THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2018

142 ADVANCED MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is pleased to issue the report on the Candidates' Item Response Analysis (CIRA) for the 2018 Advanced Mathematics Examination. The purpose is to inform teachers, parents, candidates, policy makers and other education stakeholders of how the candidates answered the questions. The report will help the stakeholders to take appropriate measures to improve the candidates' performance in future examinations.

The analysis of the candidates' responses is done in order to identify the areas in which the candidates faced certain problems, did well or somewhat well. Basically, the report highlights the candidates' strengths and weaknesses in order to determine the effectiveness of the education system.

The factors for the poor performance include the candidates' inability to use formulae, theorems, axioms, principles and special statistical distributions in answering the questions. Extracts of the candidates' responses are included in this report to illustrate the candidates' performance in the questions.

The National Examinations Council of Tanzania will highly appreciate any comments and suggestions from teachers, candidates, other education stakeholders and the public in general which show areas that need improvement so that future reports are better than the current one.

Finally, the Council would like to thank everyone who participated in the preparation of this report.

Dr. Charles E. Msonde

EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report is based on the analysis of the candidates' performance in the 2018 142 Advanced Mathematics Examination. The analysis highlights strengths and weaknesses in relation to the candidates' responses in order to provide a general overview of the candidates' performance.

The Advanced Mathematics Examination had two papers: Advanced Mathematics 1 and Advanced Mathematics 2. Advanced Mathematics 1 had ten (10) compulsory questions with 10 marks each. Advanced Mathematics 2 consisted of sections A and B. Section A consisted of four (4) compulsory questions, each carrying fifteen (15) marks. Section B consisted of four (4) questions, each carrying twenty (20) marks. The candidates were required to answer any two (2) questions. The questions were based on the 2009 Advanced Level Advanced Mathematics syllabus.

A total of 11,991 candidates sat for the Advanced Mathematics Examination, out of whom 10,041 (83.74%) candidates passed. This performance is better than that of 2017 whereby 74.78 percent of 10,553 candidates passed. This represents 13.62 percent rise in the number of candidates who sat for examination and 8.96 percent increase in the number of the candidates who passed. The candidates who passed these examinations got different grades from grade S to grade A as shown in Figure 1.

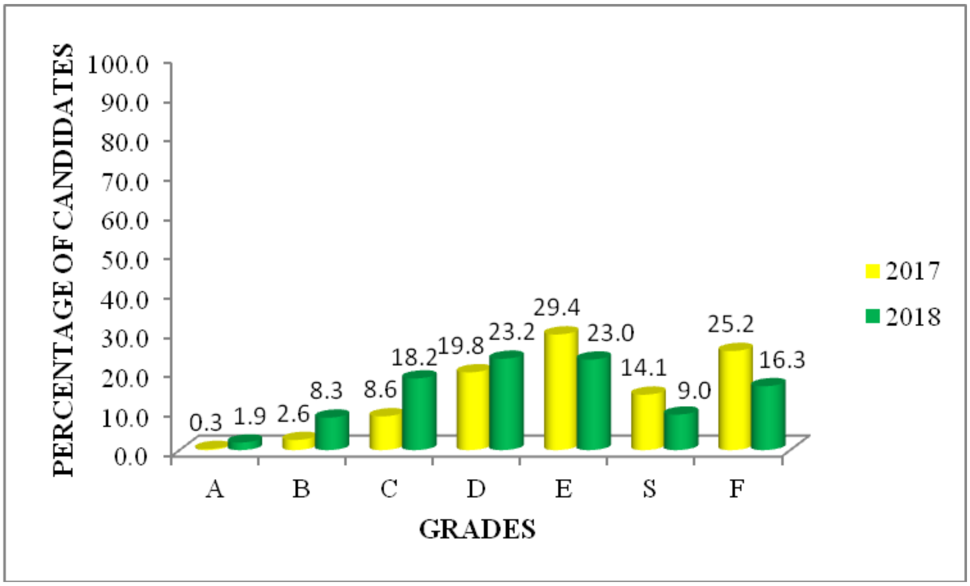


Figure 1: *Distribution of Grades for the 2017 and 2018 142 Advanced Mathematics Examination.*

The analysis of the candidates' performance in each question is presented in section 2.0. The section consists of a short description of the requirements of the questions and the analysis of how the candidates responded to the questions. Extracts showing the candidates' good and poor performance are included in the analysis. The factors for the candidates' good or poor performance in each question are given and illustrated using samples of the candidates' responses. Therefore, the analysis of each question could be used by teachers as a guide in their steps to improve teaching and learning and the candidates' future examination performance.

The analysis of the candidates' performance in each topic is shown in the appendices in which the green colour stands for good performance, the yellow colour for average performance and the red colour for poor performance. The percentage boundaries, 0 – 34, 35 – 59 and 60 – 100, are used to represent poor, average and good performance. Finally, some recommendations are given at the end of the report. The recommendations may help teachers and the government to improve the candidates' future performance in future Advanced Mathematics examinations.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 142/1 ADVANCED MATHEMATICS 1

2.1.1 Question 1: Calculating Devices

This question had parts (a) and (b). In part (a), the candidates were required to evaluate the expressions (i) $\left(\sqrt{\frac{(8.621)(27.34)}{52.18+0.0724}}\right)^{\frac{3}{5}}$ and (ii) $\sum_{x=0}^{2} xe^{x}\log(x+1)^{\frac{1}{3}}$ correct to four decimal places by using a non-programmable calculator. In part (b), the candidates were given $a=14.2,\ b=12.6,\ c=8.4$, $T=(s(s-a)(s-b)(s-c))^{\frac{1}{2}}$ and $2s=a+b+c$ and were asked to find the value of T correct to four decimal places by using a non-programmable calculator.

The analysis shows that 11,767 candidates (equivalent to 97.5 percent) attempted the question; 67.5 scored from 6 to 10 marks and 33 percent scored all 10 marks. Further analysis shows that 12.6 percent scored from 3.5 to 5.5 marks and that 19.9 percent had unsatisfactory performance, as 12.3 of them scored from 0.5 to 3 marks and the remaining 7.6 percent got 0. As Figure 2 shows, the candidates' performance was good.

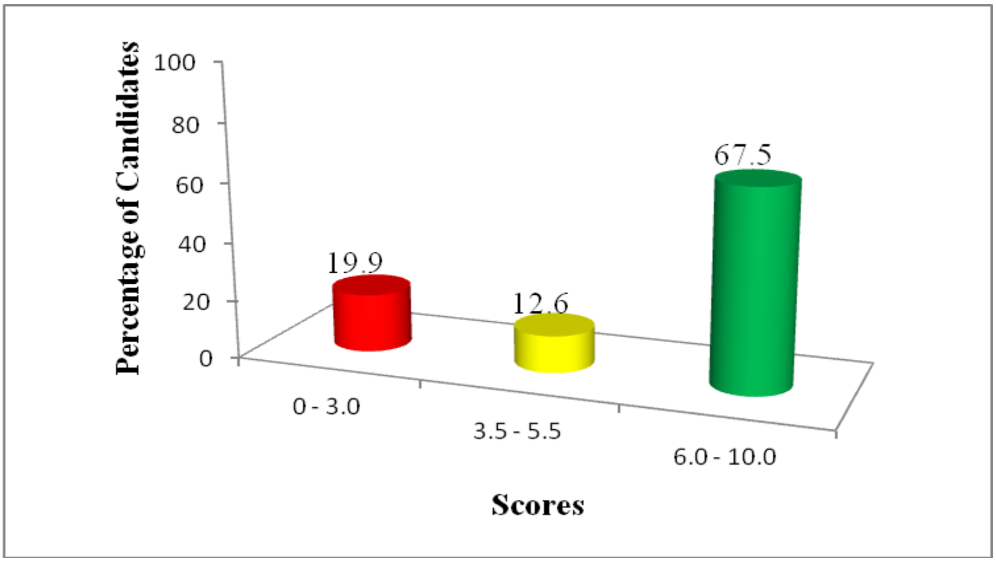


Figure 2: *Candidates' performance in question 1.*

The candidates who did part (a) (i) well used a non-programmable calculator to do the calculations involving addition, multiplication and division. They also computed the n^{th} root and the fractional exponent of a number correctly. Additionally, they wrote the final answer in four decimal places. The candidates who did part (a) (ii) well formulated a series $0+e\log(2)^{\frac{1}{3}}+2e^{2}\log(3)^{\frac{1}{3}}$ from $\sum_{x=0}^{2} xe^{x}\log_{e}(x+1)^{\frac{1}{3}}$ by substituting $x=0$, $x=1$ and $x=3$. They also used a non-programmable calculator to find the sum of terms and wrote the final answer in four decimal places. The candidates who did part (b) well could make s the subject of a, b and c from $2s=a+b+c$, enter the values of a, b and c into a non-programmable calculator to obtain the value of s, find the value of T from the values of s, a, b and c, and write the answers in four decimal places. Extract 1.1 shows the response of a candidate who answered the question correctly.

Extract 1.1

~~5~~ (e)

01 a) i) $\left(\sqrt{\frac{(8.621)(27.34)}{(52.18+0.0724)}}\right)^{3/5}$.

$=\left(\sqrt{\frac{235.69814}{52.2524}}\right)^{3/5}$.

$=\left(\sqrt{4.510761994}\right)^{3/5}$

$=(2.123855455)^{3/5}$

$=1.5714$.

$\therefore \left(\sqrt{\frac{(8.621)(27.34)}{52.18+0.0724}}\right)^{3/5}=1.5714$.

ii) $\sum_{x=0}^{2} xe^{x}\log(x+1)^{1/3} = 0e^{0}\log(0+1)^{1/3}+1e^{1}\log(1+1)^{1/3}+$
$2e^{2}\log(2+1)^{1/3}$.

$=0+2.718281828(0.1003433331)+$

$=0+0.272761455+2.350317145$

$=2.6231$

$\therefore \sum_{x=0}^{2} xe^{x}\log(x+1)^{1/3}=2.6231$.

b) $T=\left(s(s-a)(s-b)(s-c)\right)^{1/2}$

$a=14.2$, $b=12.6$, $c=8.4$, $2s=a+b+c$ that is $s=\frac{a+b+c}{2}$.

$T=\left((17.6)(17.6-14.2)(17.6-12.6)(17.6-8.4)\right)^{1/2}$

$T = (17.6\,(3.4)(5)(9.2))^{1/2}$.

$T = (2,752.64)^{1/2}$

$\therefore$ The value of $T = 52.4656$.

Extract 1.1 *A correct response from one of the candidates.*

The majority of the candidates who did not do part (a) (i) well could not use non-programmable calculators to evaluate the fractional exponent of a number. They correctly evaluated the expression $\sqrt{\frac{(8.621)(27.34)}{52.18+0.0724}}$ but got an incorrect answer of the expression $\left(\sqrt{\frac{(8.621)(27.34)}{52.18+0.0724}}\right)^{\frac{3}{5}}$.

The majority of the candidates who did not do part (a) (ii) well defined a series $\sum_{x=0}^{2} xe^{x}\log_e (x+1)^{\frac{1}{3}}$ using one or two terms by substituting $x=0$ or $x=2$ or both, ignoring $x=1$. In part (b), most of the candidates did not use non-programmable calculators to evaluate expressions with brackets and the fractional exponent as they obtained the right value of *s*, but ended up with an incorrect value of *T*. Finally, a large number of the candidates could not present the final answer to four decimals places as they wrote 52.46560778 instead of writing it as 52.4656. Extract 1.2 shows the response of a candidate who could not answer the question correctly.

Extract 1.2

nber | SUBJECT NAME..

1a i) $\left(\sqrt{\frac{(8.621)(27.34)}{52.18+0.00724}}\right)^{3/5}$.

$8.621 \times 27.34 = 235.69814$

$52.18 + 0.0724. = 52.2524$

$\left(\sqrt{\frac{235.69814}{52.2524}}\right)^{3/5}$.

$= \left(\sqrt{4.5108}\right)^{3/5}$.

$= (2.1238)^{3/5}$

$= 1.61095$

$\left(\sqrt{\frac{(8.621)(27.34)}{52.18+0.00724}}\right)^{3/5} = 1.6109$

1a ii) $\sum_{x=0}^{2} xe^x \log (x+1)^{1/3}$.

$2e^2 \log (2+1)^{1/3}$.

$14.7781 \times \log (3)^{1/3}$.

$14.7781 \times \log (1.44225)$.

$14.7781 \times 0.15904.$

$= 2.3503$.

$\therefore \sum_{x=0}^{2} xe^x \log (x+1)^{1/3} = 2.3503.$

Extract 1.2: *An incorrect response from one of the candidates.*

2.1.2 Question 2: Hyperbolic Functions

The candidates were required to (a) differentiate $\cosh^6 x$ with respect to x, (b) solve the equation $3\cosh x + \sinh x = \frac{9}{2}$ and (c) prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^{2.} + 1}\right)$ for all the values of x.

The analysis shows that 6.4 percent of the candidates who attempted this question scored from 0 to 3 marks, 11.9 percent from 3.5 to 5.5 marks and 81.6 percent from 6 to 10 marks. Generally, the candidates' performance was very high, as 93.5 percent of the candidates got more than 3 marks. Figure 3 illustrates the candidates' performance in this question.

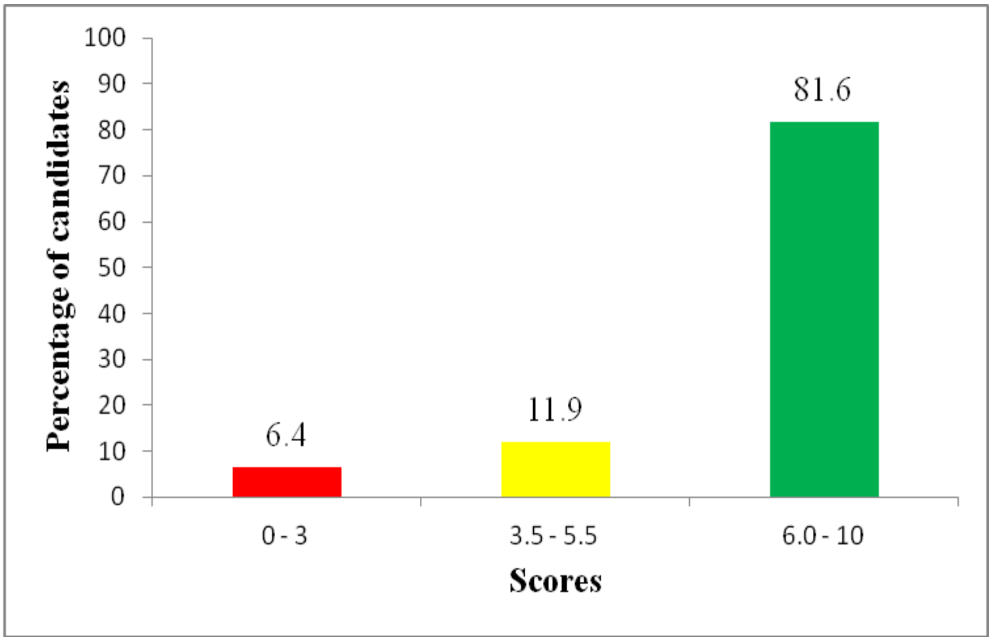


Figure 3: *Candidates' performance in question 2*

The candidates who did the question well had adequate knowledge of the concepts tested. In part (a), they differentiated the hyperbolic function $\cosh^6 x$ involving powers using the chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ and obtained $6\cosh^5 x \sinh x$. In part (b), they correctly solved the equation $3\cosh x + \sinh x = \frac{9}{2}$ by replacing $\cosh x$ and $\sinh x$ with $\frac{1}{2}\left(e^{x} + e^{-x}\right)$ and $\frac{1}{2}\left(e^{x} - e^{-x}\right)$ to produce $x = \ln 2$ or $x = -2\ln 2$. In part (c), the majority of the candidates defined the inverse hyperbolic sine function as $y = \sinh^{-1} x \Rightarrow \sinh y = x$. Furthermore, they used the definition of $\sinh y$ and the facts that $e^{y} > 0$ for all the values of y to prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^{2.} + 1}\right)$. Extract 2.1 is a solution obtained by one of the candidates who answered the question correctly.

Extract 2.1

2a. To find $f'(x)$ of $\cosh^6 x$.

$\cosh^6 x = (\cosh x)^6$.

By chain rule.

let $\cosh x = u$.

$\sinh x = \frac{dy}{dx}$.

$u^6 = y$.

$\frac{dy}{du} = 6u^5$

By chain rule.

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$.

$\frac{dy}{dx} = 6u^5 \times \cancel{\sinh}$

$\frac{dy}{dx} = 6u^5 \sinh x$.

but $u = \cosh x$.

