THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2019

141 BASIC APPLIED MATHEMATICS

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141 BASIC APPLIED MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is grateful to issue this report on Candidates' Item Response Analysis (CIRA) in the Basic Applied Mathematics examination for the Advanced Certificate of Secondary Education Examination (ACSEE) 2019. The purpose of the report is to give feedback to students, teachers, policy makers and other education stakeholders on how the candidates performed in each question and topic that were assessed in the Basic Applied Mathematics examination.

The candidates' response analysis was done so as to identify the areas in which the candidates did well, averagely or poorly; and the difficulties they faced in answering the questions. Generally, the report highlighted the candidates' strengths and weaknesses in each examination item so as to give reflection to students and teachers in their self-evaluation.

The Appendix I summarizes the analysis of candidates' performance in all topics that were assessed in the 141 Basic Applied Mathematics examinations for 2019 and 2018.

Lastly, the Council would like to thank the examiners, examination officers and all others who participated in the preparation of this report.

Dr. Charles E. Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the analysis of the candidates' performance in Basic Applied Mathematics examination for ACSEE 2019. The examination paper consisted of ten (10) compulsory questions each carrying 10 marks. The analysis mainly focused on areas in which the candidates did not perform well when answering the examination items.

The number of candidates who sat for this examination was 34,840, out of whom 19,489 (56.21%) candidates passed. This performance is higher than that of 2018 in which 18,354 (55.32%) candidates passed, implying an increase of performance by almost 0.89%.

The analysis of each question presented in section 2.0 gives a brief description of the requirements of the question and the analysis of the candidates' performance. The main reasons that contributed to good, average and weak performance, including the extracts for candidates' good and poor responses for each question are summarized in this report.

In presenting the data, three colours were used to represent the categories of performance boundaries in both Figures and Appendices.

Table of performance boundaries

Colour	Percentage boundaries for the whole examination	Score boundaries per question	Remarks
	60 - 100	6.0 - 10.0	Good
	35 - 59	3.5 - 5.5	Average
	0 - 34	0.0 - 3.0	Weak

A "pass" was rated to a candidate who scored at least 35 percent for the whole examination, which is equivalent to 3.5 marks for individual question.

Finally, section 4.0 gives useful recommendations and suggestions that would help the students, teachers and government to improve the performance of future Basic Applied Mathematics examinations.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 Question 1: Calculating Devices

This question had parts (a), (b), (c) and (d). The candidates were required to use a non-programmable calculator to: (a) compute the value of $\dfrac{\sin^{-1}(2/3)}{7.4\left(\ln\sqrt[3]{87}\right)\div 2817\log 6289}$ correct to 4 decimal places, (b) evaluate $\int_0^1(3x-2)^5\,dx$, (c) solve the equation $x^2+6x-8=0$ correct to 3 decimal places and (d) find the value of $2h(4)+t(4.5)$ correct to 4 decimal places, given that $h(x)=\dfrac{\sqrt{x+4}+(3+e^x)}{x+\sqrt[3]{x}}$ and $t(x)=\sqrt{\dfrac{(x-3)^{1/3}+(x+1)^6}{1+x}}$.

The data analysis revealed that, out of 34,286 (98.4%) candidates who answered this question, almost half, that is, 17,107 (49.9%) candidates scored from 3.5 to 10 out of 10 marks. This indicates that the performance was average. Figure 1 summarizes the candidates' performance in this question.

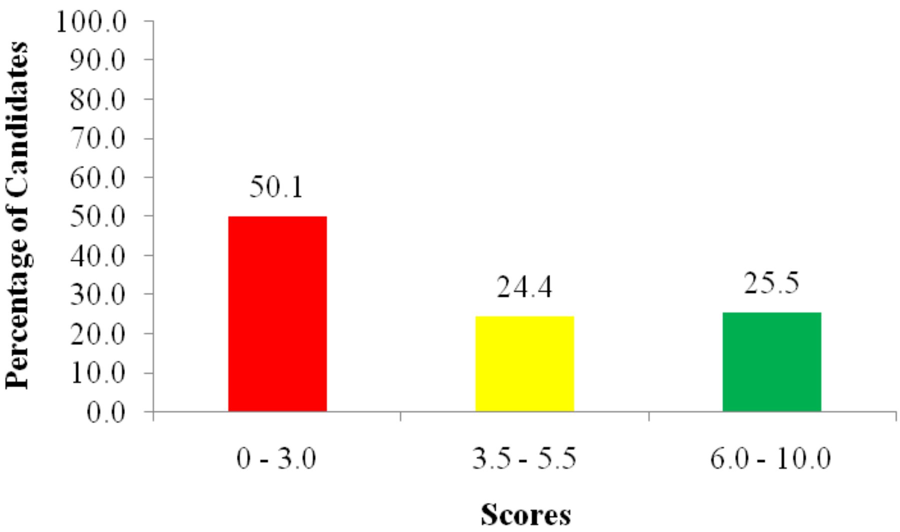
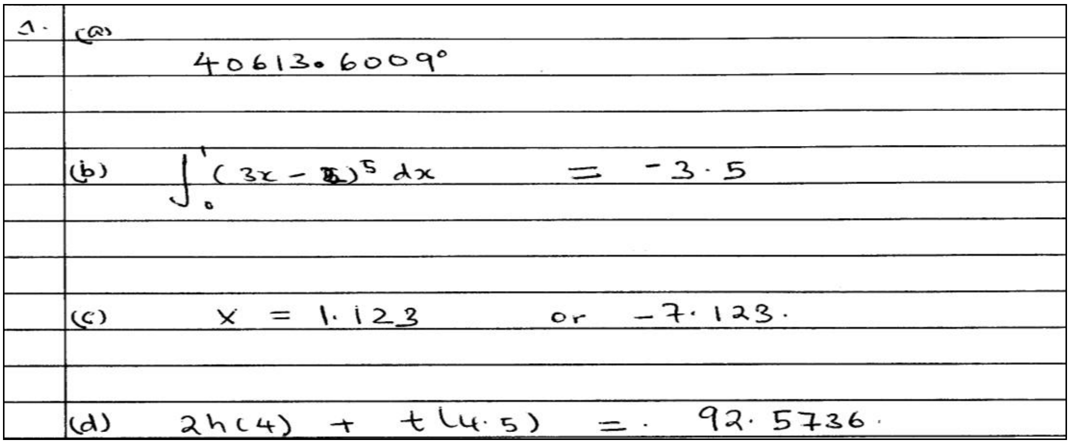


Figure 1: *Candidates' performance in question 1*

The analysis showed further that 2,035 (5.9%) candidates scored full marks. The candidates' ability to give the correct answers was contributed by several reasons, including ability to use the required functional keys of a non-programmable calculator like the *inverse of sine, natural logarithm* and *radicals* and apply the acquired knowledge and skills to calculate the value of the given expression in part (a); the "integration functional key" to evaluate the given definite integral in part (b); and the "equation mode" of the calculator to solve the given quadratic equation in part (c). In part (d), they correctly evaluated the expression $2h(4)+t(4.5)$ by using the given functions and gave final answers correctly. Extract 1.1 is a sample solution from one of the candidates.



1. (a) 40613.6009°

(b) $\int_0^1 (3x-2)^5\,dx = -3.5$

(c) $x = 1.123$ or -7.123.

(d) $2h(4) + t(4.5) = 92.5736$.

Extract 1.1: *A sample of the candidate's correct responses in question 1*

In Extract 1.1 it is evident that the candidate was able to use a non-programmable calculator to evaluate the expressions, solve the quadratic equation and evaluate the given definite integral correctly.

Despite the good performance in this question, 1,7179 (50.1%) candidates had a weak performance because they scored below 3.5 as indicated in Figure 1 while 4,270 (12.5%) candidates scored 0 mark. Failure to answer this question correctly was contributed by several reasons as follows:

In part (a), some candidates were not able to use the required functional keys of a non-programmable calculator to evaluate the given expression while others approximated the values of the terms $\sin^{-1}\left(\frac{2}{3}\right)$, $7.4\left(\ln\sqrt[3]{87}\right)$ and $2817\log 6289$ separately to four or less than four decimal ; and hence, ended up with less accurate final answer. For example, some of them wrote:

$$\frac{\sin^{-1}(2/3)}{7.4\left(\ln\sqrt[3]{87}\right)\div 2817\log 6289}=\frac{41.8103^\circ}{11.0159\div 10700.6044} = 40613.61125^\circ \approx 40613.6113^\circ.$$

Such candidates were not aware that they were supposed to type the whole expression in the calculator, command for the answer and express it to four decimal places, that is, $\dfrac{\sin^{-1}(2/3)}{7.4\left(\ln\sqrt[3]{87}\right)\div 2817\log 6289} = 40613.60086^{\circ} \approx 40613.6009^{\circ}$.

Alternatively, they were required to evaluate each term correct to more than four decimal places so as to get the required answer. For example, they could write:

$$\frac{\sin^{-1}(2/3)}{7.4\left(\ln\sqrt[3]{87}\right)\div 2817\log 6289} = \frac{41.8103149^{\circ}}{11.01590669 \div 10700.60435} \approx 40613.6009^{\circ} \text{ or}$$

$$\frac{\sin^{-1}(2/3)}{7.4\left(\ln\sqrt[3]{87}\right)\div 2817\log 6289} = \frac{0.729727656\,\text{rad}}{11.01590669 \div 10700.60435} \approx 708.8411\,\text{rad}.$$

In addition, the majority of candidates did not indicate the unit in their answer, that is, degrees or radians.

In part (b), some candidates used the normal procedures of integration by substitution method to evaluate $\int_0^1 (3x-2)^5\,dx$, instead of using a non-programmable calculator as instructed. This indicates that the candidates lacked adequate knowledge and skills of using a non-programmable calculator to evaluate definite integrals. Others used wrong functional keys; and hence, ended up with incorrect answers.

In part (c), the candidates failed to solve the equation $x^2+6x-8=0$ by using a calculator due to: inability to relate $ax^2+bx+c=0$ to $x^2+6x-8=0$ so that $a=1$, $b=6$ and $c=-8$; failure to set the calculator into the "equation mode of second degree" and omission of negative sign on the value of c. Contrary to the given instructions, other candidates applied the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ and methods of factorization instead of using a calculator as per the instruction. Furthermore, there were few candidates who were able to use the calculator to solve the given equation, but failed to express the answers correct to three decimal places.

In part (d), most of the candidates failed to compute the given equations due to improper use of brackets and wrong substitution of the given values. However, there were candidates who correctly got $h(4)=10.81479146$ and $t(4.5)=70.94400524$ but did not multiply the term $h(4)$ by 2. Instead, they just added the values of $h(4)$ and $t(4.5)$ while some of them wrote $2h(4)+t(4.5)=8h(4)+t(4.5)=8(10.81479146)+70.94400524$, instead of writing $2h(4)+t(4.5)=2(10.81479146)+70.94400524$ as required. Extract 1.2 shows a sample response of a candidate who failed to answer this question correctly.

1 a	$\dfrac{\sin^{-1}(2/3)}{7.4(\ln^3\sqrt{87}) \div 2817\log 6289}$
	$= 1.02947 \times 10^{-3}$
	To 4 decimal places
	1.02947×10^{-3}
	$= 1.0295 \times 10^{-3}$
b.	$\int_0^1 (3x-2)^5\,dx$
	soln
	let $u = 3x-2$
	$du = u^5$
	$\dfrac{dy}{du} = 3x$
	$dy/dx = (3x-2)^5$
	$dy/dx = dy/du \cdot du/dx$
	$dy/dx = 3x \cdot 3x-2$
	$dy/dx = (9x^2-6x)^5$
	$(9x^2-6x)^5$
	$(9(1)^2-6(1))^5 - (9(0)-6(0))^5$
	$= 0.5$
(c)	$x^2+6x-8=0$ to 3 decimal places
	soln
	$x^2+6x-8=0$
	$x=1$
	$x=-4$
	$(-4)^2+6(-4)-8=0$
	$16+-24-8=0$
	$16-32$ -
	-16
	To three decimal places $= 0.016$

d) Finding $2h(4) + t(4.5)$

Given that $h(x) = \frac{\sqrt{x+4} + (3+e^x)}{x + \sqrt[3]{x}}$ and $t(x) = \sqrt{\frac{(x-3)^{1/3} + (x+6)}{1-x}}$

soln

$h(x) = \frac{\sqrt{x-4} + 3+e^x}{x + \sqrt[3]{x}} \times \frac{x - \sqrt[3]{x}}{x - \sqrt[3]{x}}$

$h(x) = \frac{(x\sqrt{x-4} - \sqrt{x^2-4}) - 3x + xe^x - 3 + e^{x}\sqrt[3]{x}}{x^2(-x\sqrt[3]{x} + x\sqrt[3]{x}) - x}$

$h(x) = \frac{-3x - xe^x - 3 + e^x \sqrt[3]{x}}{x^2 - x}$

$h(x) = \frac{-3 - e^x - 3 + e^x \sqrt[3]{x}}{x}$

$h(x) = \frac{-6\sqrt[3]{x}}{x}$

$h(x) = -6$

01 d $t(x) = \sqrt{\frac{(x-3)^{1/3} + (x+1)^6}{1-x}}$

soln

$(x-3)^{1/3} + (x+1)^6$

$(x + x - 3 + 1)^{1/3 \times 6}$

$2x - 2$

$t(x) = \sqrt{\frac{(2x-2)^2}{1-x}}$

$(2x-2)(2x-2)$

$4x^2(+4x - 4x) + 4$

$t(x)$ $4x^2 - 8x + 4$

$x_1 = 1$

$x_2 = 4$

$t = 4$

$-x \overline{)4x^2 - 8x - 4}$

$2h(4) + t(4.5)$

$2 \times 6(4) + 4(4.5)$

$48 + 18$

$= 66$

66 to 4 decimal places

0.0066

Extract 1.2: *A sample of the candidate's incorrect responses in question 1*

Extract 1.2 confirms that the candidate lacked adequate knowledge and skills on the use of a non-programmable calculator to evaluate expressions, the concepts of integration and solving quadratic equations.

2.2 Question 2: Functions

This question consisted of parts (a) and (b). In part (a), the candidates were required to: (i) sketch the graph of $f(x)=\begin{cases} x^2+1 & \text{for } x>1 \\ |x| & \text{for } -2<x\le 1 \\ x+2 & \text{for } x\le -2 \end{cases}$, (ii) determine the domain and range of $f(x)$ and (iii) evaluate $f(-3)$, $f(0.5)$ and $f(2)$. In part (b), they were required to show that $[f(x)]^3 = f(x^3)+3f(x)$ if $f(x)=x+\dfrac{1}{x}$.

The analysis of data showed that this question was averagely performed because 18,520 (56.0%) candidates scored from 3.5 to 10 marks. Figure 2 illustrates the performance of candidates in this question.

