
	$10x - 8y\frac{dy}{dx} = 0$.
	$10x = +8y\frac{dy}{dx}$.
	$8y\frac{dy}{dx} = 10x$.
	$\frac{dy}{dx} = \frac{10x}{8y}$.
	$\frac{dy}{dx} = \frac{5x}{4y}$.
	But $\frac{dy}{dx}$ is the slope of the tangent to
	the hyperbola $5x^2 - 4y^2 = 1$.
	$\therefore \frac{dy}{dx} = \frac{5x}{4y} = -2$.
	$\frac{5x}{4y} = -2$.
	$5x = -2 \times 4y$
	$5x = -8y$.
	$5x + 8y = 0$.

Extract 18.2: *An incorrect response*

In Extract 18.2, the candidate was unable to find equation of a tangent.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Advanced Mathematics Examination consisted of two papers namely 142/1 Advanced Mathematics 1 and 142/2 Advanced Mathematics 2. The examination tested 18 topics, of which 10 were in paper 1 and 8 in paper 2. The topics which were tested in paper 1 are *Calculating Devices, Hyperbolic Functions, Linear Programming, Statistics, Sets, Functions, Numerical Methods, Coordinate Geometry I, Integration* and *Differentiation.* The topics which were tested in paper 2 are *Complex Numbers, Logic, Vectors, Algebra, Trigonometry, Probability, Differential Equations* and *Coordinate Geometry II.*

Based on the boundaries introduced earlier, the analysis of data shows that the candidates' performance in 12 topics was good. These topics are *Functions, Linear programming, Sets, Logic, Statistics, Vectors, Calculating Devices, Coordinate Geometry II, Algebra, Hyperbolic Functions, Complex Numbers* and *Trigonometry.* Moreover, the analysis shows that the average performance was observed in 4 topics. Such topics are *Numerical Methods, Differentiation, Coordinate Geometry II* and *Differential Equations*. The good performance in those topics was attributed to the candidates' ability to state, recall, apply and use the correct formulae, techniques, laws and identities as well as performing mathematical operations correctly.

On the other hand, the candidates' performance in 2 topics of *Integration* and *Probability* was weak. The weak performance in these topics was due to the candidate's inability to use the integration by parts to derive a reduction formula and differentiate the parametric equations in finding length of an arc. Other reasons include failure of candidates to apply the principle of combination, binomial distribution, normal distribution and the probability distribution for a continuous random variable in answering questions, see **Appendix I.**

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The CIRA report has been specifically prepared to provide the awareness to the stakeholders about the candidate's responses in ACSEE 2019. Therefore, it presents strengths and weaknesses observed from candidates' responses. The report shows that in 2019, a total of 9237 (86.74%) candidates passed showing an increase of 3.0 percent as compared to 2018 whereby 83.74 percent of the candidates passed.

The analysis of data shows that the candidates had good performance in 12 topics, average performance in 4 topics and poor performance in 2 topics see **Appendix II**. The factors for either good or poor performance are indicated in section 3.0.

4.2 Recommendations

In order to improve the candidates' performance in future examinations, it is recommended that:

(a) Teachers should teach all the topics as stipulated in the syllabus.

(b) Students should be encouraged to study in groups so that they can be able to apply the principle of combination, binomial distribution, normal distribution and probability distribution for continuous random variables in answering the examination questions.

(c) Teachers should identify slow learners and give them extra teaching on how to find integral of product of functions by using the technique of integration by parts.

(d) Teachers should give students enough exercises, tests as well as examinations on how to find the length of an arc by differentiating the parametric functions.

(e) Teachers should teach and encourage students to put more efforts in learning the topics whose performances is constantly poor, particularly *Integrations* and *Probability* (see **Appendix III**).

Appendix I

Analysis of Candidates' Performance in Each Topic for the 2019 Advanced Mathematics Examination

S/N	Topic	Number of Question	The % of Candidates who scored at 35 or above	Remarks
1	Functions	1	94.5	Good
2	Linear Programming	1	93.7	Good
3	Sets	1	91.9	Good
4	Logic	1	88.7	Good
5	Statistics	1	84.3	Good
6	Vectors	1	82.7	Good
7	Calculating Devices	1	82.3	Good
8	Coordinate Geometry II	1	79.6	Good
9	Algebra	1	78.6	Good
10	Hyperbolic Functions	1	76.9	Good
11	Complex Numbers	1	67.8	Good
12	Trigonometry	1	65.3	Good
13	Numerical Methods	1	51.1	Average
14	Differentiation	1	43.0	Average
15	Coordinate Geometry I	1	40.6	Average
16	Differential Equations	1	36.6	Average
17	Integration	1	29.3	Weak
18	Probability	1	23.4	Weak