$\frac{dy}{dx} = 6(\cosh x)^5 \sinh x$.

$\frac{dy}{dx} = 6\sinh x \cosh^5 x$.

$\therefore f'(x) = 6\sinh x \cosh^5 x$.

2b. $3\cosh x + \sinh x = 9/2$.

$6\cosh x + 2\sinh x = 9$.

$6\left(\frac{e^x + e^{-x}}{2}\right) + 2\left(\frac{e^x - e^{-x}}{2}\right) = 9$.

$3(e^x + e^{-x}) + (e^x - e^{-x}) = 9$.

$3e^x + 3e^{-x} + e^x - e^{-x} = 9$.

$4e^x + 2e^{-x} = 9$

2b.

$$4e^{x}+2e^{-x}=9$$

$$\frac{4e^{x}+2}{e^{x}}=9$$

$$4e^{2x}+2=9e^{x}$$

$$4e^{2x}-9e^{x}+2=0$$

$$e^{x}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$e^{x}=\frac{9\pm\sqrt{81-4(4\times 2)}}{2\times 4}$$

$$e^{x}=\frac{9\pm\sqrt{81-32}}{8}$$

$$e^{x}=\frac{9\pm\sqrt{49}}{8}$$

$$e^{x}=\frac{9\pm 7}{8}$$

$$e^{x}=\frac{16}{8} \text{ or } e^{x}=2/8$$

$$e^{x}=2 \text{ or } e^{x}=1/4$$

apply ln

$$x=\ln 2 \text{ or } x=\ln(1/4)$$

$$\therefore x=0.6931 \text{ or } x=-1.3863$$

2c. Required to prove for $\sinh^{-1}x=\ln(x+\sqrt{x^2+1})$

let $\sinh^{-1}x=y$.

$$x=\sinh y$$

$$x=\frac{e^{y}-e^{-y}}{2}$$

$$e^{2y}-2xe^{y}-1=0$$

$$e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$e^y = \frac{\cancel{2}x \pm \cancel{2}\sqrt{x^2+1}}{\cancel{2}}$$

$$e^y = x \pm \sqrt{x^2+1}$$

Apply ln.

$$y = \ln\left(x \pm \sqrt{x^2+1}\right).$$

but $y = \sinh^{-1} x$.

$$\sinh^{-1} x = \ln\left(x \pm \sqrt{x^2+1}\right).$$

but their is no ln of (-ve) number then.

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$ hence proved

Extract 2.1: *A correct response from one of the candidates.*

One hundred and thirteen candidates (3.1%) scored 0. The candidates did part (a) badly because they could not differentiate $\cosh^6 x$ with respect to x. For instance, in differentiating this expression, a significant number of the candidates wrote $\frac{dy}{dx} = 6\cosh^5 x$. Such candidates applied the formula $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ only because they could not use the chain rule as well. Other candidates confused differentiating trigonometric functions with hyperbolic functions. For instance, they incorrectly differentiated $\cosh x$ to give $-\sinh x$ and therefore wrote $\frac{d}{dx}\left(\cosh^6 x\right) = -6\cosh^5 x \sinh x$. In part (b), some of the candidates incorrectly defined the concepts of hyperbolic sine and cosine as $\cosh x = \frac{1}{2}\left(e^x - e^{-x}\right)$ and $\sinh x = \frac{1}{2}\left(e^x + e^{-x}\right)$ in solving $3\cosh x + \sinh x = \frac{9}{2}$. As a result, they got $x = 9.8151$ instead of $x = 0.6931$ or -1.3862. Others inserted the right expressions for $\sinh x$ and $\cosh x$ into $3\cosh x + \sinh x = \frac{9}{2}$ but could not do algebraic operations on $3\left(\frac{1}{2}\left(e^x+e^{-x}\right)\right)+\frac{1}{2}\left(e^x-e^{-x}\right)=\frac{9}{2}$ to obtain $4e^{2x}-9e^x+2=0$ which could be solved to get the required values of *x*. Despite the weaknesses mentioned many candidates obtained the equation $4e^{2x}-9e^x+2=0$ but could not solve it using one of the quadratic techniques. In part (c), some of the candidates expressed the given inverse hyperbolic sine as $x=\sinh y$ but could not replace $\sinh y$ with $\frac{1}{2}(e^y-e^{-y})$. Instead, they replaced it with $\frac{1}{2}(e^y+e^{-y})$. As a result, the candidates showed that $\sinh^{-1}x=x\pm\sqrt{x^2-1}$, instead of showing that $\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$. There were candidates who regarded $\sinh^{-1}x$ as $\frac{1}{2}\left(e^x-e^{-x}\right)$, instead of $x=\sinh y=\frac{1}{2}(e^y-e^{-y})$. Such candidates provided irrelevant proofs for the given equation and did not score any marks. Extract 2.2 is a sample response from a candidate who did the question badly.

Extract 2.2

2a. $\cosh^6 x$.

let $y = \cosh^6 x$.

$\frac{dy}{dx} = 6\sinh^5 x$

$\therefore \frac{dy}{dx} = 6\sinh^5 x$.

Extract 2.2: *An incorrect response from one of the candidates.*

2.1.3 Question 3: Linear Programming

The question read "Mama Lishe has 140, 80 and 130 units of ingredients A, B and C. A piece of bread requires 1, 1 and 2 units of A, B and C. A pancake requires 5, 2 and 1 units of A, B and C, respectively." The candidates were required to (a) formulate the linear inequalities that satisfy these conditions if *x* and *y* are the number of pieces of bread and pancakes, respectively, (b) draw a graph which shows a region representing values of *x* and *y*, (c) find the number of each snack that should be baked in order to maximize the gross income, given that the price for a piece of bread is 300/= and a pancake is 500/= and (d) find the gross income.

The question was attempted by 11,575 candidates; out of whom, 6.4 percent scored from 0 to 3 marks, 16.9 percent from 3.5 to 5.5 marks and 76.8 percent from 6.0 to 10 out of 10 marks. This is the question in the entire examination which the candidates did very well, as 93.7 percent of the candidates scored more than 3 marks. Figure 4 is a summary of the candidates' performance in this question.

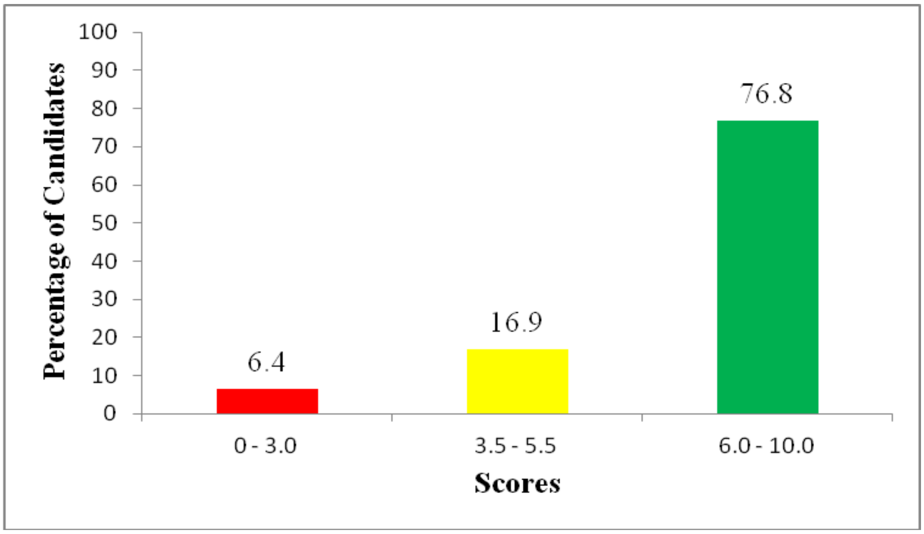


Figure 4: *Candidates' performance in question 3.*

The analysis shows that 26.6 percent of the candidates answered the question correctly and scored all 10 marks. These candidates wrote the constraints as a system of linear inequalities in part (a). The formulated linear inequalities were $x+2y\le 80$, $2x+y\le 130$ and $x+5y\le 140$. In part (b), the candidates found the set of a feasible solution that graphically represents the constraints obtained in part (a) and calculated the coordinates of the vertices from the feasible solutions and got $(0, 0)$, (65,0), (60, 10), (40, 20) and (0, 28). In part (c), the candidates wrote the objective function as $f(x,y)=300x+500y$ and computed the value of $f(x,y)=300x+500y$ at each of the vertices to determine the number of each snack to be baked in order to maximize the gross income. Extract 3.1 is taken from the answer booklet of a candidate who answered the question correctly.

Extract 3.1

3 a) Taking x be the number of pieces of bread
Taking y be the number of pieces of pancakes

Inequalities satisfying the conditions are:-

$x + 5y \leq 140$
$x + 2y \leq 80$
$2x + y \leq 130$

b) For $x + 5y \leq 140$

x	y
0	28
140	0

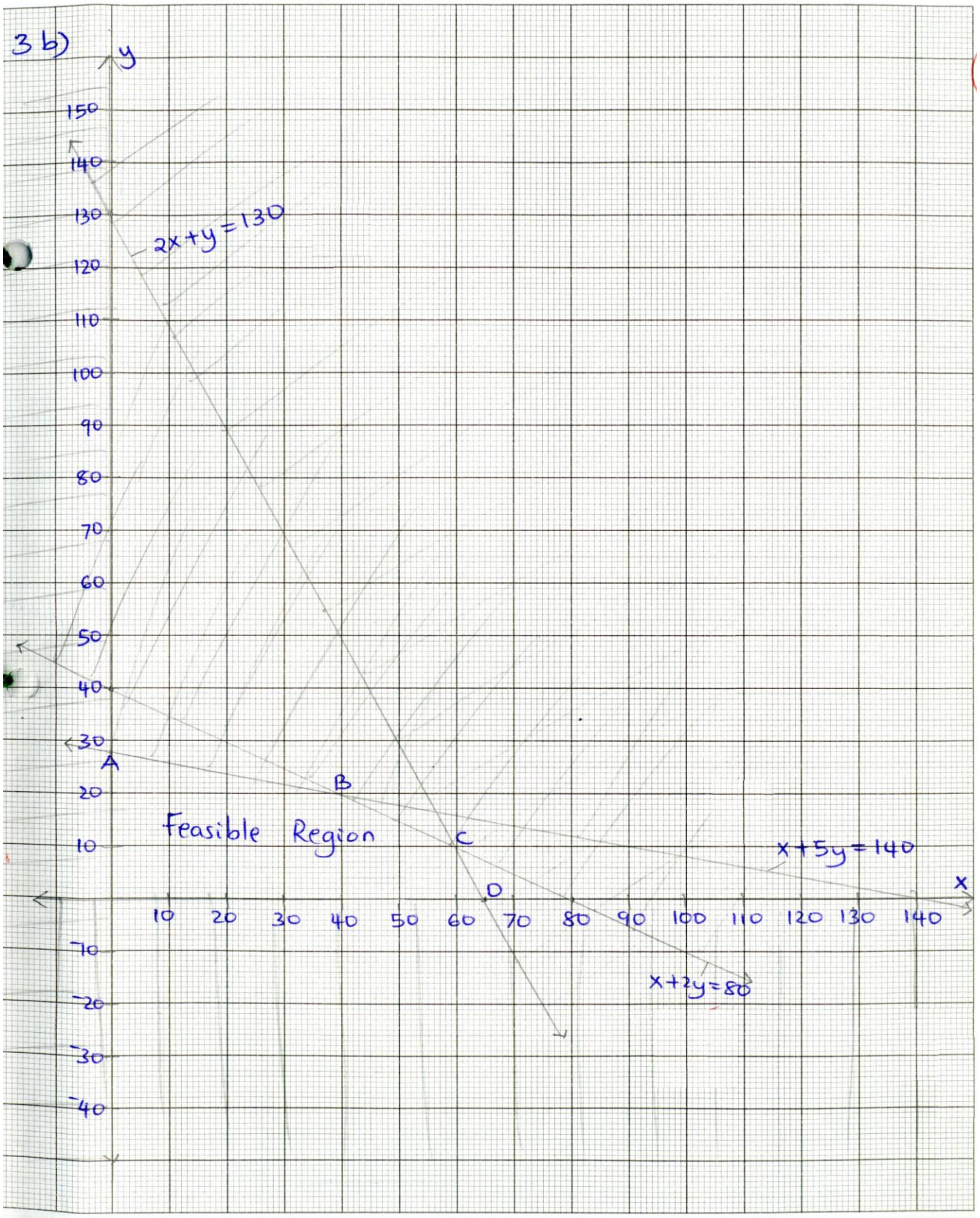
For $x + 2y \leq 80$

x	y
0	40
80	0

For $2x + y \leq 130$

x	y
0	130
65	0

3 b)


y
x
2x + y = 130
x + 5y = 140
x + 2y = 80
A
B
C
D
Feasible Region
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
-10
-20
-30
-40
10
20
30
40
50
60
70
80
90
100
110
120
130
140

3 c) Objective function

$f(x,y) = z = 300x + 500y$

Corner point	Corresponding values $Z = 300x + 500y$
$A(0,28)$	14000
$B(40,20)$	22000
$C(60,10)$	23000
$D(65,0)$	19500

In order to maximize her gross income, she should bake 60 breads and 10 pancake

d) Her gross income = 23000/=

Extract 3.1: *A correct response from one of the candidates.*

In spite of the good performance, there were candidates who got the question wrong. In part (a), they failed to identify the set of linear inequalities which satisfy the given conditions. For example, they presented the inequalities as $x+2y \geq 80$, $2x+y \geq 130$ and $x+5y \geq 140$, instead of presenting them as $x+2y \leq 80$, $2x+y \leq 130$ and $x+5y \leq 140$. Further analysis indicates that some of the candidates formulated the linear inequalities correctly but shaded them wrongly. As a result, they got an incorrect feasible region and incorrect corner points. Furthermore, some of the candidates wrote a system of linear equations $x+2y=80$, $2x+y=130$ and $x+5y=140$ instead of inequalities. Thus, they did not get a feasible region. Such candidates could not distinguish between equations and inequalities. Another serious mistake that the candidates made frequently was using inequalities as labels of lines on the graph. Extract 3.2 is a response from a candidate who got the question wrong.

Extract 3.2

3. Consider:

	Bread	Pan Cakes	Total.
A	1	5	140
B	1	2	80
C	2	1	130.

Requ ~~As~~ x, and y already defined,

(a) Three inequalities required are:-

$x + 5y \geq 140.$

$x + 2y \geq 80$

$2x + y \geq 130.$

Extract 3.2: *An incorrect response from one of the candidates.*

2.1.4 Question 4: Statistics

The candidates were given the following frequency distribution table, which represents a class of 100 students:

Class	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	t-2	1	20	t+2	t	t+3	23	11	t+4	13

In part (a), they were required to determine the value of *t*. In part (b), they were required to find (i) the mean, (ii) the standard deviation, (iii) the mean deviation and (iv) the median correctly to two decimal places.

The question was attempted by 11,938 candidates (99.0%), out of whom 62.9 percent scored from 6 to 10 marks and 3.1 percent scored all 10 marks. Further analysis shows that 29.2 percent of the candidates had average performance; their scores ranged from 3.5 to 5.5 and 7.9 percent from 0 to 3 marks. Generally, the candidates' performance was good, as Figure 5 shows.

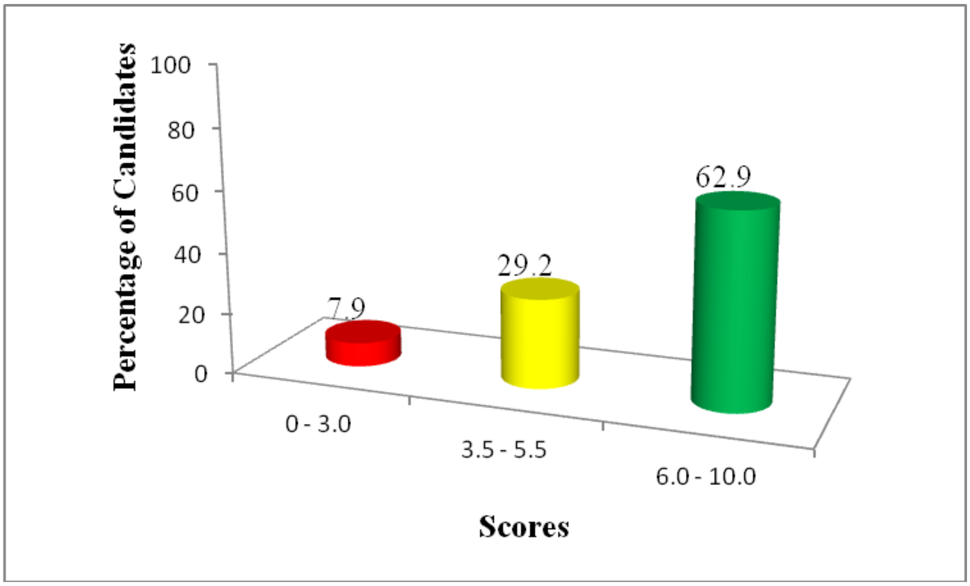


Figure 5: *Candidates' performance in question 4.*

The candidates who did part (a) well formulated the equation $(t-2)+(1)+(20)+(t+2)+(t)+(t+3)+(23)+(11)+(t+4)+(13)=100$, solved it and got $t=5$. The candidates who attempted part (b) correctly created a table with columns whose headings are class intervals, frequency (f), class marks (X), fX, fX^2 and $f\left|X-\overline{X}\right|$, and used an appropriate formulae to compute the mean, standard deviation, mean deviation and the median correct to two decimal places as 58.10, 25.56, 21.89 and 63.11. Extract 4.1 is a sample solution obtained by a candidate who answered question 4 correctly.