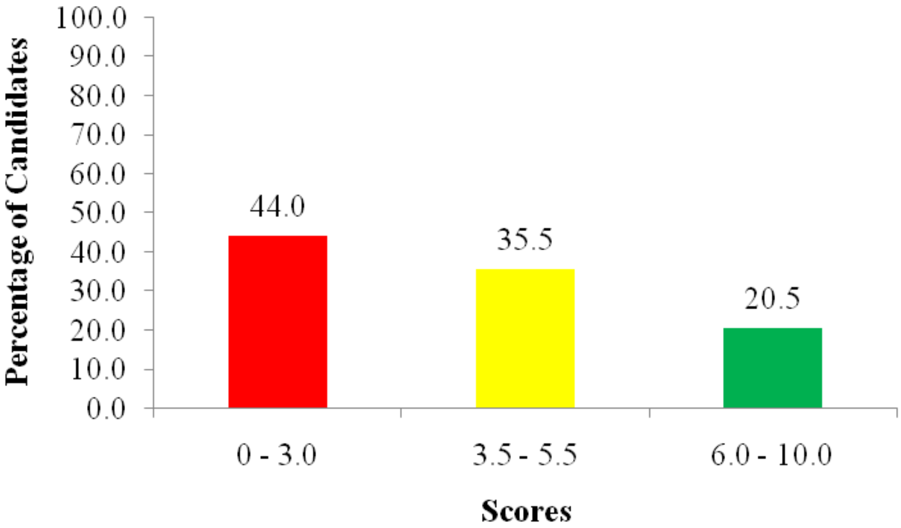


Figure 2: *Candidates' performance in question 2*

The data analysis showed further that only 240 (0.7%) candidates scored full marks. The candidates were able to answer this question correctly due to the following main reasons:

In part (a), the candidates were able to correctly sketch the graph of: $f(x)=x^2+2$ when $x>1$, $f(x)=|x|$ when $-2<x\le 1$ and $f(x)=x+2$ when $x\le -2$ on the same *xy*-plane. On that basis, they were able to state the domain, range and the correct values of $f(-3)$, $f(0.5)$ and $f(2)$.

In part (b), the candidates were able to: write $\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$, substitute $\left(x+\frac{1}{x}\right) = f(x)$ and finally get $[f(x)]^3 = f(x^3) + 3f(x)$ as shown in Extract 2.1. This indicates that the candidates had adequate knowledge on the concepts of functions, especially step functions and their properties, and the expansion of cubic functions including the skills on the use of substitution techniques in verifying polynomial functions.

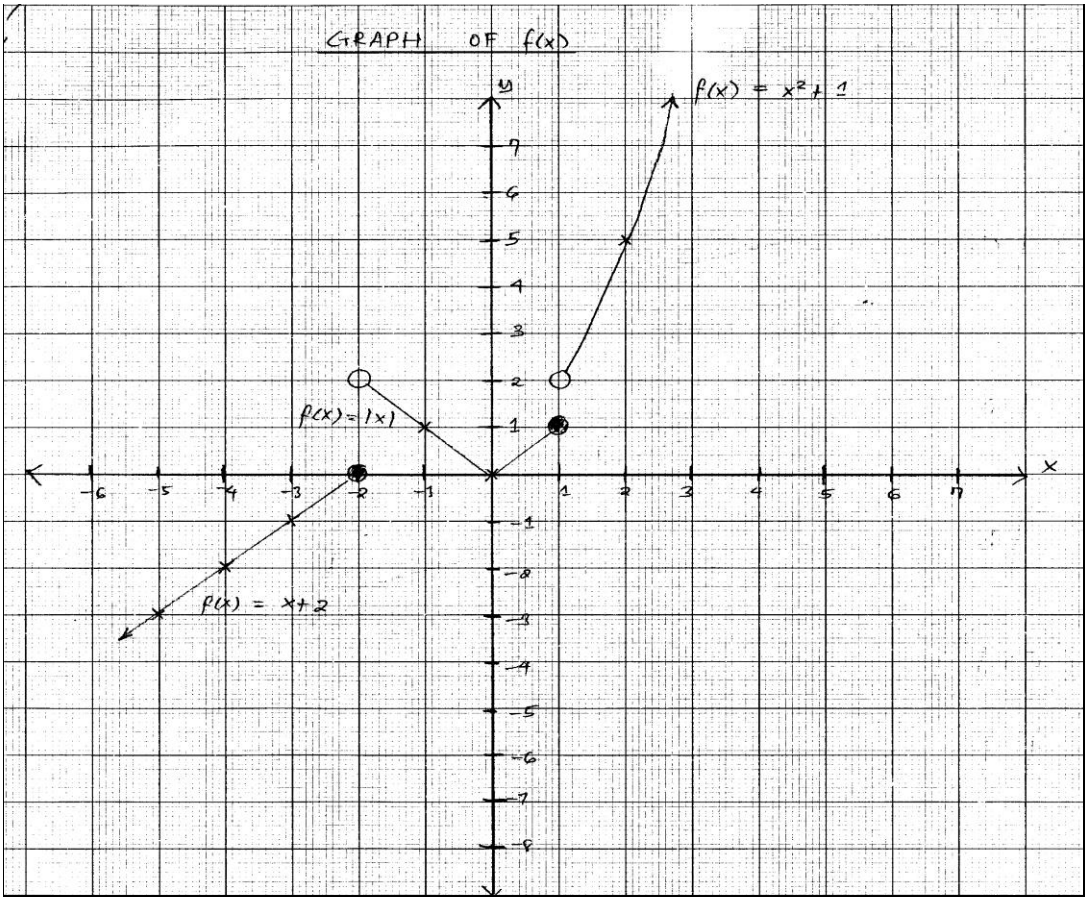
2. a/ Table of values

i/ For $f(x) = x^2 + 1$

x	1	2	3	4
x^2+1	2	5	10	17

ii/ Domain $= \{x : x \in R\}$

Range $= \{y : y \in R, y \neq 2\}$


GRAPH OF f(x)
f(x) = x² + 1
f(x) = |x|
f(x) = x + 2
y
x

2. a/ iii/ $f(-3)$ is obtained in third function,

$f(x) = x + 2$

$f(-3) = -3 + 2$

$\therefore f(-3) = -1$

$f(0.5)$ is obtained in second function,

$f(x) = |x|$

$f(0.5) = |0.5|$

$\therefore f(0.5) = 0.5$

$f(2)$ is obtained in first function,

$f(x) = x^2 + 1$

$f(2) = 2^2 + 1$

$\therefore f(2) = 5$

b/ Given, $f(x) = x + 1/x$

Then, cubing both sides,

$[f(x)]^3 = [x + 1/x]^3$

$= x^3 + 3x^2(1/x) + 3x(1/x)^2 + 1/x^3$

$[f(x)]^3 = x^3 + 3x + 3/x + 1/x^3$ — — (i)

But,

$f(x) = x + 1/x$

$f(x^3) = x^3 + 1/x^3$ — — — — (ii)

substituting (ii) into (i),

$[f(x)]^3 = (x^3 + 1/x^3) + 3[x + 1/x]$

$= f(x^3) + 3[x + 1/x]$

but $x + 1/x = f(x)$,

$\therefore [f(x)]^3 = f(x^3) + 3f(x)$

shown!

Extract 2.1: *A sample of the candidate's correct responses in question 2*

Extract 2.1 clearly shows that the candidate was able to sketch the required graph and make the necessary substitutions in verifying the polynomial function.

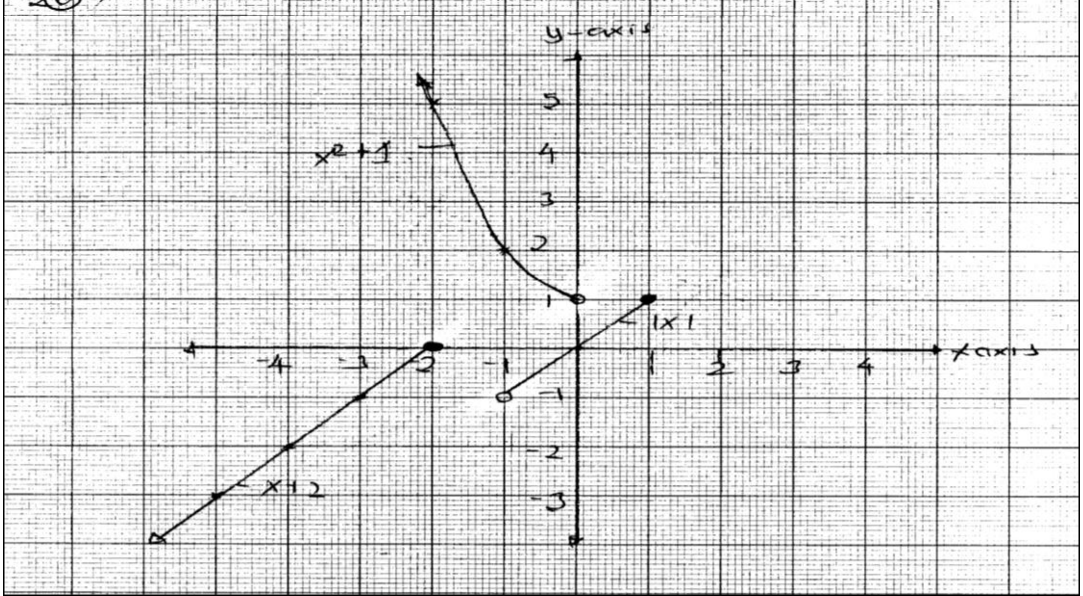
On the other hand, the analysis of data showed that 14,565 (44.0%) candidates scored marks ranging from 0 to 3.0 out of which 1,517 (4.6%) candidates got 0 mark.

In part (a) (i), the candidates failed to sketch the graph of the given step function due to inability to use the restricted domains $x > 1$, $-2 < x \leq 1$ and $x \leq -2$ for each part of the function. Others lacked adequate knowledge and skills on a function involving absolute signs. For example, there were candidates who drew curves instead of joining the points by using a ruler to get the required graph of $f(x) = |x|$. Furthermore, some of them drew a straight line for the function

$f(x)=x^2+1$ instead of drawing a curve. It was also noted that most candidates were able to get the correct values of y by using the given restricted domains but failed to sketch the correct graph. These candidates lacked enough knowledge and skills on the properties of step functions. In part (a) (ii), the majority of candidates stated wrong ranges like: $\{y:2>y<2\}$, $\{y:2<y>2\}$ and $\{y:y\geq 0\}$; and the mostly stated range was {all real numbers}. Those candidates did not understand that, since the point at $x=2$ is exclusive, then the range should be $\{y:\ y\neq 2\}$. In part (a) (iii), they could not identify which of the functions $f(x)=x^2+1$, $f(x)=|x|$ and $f(x)=x+2$ should be used to get the required values of $f(-3)$, $f(0.5)$ and $f(2)$. For example, there were candidates who wrote $f(-3)=(-3)^2+1$ or $f(-3)=|3|$. These candidates did not realize that, $x=-3$ falls in the domain $x\leq -2$ whose corresponding function $f(x)=x+2$ and should be used to get $f(-3)=-3+2=-1$. The same mistake was noted for $f(0.5)$ and $f(2)$ whereby the candidates wrote incorrect answers including: $f(0.5)=(0.5)^2+2=2.25$ and $f(2)=2+2=4$ instead of correctly writing $f(0.5)=|0.5|=0.5$ and $f(2)=2^2+2=6$.

In part (b), most of the candidates failed to expand the term $\left(x+\frac{1}{x}\right)^3$ in order to get the required result of $\left[f\left(x\right)\right]^3$. For example, some candidates wrote wrong expansions including: $x^3+\frac{1}{x^3}$ and $x^3+3+\frac{1}{x^3}$. Such candidates were supposed to: write $\left(x+\frac{1}{x}\right)^3=x^3+3x^2\left(\frac{1}{x}\right)+3x\left(\frac{1}{x^2}\right)+\frac{1}{x^3}$ that reduces to $\left(x+\frac{1}{x}\right)^3=x^3+3x+3\left(\frac{1}{x}\right)+\frac{1}{x^3}$ or rearranges as $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$; substitute $x+\frac{1}{x}=f(x)$; and make $[f(x)]^3$ the subject of the function. This indicates that the candidates lacked adequate knowledge and skills on how to apply the techniques of expansion to verify the given function. Extract 2.2 represents a sample response from one of the candidates who failed to answer the question correctly.

2(c)



2. a) (i) To determine the domain and range

solution.

$\therefore$ Domain is a set of all real numbers

$\therefore$ Range is a set of all real numbers

(iii)

value of $f(-3)$, $f(0.5)$ and $f(2)$.

(i) $f(x) = f(-3) = \begin{cases} (-3)^2+1 = 10 \\ |-3| = 3 \\ -3+2 = -1 \end{cases}$

$\therefore f(-3) = (10, 3, 1)$

(ii) for $f(0.5)$

$= \begin{cases} (0.5)^2+1 = 1.25 \\ |0.5| = 0.5 \\ 0.5+2 = 2.5 \end{cases}$

Values of $f(0.5) = 1.25, 0.5$ and 2.5.

(iii) Values for $f(2) = ?$

$f(2) = \begin{cases} (2)^2+1 = 5 \\ |2| = 2 \\ 2+2 = 4 \end{cases}$

values $f(2) = 5, 2, 4$

2. b). $f(x) = x + \frac{1}{x}$ show that $[f(x)]^3 = f(x^3) + 3f(x)$

Solution.

$f(x) = x + \frac{1}{x}$.

$[f(x)]^3 = \left[x + \frac{1}{x}\right]^3$.

$[f(x)]^3 = \frac{x^3}{1} + \frac{1}{x^3}$.

$= \frac{x^6 + 1}{x^3}$.

$[f(x)]^3 = \frac{x^6 + 1}{x^3}$.

$[f(x)]^3 = \frac{x^6}{x^3} + \frac{1}{x^3}$.

$= x^3 + 3f\,x$.

$\therefore\ [f(x)]^3 = f(x^3) + 3f(x)$.

Extract 2.2: *A sample of the candidate's incorrect responses in question 2*

As seen in Extract 2.2, the candidate failed to correctly sketch the graph of step function and verify polynomial functions.

2.3 Question 3: Algebra

This question had parts (a) and (b). In part (a), the candidates were required to find two numbers such that their difference is 1 and the difference of their squares is 7. In part (b), they were asked to find the first three terms of a geometric progression whose first term exceeds the second term by 4 and the sum of the second and third terms is $2\frac{2}{3}$.

The data analysis revealed that 10,204 (37.6%) candidates scored from 3.5 to 10 marks and among them, 8,084 (29.8%) candidates scored from 3.5 to 5.5 marks. Generally, the candidates' performance in this question was average. Figure 3 shows the percentage of candidates versus the marks they scored in this question.