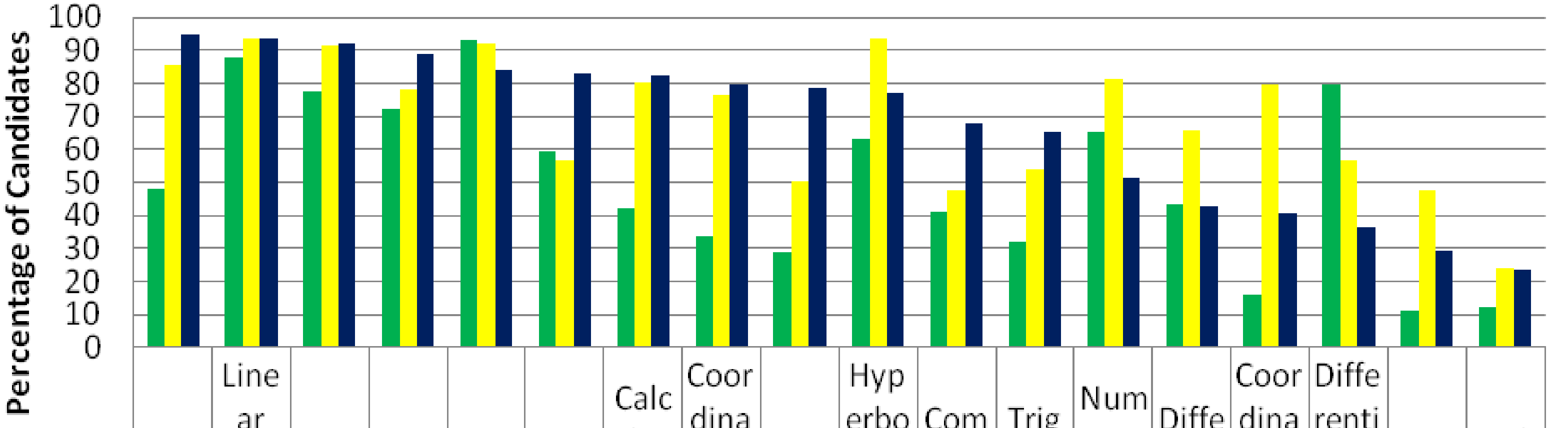
Appendix II

Analysis of Candidates' Performance in Each Topic for the 2018 & 2019 Advanced Mathematics Examination

S/N	Topic	2018			2019		
		Number of Question	The % of Candidates who scored at 35 or above	Remarks	Number of Question	The % of Candidates who scored at 35 or above	Remarks
1	Functions	1	85.5	Good	1	94.5	Good
2	Linear Programming	1	93.6	Good	1	93.7	Good
3	Sets	1	91.5	Good	1	91.9	Good
4	Logic	1	78.0	Good	1	88.7	Good
5	Statistics	1	92.1	Good	1	84.3	Good
6	Vectors	1	56.6	Average	1	82.7	Good
7	Calculating Devices	1	80.1	Good	1	82.3	Good
8	Coordinate Geometry II	1	76.4	Good	1	79.6	Good
9	Algebra	1	50.4	Average	1	78.6	Good
10	Hyperbolic Functions	1	93.5	Good	1	76.9	Good
11	Complex Numbers	1	47.8	Average	1	67.8	Good
12	Trigonometry	1	53.8	Average	1	65.3	Good
13	Numerical Methods	1	81.2	Good	1	51.1	Average
14	Differentiation	1	65.8	Good	1	43.0	Average
15	Coordinate Geometry I	1	79.7	Good	1	40.6	Average
16	Differential Equations	1	56.8	Average	1	36.6	Average
17	Integration	1	47.4	Average	1	29.3	Weak
18	Probability	1	23.9	Weak	1	23.4	Weak

Appendix III

The performance of candidates topic wise in 2017, 2018 and 2019



	Functions	Linear Programming	Sets	Logic	Statistics	Vectors	Calculating Devices	Coordinate Geometry II	Algebra	Hyperbolic Functions	Complex Numbers	Trigonometry	Numerical Methods	Differentiation	Coordinate Geometry I	Differential Equations	Integration	Probability
2017	48.1	88	77.5	72.3	93.3	59.6	42.4	33.5	28.9	62.9	41.4	32.1	65.1	43.5	15.8	79.7	11.1	12.4
2018	85.5	93.6	91.5	78.0	92.1	56.6	80.1	76.4	50.4	93.5	47.8	53.8	81.2	65.8	79.7	56.8	47.4	23.9
2019	94.5	93.7	91.9	88.7	84.3	82.7	82.3	79.6	78.6	76.9	67.8	65.3	51.1	43.0	40.6	36.6	29.3	23.4