Extract 4.1

4 a) Given $N = 100$

$100 = t - 2 + 1 + 20 + t + 2 + t + t + 3 +$
$23 + 11 + t + 4 + 13$

$100 = 5t + 75$

$\frac{25}{5} = \frac{5t}{5}$

$t = 5$

4 b)

class	f	Cum F	X	Fx	$f\lvert X-\bar{X}\rvert$	X^2	Fx^2
1 – 10	3	3	5.5	16.5	157.797	30.25	90.75
11 – 20	1	4	15.5	15.5	42.599	240.25	240.25
21 – 30	20	24	25.5	510	651.98	650.25	13,005
31 – 40	7	31	35.5	248.5	158.193	1260.25	8821.75
41 – 50	5	36	45.5	227.5	62.995	2070.25	10351.25
51 – 60	8	44	55.5	444	20.792	3080.25	24,642
61 – 70	23	67	65.5	1506.4	170.223	4290.25	98,675.75
71 – 80	11	78	75.5	830.5	191.411	5700.25	62,702.75
81 – 90	9	87	85.5	769.5	246.609	7310.25	65,792.25
91 – 100	13	100	95.5	1241.5	486.213	9120.25	118,563.25
				$\Sigma Fx = 5809.9$			$\Sigma Fx^2 = 402,885$

i) Mean

$$\bar{X} = \frac{\Sigma Fx}{N} = \frac{5809.9}{100}$$

$$\bar{X} = 58.099 \approx 58.10$$

$$\text{Mean } \bar{\bar{X}} = 58.10$$

ii) Standard deviation

$$\delta^2 = \frac{\Sigma F(x-\bar{x})^2}{N}$$

$$\delta^2 = \frac{\Sigma Fx^2}{N} - \bar{x}^2$$

$$\delta^2 = \frac{402.885}{100} - 58.099^2$$

$$\delta^2 = 653.356$$

$$\delta^2 = \sqrt{653.356}$$

$$\delta = 25.56$$

∴ Standard deviation $= 25.56$

4(b) iii) Mean deviation

$$= \frac{\Sigma f|x - \bar{x}|}{N}$$

$$= \frac{2188.812}{100}$$

$$= 21.88812 \simeq 21.89$$

$\therefore$ Mean deviation $= \underline{21.89}$

4b (iv) Median.

$$\text{Median} = L + \left(\frac{N/2 - Fb}{Fm}\right) i$$

L = Lower real limit of Median class
Fb = Summation of frequency below Median frequency
Fm = Frequency at Median class
i = Class Interval

$i = 10$, $\frac{N}{2} = \frac{100}{2} = 50$

Class = 61 − 70 , Fb = 44
L = 60.5 , Fm = 23

$$\therefore \text{Median} = 60.5 + \left(\frac{50 - 44}{23}\right) 10$$

$$= 63.10869 \simeq 63.11$$

$\therefore$ Median = 63.11

4(b) iii) Mean deviation

$$= \frac{\sum f|x - \bar{x}|}{N}$$

$$= \frac{2188.812}{100}$$

$$= 21.88812 \simeq 21.89$$

$\therefore$ Mean deviation $= 21.89$

4b (iv) Median.

$$\text{Median} = L + \left(\frac{N/2 - Fb}{Fm}\right)i$$

L = Lower real limit of Median class
Fb = Summation of frequency below Median frequency
Fm = Frequency at Median class
i = Class Interval

$i = 10$, $N/2 = 100/2 = 50$

Class = 61 – 70 , $Fb = 44$
$L = 60.5$, $Fm = 23$

$$\therefore \text{Median} = 60.5 + \left(\frac{50 - 44}{23}\right)10$$

$$= 63.10869 \simeq 63.11$$

$\therefore$ Median $= 63.11$

Extract 4.1: *A correct response from one of the candidates.*

On the other hand, the majority of the candidates who did part (a) of this question badly could not produce the equation $5t + 75 = 100$. As a result, they got an incorrect value of t, wrong data in the frequency distribution table and wrong values of the measures tested in part (b). In addition, most of the candidates used wrong formulae to calculate the median and the mean deviation. Thus,

$$Median = L + \left(\frac{\frac{N}{2} + \sum f_b}{f_m} \right) c, \quad Mean\ deviation = \frac{\sum f(X - \bar{X})}{N} \quad \text{and} \quad Mean\ deviation = \frac{\sum f(X - \bar{X})^2}{N}$$

were common in the candidates' responses. Some of the candidates calculated the upper boundaries of class intervals instead of class marks, while others used the formula of mode to calculate the median. Moreover, other candidates used the correct formula but did not identify the median class and the right class size *c*. For example, they wrote $c = 70 - 61 = 9$, instead of writing c as $70.5 - 60.5 = 10$. It is surprising, however, that several candidates did not convert the final answers to two decimal places as instructed. Extract 4.2 is an example of an incorrect response from a candidate who got the question wrong.

Extract 4.2

$F = t - 2 + 1 + 20 + t + 2 + t + t + 3 + 23 + 11 + t + 4 + 13$

$F = 5t + \cancel{69}\ 76$

$0 = 5t + 69\ 76$

$\cancel{\frac{5t}{5} = \frac{69}{5}}$

$\frac{5t}{5} = \frac{76}{5}$

$t = 15.2$

a) The value of $t = 15.2$

Extract 4.2: *An incorrect response from one of the candidates.*

2.1.5 Question 5: Sets

This question had parts (a), (b) and (c). In part (a), the candidates were required to simplify $A - (A - B)$ using properties of sets. Part (b) required the candidates to shade the set $A' \cap (B - C)$. Part (c) of the question read "In a survey of 500 movie viewers, 250 were listed as liking ‘zecomedy’, 200 as liking ‘zembwela’ and 85 were listed as liking both ‘zecomedy’ as well as ‘zembwela’.” Thus were required to find the number of people who were liking neither ‘zecomedy’ nor ‘zembwela’ by using an appropriate formula.

The question was attempted by 99.0 percent of the candidates, out of whom 8.5 percent scored from 0 to 3 marks, 22.5 percent from 3.5 to 5.5 marks and 69.0 percent from 6.0 to 10. The candidates’ performance was good, as 91.5 percent of the candidates scored more than 3 marks. Figure 6 is a summary of the candidates’ performance in this question.

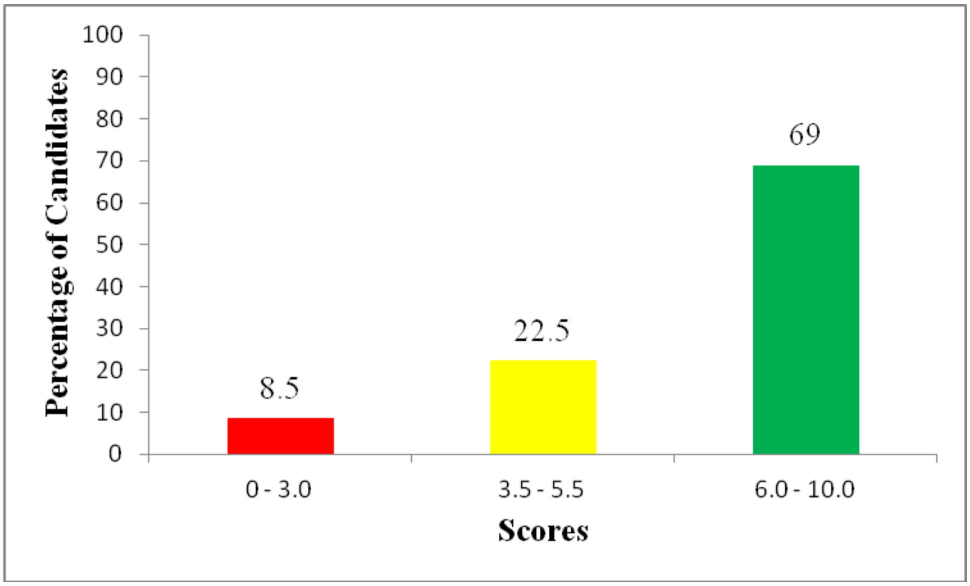


Figure 6: *Candidates’ performance in question 5.*

As indicated in Figure 6, the majority of the candidates had good performance. These candidates simplified $A-(A-B)$ into $A\cap B$ using the laws of sets in part (a). In part (b), most candidates presented accurate Venn diagrams of $A'\cap(B-C)$ which were correctly labelled and shaded. In part (c), the candidates identified the number of movie viewers, the number of people liking ‘zecomedy’, ‘zembwela’ as well as the number of people liking both ‘zecomedy’ and ‘zembwela’. They substituted the numbers correctly into the formulae $n(A'\cap B')=n(U)-n(A)-n(B)+n(A\cap B)$ and obtained 135, which was the number of people liking neither ‘zecomedy’ nor ‘zembwela’. Extract 5.1 is a sample response taken from the answer booklet of a candidate who answered the question correctly.

Extract 5.1

5 (a) Soln.

$A - (A - B)$

let $P = A - B$

$\Rightarrow P = A \cap B'$ – definition of $x - y = x \cap y'$

$\Rightarrow A - P = A \cap P'$ – definition of $x - y = x \cap y'$

$\Rightarrow A - (A - B) = A \cap (A \cap B')'$

$= A \cap (A' \cup B)$ – De morgan's law

$= (A \cap A') \cup (A \cap B)$ – Distributive law

$= \phi \cup (A \cap B)$ – Compliment law

$= A \cap B$ – Identity law

$\therefore A - (A - B) = A \cap B$

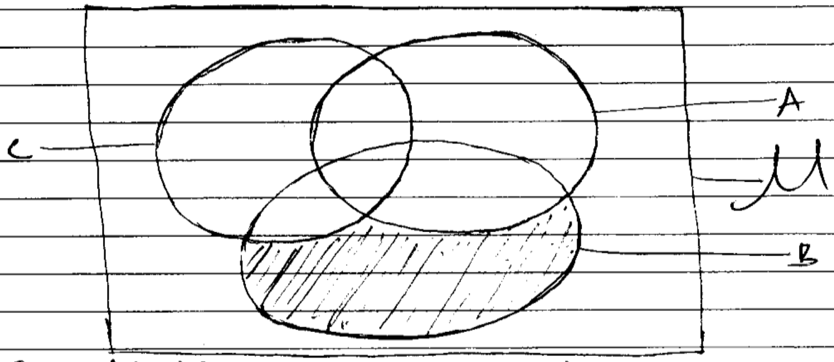
(b) Required: To shade $A' \cap (B - C)$

but

$B - C = B \cap C'$ – definition of $x - y = x \cap y'$

$\Rightarrow A' \cap (B - C) = A' \cap (B \cap C') = (A \cup C)' \cap B$

Consider a venn diagram for set A, B and C



$\therefore A' \cap (B - C)$ is shaded.

$n(B) = 200$

$n(A \cap B) = 85$

Required: $n(A \cup B)'$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 250 + 200 - 85$

$= 365$

$\Rightarrow n(A \cup B)' = n(\mu) - n(A \cup B)$

$= 500 - 365$

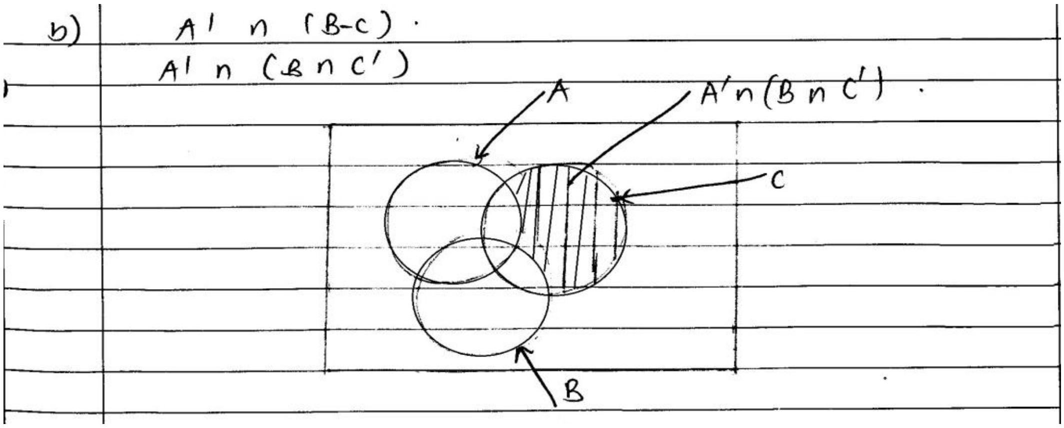
$= 135$ viewers

$\therefore$ people liking neither zeco medy nor zembela = 135 people.

Extract 5.1 *A correct response from one of the candidates.*

Despite the fact that many candidates had good performance, a total of 1016 candidates (8.5%) scored from 0 to 3 marks, out of 10 marks. These candidates faced the following challenges. In part (a), some of them presented $A-(A-B)$ as $A \cup (A \cup B')'$, instead of presenting it as $A \cap (A \cap B')'$. They should have known that $A - B = A \cap B'$. Others used logic symbols such as $A \wedge \sim(\sim A \vee B)$ to simplify $A-(A-B)$. This means they lacked knowledge of the tested topic. In part (b), most of the candidates presented $A' \cap (B-C)$ as $A' \cap (B \cap C')$ but could not shade the right region. Extract 5.2 illustrates such case. Moreover, several candidates could not draw the intersecting circles of Venn diagrams and others did not enclose the circles using either a rectangle or a square, i.e. they left them hanging. In part (c), some of the candidates used Venn diagrams to solve the problem, although the question required them to use an appropriate formula to do so. Others found the number of viewers who liked either 'zecomedy' or 'zembwela' but did not proceed further to the final answer. The candidates were required to insert the number of viewers liking either 'zecomedy' or 'zembwela' into the formula $n(A \cup B)' = n(\cup) - n(A \cup B)$ so as to obtain 135.

Extract 5.2



Extract 5.2: *An incorrect response from one of the candidates.*

2.1.6 Question 6: Functions

The question had parts (a) and (b). In part (a), the candidates were required to (i) show that $f \circ g(t)$=g$\circ f(t)$ given the functions $f(t) = e^t$ and $g(t) = \ln t$ and (ii) find the values of a and b if $f(t) = at, g(t) = bt^2 + 3$, $(f \circ g)(2) = 35$ and $(g \circ f)(3) = 75$. In part (b), the candidates were given $f(x) = \dfrac{x^3}{1-x^2}$ and were required to (i) find the horizontal and vertical asymptotes of $f(x)$, (ii) sketch the graph of $f(x)$ and (iii) state the domain and range of the function $f(x)$.

The question was attempted by 98.9 percent of the candidates, out of whom 14.5 percent scored from 0 to 3 marks and 0.8 percent scored a 0 mark. Notably, the percentage of the candidates who scored more than 3 marks is 85.5, which indicates that the candidates' general performance in this question was good.

This performance is a result of the candidates' ability to evaluate the expressions for composite functions $fog(t)$ and $gof(t)$ from $f(t) = e^t$ and $g(t) = \ln t$ to get t and to write the statement $fog(t) = gof(t) = t$ in order to conclude the verification. The candidates who did part (a) (ii) well formulated the composite function $fog(t) = abt^2 + 3a$ from $f(t) = at$ and $g(t) = bt^2 + 3$, formulated two equations

$35 = 4ab + 3a$ and $3ab + a = 25$ from $fog(2) = 35$ and $gof(3) = 75$, and solved the equations simultaneously to get $a = 1$ and $b = 8$. The candidates who attempted part (b) (i) could divide the numerator by the denominator of the rational function $\frac{x^3}{1-x^2}$ to get an oblique asymptote of $y = -x$ and solve the quadratic equation $1 - x^2 = 0$ to get vertical asymptotes $x = 1$ and $x = -1$. In part (b) (ii), the candidates sketched the dotted lines of asymptotes on an xy plane and traced the path of the $f(x)$ in all three regions of the xy plane, namely $x < -1$, $-1 < x < 1$ and $x > 1$. In part (b) (iii), they stated the domain and range of the function correctly.