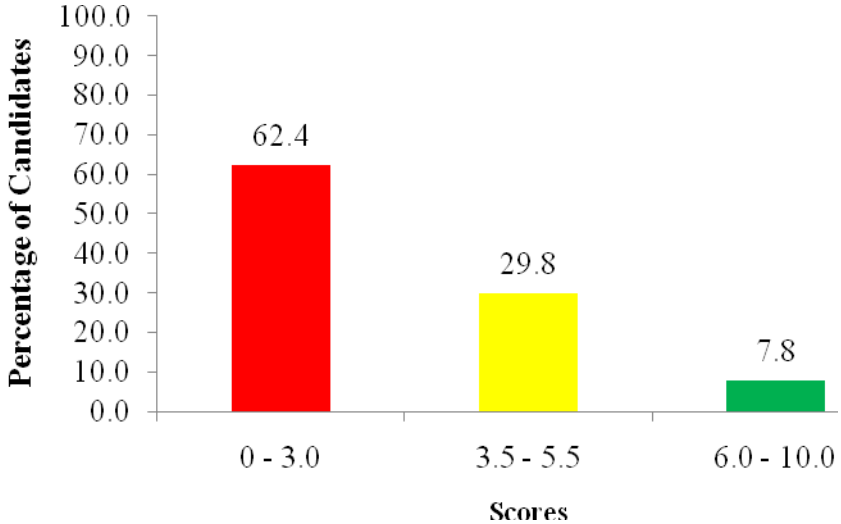


Figure 3: *Candidates' performance in question 3*

The analysis showed further that 705 (2.6%) candidates answered this question correctly and scored full marks. The candidates' ability to answer the question correctly was contributed by several reasons including the following:

In part (a), the candidates were able to formulate mathematical equations $x-y=1$ and $x^2-y^2=7$, where x and y denote the first number and second number, respectively. Then, they solved these equations simultaneously by using proper substitution method. In part (b), they were able to formulate the equations $G_1-G_1r=4$ and $G_1r+G_1r^2=2\frac{2}{3}$, where G_1 and r represent the first term and the common ratio, respectively. Furthermore, the candidates were able to find the values of G_1 and r that were used to determine the first three terms of the geometric progression. This indicates that these candidates had adequate knowledge and skills on simultaneous equations and geometric progression. Extract 3.1 illustrates a sample of the candidate's correct answers.

3.	a) Let the numbers be x and y
	Then,
	The difference is 1, $x - y = 1$ - - - - (i)
	Also,
	The difference of their squares is 7
	$x^2 - y^2 = 7$ — — — (ii)
	solving (i) and (ii)
	(i) $x - y = 1$
	(ii) $x^2 - y^2 = 7$, by difference of two squares,
	From (ii) $(x+y)(x-y) = 7$
	$(x+y)\ 1 = 7$
	$x + y = 7$ - - - (iii)
	solving (i) and (iii)
	(i) $x - y = 1$
	(iii) $x + y = 7$
	$2y = 6$
	$y = 3$
	substitute into (i), $x = y + 1$
	$x = 4$
	$\therefore$ The two numbers are 4 and 3
	b) Given; Let first term $= G_1$
	second term $= G_2$
	Then, third term $= G_3$
	$G_1 - G_2 = 4$ — — — - (i)
	$G_2 + G_3 = 2\frac{2}{3}$ — —— (ii)

3. b/

(i) $G_1 - G_1 r = 4$

(ii) $G_1 r + G_1 r^2 = 2\frac{2}{3}$

Dividing (ii) by (i)

$$\frac{(ii)}{(i)} = \frac{G_1(r + r^2)}{G_1(1 - r)} = \frac{2\frac{2}{3}}{4}$$

$$4r + 4r^2 = 2\tfrac{2}{3} - 2\tfrac{2}{3}r$$

$$4r^2 + 6\tfrac{2}{3}r - 2\tfrac{2}{3} = 0$$

solving quadratically,

$$r = \tfrac{1}{3} \text{ or } -2$$

Then,

First term,

From (i) $G_1(1 - r) = 4$

$$G_1 = \frac{4}{1 - r}$$

when $r = \frac{1}{3}$, $G_1 = 6$

$r = -2$, $G_1 = \frac{4}{3}$

Second term

$G_2 = G_1 r$

when $r = \frac{1}{3}$, $G_2 = 2$

$r = -2$, $G_2 = -\frac{8}{3}$

Third term,

$G_3 = G_2 r$

when $r = \frac{1}{3}$, $G_3 = \frac{2}{3}$

$r = -2$, $G_3 = \frac{16}{3}$

$\therefore$ First term, ~~is~~ second term and third terms are $(6, 2 \text{ and } \frac{2}{3})$ or $(\frac{4}{3}, -\frac{8}{3} \text{ and } \frac{16}{3})$

Extract 3.1: *A sample of the candidate's correct responses in question 3*

In Extract 3.1 it is obvious that the candidate was able to formulate the equations from the given word problems and obtained the required values by using the appropriate methods.

Contrarily, the analysis showed that 16,913 (62.4%) candidates scored below 3.5 marks in this question. It was further noted that 2,392 (8.8%) candidates scored 0 mark. Observation from the scripts showed that the candidates' failure in this question was attributable to the following main mistakes:

In part (a), the candidates failed to formulate the correct mathematical equations from the given word problem. The wrong equations that were frequently noted include: $x+y=1$, $x^2+y^2=7$ and $(x-y)^2=7$ instead of $x-y=1$ and $x^2-y^2=7$. Other candidates wrote wrong equation $\sqrt{x}-\sqrt{y}=7$ due to confusion between the terms "square" and "square root". On the other hand, few candidates expressed the word problem correctly into mathematical equations and made proper substitution of $x=(1+y)$ into $x^2-y^2=7$ but failed to expand the term $(1+y)^2$ when solving $(1+y)^2-y^2=7$. For example, a wrong expansion like $(1+y)^2=1+y^2$ was noted from the responses of some candidates. Others wrote $(1+y)-y^2=7$ instead of expanding $(1+y)^2-y^2=7$ as $1+2y+y^2-y^2=7$; and then solving for y.

In part (b), some candidates considered the given problem an arithmetic progression (AP) as they applied the formula $\mathrm{A}_n=\mathrm{A}_1+(n-1)d$ instead of $G_n=G_1r^{n-1}$ for the n^{th} terms. Others formulated the incorrect equations like: $G_1=4G_2$, $G_1=G_2-4$ and $G_1+4=G_2$ instead of $G_1-G_2=4$. Similarly, in formulating the equation for the sum of the second and third terms, there were candidates who wrote wrong equations like $S_2+S_3=2\frac{2}{3}$, where S_2 stands for the sum of the first two terms and S_3 stands for the sum of the first three terms. Principally, these candidates were supposed to write $G_2+G_3=2\frac{2}{3}$, where $G_2=G_1r$ and $G_3=G_1r^2$.

However, some candidates were able to formulate the correct equations but failed to solve them simultaneously so as to get the correct values of G_1 and r. This indicates that these candidates lacked adequate knowledge and skills on algebra. Extract 3.2 shows a sample work of a candidate who failed to answer this question correctly.

3(a) $x + y = 1$

$x^2 + y^2 = 7.$

Soln.

$x + y = 1$

$x^2 + y^2 = 7$

$x = 1 - y$

$(1-y)^2 + y^2 = 7$

$(1-y)(1-y) + y^2 = 7$

$1 - y - y + y^2 + y^2 = 7$

$2y^2 - 2y - 6 = 0$

$\underline{y = 2.3}$

$x + y = 1$

$x = 1 - 2.3$

$\underline{x = -1.3}$

$x - y = 1$

$x + y = 7$

$-2y = -6$

$y = 3$

$\therefore$ These numbers are 3 and 4.

b) $A_1 = 4 + A_2.$

$A_2 + A_3 = 8/3.$

From

$d = A_2 - A_1$

$d = -4.$

$A_n = A_1 + (n-1)d.$

$A_2 = A_1 + (2-1)d$

$A_2 = A_1 + d$

$A_3 = A_1 + (3-1)d$

$A_3 = A_1 + 2d.$

$A_1 + d + A_1 + 2d = 8/3$

$2A_1 + 3d = 8/3$

$2A_1 = \frac{8}{3} - 3d.$

$2A_1 = 8/3 - 3(-4)$

$2A_1 = 44/3$

$A_1 = \frac{22}{3}$

$A_2 = A_1 + d$

$= \frac{22}{3} - 4$

$A_2 = \frac{10}{3}$

$A_3 = A_1 + 2d$

$= \frac{22}{3} - 2 \times 4$

$A_3 = -\frac{2}{3}$

$\therefore$ 1st three terms $\frac{22}{3}$, $\frac{10}{3}$ and $-\frac{2}{3}$

Extract 3.2: *A sample of the candidate's incorrect responses in question 3*

As seen in Extract 3.2 the candidate formulated the incorrect equations from the word problems in both parts (a) and (b) indicating lack of algebraic skills in formulating equations with two unknowns.

2.4 Question 4: Differentiation

This question had parts (a), (b) and (c). The candidates were required to: (a) differentiate with respect to x the function $f(x) = e^{x^2+3x+2}$, (b) use the implicit differentiation to find the derivative of $x^2 + y^2 - 6xy + 3x - 2y + 5 = 0$ and (c) find the stationary points of the function $f(x) = 2x^3 - 3x^2 - 36x + 14$ and state their nature.

According to the data analysis, this question had an average performance because 10,914 (42.0%) candidates scored from 3.5 to 10 marks, out of them 5,925 (22.8%) candidates scored from 6.0 to 10 marks. The summary of performance is presented in Figure 4.

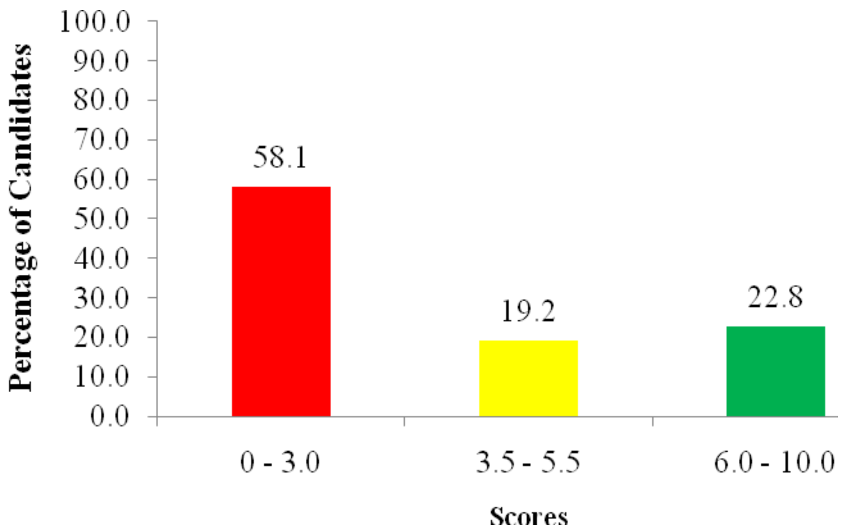


Figure 4: *Candidates' performance in question 4*

In this question, only 808 (3.1%) candidates got full marks. These candidates were able to answer the question correctly because of the following main reasons:

In part (a), the candidates were able to either use substitution techniques and chain rule or apply logarithm on both sides to differentiate the exponential function $f(x) = e^{x^2+3x+2}$. In part (b), they were able to differentiate the given function implicitly, that is, $2x + 2y\frac{dy}{dx} - 6y - 6x\frac{dy}{dx} + 3 - 2\frac{dy}{dx} = 0$; and then, rearrange it as $\frac{dy}{dx} = \frac{6y - 2x - 3}{2y - 6x - 2}$. This implies that the candidates had adequate knowledge and skills on derivative of implicit functions. In part (c), the candidates were able to apply the concept of differentiation to get the maximum and minimum points of the curve of $f(x) = 2x^3 - 3x^2 - 36x + 14$. Extract 4.1 represents the best answer from one of the candidates.

4.(a). $f(x) = e^{x^2+3x+2}$

$y = e^{x^2+3x+2}$

let $x^2+3x+2 = u$.

$\frac{du}{dx} = 2x+3$.

$y = e^u$.

$\frac{dy}{du} = e^u$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

(a). $\frac{dy}{dx} = e^u \cdot (2x+3)$.

But $u = x^2+3x+2$

$\frac{dy}{dx} = (2x+3)\ e^{x^2+3x+2}$.

(b). $x^2+y^2-6xy+3x-2y+5=0$.

On differentiating the above eqn with respect to x.

$2x + 2y\frac{dy}{dx} - \left(6y + 6x\frac{dy}{dx}\right) + 3 - 2\frac{dy}{dx} + 0 = 0$.

$2x + 2y\frac{dy}{dx} - 6y + 6x\frac{dy}{dx} + 3 - 2\frac{dy}{dx} = 0$

$2x+3-6y + 2y\frac{dy}{dx} - 6x\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$2x+3-6y - (-2y+6x+2)\frac{dy}{dx} = 0$

$2x+3-6y - (6x+2-2y)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2x+3-6y}{6x+2-2y}$

(c). $f(x) = 2x^3-3x^2-36x+14$.

$y = 2x^3-3x^2-36x+14$.

$\frac{dy}{dx} = 6x^2-6x-36$

c) But at stationary points $\frac{dy}{dx} = 0$

$0 = 6x^2 - 6x - 36$

$x = 3$ and -2.

Thus the stationary points are obtained from:

$y = 2x^3 - 3x^2 - 36x + 14$

When $x = 3$.

$y = 2(3)^3 - 3(3)^2 - 36(3) + 14$

$y = -67$.

When $x = -2$

$y = 2(-2)^3 - 3(-2)^2 - 36(-2) + 14$

$y = 58$.

The stationary points are $(3, -67)$ and $(-2, 58)$

Nature of stationary points.

from:

$\frac{dy}{dx} = 6x^2 - 6x - 36$

$\frac{d^2y}{dx^2} = 12x - 6$, But $x = 3$.

$\frac{d^2y}{dx^2} = 12(3) - 6 = 30$.

When $x = -2$.

$\frac{d^2y}{dx^2} = 12(-2) - 6 = -30$.

Thus the stationary points $(3, -67)$ are minimum.

and $(-2, 58)$ are maximum

Extract 4.1: *A sample of the candidate's correct responses in question 4*

Extract 4.1 reveals that the candidate correctly applied the techniques and methods of differentiation, including the proper use of chain rule and substitution techniques in finding the derivatives and the stationary points.

Despite the average performance, 15,112 (58.1%) candidates had a weak performance as they scored below 3.5 marks, including 7,970 (30.6%) candidates who scored 0 mark. The candidates' inability to answer the question correctly was mainly due to the following reasons:

In part (a), they failed to use the chain rule to find the derivative of the given function. Some of incorrect derivatives noted from their responses include: $\frac{dy}{dx} = e^{2x+3}$, $f^{'}(x) = \frac{1}{2x+3}e^{x^2+3x+2}$ and $\frac{dy}{dx} = e^{x^2+3x+2}$, instead of writing $\frac{dy}{dx} = (2x+3)e^{x^2+3x+2}$. There were some candidates who applied the chain rule correctly, but did not put the brackets to the final answer as they incorrectly wrote

$\frac{dy}{dx}=2x+3e^{x^2+3x+2}$. Furthermore, other candidates applied logarithm on one side rather than on both sides of the given function before differentiating. They did as follows:

$y=\ln e^{x^2+3x+2} \Rightarrow y=x^2+3x+2 \Rightarrow \frac{dy}{dx}=2x+3$. Instead, they were supposed to write $\ln y=\ln e^{x^2+3x+2} \Rightarrow \ln y=x^2+3x+2 \Rightarrow \frac{1}{y}\frac{dy}{dx}=2x+3 \Rightarrow \frac{dy}{dx}=(2x+3)y$ and finally substitute $y=e^{x^2+3x+2}$.

In part (b), the candidates were not able to correctly differentiate $x^2+y^2-6xy+3x-2y+5=0$ implicitly. Most of them failed to differentiate the terms: y^2, $6xy$ and $2y$ with respect to x. For example, they incorrectly wrote $\frac{d}{dx}(-6xy)=-6x\frac{dy}{dx}+6y$ or $\frac{d}{dx}(-6xy)=-6x\frac{dy}{dx}+y$ instead of $\frac{d}{dx}(-6xy)=-6x\frac{dy}{dx}-6y$ due to lack of adequate knowledge and skills on application of the product rule and omission of negative sign. Others were able to differentiate each term correctly and get $2x+2y\frac{dy}{dx}-6y-6x\frac{dy}{dx}+3-2\frac{dy}{dx}=0$, but failed to correctly make $\frac{dy}{dx}$ the subject of the equation.

In part (c), the candidates were unable to determine the stationary points of the function $f(x)=2x^3-3x^2-36x+14$ as they were not aware that, at maximum or minimum, the derivative $f'(x)=6x^2-6x-36=0$ was an essential step in answering this part. However, some candidates were able to obtain $6x^2-6x-36=0$ and to correctly solve for x but failed to get the corresponding values of y. This is because they substituted the values of x in the derivative $f'(x)=6x^2-6x-36$ instead of substituting them in the function $f(x)=2x^3-3x^2-36x+14$. As a result, such candidates got incorrect stationary points $(3,0)$ and $(-2,0)$ instead of $(3,-67)$ and (–2,58). Furthermore, other candidates got the correct stationary points, but failed to test the points in order to determine their nature while others just let $2x^3-3x^2-36x+14=0$ and solved for x. This indicates that the candidates lacked adequate knowledge and skills on how to determine the nature of stationary points of the given polynomial function. Extract 4.2 shows a sample response of a candidate who failed to answer this question correctly.

4a	$f(x) = e^{x^2+3x+2}$
	let $x^2+3x+2 = t$
	$t = x^2+3x+2$
	$y = e^t$
	$dt/dx = 2x+3$
	$dy/dx = e^t$
	$dy/dx = e^t$
	But $t = 2x+3$
	$dy/dx = e^{2x+3}$
04. (b).	Differentiation.
	Solution,
	$x^2+y^2-6xy+3x-2y+5=0$
	Differentiate with respect to x.
	$\frac{dy}{dx} = ?$ $x^2+y^2-6xy+3x-2y=0$
	$\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y - 6x\left(\frac{dy}{dx}\right) + 6y + 3 - 2 = 0$
	$\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y - 6x\left(\frac{dy}{dx}\right) + 6y = 1$
	$\frac{dy}{dx} = 2x\frac{dy}{dx} - 6\left(\frac{dy}{dx}\right) = 1-8y$
	$\frac{= 2x - 6\left(\frac{dy}{dx}\right)}{2x-6} = \frac{1-8y}{2x-6}$
	$\frac{1-8y}{2x-6} = \frac{dy}{dx}$
	$\therefore \frac{dy}{dx}$ of $x^2+y^2-6xy+3x-2y+5=0$ is $\frac{dy}{dx} = \frac{1-8y}{2x-6}$
c.	$f(x) = 2x^3+3x^2-36x+14$
	$dy/dx = 3x^2+3x-36$
	$x_1 = 3, \; x_2 = -4$
	When x_1 is considered.
	$2(3)^3+3(3)^2-36(3)+14$
	$d'y/dx = 6x+3$
	$6x+3 = 0$
	$6x = -3$
	$x_{21} = -13, \; x_{22} = 78$

Extract 4.2: *A sample of the candidate's incorrect responses in question 4*

In Extract 4.2 the candidate failed to correctly apply the chain rule and substitution techniques in finding the derivatives and the stationary points.

2.5 Question 5: Integration

This question had parts (a) and (b). The candidates were required to: (a) use the substitution method to find: (i) $\int x\sqrt{x^2+1}\,dx$ and (ii) $\int \tan x\,dx$) and (b) find the area bounded by the curve $y = x^2 - 4x + 3$ and the x-axis.

The analysis of data showed that 8,977 (38.6%) candidates scored 3.5 marks and above. Out of them, 4,391 (18.9%) candidates scored from 3.5 to 5.5 marks while 4,586 (19.7%) candidates scored from 6.0 to 10 marks. This shows that the performance was average. Figure 5 represents a summary of performance in this question.

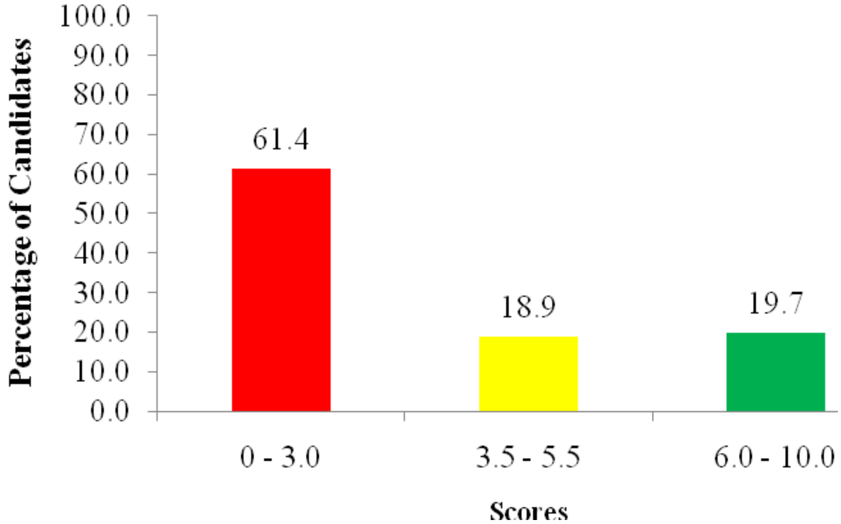


Figure 5: *Candidates' performance in question 5*

The data analysis showed further that 443 (1.9%) candidates were able to answer this question correctly and scored 10 marks. These candidates were able to give the required answers because of the following major reasons:

The candidates correctly applied the substitution techniques to solve the integrals given in part (a). They were also able to determine the limits of integration and apply the appropriate formula to find the area bounded by the given curve and x - axis in part (b) as shown in Extract 5.1.

5. a) i) $\int x\sqrt{x^2+1}\,dx$

let $u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \int \left(x\left(u^{1/2}\right)\right)\frac{du}{2\cancel{x}}$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} + c$

$= \frac{u^{3/2}}{3} + c$

But $u = x^2+1$

$\therefore \int x\sqrt{x^2+1}\,dx = \frac{\left(\sqrt{x^2+1}\right)^3}{3} + c$

5. a) ii) $\int \tan x\,dx$

$= \int \frac{\sin x}{\cos x}\,dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$= -\int \frac{\cancel{\sin x}}{u}\,\frac{du}{\cancel{\sin x}} = -\int \frac{du}{u} = -\ln u + c$

but $u = \cos x$

$= -\ln(\cos x) + c$

$\therefore \int \tan x\,dx = -\ln(\cos x) + c$

b)

$\text{Area} = \left|\int_a^b f(x)\,dx\right|$

Given; $y = x^2 - 4x + 3$ and $y = 0$

$x^2 - 4x + 3 = 0$

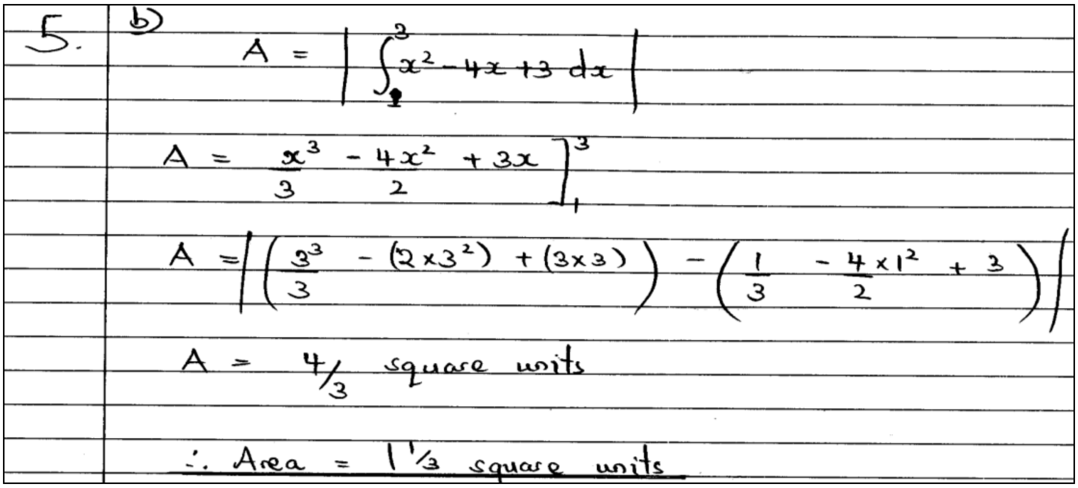
$x^2 - x - 3x + 3 = 0$

$(x^2 - x) \mp (3x - 3) = 0$

$x(x-1) + 3(x-1) = 0$

$x - 1 = 0$ or $x + 3 = 0$

$x = +1$ or $x = +3$



5. b)

$$A = \left| \int_1^3 x^2 - 4x + 3\, dx \right|$$

$$A = \frac{x^3}{3} - \frac{4x^2}{2} + 3x \Big]_1^3$$

$$A = \left| \left(\frac{3^3}{3} - (2 \times 3^2) + (3 \times 3) \right) - \left(\frac{1}{3} - \frac{4 \times 1^2}{2} + 3 \right) \right|$$

$A = 4/3$ square units

$\therefore$ Area $= 1\,{}^1\!/_3$ square units

Extract 5.1: *A sample of the candidate's correct responses in question 5*

In Extract 5.1 the candidate was able to correctly solve the given integrals by using proper substitution techniques and determine the area bounded by the curve and the x-axis using the proper limits.

On the other side, 14,296 (61.4%) candidates scored below 3.5 marks and among them, 5,786 (24.9%) candidates scored 0 mark. These candidates failed to get the required answers due to various reasons including the following:

In part (a) (i), the candidates failed to get the correct answer because they applied incorrect substitutions as some of them wrote: $\int x\sqrt{x^2+1}dx = \int \sqrt{x^4+x^2}\,dx$, then made the substitution $u = x^4 + x^2$, a step which did not enable them to proceed. Others applied the correct substitution $u = x^2 + 1$ and got $\int x\,\sqrt{x^2+1}\,dx = \int \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int u^{1/2}\,du$ but failed to integrate to get the required solution. This shows that the candidates lacked enough knowledge and skills of integrating polynomial expressions involving radicals or terms with fractional exponents. However, some candidates were able to correctly integrate $\frac{1}{2}\int u^{1/2}du$ but they ignored the coefficient $\frac{1}{2}$ when writing the final answer. That is, they incorrectly wrote $\frac{1}{2}\int u^{1/2}du = \left(\frac{2}{3}\right)u^{3/2} + C$ instead of $\frac{1}{2}\int u^{1/2}du = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)u^{3/2} + C = \left(\frac{1}{3}\right)u^{3/2} + C$. It was also noted that some of the candidates were able to integrate correctly but forgot to add the arbitrary constant of integration to the answer while others wrote the final answer with the integral sign " $\int$ ", that is, $\left(\frac{1}{3}\right)\int\left(x^2+1\right)^{3/2}+C$ indicating lack of understanding on the use of integral sign.

In part (a) (ii), the candidates were unable to integrate $\int \tan x \, dx$. Some candidates who opted to use the substitution method failed to proceed, because they were unable to differentiate $u=\tan x$ or $u=\sin x$ to express the given integral in terms of u. This observation indicates that the candidates lacked enough knowledge and skills of differentiation and application of trigonometric identities. Other candidates correctly used the substitution $u=\cos x$ and got $-\int \frac{1}{u} du$ but failed to proceed because they were unable to recall the identity $\int \frac{1}{u} du = \ln u + C$. Others confused between the concepts of integration and differentiation. For example, there were candidates who applied the quotient rule $\frac{dy}{dx}=\frac{vdu-udv}{v^2}$ to differentiate $\tan x$ by letting $u=\sin x$ and $v=\cos x$ instead of performing the integration.

In part (b), it was noted that, many candidates used wrong formulae, such as $\text{Area}=\int_a^b y^2 \, dx$ and $\text{Area}=\int_a^b \pi y^2 \, dx$ instead of $\text{Area}=\int_a^b \mathrm{f}(x) \, dx$ when finding the area bounded by the curve and the x-axis. It was further noted that there were some candidates who failed to find the limits of integration correctly. A good example is from those who solved the equations $y=x^2-4x+3$ and $y=x$. Others could neither sketch the curve for $y=x^2-4x+3$ nor solve it when $y=0$ by any of the quadratic methods in order to get the lower and upper limits of integration, that is, $a=1$ and $b=3$. Had they got to this point, they could proceed with the step "$\text{area} = \int_1^3 (x^2-4x+3)dx$". Those candidates lacked enough knowledge and skills on the application of integration in finding the area bounded by the curve of a polynomial function and a line. Also, some of the candidates were able to integrate correctly and apply the limits but did not omit the negative sign from the final answer. Thus, they concluded that the $\text{area} = -\frac{4}{3}$ instead of area $= \left|-\dfrac{4}{3}\right| = \dfrac{4}{3}$ square unit. These candidates did not understand that the negative sign shows that the area to be determined is below the x-axis. Extract 5.2 shows a sample work of a candidate who failed to answer this question correctly.

5a

i/ $\int x\sqrt{x^2+1}\,dx$

Now

$= \int \sqrt{(x^2+1)\,x^2}\,dx$

$= \int \sqrt{x^4+x^2}\,dx$

$= \int (x^4+x^2)^{1/2}\,dx$

Let

$x^4+x^2 = u$

$= \int u^{1/2}\,dx$

but

$\frac{du}{dx} = (4x^3+2x)$

$dx = du/(4x^3+2x)$

Now

$= \int u^{1/2} \times du/(4x^3+2x)$

$= \frac{1}{(4x^3+2x)} \int u^{1/2}\,du$

5a.