Extract 6.1

6 (a)	(ii) $f(t) = at$, $g(t) = bt^2 + 3$
	find fog.
	$fog = a(bt^2 + 3)$
	$= abt^2 + 3a$.
	but $fog(2) = 35$.
	$35 = ab(2)^2 + 3a$.
	$35 = 4ab + 3a$. - - (i)
	also $fog(3) = 75$.
	$75 = ab(3)^2 + 3a$.
	$75 = 9ab + 3a$ - - (ii)
	Solve simultaneously
	$- \begin{cases} 9ab + 3a = 75 \\ 4ab + 3a = 35 \end{cases}$
	$5ab = 40$
	$ab = 8$.
	$a = 8/b$. take eqn (i)
	$35 = 4(2/b)\,b + 3(8/b)$
	$35 = 32 + \frac{24}{b}$.
	$35b = 32b + 24$
	$3b = 24$
	$b = 8$ - - (iii)
	and $a = 8/b$
	$a = 8/8 = 1$ - - (iv)
	$\therefore a = 1$ and $b = 8$.

6(b) $f(x) = \frac{x^3}{1-x^2}$

i) Horizontal asymptotes:

$y = \frac{x^3}{1-x^2}$

$y = \frac{x^3/x^2}{1/x^2 - x^2/x^2}$

$y = \frac{x}{1/x^2 - 1}$

as $x \to \infty$

$y = \frac{x}{0-1}$

$\underline{y = -x}$

vertical asymptotes:

let D(denominator (x)) = 0

$1 - x^2 = 0$

$x^2 = 1$

$x = \pm 1$

$\underline{x = 1 \text{ or } x = -1}$

find x and y intercept

x-intercept, $y = 0$.

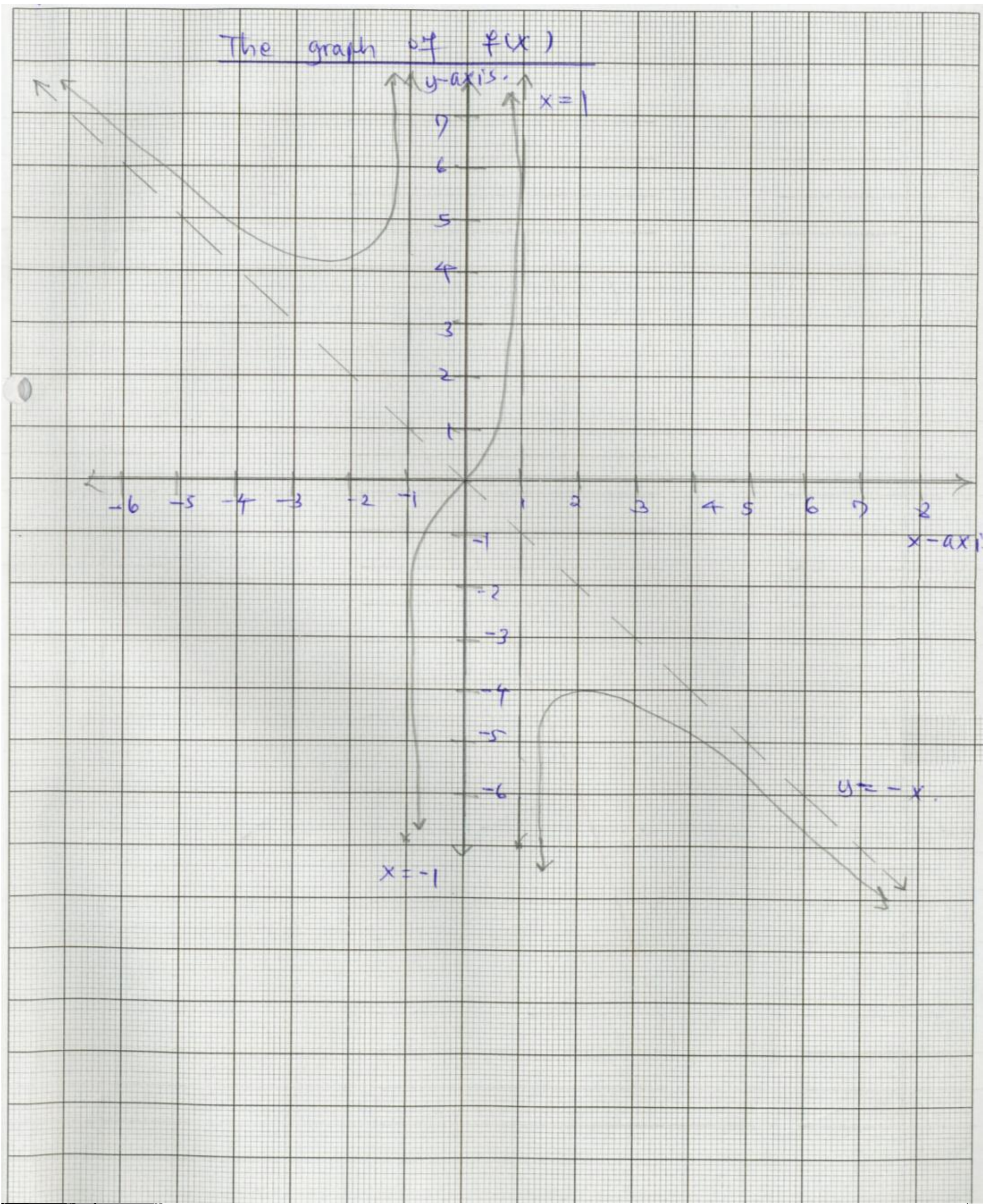
$0 = \frac{x^3}{1-x^2}$

$x = 0$ x-intercept, $(0, 0)$

y-intercept, $x = 0$

$y = \frac{0}{1-0} = 0$.

y intercept $(0, 0)$

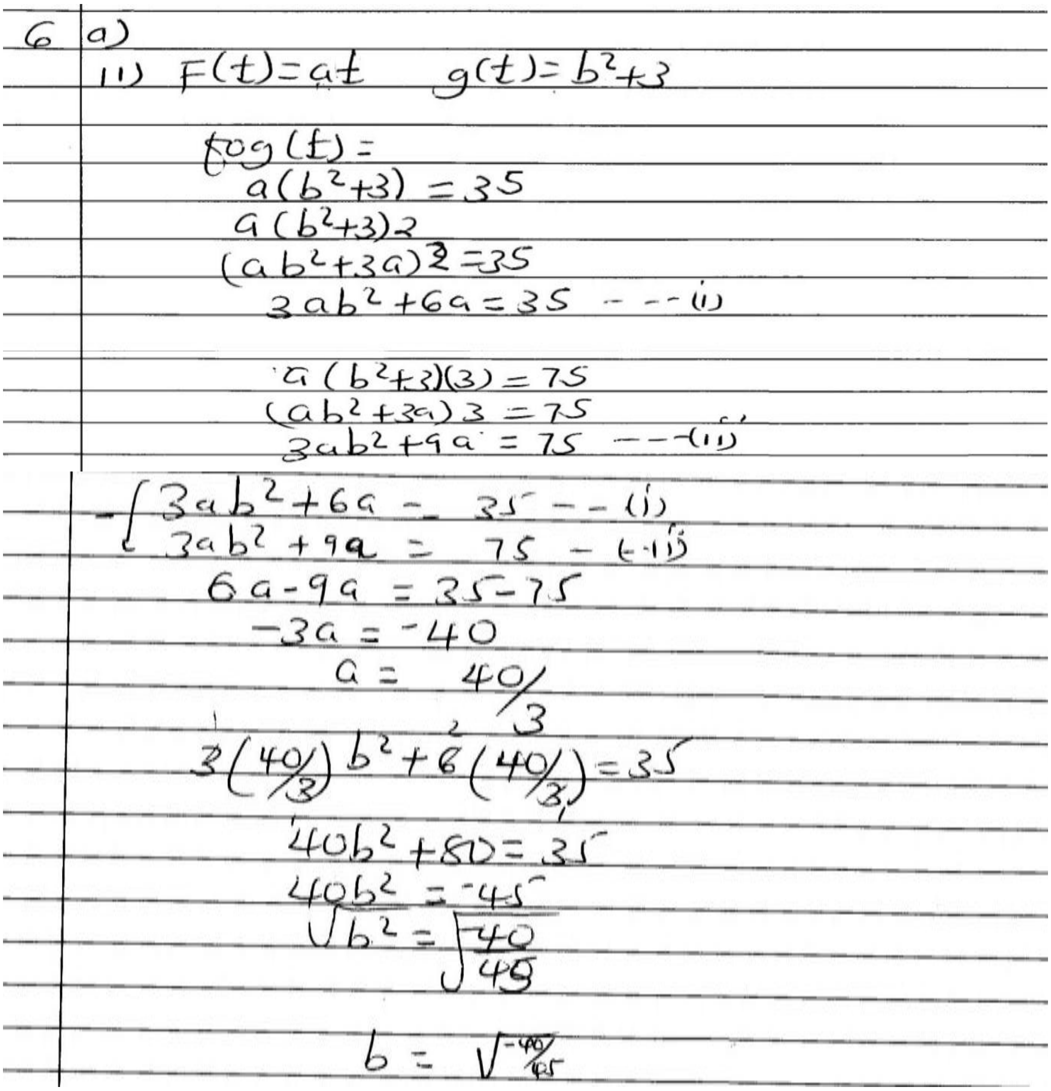


6 (b)	(ii) A graph of $f(x)$
	(iii) Domain = {all real numbers, $x \neq 1$ and $x \neq -1$}
	Range = {all real numbers}.

Extract 6.1 *A correct response from one of the candidates.*

On the other hand, 91 candidates (0.8%) scored zero. The analysis shows that such candidates committed various serious errors. In part (a) (i), a significant number of the candidates arrived at the expressions $e^{\ln t}$ and $\ln e^t$ but could not simplify them so as to get the lowest term of t. These candidates could not apply the laws of exponents and logarithms. Further analysis shows that other candidates expressed $f \circ g(t)$ as $(e^t)(\ln t)$ and $g \circ f(t)$ as $(\ln t)(e^t)$, which means they could not manipulate composite functions. Several candidates lost some or all the marks in part (b) (ii) because they committed minor errors in formulating $f \circ g(t)$ or substituting $f \circ g(2) = 35$ or $f \circ g(3) = 75$ with $f \circ g(t)$ and in solving the simultaneous equations $35 = 4ab + 3a$ and $9ab + 3a = 75$. The poor performance in part (b) (i) was due to the candidates' inability to divide the numerator and the denominator by x^2, to understand the concept of limits of $f(x)$ as x becomes positively or negatively large or to treat $1 - x^2 = 0$ as a quadratic equation. Hence, they ended up with $x = 1$ and ignored $x = -1$. Some of the candidates did not appreciate the classification of asymptote because they classified the oblique asymptote $y = -x$ as a horizontal asymptote. The errors in part (b) (ii) were caused by the unclear answers in part (b) (i). Furthermore the candidates could not plot the dotted lines of asymptotes. In part (b) (iii), the mathematical language usually used to state the domain and range of $f(x)$ was a major challenge among the candidates. Extract 6.2 is a solution of one of the candidates who got the question wrong.

Extract 6.2



6 a)

ii) $F(t)=at$ $\quad g(t)=b^2+3$

$fog(t)=$

$a(b^2+3)=35$

$a(b^2+3)2$

$(ab^2+3a)2=35$

$3ab^2+6a=35$ ---(i)

$a(b^2+3)(3)=75$

$(ab^2+3a)3=75$

$3ab^2+9a=75$ ---(ii)

$-\begin{cases}3ab^2+6a=35 \text{ --(i)}\\ 3ab^2+9a=75 \text{ --(ii)}\end{cases}$

$6a-9a=35-75$

$-3a=-40$

$a=\frac{40}{3}$

$3\left(\frac{40}{3}\right)b^2+6\left(\frac{40}{3}\right)=35$

$40b^2+80=35$

$40b^2=-45$

$\sqrt{b^2}=\sqrt{\frac{40}{45}}$

$b=\sqrt{\frac{-40}{45}}$

Extract 6.2: *An incorrect response from one of the candidates.*

2.1.7 Question 7: Numerical Methods

This question had parts (a) and (b). In part (a), the candidates were required to approximate the value of $\int_3^7 \frac{1}{x-2}\,dx$ correct to four decimal places by using (i) the Trapezoidal rule with five ordinates and (ii) the Simpson's rule with five ordinates. In part (b), the candidates were required to evaluate the exact value of the integral $\int_3^7 \frac{1}{x-2}\,dx$ and then compare the answer with those found in part (a).

The question was attempted by 94.7 percent of the candidates, out of whom 19.0 percent scored from 0 to 3 marks, 13.1 percent from 3.5 to 5.5 marks and 67.9 percent from 6 to 10 marks. Generally, the candidates' performance was good, as 81 percent scored from 3.5 to 10 marks. Figure 7 is a summary of the candidates' performance in this question.

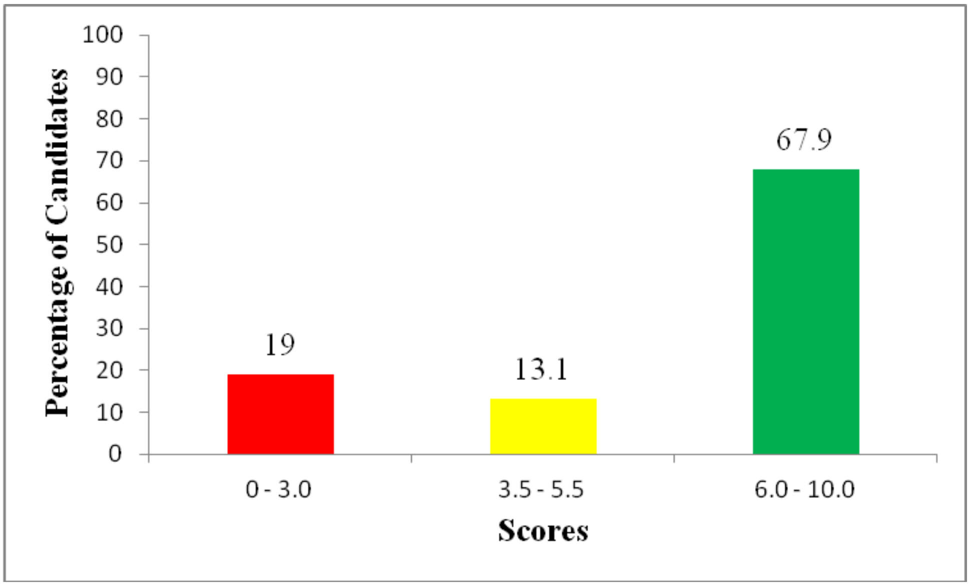


Figure 7: *Candidates' performance in question .*

The analysis shows that there were 2592 candidates who scored full marks. This indicates that they had good knowledge of numerical integration. These candidates demonstrated the following strengths: In part (a), most of them substituted $a = 3$, $b = 7$ and $n = 5$ into the formula $h = \frac{b-a}{n-1}$ to obtain the value of h as 1, constructed a table of values with correct entries for x and y, and used a table of values and the Trapezoidal rule $A = \frac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$ to approximate the value of $\int_3^7 \frac{1}{x-2}dx$ to four decimal places. Similarly, in part (b), most of the candidates were familiar with the Simpson's rule. Thus, they used the table of values in part (a) and the formula $A = \frac{h}{3}\left[y_0 + y_4 + 4\left(y_1 + y_3\right) + 2y_2\right]$ to approximate $\int_3^7 \frac{1}{x-2}\,dx$ to get 1.6222. In part (c), most of the candidates replaced $x-2$ with *u* and therefore it was easy for them to obtain 1.6094 as the exact value of $\int_3^7 \frac{1}{x-2}\,dx$. The candidates then evaluated the error in relation to the Simpson's rule and the Trapezoidal rule. Finally, they concluded that the Simpson's rule is more accurate than the Trapezoidal rule (see Extract 7.1).