$= 1/4x^3+2x \int u^{1/2}\,du$

$= \frac{1}{4x^3+2x}\left[\frac{u^{-1/2}}{1/2-1}\right]$

$= \frac{1}{4x^3+2x}\left[-2/u^{1/2}\right]$

but

$u = x^4+x^2$

$= \frac{-2}{4x^3+2x}\left[1/\sqrt{x^4+x^2}\right]$

$= \frac{-2}{4x^3+2x}\left[\frac{1}{x\sqrt{x^2+1}}\right]$

$= \frac{-2}{4x^3+2x\,(\sqrt{x^4+x^2})}$

Now

i/ $\int x\sqrt{x^2+1}\,dx = \frac{-2}{(\sqrt{x^4+x^2})\;4x^3+2x}$

5 (a) ii. $\int \tan x \, dx$

Let $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int \tan x \, dx$

$\int u \frac{du}{\sec^2 x}$

$= -\sec^2 x + C.$

(b) $y = x^2 - 4x + 3$

by solving

$y = x^2 - 4x + 3$

$0 = x^2 - 4x + 3$

$(x-3)(x-1) = 0$

$x - 3 = 0$

$x = 3$

$x - 1 = 0$

$x = 1.$

From $\int_a^b \pi y^2 \, dx$

$\int_1^3 \pi (x^2 - 4x + 3)^2 \, dy$

5 (b) $\int_1^3 \pi (x^4 - 8x^3 + 21x^2 - 24x + 9) \, dx.$

$\pi \int_1^3 x^4 dx - \int_1^3 8x^3 dx + \int_1^3 21x^2 dx - \int_1^3 24x \, dx$

$+ \int_1^3 9x^0 dx.$

$\pi \left[\frac{x^5}{5} - \frac{8x^4}{4} + \frac{21x^3}{3} - \frac{24x^2}{2} + 9x \right]_1^3$

$\pi \left[\frac{3^5}{5} - \frac{8(3)^4}{4} + \frac{21(3)^3}{3} - \frac{24(3)^2}{2} + 9(3) \right]$

$- \left[\frac{(1)^5}{5} - \frac{8(1)^4}{4} + \frac{21(1)^3}{3} - \frac{24(1)^2}{2} + 9(1) \right]$

$= -5.4 - 2.2$

$= -7.6$

$= 7.6\pi$ cubic units.

The volume is 7.6π cubic units.

Extract 5.2: *A sample of the candidate's incorrect responses in question 5*

As seen in Extract 5.2 the candidate failed to apply the correct substitution in part (a) (i); use proper trigonometric identity in part (a) (ii); and to apply the correct formula in part (b).

2.6 Question 6: Statistics

The question was preceded by the following descriptions: The scores of 22 students in one of the Basic Applied Mathematics tests are as follows: 49, 64, 38, 46, 60, 68, 46, 42, 62, 38, 68, 57, 63, 76, 51, 55, 66, 63, 58, 47, 59 and 54. The candidates were required to: (a) summarize the data in a frequency distribution table by using the class interval of 5 and the lowest limit of 35; (b) find the mean score by using the assumed mean, $A = 57$, and give the answer correct to five significant figures and (c) find the inter-quartile range correct to one decimal place.

According to the data analysed, 8,919 (26.3%) candidates got scores ranging from 3.5 to 5.5 marks while 8,737 (25.7%) candidates scored 6.0 marks and above. Since a total of 17,656 (52.0%) candidates scored at least 3.5 marks, it is evident that the performance was average. Figure 6 shows a summary of candidates' performance in this question.

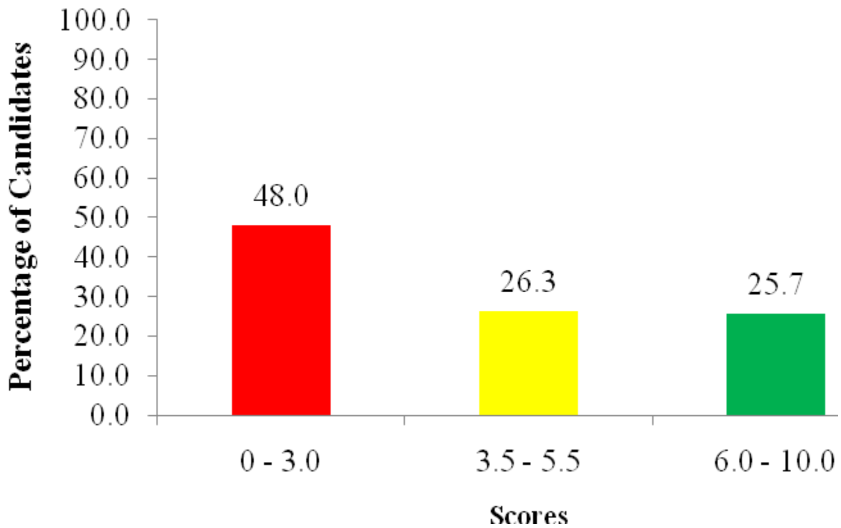


Figure 6: *Candidates' performance in question 6*

The analysis showed further that 2,141 (6.3%) candidates were able to answer this question correctly and scored full marks due to the following main factors:

In part (a), these candidates were able to use the given statistical information to identify the correct class intervals and their corresponding frequencies to construct a frequency distribution table. In part (b), they were able to apply the correct formula to find the mean by using the correct class marks, frequencies, the given assumed mean, $A = 57$. They were finally able to express the answer in five significant figures. In part (c), they correctly applied the appropriate formulae to get the inter-quartile range and gave the answer in one decimal place. A sample of a correct work is represented in Extract 6.1.

6. a) Frequency distribution table

class interval	f	x	d	fd	c.f
35 – 39	2	37	-20	-40	2
40 – 44	1	42	-15	-15	3
45 – 49	4	47	-10	-40	7
50 – 54	2	52	-5	-10	9
55 – 59	4	57	0	0	13
60 – 64	5	62	5	25	18
65 – 69	3	67	10	30	21
70 – 74	0	72	15	0	21
75 – 79	1	77	20	20	22
	$\Sigma f = 22$			$\Sigma fd = -30$	

6. b) Mean $(\bar{x}) = A + \frac{\Sigma fd}{\Sigma f}$

$$\bar{x} = 57 + \frac{-30}{22}$$

$$\bar{x} = 55.636$$

$\therefore$ Mean $= 55.636$

c) Interquartile Range $= Q_3 - Q_1$

$$Q_3 = L + \left(\frac{\frac{3N}{4} - N_b}{N_w}\right)i$$

$$Q_3 = 59.5 + \left(\frac{16.5 - 13}{5}\right)5$$

$$Q_3 = 63.$$

$$Q_1 = L + \left(\frac{N/4 - N_b}{N_w}\right)i$$

$$Q_1 = 44.5 + \left(\frac{5.5 - 3}{4}\right)5$$

$$Q_1 = 47.625$$

$$I.Q.R = 63 - 47.625 = 15.4$$

$\therefore$ Interquartile Range $= 15.4$

Extract 6.1: *A sample of the candidate's correct responses in question 6*

Extract 6.1 shows the work of a candidate who correctly used the given data to construct a frequency distribution table, hence find the mean and inter-quartile range by using the appropriate formulae.

Despite the fact that most candidates performed averagely, the analysis revealed that 16,310 (48.0%) candidates had a weak performance since they scored below 3.5 marks, out of which, 2,997 (8.8%) candidates scored 0 mark. The candidates' failure to give the correct answers was mainly contributed by the following reasons:

In part (a), the candidates could not write the correct classes by using the class size and the given lowest limit, hence led to incorrect frequencies. In identifying the upper limit of the lowest class interval, they did not know exactly what should be added to 35 so as to maintain a class interval of 5. Others managed to identify the correct class intervals, but failed to get the correct frequencies. This shows that these candidates lacked adequate knowledge and skills on using class intervals or class boundaries to construct the frequency distribution tables.

In part (b), some candidates applied the wrong formulae like: $\overline{X} = \text{A}+\frac{\sum fx}{\sum f}$ or $\overline{X} = \frac{\sum fd}{\sum f}$ instead of $\overline{X} = A+\frac{\sum fd}{\sum f}$ where $d = X - A$, f is the frequency and A is the assumed mean. Others were able to apply the correct formula but ended up with incorrect answers due wrong frequency distribution tables. Also, there were candidates who wrongly applied the formula $\overline{X} = \frac{\sum fx}{\sum f}$ as the formula for assumed mean method whereas others did not express the final answer correct to five significant figures.

In part (c), it was noted that some candidates were able to recall the correct formulae for calculating the quartiles, that is, $Q_1 = \text{L}+\left(\frac{\frac{1N}{4}-n_b}{n_w}\right)C$ and $Q_3 = \text{L}+\left(\frac{\frac{3N}{4}-n_b}{n_w}\right)C$, where n is the quartile position, L is the lower real limit of the quartile class, N is the sum of frequencies, n_b is the sum of all frequencies below the quartile class, n_w is the quartile frequency and C is the class size, but failed to identify the correct values of the terms involved in the formulae. However, others were able to calculate Q_1 and Q_3 correctly but applied wrong formulae when finding the inter-quartile range, such as *inter-quartile range* $= L + \left(\dfrac{\frac{N}{2} - f_b}{f_w}\right)i$. Such candidates were not aware that the expression

$$L + \left(\dfrac{\frac{N}{2} - f_b}{f_w}\right)i$$

is the formula for finding the median. They also gave the $\text{inter-quartile range as } \dfrac{Q_3 - Q_1}{2}$ while it is the formula for finding the semi-inter-quartile range. Basically, they were supposed to use the formula, inter-quartile range = $Q_3 - Q_1$. Others were able to get the inter-quartile range but did not express the final answer correct one decimal place as required.

In another observation, there were candidates who considered the inter-quartile range as the difference between the highest score and the lowest score of the given data. For example, some candidates wrote: inter-quartile range = $76 - 38 = 38$. Extract 6.2 represents the work of a candidate who failed to answer this question correctly.

06	(a)	Class· mark	frequency	X.	d=X−Ā	Σfd
		35·5 − 39·5	~~37·5~~ 2	37·5	−19·5	−39
		40·5 − 44·5	~~4~~ 1	42·5	−14·5	−14·5
		45·5 − 49·5	4	47·5	−9·5	−38
		50·5 − 54·5	~~8~~ 3	52·5	−4·5	−12·5
		55·5 − 59·5	4	57·5	0·5	2
		60·5 − 64·5	4	62·5	5·5	22
		65·5 − 69·5	3	67·5	10·5	31·5
		70·5 − 74·5	0	72·5	15·5	0
		75·5 − 79·5	1	77·5	20·5	20·5
						−5.857

Then mean

$$x = A + \frac{\Sigma fd}{\Sigma f} i$$

$$x = 57 + \frac{-5.852 \times 5}{22}$$

$$x = 55.68$$

c/ interquartile ran

$$\cancel{Q_3} = L + \frac{N(\tau - fb)}{fw} i)$$

$L = 55$. $fb = 7$ $fw = 3$ $i = 5$

∴ interquartile range in Q3 = 61.70

Extract 6.2: *A sample of the candidate's incorrect responses in question 6*

In Extract 6.2 the candidate constructed an incorrect frequency distribution table that consequently led to a wrong mean. The candidate also applied a wrong formula for inter-quartile range of the given data.

2.7 Question 7: Probability

This question had parts (a), (b) and (c). In part (a), the candidates were required to show that ${}^{n}C_{r} = {}^{n}C_{n-r}$. In part (b), they were required to use the appropriate formula to find $P(A' \cap B')$, given that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.1$. In part (c), they were asked to find the probability of drawing a red marble and a blue marble in any order from a bag containing 8 marbles of which 3 are red and 5 are blue when two marbles are drawn at random one at a time with replacement.

The analysis of data showed that this question had the lowest performance, since 6,479 (23.5%) candidates scored from 3.5 to 10 marks and 21,087 (76.5%) candidates scored below 3.5 marks. Generally, the candidates had a weak performance. The summary of performance is presented in Figure 7.

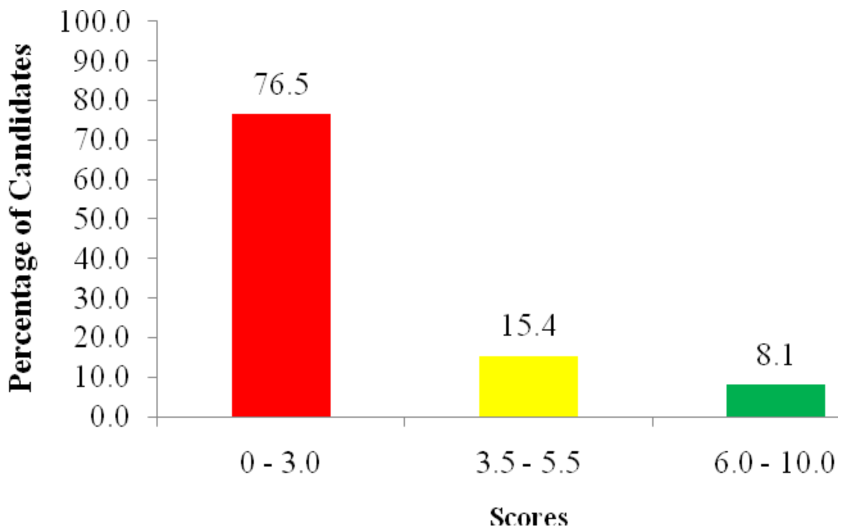


Figure 7: *Candidates' performance in question 7*

The analysis revealed further that 6,171 (22.4%) candidates scored 0 mark. The candidates' inability to answer the question correctly was caused by the following reasons:

In part (a), the candidates applied incorrect formulae like ${}^{n}C_{r}=\dfrac{n!}{r!(r-n)!}$ or ${}^{n}C_{r}=\dfrac{n!}{(n-r)!}$ instead of ${}^{n}C_{r}=\dfrac{n!}{r!(n-r)!}$ that could be used to show that ${}^{n}C_{r}={}^{n}C_{n-r}$. Others wrote the formula correctly but failed to apply it appropriately to define ${}^{n}C_{n-r}$. For example, some of them wrote: ${}^{n}C_{n-r}=\dfrac{n!}{(n-n-r)!\text{r}!}$ while others wrote ${}^{n}C_{n-r}=\dfrac{n}{(n-(n-r))n-r}$ instead of ${}^{n}C_{n-r}=\dfrac{n!}{(n-(n-r))!(n-r)!}$. This indicates that, those candidates had insufficient knowledge and skills on the principles of combinations and permutations; and the definition of factorial notation.