Extract 7.1

7(a).

x	3	4	5	6	7
y	1	0.5	0.3333	0.25	0.2
	y_1	y_2	y_3	y_4	y_n

a (i) (i) By Trapezoidal rule.

$$A = \frac{d}{2}\left(y_1 + y_n + 2\sum \text{remaining ordinates}\right)$$

$$d = \frac{7-3}{\text{number of strips} = 4} = \frac{4}{4} = 1.$$

$d = 1$.

$$A = \frac{1}{2}\left(1 + 0.2 + 2\left(0.5 + 0.3333 + 0.25\right)\right)$$

$$A = \frac{1}{2}\left(3.3666\right) =$$

$A = 1.6833$.

$$\therefore \int_3^7 \frac{1}{x-2} = 1.6833.$$

a (ii) By simpson rule.

$$A = \frac{d}{3}\left(y_1 + y_n + 2\sum \underset{\text{ordinates}}{\text{odd}} + 4\sum \underset{\text{ordinates}}{\text{even}}\right)$$

$$A = \frac{1}{3}\left(1 + 0.2 + 2\left(0.3333\right) + 4\left(0.5 + 0.25\right)\right)$$

$A = \frac{1}{3}(1.2 + 0.6666 + 3)$.

$= \frac{1}{3}(4.8666)$.

$= 1.6222$.

$\therefore \int_3^7 \frac{1}{x-2} = 1.6222$.

(b) $\int_3^7 \frac{1}{x-2}\,dx$ let $u = x-2$. $\frac{du}{dx} = 1$.

$\int_3^7 \frac{1}{u}\,du = \left[\ln u\right]_3^7 = \left[\ln(x-2)\right]_3^7$

$= \left[\ln(5) - \ln 1\right] = 1.6094$.

$\therefore$ when Trapezoidal rule was used. the difference between Exact value. and that obtained by Trapezoidal rule was $1.6833 - 1.6094 = 0.0739$.

$\therefore$ When simpson rule was used the difference between Exact value and that obtained by simpson rule was $1.6222 - 1.6094 = 0.0128$.

Since deviation of the value obtained by simpson rule from Exact value is less than the ~~difference~~ deviation of Trapezoidal rule.

Simpson rule is more accurate than Trapezoidal rule.

Extract 7.1 *A correct response from one of the candidates.*

In spite of the good performance, 2169 candidates did not answer the question correctly. In parts (a) and (b), most of them did not subtract 1 from the number of ordinates so as to get the number of strips $(5–1=4)$. Thus, they used a formula like $h = \frac{b-a}{n}$ or $h = \frac{b-a}{n+1}$ instead of $h = \frac{b-a}{n-1}$ to find the width of the strips. Such candidates ended up getting either $h = 0.8$ or 0.6667, instead of getting $h = 1$.

Hence, they got incorrect approximations for $\int_3^7 \frac{1}{x-2}\,dx$ in both the Simpson's and the Trapezoidal computations, as Extract 7.2 shows. Further analysis indicates that some of the candidates used wrong formulae, such as $A = \frac{h}{2}[y_0 + y_4 + (y_1 + y_2 + y_3)]$ and $A = \frac{h}{3}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$, to approximate the value of $\int_3^7 \frac{1}{x-2}\,dx$, and that others computed the value of *h* and the table of values correctly but interchanged the Simpson's rule with the Trapezoidal rule during the computations. Also, several candidates computed $\int_3^7 \frac{1}{x-2}\,dx$ correctly but failed to approximate their answers to four decimal places as instructed. In addition, many candidates did not evaluate the value of *h* between the indicated limits of integration. This is because they started at $x = 0$, instead of starting at $x = 3$, possibly from the y_0 in the formulae. In part (c), some of the candidates could not find the actual value of $\int_3^7 \frac{1}{x-2}\,dx$. This means they lacked the knowledge and skills for doing integration using substitution techniques. Moreover, some of the candidates compared the value obtained by using the Trapezoidal rule with the value obtained by using the Simpson's rule, instead of comparing both values to the exact value of the definite integral.

Extract 7.2

7(a) $\int_3^7 \frac{1}{x+2}\,dx$

X	3	3.8	4.6	5.4	6.2	7.
y	1	0.5556	0.3846	0.2941	0.2381	0.2.

$d = \frac{7-3}{5} = 0.8.$

$d = 0.8$

(i) By trapezoidal Method

$= \frac{1}{2} d\left[y_0 + y_5 + 2(y_2 + y_3 + y_4)\right]$

$= \frac{1}{2} \times 0.8 \times \left[1 + 0.2 + 2(0.5556 + \ldots 0.2381)\right]$

$= 1.6579$

By Simpson's Method

$= \frac{1}{3} d\left[y_0 + y_5 + 4(y_1 + y_3) + 2(y_2 + y_4)\right]$

$= \frac{1}{3} \times 0.8\left[1 + 0.2 + 4(0.5556 + 0.2941) + 2(0.2381\right]$

$= 1.5584$

Extract 7.2: *An incorrect response from one of the candidates.*

2.1.8 Question 8: Coordinate Geometry I

This question had parts (a), (b) and (c). In part (a), the candidates were instructed that the circles $x^2 + y^2 - 2y - 8 = 0$ and $x^2 + y^2 - 24x + hy = 0$ cut orthogonally and were asked to determine the value of h. In part (b), the candidates were required to find the equation of normal to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ passing through the point $K(7,4)$. In part (c), the candidates were asked to calculate the area of the triangle whose vertices were the points $K(3,5)$, $K(4,2)$ and $K(6,3)$.

The question was attempted by 10,894 candidates (90.3%). Of them, 50.0 percent did well, 4.6 percent scored all 10 marks, 29.9 percent had average performance with scores ranging from 3.5 to 5.5, 20.2 percent did badly and 6.0 percent got 0. Generally, the candidates had good performance in this question. Figure 8 is a summary of the candidates' performance.

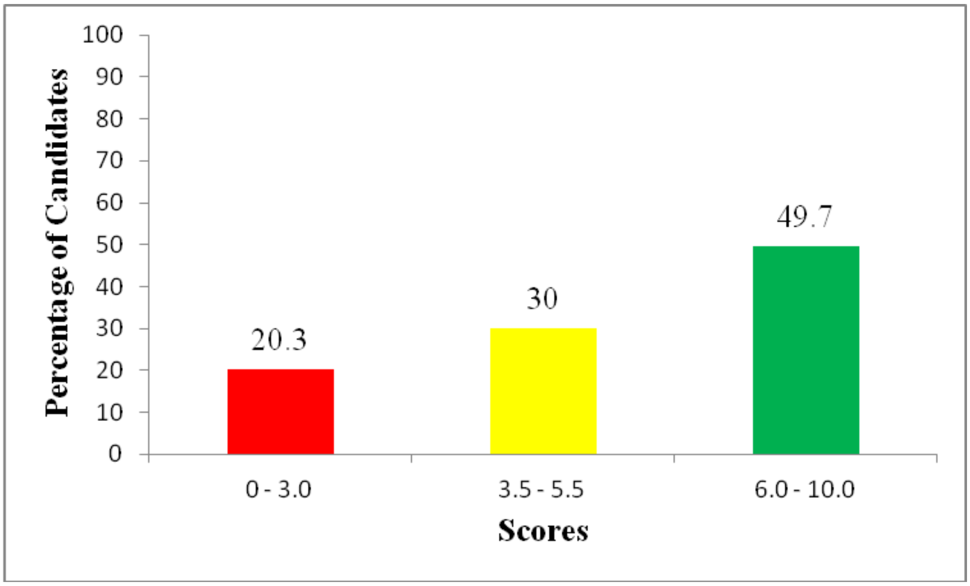


Figure 8: *Candidates' performance in question 8.*

Part (a) was attempted by a large number of candidates. The two formulae for finding h which were given in the marking scheme were $d^2 = r_1^2 + r_2^2$ and $c_1 + c_2 = 2f_1 f_2 + 2g_1 g_2$. The majority of the candidates preferred the latter to the former. The candidates who attempted part (a) correctly determined the values of c_1, g_1, f_1 and c_2, g_2, f_2 from circles $x^2 + y^2 - 2y - 8 = 0$ and $x^2 + y^2 - 24x + hy = 0$. After appropriate substitution, they got $h = 8$. The candidates who did part (b) very well could find the centre of the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ as (2, 3). They also obtained the equation of normal using centre (2, 3) and the point $K(7,4)$ as $x - 5y + 13 = 0$. The candidates who did part (c) well used either the formula $A = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$ or

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} \quad \text{or} \quad A = \frac{1}{2}\left\|\begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix}\right\|$$

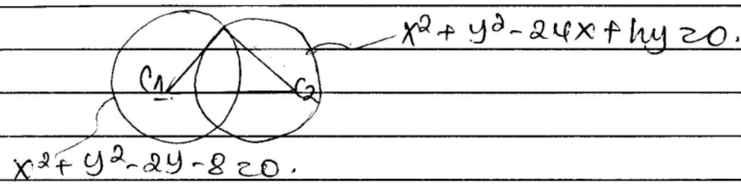
to calculate the area of a triangle with $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. A sample solution from a candidate who performed the question well is given in Extract 8.1.

Extract 8.1

8. a)   soln.

Circle 1 : $x^2 + y^2 - 2y - 8 = 0$

circle 2 : $x^2 + y^2 - 24x + hy = 0$.



For onthogonal circles.

$C_1 + C_2 = 2(g_1g_2 + f_1f_2)$.

From circle 1 : $x^2 + y^2 - 2y - 8 = 0$

$-2fy = -2y$

$2f_1 = 2$.

$f_1 = 1$.

$g_1 = 0$.

$2f_1 = 2.$

$f_1 = 1.$

$g_1 = 0.$

$c_1 = -8$

From circle 2; $x^2+y^2-24x+hy=0$

$-2gx = -24x$

$2g_2 = 24$

$g_2 = 12$

$f_2 =$

$-2fy = hy.$

$-2f_2 = h$

$f_2 = -h/2.$

$c_2 = 0$

From;

$c_1 + c_2 = 2(g_1g_2 + f_1f_2)$

$-8 + 0 = 2(0\times 12 + (1\times -h/2))$

8.(a) $-8 = 2(-h/2)$

$-8 = -h$

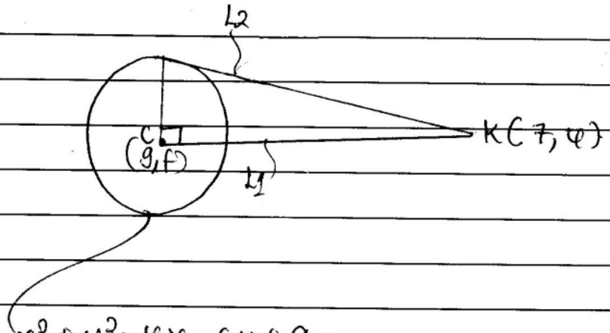
$8 = h$

$\therefore$ value of h is 8.

8 (b) Data

point $K(7,4)$

equation circle $= (x^2+y^2-4x-6y+9) = 0.$



$x^2+y^2-4x-6y+9=0.$

$-2fy = -6y.$

$-f = -3.$

$-f = -3.$

$f = 3.$

$-2gx = -4x$

$-2g = -4$

$g = 2.$

$m = \frac{\Delta y}{\Delta x}.$

$m_1 = \frac{\Delta y}{\Delta x}.$

points $C(2,3)$ and $R(7,4)$

$m_1 = \frac{4-3}{7-2}$

$m_1 = \frac{1}{5}.$

8 (b). From linear equation.

$m = \frac{y_0 - y_1}{x_0 - x_1}$

$\frac{1}{5} = \frac{y-4}{x-7}$

$\frac{1}{5} = \frac{y-4}{x-7}$

$x - 7 = 5y - 20$

$5y - x = 20 - 7$

$5y - x = 13.$

$5y - x = 13$

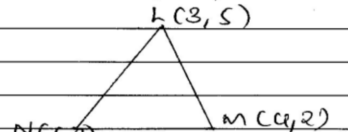
$x + 13 - 5y = 0$

$\underline{x - 5y + 13 = 0}$

$\therefore$ the equation of normal line is $x - 5y + 13 = 0$

8 (c). Required triangle's area.

points $L(3,5)$, $M(4,2)$ and $N(6,3)$.



$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} 3 & 5 \\ 4 & 2 \\ 6 & 3 \\ 3 & 5 \end{vmatrix}$$

8 (c) $A = \frac{1}{2}\left[\left((4\times5)+(6\times2)+(3\times3)\right] - \left((3\times2)+(4\times3)+(6\times5)\right)\right]$

$A = \frac{1}{2}\left[(20+12+9) - (6+12+30)\right]$

$A = \frac{1}{2}\left[41 - 48\right]$.

$A = \frac{1}{2}\left|-7\right|$

$A = \frac{1}{2}\times 7$

$A = 3.5$ square units

$\therefore$ Area of triangle is 3.5 square units.

Extract 8.1: *A correct response from one of the candidates.*

On the other hand, a significant number of the candidates got low marks. In part (a), they used wrong formulae, such as $c_1c_2 = r_1^{\,2} + r_2^{\,2}$, $c_1c_2 = r_1 + r_2$, $r_1 = r_2$, $c_1c_2 = 2f_1f_2 + 2g_1g_2$ and $c_1 - c_2 = 2g_1g_2 - 2f_1f_2$, to find *h*. Though the sub-section of the question was not too demanding, some of the candidates derived the formula $r_1^{\,2} + r_2^{\,2} = \overline{c_1c_2}^{\,2}$ from the general equation of the circle (this was wastage of time!) and others subtracted the equation of one circle from the other. Hence, they ended up with the equation of the common chord, which was not required. In part (b), most of the candidates substituted $\text{K}(7,4)$ to the expression $\frac{dy}{dx} = \frac{4-2x}{2y-6}$ which they had obtained by differentiating the equation of the circle $x^2 + y^2 - 4x - 6y + 9 = 0$. This might have been correct if $\text{K}(7,4)$ satisfied the equation $x^2 + y^2 - 4x - 6y + 9 = 0$. But the point (7,4) did not do so. The candidates should have known that the normal lines to a circle pass through the centre of the circle. In part (c), several candidates scored low marks because they did not relate the distance between two points and the perpendicular distance of a point to a line so as to compute the area of the given triangle. Furthermore, most of the candidates regarded the given triangle as being right-angled, although it was not. Such candidates used only the distance formula and $Area = \frac{1}{2}\times base \times height$

to find the area of the triangle. Hence, they got incorrect values of the area, including 5.7009 square units, $2\sqrt{5}$ square units and $\sqrt{26}$ square units, rather than getting 3.5 square units. Other candidates used the following incorrect formulae:

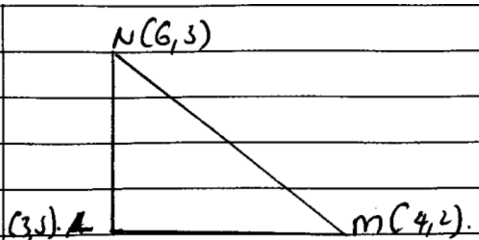
$$A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}, \qquad A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ 1 & 1 \end{vmatrix}, \qquad A=\begin{Vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{Vmatrix} \text{ and}$$

$$A=\frac{1}{2}\left|x_1(y_2-y_3)+y_1(x_2-x_3)+1(x_2y_3-x_3y_2)\right|.$$

Extract 8.2 is a sample solution from a candidate who got the question wrong.

Extract 8.2

8(c).



Area = $\frac{1}{2}$ (base x height).

Base = ~~AB~~ $\overline{AM}$

height = $\overline{AN}$

Distance = $(x_1-x_2)^2 + (y_1-y_2)^2$

$\overline{AM} = (4-1)^2 + (2-5)^2$

$\overline{AM} = 9 + 9$

$\overline{AM} = 18$.

$\overline{AN} = (6-3)^2 + (3-5)^2$.

$= 9 + 4$

$\overline{AN} = 13$

Area = $\frac{1}{2}(18 \times 13)$.

Area = 117 unit^2.

Extract 8.2: *An incorrect response from one of the candidates.*

2.1.9 Question 9: Integration

The question required the candidates to (a) find the value of $\int \frac{x-2}{(x^2+2)(x+1)} dx$, (b) evaluate $\int_0^{\frac{5}{3}\pi} \frac{\tan x + \sin x}{\cos x} dx$, (c) (i) derive the arc length formula for the curve AB from $x = a$ to $x = b$, given that A and B are any two points on the graph of $y = f(x)$ and (c) (ii) find the length of a curve $y = \frac{3}{4}x$ from $x = 0$ to $x = 4$.

The analysis shows that almost half of the candidates (47.4%) got more than 3, out of 10 marks. Of these candidates, 24.3 percent scored from 3.5 to 5.5 marks. This indicates that the performance was average. Figure 9 is a summary of the candidates' performance in this question.