In part (b), the candidates applied wrong formulae including the following: $P(\text{A}'\cap\text{B}')=P(A\cap B)'$, $P(\text{A}'\cap\text{B}')=P(\text{A}')\times P(\text{B}')$, $P(\text{A}'\cap\text{B}')+P(A\cap B)=1$ and $P(\text{A}'\cap\text{B}')=P(\text{A})+P(\text{B})-P(\text{A}\cap\text{B})$ instead of writing: $P(A'\cap B')=1-P(A\cup B)$ where $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. However, there were candidates who were able to get $P(\text{A}\cup\text{B})=0.6$ but failed to use it in order to obtain the required value of $P(A' \cap B')$ because they failed to realize that $P(A' \cap B') = P(A \cup B)'$ where $P(A \cup B)' = 1 - P(A \cup B)$.

In part (c), the majority of candidates considered the experiment as if the selection was done without replacement and represented the information incorrectly on a tree diagram. Others drew the correct tree diagram but failed to get the correct answer because they used the wrong formulae in finding the probability. The mostly noted wrong formula was $P(RB) = P(R) + P(B)$ instead of using the correct formula $P(RB) = P(R) \times P(B)$. This observation indicates lack of adequate knowledge and skills on the rules of probability. Extract 7.1 is a sample of an incorrect response.

7(a) Show. ${}^{n}C_{r} = {}^{n}C_{n-r}$.

For Combinations.

$${}^{n}C_{r} = \frac{n!}{(n-r)!}$$

$${}^{n}C_{n-r} = \frac{n!}{n-(n-r)!}$$

So:

$$\frac{n!}{n-(n-r)!}$$

$$= \frac{(n)(n-1)(n-2)!}{n-(n-1)(n-r-1)(n-r-2)!}$$

$$C = \frac{n!}{(n-r)!}$$

hence ${}^{n}C_{r} = {}^{n}C_{n-r}$

7b $P(A\cap B) + P(A'\cap B') = 1$

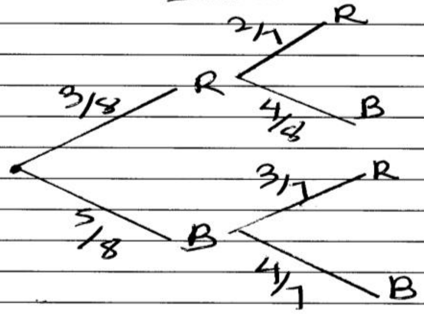
$P(A'\cap B') = 1 - P(A\cap B)$

But $P(A\cap B) = 0.1$

$P(A'\cap B') = 1 - 0.1$

$P(A'\cap B') = 0.9$

7c



$P(R\cap B) = P(R\cap B) + P(B\cap R)$

$= \left(\frac{3}{8}\times\frac{4}{8}\right) + \left(\frac{5}{8}\times\frac{3}{7}\right)$

$= \frac{3}{16} + \frac{15}{56}$

$P(R\cap B) = \frac{51}{112}$

Extract 7.1: *A sample of the candidate's incorrect responses in question 7*

The responses given in Extract 7.1 show that the candidate failed to apply the principles of combination in showing that ${}^{n}C_{r} = {}^{n}C_{n-r}$ in part (a). Also, the candidate was unable to apply the appropriate rules in finding the required probabilities in parts (b) and (c).

Although the performance was weak, the analysis showed further that 233 (0.8%) candidates were able to answer this question correctly and scored full marks. These candidates were able to do the following:

In part (a), they applied the correct formula of combination when proving the given equation. In part (b), they were also able to find the probability of $P(A'\cap B')$ by using the appropriate formula. In part (c), the candidates used the correct diagram and formula to find the probability of the given problem as shown in Extract 7.2.

7. (a) Required to show

$$^nC_r = {}^nC_{n-r}$$

from L.H.S of equation:

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

from R.H.S of equation

$$^nC_{n-r} = \frac{n!}{(n-n+r)!\,(n-r)!}$$

$$^nC_{n-r} = \frac{n!}{r!\,(n-r)!}$$

$$^nC_{n-r} = \frac{n!}{(n-r)!\,r!}$$

$\therefore$ Thus L.H.S = R.H.S

$$^nC_r = {}^nC_{n-r}$$

b) $P(A) = 0.3$ $P(B) = 0.4$ $P(A \cap B) = 0.1$

Required

$$P(A' \cap B') = P(A \cup B)'$$

but

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.4 - 0.1$$

$$= 0.7 - 0.1$$

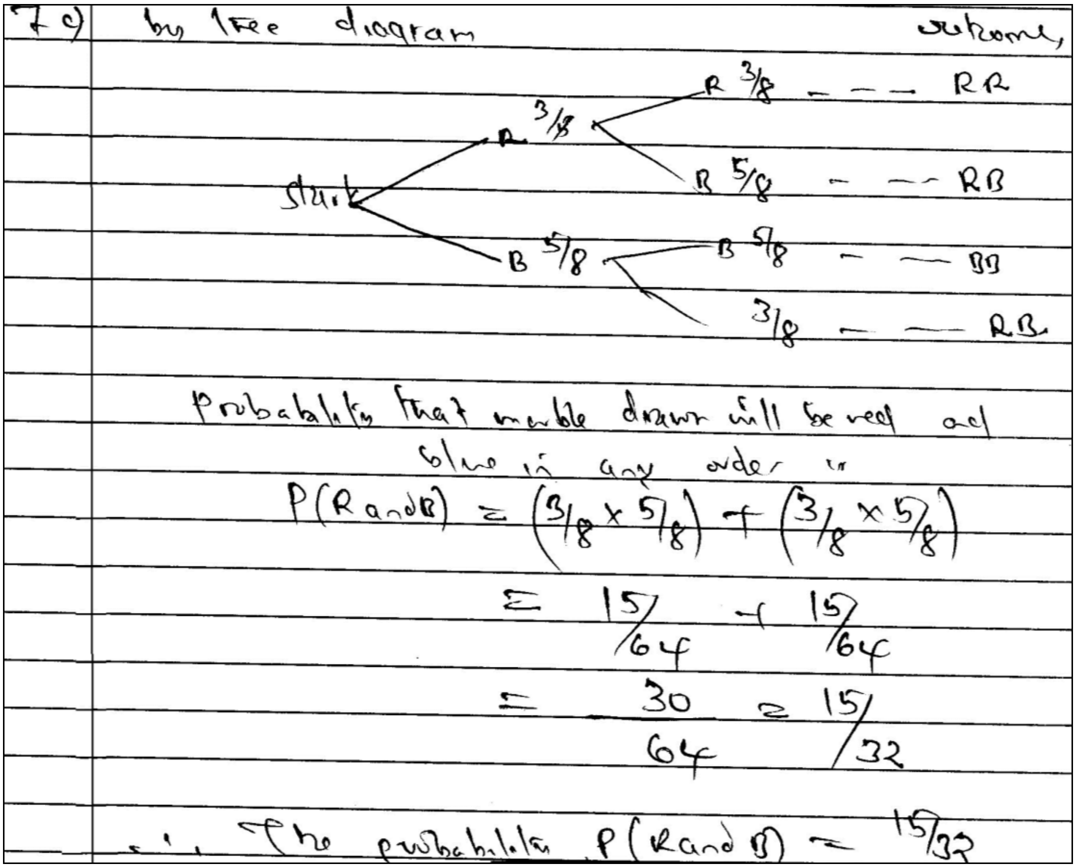
$$P(A \cup B) = 0.6$$

Then

$$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$

$$P(A' \cap B') = P(A \cup B)' = 1 - 0.6 = 0.4$$

$\therefore$ $P(A' \cap B') = 0.4$


7 c) by Tree diagram outcome,

R 3/8 — — — RR

R 3/8

R 5/8 — — — RB

start

B 5/8

B 5/8 — — — BB

3/8 — — — RB.

Probability that marble drawn will be red and blue in any order is

$P(R \text{ and } B) = \left(\frac{3}{8} \times \frac{5}{8}\right) + \left(\frac{3}{8} \times \frac{5}{8}\right)$

$= \frac{15}{64} + \frac{15}{64}$

$= \frac{30}{64} = \frac{15}{32}$

$\therefore$ The probability $P(R \text{ and } B) = \frac{15}{32}$

Extract 7.2: *A sample of the candidate's correct responses in question 7*

Extract 7.2 reveals that the candidate was able to correctly apply the principles of combination in showing that ${}^{n}C_{r} = {}^{n}C_{n-r}$ and find the required probabilities by using the appropriate rules.

2.8 Question 8: Trigonometry

This question consisted of parts (a) and (b). In part (a), the candidates were required to calculate the size of angle Y from the given triangle XYZ, such that $\overline{XY} = 3.5$, $\overline{YZ} = 4.5$ and $\overline{ZX} = 6.5$. In part (b), they were required to solve the equation $1 + \cos\theta = 2\sin^2\theta$ for values of θ ranging from 0 to 2π.

The data analysis showed that 7,455 (34.2%) candidates scored at least 3.5 marks whereas 14,326 (65.8%) candidates scored below 3.5 marks as shown in Figure 8. According to this analysis, the overall candidates' performance was weak.

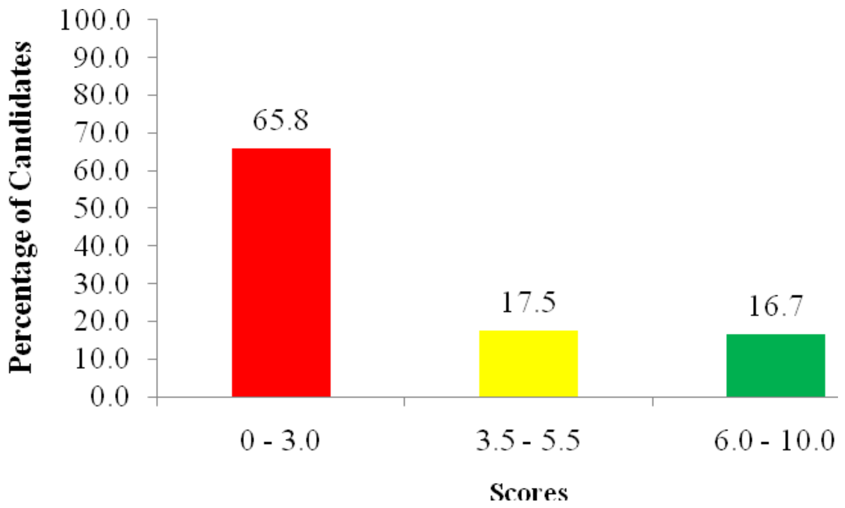


Figure 8: *Candidates' performance in question 8*

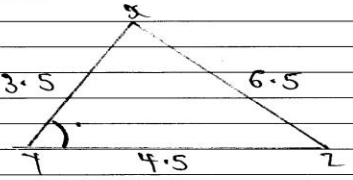
It was noted that 11,021 (50.6%) candidates scored 0 mark in this question as they committed the following errors:

In part (a), these candidates were not able to recall the *cosine rule* correctly when finding the size of angle Y. For example, some of them wrote wrong formulae like $\overline{XZ}^2 = \overline{YZ}^2 + \overline{XY}^2 - (\overline{YZ})(\overline{XY})\cos\hat{Y}$ and $\overline{XZ}^2 = \overline{YZ}^2 \times \overline{XY}^2 - 2(\overline{YZ})(\overline{XY})\cos Y$ instead of writing $\overline{XZ}^2 = \overline{YZ}^2 + \overline{XY}^2 - 2(\overline{YZ})(\overline{XY})\cos\hat{Y}$.

However, some candidates applied inappropriate formulae such as: $\sin\hat{Y} = \frac{\overline{XZ}}{\overline{YZ}}$ or $\tan\hat{Y} = \frac{\overline{XZ}}{\overline{XY}}$ as they considered the given triangle a right angled triangle; and the *sine rule*, $\frac{\sin\hat{X}}{x} = \frac{\sin\hat{Y}}{y} = \frac{\sin\hat{Z}}{z}$. Also, few candidates were able to write the *cosine rule* correctly but failed to correctly substitute the given values. Some of these candidates wrongly interchanged the given lengths $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$; and hence, ended up with wrong answers. Also, there were candidates who could substitute the given values in the formula by writing $6.5^2 = 4.5^2 + 3.5^2 - 2(4.5)(3.5)\cos\hat{Y}$, but failed to evaluate $\cos\hat{Y}$. In addition, some candidates were able to write $\cos\hat{Y} = (-0.3095)$ but could not evaluate angle Y while others ignored the negative sign and ended up with wrong angles. For example, they wrote $\hat{Y} = \cos^{-1}(0.3095)$ instead of $\hat{Y} = \cos^{-1}(-0.3095)$ that could give the required angle, $Y = 108^0$.

In part (b), the candidates were unable to use the correct procedures in finding the required values of θ. For instance, some candidates split the given equation into parts such as $2\sin^2\theta = 0$ or $-\cos\theta = 0$ indicating lack of knowledge and skills on application of trigonometric identities to factorize trigonometric equations. In principle, they were required to substitute the identity $\sin^2\theta = 1 - \cos^2\theta$ to express the given equation in the quadratic form $2\cos^2\theta + \cos\theta - 1 = 0$. This equation was an important step in obtaining the required angles. They were also supposed to write the equation as $(\cos\theta + 1)(2\cos\theta - 1) = 0$ so that they could get the equations $\cos\theta + 1 = 0$ and $2\cos\theta - 1 = 0$ and ultimately solve for θ in the range 0 to 2π. Extract 8.1 represents a poor solution from one of the candidates.

8. a)



From cosine rule.

$y^2 = x^2 z^2 - 2xz\cos y$

$2xz\cos y = x^2 z^2 - y^2$

$\cos y = \frac{x^2 z^2 - y^2}{2xz}$

$\cos y = \frac{4.5^2 \times 3.5^2 - 6.5^2}{2 \times 4.5 \times 3.5}$

$\cos y = \frac{205.8125}{31.5}$

$\cos y = 6.533730159$

$y = \cos^{-1}(6.533)$

$y =$

b) $1 + \cos\theta = 2\sin^2\theta$

$2\sin^2\theta - \cos\theta - 1 = 0.$

$2\sin^2\theta = 0$ or $-\cos\theta = 0$ or $-1 = 0$

$\frac{2}{2}\sin\theta\sin\theta = \frac{0}{2}$

$\sin^2\theta = 0$

$\theta = \sin^{-1} 0 = 0°$

8. b) $-\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$= 90°$

sin is +ve in 2nd quadrant

$\theta = 180 - \theta$

$\theta = 180 - 0$

$\theta = 180°$

cos in −ve in 2nd and 3rd.

2nd $\theta = 180 - \theta$

$\theta = 180 - 90$

$\theta = 90$

3rd $\theta = 180 + \theta$

$\theta = 180 + 90$

$\theta = 270$

$\therefore \theta = 0°, 90°, 180°, 270°$

Extract 8.1: *A sample of the candidate's incorrect responses in question 8*

In Extract 8.1 the candidate failed to correctly apply the *cosine rule* and solve the given equation by using the appropriate trigonometric identity.