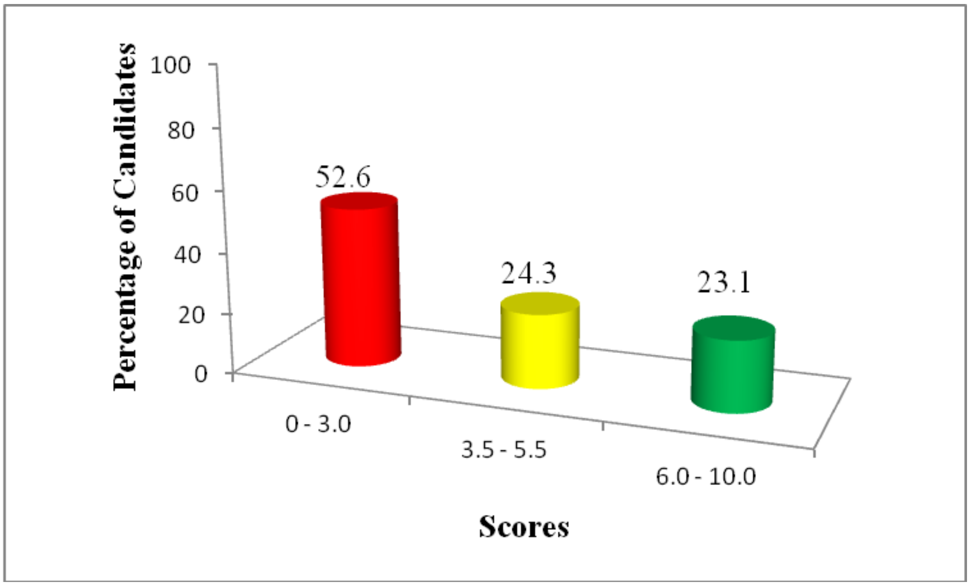


Figure 9: *Candidates' performance in question 9.*

The analysis shows that some of the candidates did the question well. In part (a), they decomposed a rational function $\frac{x-2}{\left(x^2+2\right)\left(x+1\right)}$ into the difference of its partial fractions $\frac{x}{x^2+2}$ and $\frac{1}{x+1}$. The candidates also used the substitutions x^2+1 and $x+1$ to integrate the integrals $\int \frac{x}{x^2+2}dx$ and $\int \frac{1}{x+1}dx$, thus ending up with $\ln\left(\frac{\sqrt{x^2+2}}{x+1}\right)+C$ as the final answer. In part (b), the candidates presented $\int_0^{\frac{5}{3}\pi} \frac{\tan x+\sin x}{\cos x}dx$ as $\int_0^{\frac{5}{3}\pi} \frac{\sin x}{\cos^2 x}dx+\int_0^{\frac{5}{3}\pi} \frac{\sin x}{\cos x}dx$. In the next step, they used the substitution method by replacing $\cos x$ with u to find $\int_0^{\frac{5}{3}\pi} \frac{\sin x}{\cos^2 x}dx+\int_0^{\frac{5}{3}\pi} \frac{\sin x}{\cos x}dx$. Thus, they got 1.693 as the final answer. In part (c) (i), the candidates used the graphical sketch of the continuous function $y=f(x)$ between $x = a$ and $x = b$ and the Pythagoras theorem to derive the length of a curve in cartesian form as $s=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$. Moreover, the use of the formula from part (c) (i) and careful differentiation of $y=\frac{3}{4}x$ with respect to x produced 5 units as the length of the curve in part (c) (ii). Extract 9.1 is a sample answer from one of the candidates.

Extract 9.1

9. (a) let $I = \int \frac{x-2}{(x^2+2)(x+1)}dx$

let $\frac{x-2}{(x^2+2)(x+1)} = \frac{Ax+B}{x^2+2} + \frac{C}{x+1}$

$\Rightarrow x-2 = (Ax+B)(x+1) + C(x^2+2)$

9. $x-2 = (Ax+B)(x+1) + C(x^2+2)$

put $x = -1$, $-3 = 0 + 3C$

$\Rightarrow C = -1$

put $x = 0$, $-2 = B + 2C$

$\Rightarrow -2 = B - 2$

$\Rightarrow B = -2+2 = 0$

Comparing coefficients of x^2

$0 = A + C \Rightarrow A = -C = 1$

$\therefore \dfrac{x-2}{(x^2+2)(x+1)} = \dfrac{x}{x^2+2} - \dfrac{1}{x+1}$

$I = \int \left(\dfrac{x}{x^2+2} - \dfrac{1}{x+1} \right) dx$

$I = \dfrac{1}{2} \int \dfrac{2x}{x^2+2}\, dx - \int \dfrac{1}{x+1}\, dx$

$I = \dfrac{1}{2} \ln (x^2+2) - \ln (x+1) + C$

$\therefore \int \dfrac{x-2}{(x^2+2)(x+1)}\, dx = \dfrac{1}{2} \ln (x^2+2) - \ln (x+1) + C$

(b) let $I = \int_0^{5/3\pi} \dfrac{\tan x + \sin x}{\cos x}\, dx$

$I = \int_0^{5/3\pi} \left(\dfrac{\tan x}{\cos x} + \dfrac{\sin x}{\cos x} \right) dx$

$I = \int_0^{5/3\pi} \left(\sec x \tan x + \dfrac{\sin x}{\cos x} \right) dx$

9. $I = \Big[\sec x - \ln (\cos x) \Big]_0^{5/3\pi}$

$I = \left(2 - \ln \dfrac{1}{2} \right) - (1 - \ln 1)$

$I = (2 + \ln 2) - (1 - 0)$

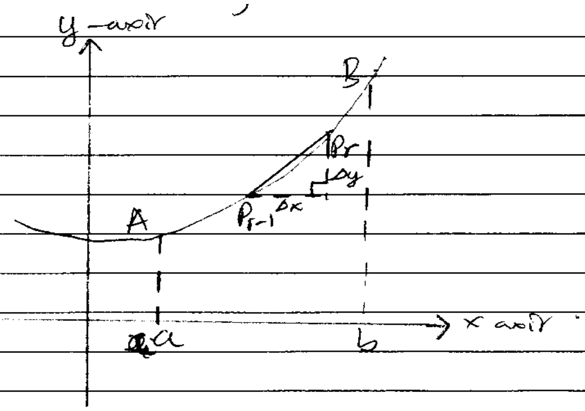
$I = 1 + \ln 2 \simeq 1.6931$

$I = 1.6931$

$\therefore \int_0^{5/3\pi} \left(\dfrac{\tan x + \sin x}{\cos x} \right) dx = \underline{1.6931}$

(c) (i) consider the diagram below

y-axis



Suppose ~~am~~ an arc AB is divided into ~~small~~ large number of ~~ss~~ chords with very small length

Then length of arc AB $\longrightarrow$ Summation of length of chords

9. (c) (i) from the figure above by using pythagoras theorem length of chord $P_{r-1}P_r$ is given by $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

$$\Rightarrow P_{r-1}P_r^2 = (\Delta x)^2 + (\Delta y)^2$$

$$\cancel{\Delta r} \; P_{r-1}P_r^2 = \left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right](\Delta x)^2$$

$$\Rightarrow P_{r-1}P_r = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \cdot \Delta x$$

as $\Delta x \to 0$, $\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$.

length arc ~~a~~ AB is given by summation of these small chords from $x = a$ to $x = b$. This is done by integration (since $\Delta x \to 0$).

$\cancel{b}$ $\therefore$ length arc AB $(S) = \displaystyle\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

but $\frac{dy}{dx} = f'(x)$

$$\Rightarrow S = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

(ii) $y = \frac{3}{4}x \Rightarrow \frac{dy}{dx} = \frac{3}{4}$.

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{\frac{4^2+3^2}{4^2}}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \frac{5}{4}$$

9. (c) (ii)

$$\therefore S = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$S = \int_0^4 \frac{5}{4}\,dx$$

$$S = \left[\frac{5}{4}x\right]_0^4$$

$$S = 5 \text{ units}$$

$\therefore$ length of the curve is 5 units

Extract 9.1: *A correct response from one of the candidates.*

On the other hand, a total of 1388 candidates got 0 in this question. The responses of these candidates contained several mistakes. In part (a), the candidates expressed the integrand $\frac{x-2}{(x^2+2)(x+1)}$ in partial fractions, but failed to integrate it with respect to x. For example, some of the candidates wrote either $\frac{x-2}{(x^2+2)(x+1)} = \frac{A}{x^2+2} + \frac{B}{x+1}$ or $\frac{x-2}{(x^2+2)(x+1)} = \frac{Ax}{x^2+2} + \frac{B}{x+1}$ instead of $\frac{x-2}{(x^2+2)(x+1)} = \frac{Ax+B}{x^2+2} + \frac{C}{x+1}$. Surprisingly, many candidates split the integrand into its component fractions but failed to solve for the coefficients. Others worked out the value of the coefficients A, B and C successfully but could not integrate the resulting integrands using the substitution method. This indicates that they lacked adequate and concrete knowledge and skills pertaining to algebra as well as integration techniques. In part (b), most of the candidates did not simplify the integrand $\frac{\tan x + \sin x}{\cos x}$ before integrating it. Many candidates skipped part (c) (i) of the question. The candidates who attempted part (c) (ii) used incorrect formulae for the arc length, such as $s=\int_a^b\left(\sqrt{1-\left(\frac{dy}{dx}\right)^2}\right)dx$, $s=\sqrt{1-\left(\frac{dy}{dx}\right)}dx$ and $s=\int_a^b ydx$,

instead of using $s=\int_a^b\left(\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right)dx$ to find the length of an arc. Extract 9.2 illustrates one of such cases.

Extract 9.2

9 c 2) Arc length.

$\int_a^b y$.

Length $= \int_0^4 \frac{3}{4}x \cdot dx$.

Length $= \left[\frac{3}{4}\right]_4^0 \left\{\left[\frac{3}{4}\right]^4 - \left[\frac{3}{4}\right]^0\right.$

Length $= 3m$.

Extract 9: *An incorrect response from one of the candidates.*

2.1.10 Question 10: Differentiation

This question required the candidates to (a) find $\frac{dy}{dx}$ of the curve $x\sin y + y\cos x = 2$ at $x=\frac{\pi}{2}$ and $y=\pi$, (b) use the second derivative test to investigate the stationary values of $f(x)=2x^2-8x+5$ and (c) differentiate $f(x)=\frac{1}{2}\cos 3x$ from first principles.

The analysis shows that 10,124 candidates (83.9%) attempted this question. Out of these candidates, 34.2 percent scored from 0 to 3 marks, 29.7 percent from 3.5 to 5.5 marks and 36.1 percent from 6 to 10 marks. The candidates' performance was good, as 65.9 percent of the candidates scored more than 3 marks. Figure 10 is a summary of the candidates' performance in this question.

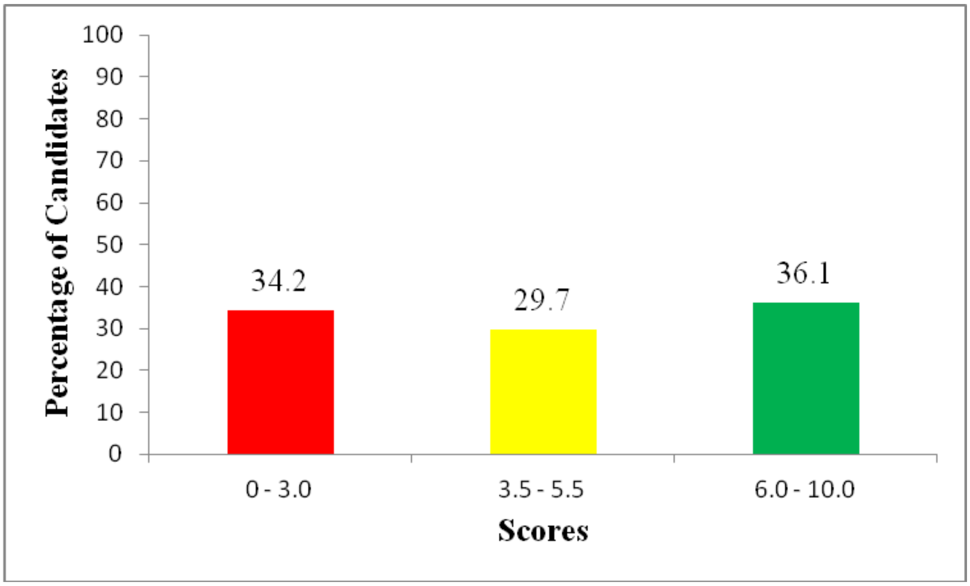


Figure 10: *Candidates' performance in question 10.*

The candidates with good performance answered two or three parts of the question correctly. In part (a), implicit differentiation was well handled as evidenced by most of the candidates' appreciation of the need to apply the product rule to differentiate $x\sin y + y\cos x = 2$ to get $\frac{dy}{dx} = \frac{y\sin x - \sin y}{x\cos y + \cos x}$. The candidates also substituted $x = \frac{\pi}{2}$ and $y = \pi$ into an expression for $\frac{dy}{dx}$ to give the gradient equal to -2. In part (b), most of the candidates differentiated $f(x)$ to get $f'(x) = 4x - 8$, set the derivative equal to zero and solved the equation $4x - 8 = 0$ so as to obtain the value of $x = 2$, substituted the value of *x* back into the original function $f(x)$ to obtain the corresponding y – coordinate as -3 and differentiated the derivative of $f(x)$ so as to arrive at $f''(x) = 4$. Since the second derivative was positive, the candidates concluded that $f(x)$ had the least value of -3. In part (c), many candidates noted that the definition of the derivative of a function $f(x)$ from first principles is written as $f'(x) = \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$. They used this formula and the factor formula to differentiate $f(x)=\frac{1}{2}\cos 3x$ and got $f'(x)=-\frac{3}{2}\sin 3x$.

Extract 10.1 is a sample solution from a candidate who did parts (a) and (b) of the question well.

Extract 10.1

10 a)

$$x\sin y + y\cos x = 2$$

$$x\cos y\frac{dy}{dx} + \sin y(1) + {}^{-}y\sin x + \cos x\frac{dy}{dx} = 0$$

$$x\cos y\frac{dy}{dx} + \cos x\frac{dy}{dx} = y\sin x - \sin y$$

$$\frac{dy}{dx}(x\cos y + \cos x) = y\sin x - \sin y$$

$$\frac{dy}{dx} = \frac{y\sin x - \sin y}{x\cos y + \cos x}$$

when $x = \pi/2$ and $y = \pi$

$$\frac{dy}{dx} = \frac{\pi\sin\pi/2 - \sin\pi}{\pi/2\cos\pi + \cos\pi/2}$$

$$\frac{dy}{dx} = \frac{\pi(1) - 0}{\pi/2(-1) + 0}$$

$$\frac{dy}{dx} = \frac{\pi}{-\pi/2}$$

$$\frac{dy}{dx} = -2$$

b)

$$f(x) = 2x^2 - 8x + 5$$

Let $y = f(x)$

then,

$$\frac{dy}{dx} = 4x - 8$$

For stationary point $\frac{dy}{dx} = 0$

$$0 = 4x - 8$$

$$4x = 8$$

$$x = 2$$

$$y = 2(2)^2 - 8(2) + 5$$

$$y = -3$$

$\therefore$ Stationary point $= (2, -3)$

10 b) $\frac{dy}{dx} = 4x - 8$

$\frac{d^2y}{dx^2} = 4$

$\frac{d^2y}{dx^2}$ is positive then the point is minimum

Extract 10.1: *A correct response from one of the candidates.*

On the other hand, there were several candidates who scored low marks. In part (a), some of such candidates did not know the steps followed in implicit differentiation because the function $x\sin y + y\cos x = 2$ cannot be defined explicitly as a function of *x*. Other candidates failed to substitute $x = \frac{\pi}{2}$ and $y = \pi$ into the gradient expression $\frac{dy}{dx} = \frac{y\sin x - \sin y}{x\cos y + \cos x}$ so as to get -2. In addition, a substantial number of the candidates did not differentiate the constant term, as Extract 10.2 shows. In part (b), a significant number of the candidates regarded the second derivative $f''(x) = 4$ as the y-coordinate. These candidates did not realise that they were required to substitute the value of *x* obtained previously back into the original equation in order to find the value of y. Others did not understand the question and as a result they ended up with the computations of the first and second derivatives only. This shows that most of the candidates did not know the steps of the second derivative method to investigate stationary values. In part (c), many candidates were not adequately knowledgeable about the steps for differentiating a function from first principles. These candidates did not know the appropriate formula for differentiating $f(x) = \frac{1}{2}\cos 3x$. They used the following formulae $f'(x) = \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$ or $f'(x) = \lim_{h\to 0}\left(\frac{f(x) - f(x+h)}{h}\right)$, rather than $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$. Others wrote the formula correctly but failed to manipulate the statement $f'(x)=\lim_{h\to 0}\frac{1}{2}\left(\frac{(\cos 3x+3h)-\cos 3x}{h}\right)$ to get $\lim_{h\to 0}\left(\frac{-\left(\sin\left(3x+\frac{3h}{2}\right)\sin\left(\frac{3h}{2}\right)\right)}{h}\right)$ because they were poor in using the factor formula. These candidates did not provide the derivative of $f(x)=\frac{1}{2}\cos 3x$ as $f'(x)=-\frac{3}{2}\sin 3x$.

Extract 10.2

10(a) solution.

Given: $x\sin y + y\cos x = 2 \quad x = \pi/2 \quad y = \pi$

$\cancel{x}$

$x\cos y\frac{dy}{dx} + \sin y + y(-\sin x) + \frac{dy}{dx}\cos x = 2.$

$x\cos y\frac{dy}{dx} + \frac{dy}{dx}\cos x + \sin y - y\sin x = 2.$

$\frac{dy}{dx}\frac{(x\cos y + \cos x)}{x\cos y + \cos x} = \frac{2 - \sin y + y\sin x}{x\cos y + \cos x}.$

$\frac{dy}{dx} = \frac{2 - \sin y + y\sin x}{x\cos y + \cos x}.$

But $x = \pi/2$ and $y = \pi$

$dy/dx = \frac{2 - \sin\pi + \cancel{y}\,\pi\sin\pi/2}{\pi/2\cos\pi + \cos\pi}.$

$= \frac{2 - 0 + \pi(1)}{\pi/2(-1) - 1}.$

$= \frac{2+\pi}{\pi/2 - 1}$

$= \frac{2+\pi}{\frac{\pi-2}{2}}.$

$= (2+\pi)2$

$$\frac{dy}{dx} = \frac{2 - \sin y + y\sin x.}{x\cos y + \cos x}$$

But $x = \pi/2$ and $y = \pi$

$$dy/dx = \frac{2 - \sin\pi + y\ \pi\sin\pi/2}{\pi/2\cos\pi + \cos\pi.}$$

$$= \frac{2 - 0 + \pi(1).}{\pi/2(-1) - 1.}$$

$$= \frac{2+\pi}{\pi/2 - 1}$$

$$= \frac{2+\pi}{\frac{\pi - 2.}{2}}$$

$$= \frac{(2+\pi)2}{\pi - 2}$$

$$= \frac{4 + 2\pi}{\pi - 2.}$$

$$\therefore \frac{dy}{dx} = \frac{4 + 2\pi}{\pi - 2.}$$

Extract 10.2: *An incorrect response from one of the candidates.*

2.2 142/2 ADVANCED MATHEMATICS 2

2.2.1 Question 1: Complex Numbers

This question comprised parts (a), (b), (c) and (d). In part (a), the candidates were required to prove that $\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$ using De Moivre's theorem. In part (b) (i), the candidates were instructed that the equation $6 - z^2 = 8i - (2+4i)z$ had roots z_1 and z_2. They were required to find z_2 in the form $a+bi$. In part (b) (ii), they were required to express $\frac{6}{x^2 - 2x + 10}$ in partial fractions with complex linear denominators. In part (c), the candidates were required to use mathematical induction to prove that $\mathrm{z}^{\mathrm{n}} = \mathrm{r}^{\mathrm{n}}(\mathrm{cosn\theta} + \mathrm{isinn\theta})$ where z stands for any complex number $r(\cos\theta + i\sin\theta)$ for all positive integers n. In part (d), the candidates were required to prove that z lies on a circle whose radius is $\frac{4}{3}$ if $|\mathrm{z}+1| = 2|\mathrm{z}-1|$.

The question was attempted by 11,536 candidates (95.6%), out of 12,065 candidates. Of such candidates, 18.3 percent had good performance. Out of these candidates, 0.5 percent scoring all 15 marks. Furthermore, 29.5 percent of the candidates who attempted the question scored from 5.5 to 8.5 marks and 52.2 percent did badly. Of the latter candidates, 7.0 percent got 0. Generally, the candidates' performance was average, as 47.8 scored more than 5 marks. Figure 11 summarizes some of the statistics for the candidates' good performance in this question.

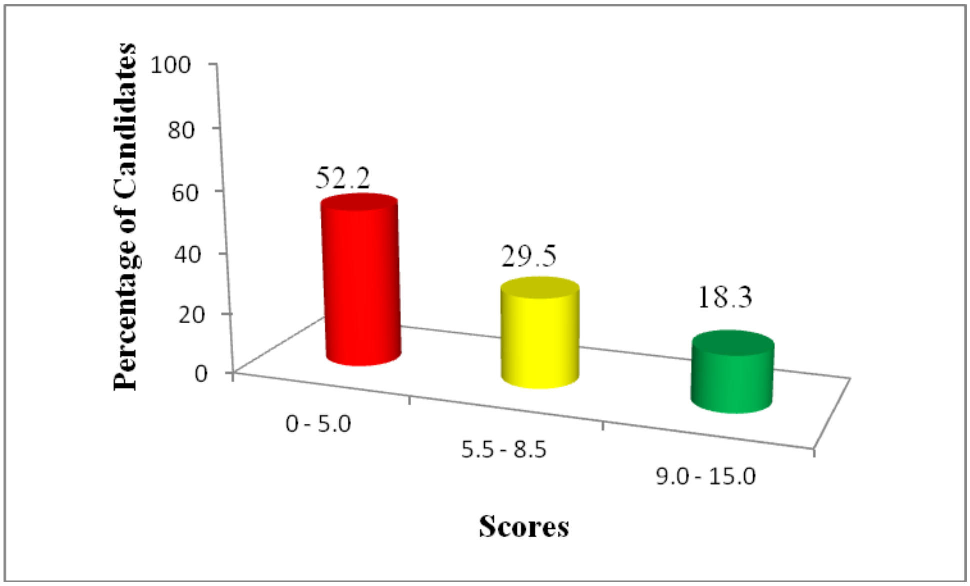


Figure 11: *Candidates' performance in question 1.*

The candidates with exemplary performance in part (a) correctly wrote the statement $\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$, applied binomial expansion to $(\cos\theta + i\sin\theta)^5$ with $i^2 = -1$ and equated imaginary parts to get the required expression for $\sin 5\theta$. By appreciating the substitution of $1-\cos^2\theta$ to $\sin^2\theta$, the candidates obtained $16\cos^4\theta - 12\cos^2\theta + 1$ from $\dfrac{\sin 5\theta}{\sin\theta}$ as required. In part (b) (i), some of the candidates rearranged the terms of $6 - z^2 = 8i - (2+4i)z$ to get the quadratic equation $z^2 - (2+4i)z + 8i - 6 = 0$. Then they used the idea of sum and product of the roots of the quadratic equation to get $z_1 + z_2 = -\dfrac{b}{a}$ given $z_1 = 3 + i$, $a = 1$ and $b = -2 - 4i$. After making necessary simplifications, they got $z_2 = -1 + 3i$. In part (b) (ii), they factorized $x^2 - 2x + 10$ to $(x-(1+3i))\,(x-(1-3i))$ and decomposed $\dfrac{6}{x^2 - 2x + 10}$ into $\dfrac{A+Bi}{(x-(1+3i))} + \dfrac{C+Di}{(x-(1-3i))}$. The candidates then used the method of undetermined coefficients to arrive at $A=0$, $B=-1$, $C=0$ and $D=1$. They ended up with $\frac{-i}{(x-(1+3i))}+\frac{i}{(x-(1-3i))}$ as the partial fractions of $\frac{6}{x^2-2x+10}$. In part (c), they verified the statement for the integer, $n=1$, formulated the general statement for proof, i.e $z^k = r^k(\cos k\theta + i\sin k\theta)$; and obtained $z^{k+1} = r^{k+1}\left(\cos(k+1)\theta + i\sin(k+1)\theta\right)$ from expressions of z and z^k, while at the same time appreciating the laws of exponents and the compound angle formulae of sine and cosine. The candidates with good performance in part (d) defined z in polynomial form as $z = x+iy$ such that $|z+1| = 2|z-1| \Rightarrow \sqrt{(x+1)^2+y^2} = 2\sqrt{(x-1)^2+y^2}$ and expressed the resulting equation in the form $\left(x-\frac{5}{3}\right)^2 + (y-0)^2 = \left(\frac{4}{3}\right)^2$, which enabled them to prove that it is an equation of a circle with a radius of $\frac{4}{3}$. Extract 11.1. is a sample answer from a candidate who got the question right.

Extract 11.1

Q1. (a) $\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$

from Demoivre's theorem

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

$(\cos\theta+i\sin\theta)^5 = \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2 + 10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$

$= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta + -10\cos^2\theta\sin^3\theta i + i\sin^5\theta + 5\cos\theta\sin^4\theta$

$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta + i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$

Q1. then:

$\cos 5\theta + i\sin 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta + i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$

Equating imaginary part

$$\frac{\sin 5\theta}{\sin\theta} = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\sin\theta}$$

$$= 5\cos^4\theta - 10\cos^2\theta\sin^2\theta + \sin^4\theta$$

$$= 5\cos^4\theta - 10\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$$

$$= 5\cos^4\theta - 10\cos^2\theta + 10\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$$

$$= 16\cos^4\theta - 12\cos^2\theta + 1$$

Hence

$$\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$$

(b) (i) $6 - z^2 = 8i - (2+4i)z$ roots z_1 and z_2

$z^2 - (2+4i)z + 8i - 6 = 0$

$z^2 - (2+4i)z + (8i-6) = 0$

$z^2 - (\text{sum of roots})z + (\text{product of roots}) = 0$

$z^2 - (z_1+z_2)z + z_1z_2 = 0$

$z_1 + z_2 = 2+4i$

$z_1z_2 = 8i - 6$

let $z_2 = x+iy$, $z_1 = 3+i$

$3+i + x+iy = 2+4i$

$3+x + (1+y)i = 2+4i$

$3+x = 2$, $1+y = 4$

$x = 2-3$ $\quad$ $y = 4-1$

$x = -1$ $\quad$ $y = 3$

$z_2 = -1+3i$

$z_1z_2 = (-1+3i)(3+i)$

01. (b) (i) $z_1z_2 = -3 - i + 9i + 3i^2$

$z_1z_2 = -3 - i + 9i - 3$

$z_1z_2 = 8i - 6$ Hence shown

$\therefore z_2 = -1+3i$

where $a = -1$, $b = 3$

(ii) $$\frac{6}{x^2-2x+10} = \frac{6}{(x-(1+3i))(x-(1-3i))}$$

$$\frac{6}{(x-(1+3i))}\cdot\frac{1}{x-(1-3i)} = \frac{A}{(x-1-3i)} + \frac{B}{(x-1+3i)}$$

$$\frac{6}{x^2-2x+10} = \frac{A(x-1+3i)+B(x-1-3i)}{\cancel{(x-1-3i)(x-1+3i)}}$$

$6 = A(x-1+3i)+B(x-1-3i)$

put $x = 1-3i$

$6 = A(\cancel{1-3i}-\cancel{1}+3i)+B(\cancel{1}-3i-\cancel{1}+3i)$

$6 = A(0)+B(-6i)$

$1 = -Bi$

$B = 1/i$

$\underline{\underline{B = 1i}}$

put $x = 1+3i$

$6 = A(\cancel{1}+3i-\cancel{1}+3i)+B(1+\cancel{3i}-1-3i)$

$6 = 6(Ai)+B(0)$

$A = 1/i$

$A = -1i$

then into partial fractions.

Q1. (b) (ii)

$$\frac{6}{x^2-2x+10} = \frac{-1i}{(x-1-3i)} + \frac{1i}{(x-1+3i)}$$

(c) By mathematical induction

$z^n = r^n(\cos n\theta + i\sin n\theta)$

1. proof for $n=1$

$z^1 = r^1(\cos 1\theta + i\sin 1\theta)$

$z = r(\cos\theta + i\sin\theta)$

Hence it is true for $n=1$.

2. Assume it is true for $n=k$

$z^k = r^k(\cos k\theta + i\sin k\theta)$ ---------- (i)

3. proof for $n=k+1$

$z^{k+1} = r^{k+1}(\cos(k+1)\theta + i\sin(k+1)\theta)$

consider r.H.S

$z^{k+1} = z^k \cdot z^1$

$= r^k(\cos k\theta + i\sin k\theta)(r(\cos\theta + i\sin\theta))$

$= r^k\cdot r[\cos k\theta\cos\theta + i\sin\theta\cos k\theta + i\sin k\theta\cos\theta + i^2\sin k\theta\sin\theta]$

$= r^k\cdot r[\cos k\theta\cos\theta + i\sin\theta\cos k\theta + i\sin k\theta\cos\theta - \sin k\theta\sin\theta]$

$= r^k \cdot r\,[(\cos k\theta \cos\theta - \sin k\theta \sin\theta) + i(\sin k\theta \cos\theta + \cos k\theta \sin\theta)]$

$= r^k \cdot r\,[\cos(k\theta + \theta) + i\sin(k\theta + \theta)]$

$= r^k \cdot r\,[\cos(k+1)\theta + i\sin(k+1)\theta]$

$= r^{k+1}(\cos(k+1)\theta + i\sin(k+1)\theta)$ L.H.S

Hence proved!

Extract 11.1: *A correct response from one of the candidates.*

The candidates who scored low marks didn't write correct answers. In part (a), several candidates did not expand $(\cos\theta + i\sin\theta)^5$. The majority of these candidates used inappropriate formulae such as $2i\sin\theta = z^n - \frac{1}{z^n}$ to find $\sin 5\theta$. In part (b) (i), a considerable number of candidates did not use the concept of sum and the product of roots of quadratic equations to find z_2. Others did not re-arrange $6 - z^2 = 8i - (2+4i)z$ in quadratic form as $z^2 - (2+4i)z + 8i - 6 = 0$. Some of the candidates used the idea that a conjugate of z_1 is also the root of $z^2 - (2+4i)z + 8i - 6 = 0$ to get z_2. This was not correct, as the quadratic equation contained complex coefficients. In part (b) (ii), most of the candidates failed to represent $x^2 - 2x + 10$ as the product of two complex factors, that is, $(x-1-3i)(x-1+3i)$. Hence, they got incorrect partial fractions. Though part (c) was done well by the majority of the candidates, a few candidates used $n = 0$, although 0 is not a positive integer. As a result, they did not reach at $z^{k+1} = r^{k+1}(\cos(k+1)\theta + i\sin(k+1)\theta)$. In part (d), some of the candidates did not get $\left(x - \frac{5}{3}\right)^2 + y^2 = \frac{16}{9} = \left(\frac{4}{3}\right)^2$, regardless of their good start. Some of the candidates committed simplification errors; for example, they obtained $(x+1)^2 + y^2 = 2((x-1)^2 + y^2)$ from $\sqrt{(x+1)^2 + y^2} = 2\sqrt{(x-1)^2 + y^2}$, forgetting to square 2. Extract 11.2 shows a sample answer from one of the candidates who got the question wrong.

Extract 11.2

15(i) Solns:

from $\frac{6}{x^2+10-2x} = \frac{6}{(1-3i)(1+3i)}$

let $\frac{6}{(1-3i)(1+3i)} = \frac{A}{(1-3i)} + \frac{B}{(1+3i)}$

$6 = A(1+3i) + B(1-3i)$

$6 = A + 3Ai + B - 3Bi$

$6 + 0i = A + B + (3A - 3B)i$

$A + B = 6$

$3A - 3B = 0$

$A = 3$ and $B = 3$

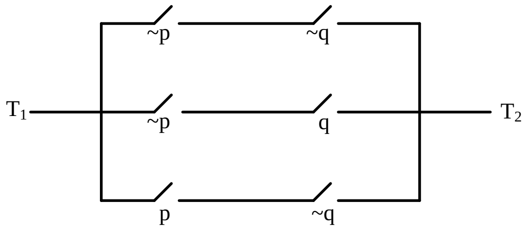
from $\frac{6}{(1-3i)(1+3i)} = \frac{3}{(1-3i)} + \frac{3}{(1+3i)}$

$\therefore$ The answer is $\frac{3}{(1-3i)} + \frac{3}{(1+3i)}$

Extract 11.2: *An incorrect response from one of the candidates.*

2.2.2 Question 2: Logic

The question had parts (a), (b) and (c). In part (a) (i), the candidates were required to draw a simplified electrical network using the following circuit.



In part (a) (ii), they were required to simplify the proposition $\sim((p \wedge q) \rightarrow (\text{p} \vee q))$ using the laws of algebra. In part (b), the candidates were required to determine whether the proposition $[(\text{p} \rightarrow \sim \text{q}) \wedge (q \vee r) \wedge p] \rightarrow r$ is a tautology or not using the truth table. In part (c), the candidates were required to use the laws of algebra to prove that the propositions $p \wedge (q \vee r)$ and $[p \rightarrow (\text{q} \vee \sim \text{r})] \rightarrow (p \wedge q)$ are equivalent.