Though the performance was generally weak, the analysis also showed that 1,057 (4.9%) candidates answered this question correctly and scored full marks. These candidates were able to do the following:

In part (a), these candidates were able to apply the *cosine rule* correctly and get the required size of angle Y. In part (b), they were able to solve the equation $1+\cos\theta = 2\sin^2\theta$ by using the appropriate identity and get the required values of θ in the range 0 to 2π as shown in Extract 8.2.

8. a/ From cosine rule,

$$a^2 = b^2 + c^2 - 2bc.\cos\hat{A}.$$

From the triangle,

$$y^2 = x^2 + z^2 - 2xz.\cos\hat{Y}$$

$$\overline{XZ}^2 = \overline{YZ}^2 + \overline{XY}^2 - 2(\overline{YZ})(\overline{XY})\cos y^\circ$$

$$6.5^2 = 4.5^2 + 3.5^2 - 2(4.5)(3.5)\cos y^\circ$$

$$2(4.5)(3.5)\cos y^\circ = 4.5^2 + 3.5^2 - 6.5^2$$

$$y = \cos^{-1}\left[\frac{4.5^2 + 3.5^2 - 6.5^2}{2(4.5)(3.5)}\right]$$

$\therefore$ Angle $y = 108.03^\circ$

b/ Given $1 + \cos\theta = 2\sin^2\theta$, $\sin^2\theta + \cos^2\theta = 1$,

$$1 + \cos\theta = 2[1 - \cos^2\theta]$$

$$1 + \cos\theta = 2 - 2\cos^2\theta$$

$$2\cos^2\theta + \cos\theta - 1 = 0, \text{ solving,}$$

$$\cos\theta = 0.5 \text{ or } -1$$

$\Rightarrow$ when $\cos\theta = 0.5$

$$\theta = \cos^{-1} 0.5$$

$$\theta_1 = 60^\circ \; (= \pi/3 \text{ rad})$$

Since cosine is positive in fourth quadrant,

$$\theta_2 = 360^\circ - 60^\circ$$

$$\theta_2 = 300^\circ \; (= 15/9\,\pi \text{ rad})$$

$\Rightarrow$ when $\cos\theta = -1$

$$\theta = \cos^{-1} -1$$

$$\theta_3 = 180^\circ \; (= \pi \text{ rad})$$

$\therefore$ Values of θ are π, $15/9\,\pi$ and $\pi/3$ rad.

Extract 8.2: *A sample of the candidate's correct responses in question 8*

Extract 8.2 shows the work of a candidate who correctly applied the *cosine rule* and obtained the size of angle Y and solved the given equation by using the appropriate trigonometric identity.

2.9 Question 9: Matrices

This question consisted of parts (a) and (b). In part (a), the candidates were required to find (i) $|A|$ and (ii) A^{-1}, given that $A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}$. In part (b), they were required to use the result obtained in part (a) (ii) to solve $\begin{cases} x+y+z=7 \\ x-y+2z=9 \\ 2x+y-z=1 \end{cases}$ simultaneously.

The data analysis showed that 21,962 (65.9%) candidates scored from 3.5 to 10 marks while 11,353 (34.1%) candidates scored below 3.5 marks as shown in Figure 9. This shows that the performance was good.

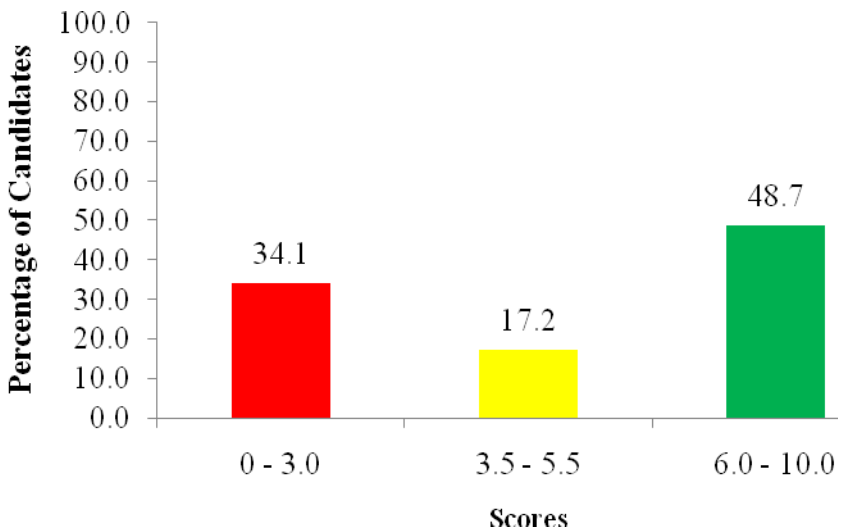


Figure 9: *Candidates' performance in question 9*

The analysis showed further that 4,446 (13.3%) candidates scored full marks. The candidates were able to get the required answers due to the following main observations:

In part (a), these candidates were able to use the correct procedures to find: the determinant $|A|$; the cofactors; and the adjoint of the given matrix so as to get the required inverse matrix A^{-1}. This indicates that the candidates had adequate knowledge and skills on how to find the inverse of a 3×3 matrix. In part (b), they were able to write the given simultaneous equations in matrix form $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$ and solve for x, y and z by using the inverse matrix they obtained in part (a) (ii). These candidates had sufficient knowledge and skills on how to solve simultaneous equations involving three unknowns by using the inverse matrix method as shown in Extract 9.1.

9ai) Solution

Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}$

$|A| = 1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$

$|A| = (1-2) - (-1-4) + (1-(-2))$

$|A| = -1+5+3$

$|A| = 7$

9aii) Minors of $A = \begin{pmatrix} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \end{pmatrix}$

Minor of $A = \begin{pmatrix} -1 & -5 & 3 \\ -2 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix}$

Cofactor of $A = \begin{pmatrix} -1 & -5 & 3 \\ -2 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix} \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

Cofactor of $A = \begin{pmatrix} -1 & 5 & 3 \\ 2 & -3 & 1 \\ 3 & -1 & -2 \end{pmatrix}$

9a ii) Adjoint ~~of~~ $A = \begin{pmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix}$

$$A^{-1} = \frac{1}{|A|} \times \text{Adjoint matrix}$$

$$A^{-1} = \frac{1}{7}\begin{pmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}$$

9b) solution

Using:

$$A^{-1} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}$$

$$\begin{cases} x + y + z = 7 \\ x - y + 2z = 9 \\ 2x + y - z = 1 \end{cases}$$

In matrix form: $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$

Premultiplication:

$$\begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}\begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$$

9b.) But: $\begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}\begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/7 \times 7 + 2/7 \times 9 + 3/7 \times 1 \\ 5/7 \times 7 + -3/7 \times 9 + -1/7 \times 1 \\ 3/7 \times 7 + 1/7 \times 9 + -2/7 \times 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 + 18/7 + 3/7 \\ 5 + -27/7 + -1/7 \\ 3 + 9/7 + -2/7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$\therefore x = 2,\ y = 1$ and $z = 4$

Extract 9.1: *A sample of the candidates' correct responses in question 9*

As seen in Extract 9.1 the candidate used the correct procedures to find the determinant and the inverse of a 3×3 matrix as well as using the inverse matrix method to solve the given system of simultaneous equations involving three unknowns.

Despite the good performance, it was noted that 3,093 (9.3%) candidates scored 0 mark in this question. Some of the factors that led to candidates' failure to get the required answers are as follows:

In part (a) (i), the candidates failed to use the correct procedures to get the determinant of the given matrix. For example, in finding the determinant of the matrix, some of them wrote wrong steps including: $|\mathrm{A}|=1\begin{pmatrix}-1 & 2\\ 1 & -1\end{pmatrix}+1\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}+1\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ and $|A|=1\begin{vmatrix}-1 & 2\\ 1 & -1\end{vmatrix}+1\begin{vmatrix}1 & 2\\ 2 & -1\end{vmatrix}+1\begin{vmatrix}1 & -1\\ 2 & 1\end{vmatrix}$. In order to arrive at the required answer, they were supposed to write $|A|=1\times\begin{vmatrix}-1 & 2\\ 1 & -1\end{vmatrix}-1\times\begin{vmatrix}1 & 2\\ 2 & -1\end{vmatrix}+1\times\begin{vmatrix}1 & -1\\ 2 & 1\end{vmatrix}$. Such candidates lacked adequate knowledge and skills on finding the determinant of a 3×3matrix.

In part (a) (ii), the candidates were not able to use the correct procedures to get the required inverse matrix after failing to find the *adjoint* and *transpose* which were the important steps to get the inverse of the given matrix. Others failed to differentiate between the *determinant* and the *inverse* of matrix A. However, in finding the inverse matrix, some candidates applied wrong formulae like $A^{-1}=\frac{1}{|A|}$ instead of using the formula $\mathrm{A}^{-1}=\frac{1}{|\mathrm{A}|}\mathrm{adjoint}(\mathrm{A})$.

In part (b), the candidates failed to correctly solve the given simultaneous equations because they used the incorrect determinants or the wrong inverse of the given matrix. Furthermore, some candidates used the Cramer's rule while others calculated the values of x, y and z by using a scientific calculator, which was contrary to the instructions. Extract 9.2 illustrates the incorrect answers of one of the candidates.

9. a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

i) $|A|$

$$|A| = 1\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} + 1\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} + 1\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$$

$$+1(-1) + 1(-3) + (-1)$$

$$-1 + -3 + 1 = -3$$

9. b) $\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 2 & & -1 \end{pmatrix} \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 7 & 1 & 1 \\ 9 & -1 & 2 \\ 1 & 1 & -1 \end{pmatrix} =$$

$$7(-1-2) + 1(-7-1) + 1(-7-9)$$

$$-21 + {}^{-}8 + 2$$

$$= -27$$

$$\begin{pmatrix} 1 & 7 & 1 \\ 1 & 9 & 2 \\ 2 & 1 & -1 \end{pmatrix}$$

$$1(-9-2) + 1(-1-2) + 1(9-7)$$

$$-11 + {}^{-}3 + 2$$

$$= -12.$$

$$\begin{pmatrix} 1 & 1 & 7 \\ 1 & -1 & 9 \\ 2 & 1 & 1 \end{pmatrix}$$

$$1(-1-9) + 1(1-14) + 7(-1-1)$$

$$-10 + {}^{-}13 + {}^{-}2$$

$$= -25.$$

Extract 9.2: *A sample of the candidate's incorrect responses in question 9*

In Extract 9.2 the candidate failed to use the correct procedures to get the determinant of the given matrix in part (a), as a result he/she was unable to solve the given simultaneous equations by using inverse matrix method in part (b).

2.10 Question 10: Linear Programming

This question had parts (a) and (b). The question started with the following descriptions: In a workshop, each carpenter makes chairs and tables. Carpenter I is limited to 10 days a month whereas carpenter II is limited to 15 days a month. The following table shows the number of days it takes to manufacture a chair and a table and the profit on each item:

	Chair(s)	Table(s)
Carpenter I	2	1
Carpenter II	1	3
Profit (USD)	30	45

The candidates were required to: (a) write the four inequalities by taking x and y to be the respective number of chairs and tables that should be made and (b) find the number of chairs and tables each carpenter should make in a month so as to maximize the income.

The data analysis revealed that this question had the highest performance, since 27,372 (83.2%) candidates scored from 3.5 to 10 marks, and out of these, 9,572 (29.1%) candidates scored full marks. Generally, the performance was good. Figure 10 shows a summary of candidates' performance.

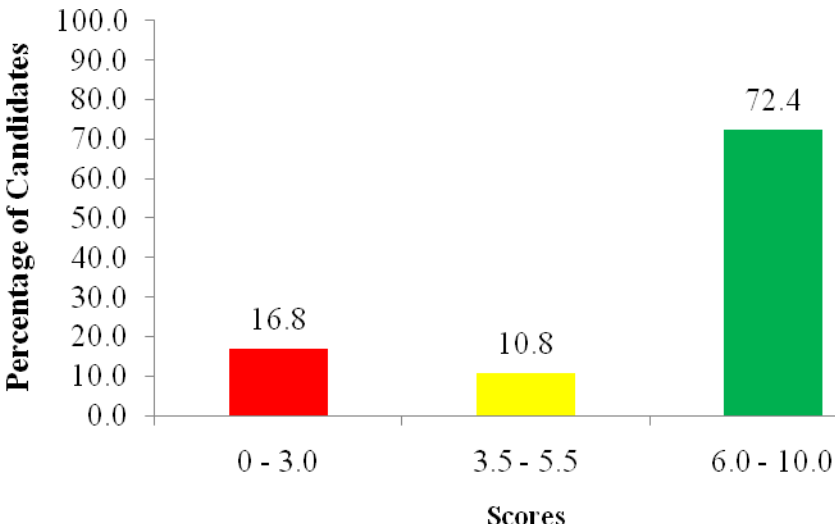


Figure 10: *Candidates' performance in question 10*

The candidates who gave correct answers were able to: formulate the objective function, that is, $f(x, y) = 30x + 45y$ and the inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x \geq 0$ and $y \geq 0$; draw the graph of the obtained inequalities; identify the corner points of the feasible region and finally determine the number of chairs and tables required as per task of the question as shown in Extract 10.1.

10. Soln:

From the table,

	Chair (days)	Table (days)	Maximum no of days
Carpenter I	2	1	10
Carpenter II	1	3	15
Profit (USD)	30	45	

a/ Constraints (inequalities)

a/ $2x + y \leq 10$

b/ $x + 3y \leq 15$

c/ $x \geq 0$

d/ $y \geq 0$

where x is number of chairs
y is number of tables

10. b/ From the data,

Objective function, $f(x,y) = 30x + 45y$

Required :

In order to maximize the income,

⇒ Maximize the value of $f(x,y) = 30x + 45y$

Tables of values

i/ $2x + y = 10$

x	0	5
y	10	0

ii/ $x + 3y = 15$

x	0	15
y	5	0

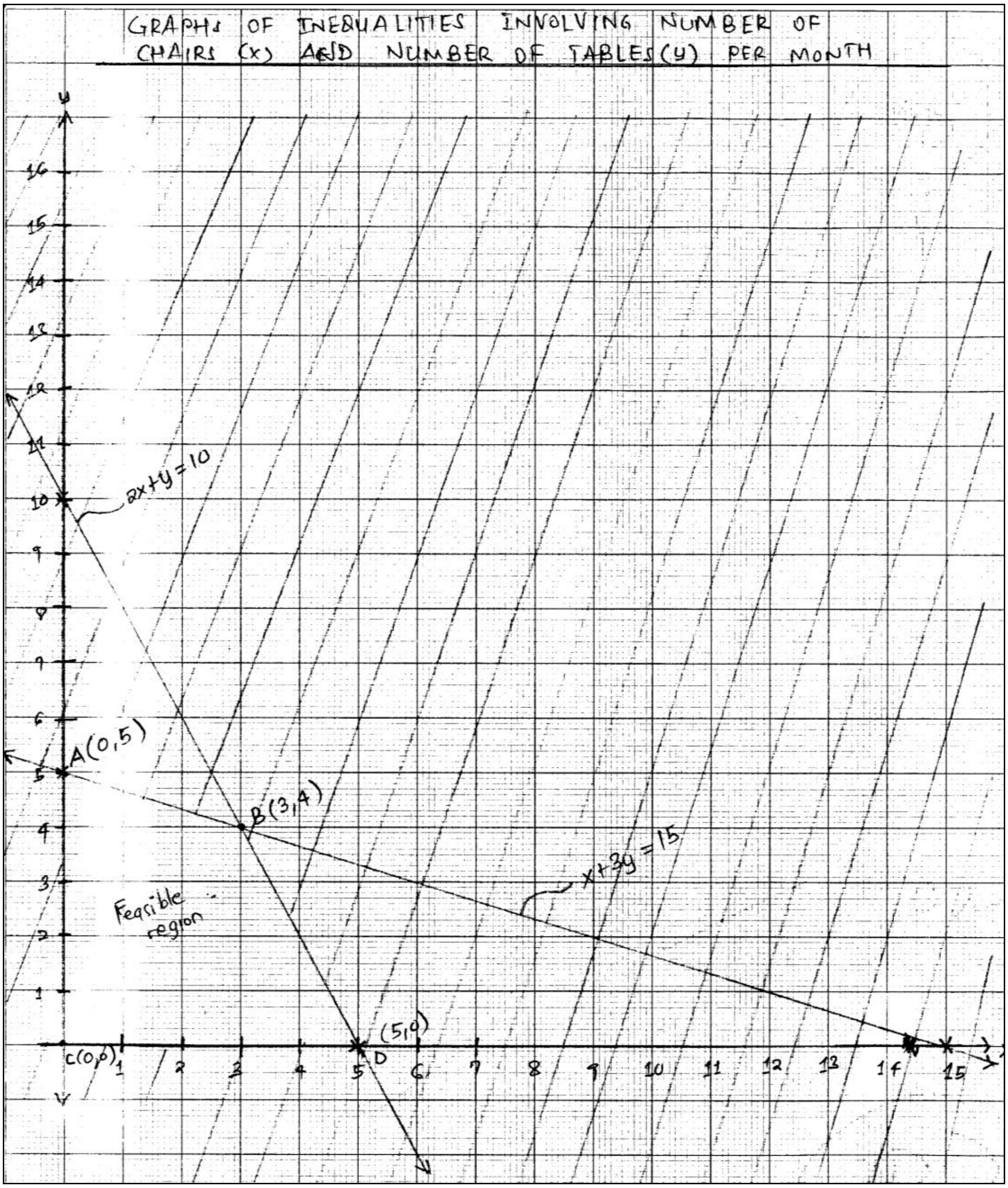
Table of results

Objective function, $f(x,y) = 30x + 45y$

Point	Objective function	Value (USD)
A (0,5)	$f(0,5) = 30(0) + 45(5)$	225
B (3,4)	$f(3,4) = 30(3) + 45(4)$	270
C (0,0)	$f(0,0) = 30(0) + 45(0)$	0
D (5,0)	$f(5,0) = 30(5) + 45(0)$	150

In order to maximize income, maximum of income obtained should be 270 USD.

∴ 3 chairs and 4 tables should be made in a month so as to maximize income.



Extract 10.1: *A sample of the candidate's correct responses in question 10*

In Extract 10.1 the candidate correctly formulated the required inequalities, drew a graph of the obtained inequalities and determined the number of chairs and tables as required.

Referring to Figure 10, a total of 5,515 (16.8%) candidates scored from 0 to 3.0 marks, including 1,568 (4.8%) candidates who scored 0 mark. The candidates' inability to get high scores in this question was mainly contributed by the following reasons:

Formulation of incorrect inequalities in part (a) by giving wrong answers like: $2x+y\geq 10$ and $x+3y\geq 15$ instead of $2x+y\leq 10$ and $x+3y\leq 15$. This observation confirms the lack of enough knowledge and skills on how to formulate linear inequalities from a linear programming problem. As a result, the candidates ended up drawing the incorrect graphs.

In part (b), the candidates failed to identify the correct objective function. For example, some of them wrote wrong objective functions like: $f(x,y)=x+y$ and $f(x,y)=10x+15y$ instead of $f(x,y)=30x+45y$. Extract 10.2 shows a sample of such an incorrect response.

10 a) The inequalities are

$x \geq 0$

$y \geq 0$

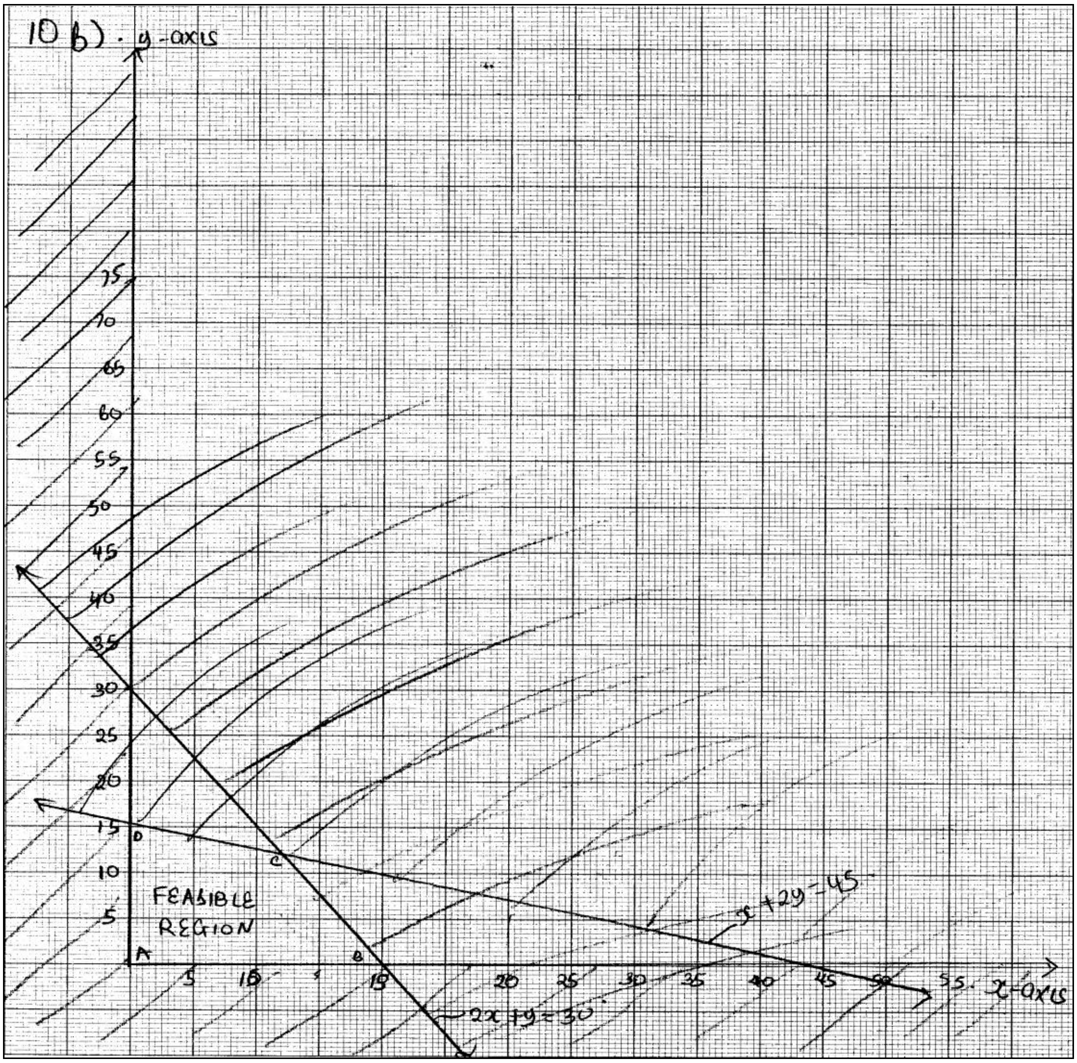
$2x + y \leq 30$

$x + 3y \leq 45$

b) Table of results.

corner points	maximum $f(x,y) = 10x+15y$
A (0, 0)	0
B (15, 0)	150
C (9, 12)	270
D (0, 15)	225.

The number of chairs and table each carpentner should make inorder to maximize the income is 9 chairs and 12 tables.



Extract 10.2: *A sample of the candidate's incorrect responses in question 10*

In Extract 10.2 the candidate drew the incorrect graph due to failure to formulate the correct inequalities.

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Basic Applied Mathematics examination consisted of ten (10) questions. The analysis showed that, out of ten (10) topics that were tested, *Linear Programming (83.2%)* and *Matrices (65.9%)* had a good performance. The topics that had average performance were: *Functions (56.0%), Statistics (52.0%), Calculating Devices (49.9%), Differentiation*

(42.0%), *Integration (38.6%)* and *Algebra (37.6%)*. However, *Trigonometry (34.2%)* and *Probability (23.5%)* had a weak performance.

The main factors that contributed to average and good performance include the candidates' ability to: use a non-programmable scientific calculator to solve mathematical problems; apply the correct identities, rules and formulae in verifying and solving problems as well as sketching and interpreting graphs correctly.

Contrarily, the main reasons that made the candidates to score low marks include the failure to: use the required functional keys of a non-programmable scientific calculator to solve mathematical problems; sketch correct graphs; formulate mathematical statements, equations and inequalities from word problems; differentiate exponential, implicit and polynomial functions; and integrate by using substitution techniques; The other reason was inability to summarize ungrouped data in a frequency distribution table.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

Generally, the Candidates' Item Response Analysis (CIRA) showed that 56.21 percent of the candidates passed the 141 Basic Applied Mathematics examination in ACSEE 2019. The CIRA showed that out of ten (10) tested topics, eight (08) of them had a performance of at least 35 percent while only two (02) topics had a performance below the average as summarized in section 3.0. It was further noted that the topic that had the highest performance was *Linear Programming* followed by *Matrices*. On the other hand, *Probability* was the topic that had the lowest performance.

Lastly, it is hoped that this report will be used as a special tool by students, teachers, policy makers, curriculum developers and other education stakeholders to improve the performance of Basic Applied Mathematics subject, especially the topics that had the weak and average performance in the future examinations.

4.2 Recommendations

In order to improve the performance of 141 Basic Applied Mathematics in future ACSEE, the students and teachers should play their roles effectively in the teaching and learning process. To achieve a good performance, the following are recommended to both students and teachers:

4.2.1 Role of students

- Doing as many exercises as possible in order to improve the ability to apply theorems, formulae and other concepts in solving mathematical problems.
- Working in groups so as to share knowledge and enhance good understanding of a particular concept as well as mastering of the subject.
- Consulting teachers for any difficulties or challenges faced during the learning process so as to get timely assistance.
- Using different Basic Applied Mathematics books in order to improve the ability in answering the examination questions.
- Making correction for the tests and examinations including revision on various topics or concepts that seem to be difficult.

4.2.2 Role of teachers

- Teaching effectively all the topics outlined in the syllabus in time.
- Teaching students on how to use a non-programmable scientific calculator to solve different mathematical problems.
- Identifying students with learning difficulties with a view to assisting them through remedial classes and consultations.
- Making a good use of different teaching and learning materials like books, internet resources and other teaching aids so as to improve the students' competence in the subject.
- Providing frequent exercises and tests to students, marking them and giving feedback to students on time.

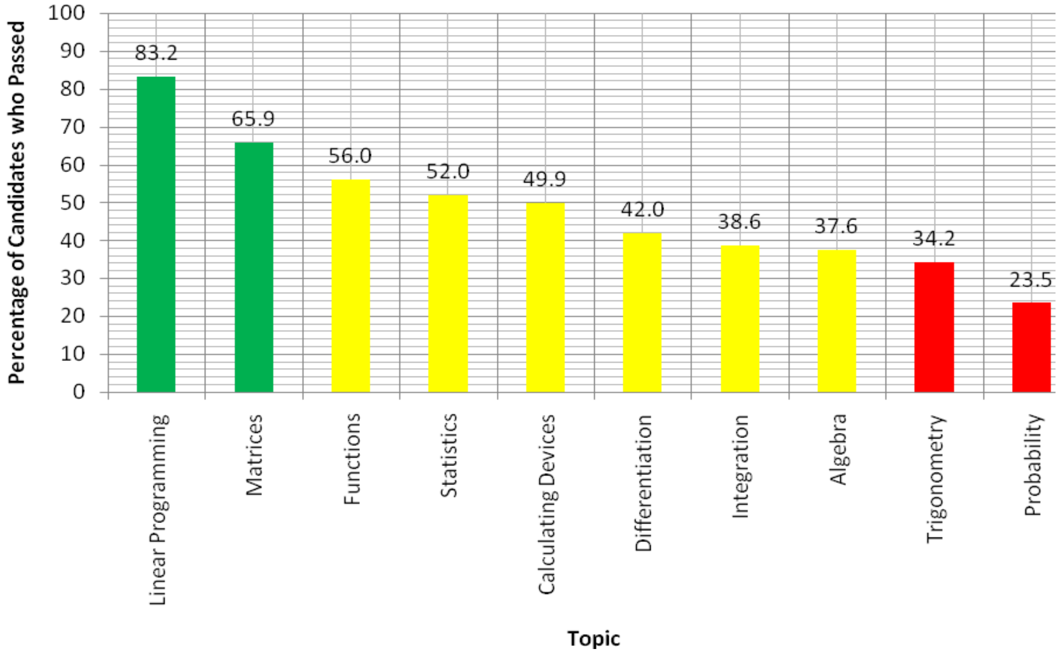
APPENDICES

Appendix I

Analysis of Candidates' Performance in each Topic for ACSEE 2019 & 2018

S/N	Topic	Question Number	2019		2018	
			Percentage of Candidates who Passed (Scored at least 3.5 marks)	Remarks	Percentage of Candidates who Passed (Scored at least 3.5 marks)	Remarks
1	Linear Programming	10	83.2	Good	64.2	Good
2	Matrices	9	65.9	Good	72.9	Good
3	Functions	2	56.0	Average	50.2	Average
4	Statistics	6	52.0	Average	50.4	Average
5	Calculating Devices	1	49.9	Average	65.5	Good
6	Differentiation	4	42.0	Average	44.7	Average
7	Integration	5	38.6	Average	40.1	Average
8	Algebra	3	37.6	Average	44.9	Average
9	Trigonometry	8	34.2	Weak	36.0	Average
10	Probability	7	23.5	Weak	64.7	Good
Average Performance per Topic			48.29	Average	53.36	Average

Appendix II



Percentage of Candidates who Passed
100
90
80
70
60
50
40
30
20
10
0
83.2
65.9
56.0
52.0
49.9
42.0
38.6
37.6
34.2
23.5
Linear Programming
Matrices
Functions
Statistics
Calculating Devices
Differentiation
Integration
Algebra
Trigonometry
Probability
Topic