The analysis shows that 22 percent of the candidates who attempted this question scored from 0 to 5 marks, 26.8 percent from 5.5 to 8.5 marks and 51.2 percent from 9.0 to 15 marks. Further analysis shows that 78 percent of candidates scored more than 5 marks; 969 (8.1%) of such candidates scored full marks. On the basis of this analysis, it can be argued that the candidates' performance in this question was good. Figure 12 illustrates the candidates' performance in question 2.

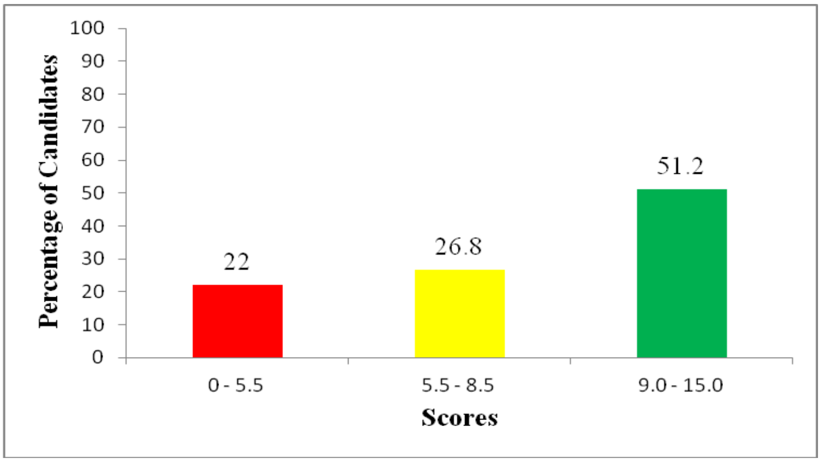


Figure 12: *Candidates' performance in question 2.*

The analysis of the candidates' responses shows that the majority of the candidates had good performance. In part (a) (i), the candidates identified the compound statement that represented the given network diagram, used the algebraic laws of logic to simplify the preceding statement into $\sim p \vee \sim q$ and drew the simplified electric network that corresponded to the simplified statement. In part (a) (ii), the candidates used the algebraic laws of logic expressions, such as demorgan's, commutative, compliment and identity law, to simplify the statement $\sim((p \wedge q) \rightarrow (\mathrm{p} \vee q))$ into an F statement. In part (b), the candidates prepared a truth table with the columns p, q, r, ~q, $p \rightarrow \sim q$, $q \vee r$, $(p \rightarrow \sim q) \wedge (q \vee r)$, $\left[(p \rightarrow \sim q) \wedge (q \vee r)\right] \wedge p$ and $\left[(p \rightarrow \sim q) \wedge (q \vee r) \wedge p\right] \rightarrow r$. On the basis of the information in the table, they concluded that $\left[(p \rightarrow \sim q) \wedge (q \vee r) \wedge p\right] \rightarrow r$ is a tautology. Similarly, in part (c), the candidates used the laws of propositions correctly to prove that the statement $p \wedge (q \vee r)$ is logically equivalent to $[p \rightarrow (\mathrm{q} \vee \sim \mathrm{r})] \rightarrow (p \wedge q)$. Extract 12.1 shows the responses of a candidate who answered the question correctly.

Extract 12.1

From the circuit

2(a) (i) $(\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$

Using law of algebra

$\sim p \wedge (\sim q \vee q) \vee (p \wedge \sim q)$ distributive law

$(\sim p \wedge T) \vee ((p \wedge \sim q)$ Complement law

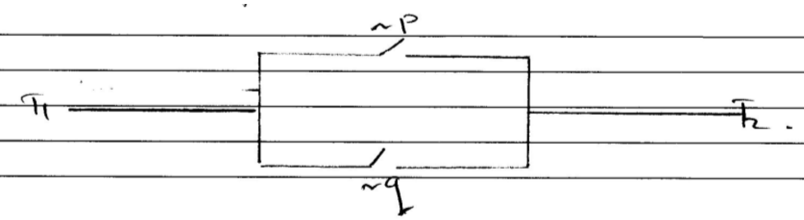
$\sim p \vee (p \wedge \sim q)$ Identity law

$(\sim p \vee p) \wedge (\sim p \vee \sim q)$ Distributive law

$T \wedge (\sim p \vee \sim q)$ complement law

$(\sim p \vee \sim q)$ Identity law

Simplified electrical network



2a (ii) $\sim((p \wedge q) \rightarrow (p \vee q))$

$\sim(\sim(p \wedge q) \vee (p \vee q))$ $\quad p \rightarrow q \equiv \sim p \vee q$

$((p \wedge q) \wedge (\sim(p \vee q)))$ $\quad$ Demorgans law

$(p \wedge q) \wedge (\sim p \wedge \sim q)$ $\quad$ Demogans law

$p \wedge \sim p \wedge q \wedge \sim q$ $\quad$ Associative property

$F \wedge F$ $\quad$ Complement law

$\underline{\underline{F}}$ $\quad$ Identity law

~~let~~

let $A = p \rightarrow \sim q$ $\quad B = q \vee r$ $\quad C = A \wedge B$ $\quad D = (C \wedge p)$

(b)

p	q	r	$\sim q$	A $p \rightarrow \sim q$	B $q \vee r$	C $A \wedge B$	D $C \wedge p$	$D \rightarrow r$
T	T	T	F	F	T	F	F	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	F	F	T
F	T	T	F	T	T	T	F	T
F	T	F	F	T	T	T	F	T
F	F	T	T	T	T	T	F	T
F	F	F	T	T	F	F	F	T

The proposition is TAUTOLOGY since the last column ends with Truth value only.

2 (c) Using laws of algebra to prove $p \wedge (q \vee r)$ and $[p \rightarrow (q \vee \sim r)] \rightarrow (p \wedge q)$ are equivalent

Starting with

$p \wedge (q \vee r)$

Simplifying $[p \rightarrow (q \vee \sim r)] \rightarrow p \wedge q$

$[\sim p \vee (q \vee \sim r)] \rightarrow p \wedge q$ $\quad p \rightarrow q \equiv \sim p \vee q$,

$[p \wedge \sim(q \vee \sim r)] \vee p \wedge q$ Dermogans law

$[p \wedge (\sim q \wedge \sim\sim r)] \vee p \wedge q$ Dermogan's law

$[p \wedge \sim q \wedge r] \vee [p \wedge q]$ Associative law

$p \wedge [(\sim q \wedge r) \vee q]$ Distributive law

$p \wedge [\sim q \vee q \wedge q \vee r]$ Distributive law

$p \wedge [T \wedge q \vee r]$ Complement law

$p \wedge (q \vee r)$ Identity law

Hence shown.

Hence $[p \rightarrow (q \vee \sim r)] \rightarrow p \wedge q$ is equivalent to $p \wedge (q \vee r)$

Extract 12.1: *A correct response from one of the candidates.*

In spite of the candidates' good performance, 372 candidates (22%) scored low marks. In part (a) (i), some of the candidates failed to formulate the appropriate statement from the network diagram. The common mistake had to do with seeing the statement such as $(\sim p \vee \sim q) \wedge (\sim p \vee q) \wedge (p \vee \sim q)$, rather than $(\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$ indicating improper understanding of conjunction and disjunction statements. Others formulated the required statement but could not apply the laws of propositions to simplify it to $\sim p \vee \sim q$. Other candidates did not know that the symbol $\vee$ is used with a parallel connection of switches. Such candidates presented a series connection for ~p and ~q instead of a parallel connection of an electric network. In part (a) (ii), several candidates presented $\sim((p \wedge q) \rightarrow (p \vee q))$ as $\sim\sim(p \wedge q) \vee (p \vee q)$. This means that they had insufficient knowledge of the implication symbol in terms of $\vee$. The candidates were supposed to represent the given statement as $\sim(\sim(p \wedge q) \vee (p \vee q))$ and then simplify it using appropriate laws. Several candidates had difficulty writing the laws in front of the statements. For instance, writing associative or distributive instead of commutative and idempotent instead of identity. Other candidates did not write the laws that they used in front of the statements. Such candidates were awarded one and a half marks. In part (b), some of the candidates did not use truth tables to determine whether the given statement was tautology. The candidates should have known that the conjunction statement is true only if both values are true, otherwise the statement is false, that the disjunction statement is true if at least one value is true and that the conditional statement is true if both values are true and false if the first value is false. In addition, some of the candidates could not identify the number of the rows in the truth table for a compound statement involving three statement letters, as Extract 12.2 shows. Such candidates should have known that the number of the rows in the truth table depends upon the number of the different components in the statement and can be written as 2^n, where n is the number of the components forming the statement. The number of the rows was 8. In part (c), a considerable number of candidates could not prove that $p \wedge (q \vee r)$ and $[p \rightarrow (\mathrm{q} \vee \sim \mathrm{r})] \rightarrow (p \wedge q)$ are equivalent. Some of these candidates failed to pair expressions with their respective laws of propositions of logic, while others used truth tables contrary to what they were asked to do.

Extract 12.2

2.(b) $\langle \{p \rightarrow \sim q\} \wedge \{q \vee r\} \wedge p \rangle \rightarrow r$

				A	B	C	D	
p	q	r	$\sim q$	$p\rightarrow\sim q$	$q \vee r$	$A \wedge B$	$C \wedge p$	$D \rightarrow r$
T	T	T	F	F	T	F	F	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	F	T
F	T	T	F	T	T	T	F	T
F	T	F	F	T	T	T	F	T

$\therefore [\langle p \rightarrow \sim q\rangle \wedge \langle q \vee r \rangle \wedge p]$ is a Tautology

Extract 12.2: *An incorrect response from one of the candidates.*

2.2.3 Question 3: Vectors

This question had parts (a), (b) and (c). In part (a), the candidates were required to find (i) the work done in moving an object along a straight line from $(3, 2, -1)$ to $(2, -1, 4)$ in a force field given by $\underline{F} = 4\underline{i} - 3\underline{j} + 2\underline{k}$ and (ii) the possible values of the constant t if $\underline{a} = (3t + 1)\underline{i} - \underline{j} - \underline{k}$ is perpendicular to $\underline{b} = (t + 3)\underline{i} - 3\underline{j} - 2\underline{k}$. In part (b), the candidates were asked to (i) show that quadrilateral is a parallelogram and (ii) find the actual area of the parallelogram in (b) (i), given that the vertices of a quadrilateral are A (5, 2, 0), B (2, 6, 1), C (2, 4, 7) and D (5, 0, 6). In part (c), the candidates were given that the vertices A, B and C of a triangle are at the points with position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively and were required to show that the area of the triangle is equal to $\frac{1}{2}|\underline{a}\times\underline{b}+\underline{b}\times\underline{c}+\underline{c}\times\underline{a}|$ square units.

A total of 11,550 candidates (95.7%) attempted the question. The analysis shows that 12.5 percent performed well, as they scored marks ranging from 9 to 15. Only 0.3 percent scored all 15 marks, 44.1 percent scored from 5.5 to 8.5 marks and 43.4 percent from 0 to 5 marks, with 5.5 percent scoring 0. Generally, the candidates' performance was average, as 56.6 percent scored more than 5 marks. Figure 13 illustrates the candidates' performance in question 3.

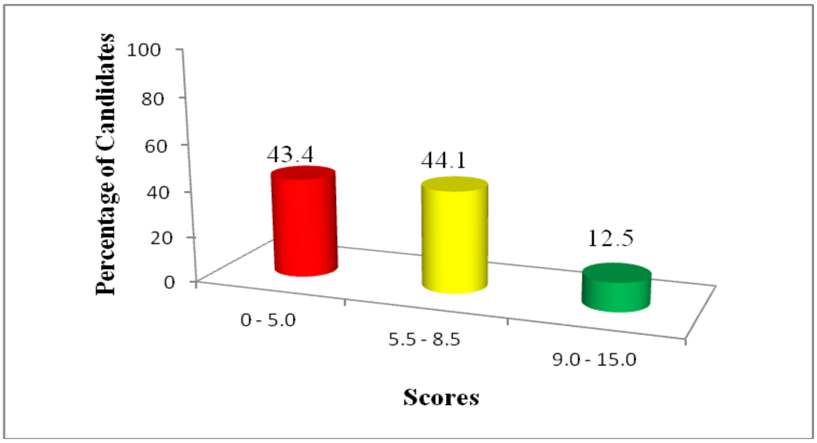


Figure 13: *Candidates' performance in question 3.*

The impressive performance in part (a) is a result of the candidates' ability to (i) determine the displacement vector $\underline{d}$ from $(3,2,-1)$ to $(2,-1,4)$ and use the formula $Workdone = |\underline{F}.\underline{d}|$ to get 15 units, and (ii) use the property of a dot product $\underline{a}.\underline{b}=0$ to formulate an equation $3t^2+10t+8=0$ that was solved to obtain $t=\frac{-4}{3}$ or $t=-2$. In part (b) (i) , the candidates determined displacement vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{CB}$ and $\overrightarrow{CD}$, and correctly showed that $\overrightarrow{BC}\times\overrightarrow{AD}$ and $\overrightarrow{AB}\times\overrightarrow{CD}$ resulted in zero vectors. In part (b) (ii), they used the formula $Area = |\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AD}}|$ or

$Area = \left|\overrightarrow{CB} \times \overrightarrow{CD}\right|$ to obtain 32.187 square units as the actual area of the parallelogram. The candidates who answered part (c) correctly expressed $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as $\underline{b}-\underline{a}$ and $\underline{c}-\underline{a}$, and used the formula $Area = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$ and the properties $\underline{a}\times\underline{a}=0$ and $-\underline{a}\times\underline{b}=\underline{b}\times\underline{a}$ to prove that $Area = \frac{1}{2}\left|\underline{a}\times\underline{b}+\underline{b}\times\underline{c}+\underline{c}\times\underline{a}\right|$ square units. Extract 13.1 is a sample response of the candidate(s) with exemplary performance.

Extract 13.1

3 (a) (i) $W = F \cdot d.$

$$d = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 5 \end{pmatrix}$$

$d = -i - 3j + 5k.$

$F = 4i - 3j + 2k.$

Work done $= F \cdot d.$

$$F \cdot d = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -3 \\ 5 \end{pmatrix} = -4 + 9 + 10$$

$= 15$ units

Work done $= 15$ units.

3(a) (ii) $a = (3t+1)i - j - k.$

$b = (t+3)i - 3j - 2k.$

$a \cdot b = |a|\,|b| \cos \theta.$

since a and b are perpendicular angle between them (θ) is 90°.

$a \cdot b = |a||b| \cos 90^\circ.$

$a \cdot b = 0.$

$$a \cdot b = \begin{pmatrix} 3t+1 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} t+3 \\ -3 \\ -2 \end{pmatrix} = 0.$$

$(3t+1)(t+3) + 3 + 2 = 0$

$$(3t+1)(t+3)+5=0.$$

$$(3t+1)(t+3)=-5.$$

$$3t^2+9t+t+3=-5.$$

$$3t^2+10t+3=-5.$$

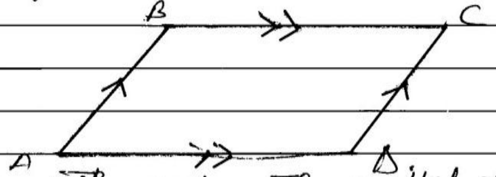
$$3t^2+10t+3+5=0.$$

$$3t^2+10t+8=0.$$

$$t=-4/3 \qquad t=-2.$$

$\therefore$ $t=-4/3$ and $t=-2$.

3 (b)(i) condition for parallelogram.



$\overrightarrow{AB}$ parallel to $\overrightarrow{CD}$ and $\overrightarrow{BC}$ parallel to $\overrightarrow{AD}$.

and $|AB|=|CD|$ also $|BC|=|AD|$.

$$\overrightarrow{AB}=\begin{pmatrix}2\\6\\1\end{pmatrix}-\begin{pmatrix}5\\2\\0\end{pmatrix}=\begin{pmatrix}-3\\4\\1\end{pmatrix}. \quad |AB|=\sqrt{26}.$$

$$\overrightarrow{BC}=\begin{pmatrix}2\\4\\7\end{pmatrix}-\begin{pmatrix}2\\6\\1\end{pmatrix}=\begin{pmatrix}0\\-2\\6\end{pmatrix} \quad |BC|=\sqrt{40}$$

$$\overrightarrow{DC}=\begin{pmatrix}2\\4\\7\end{pmatrix}-\begin{pmatrix}5\\0\\6\end{pmatrix}=\begin{pmatrix}-3\\4\\1\end{pmatrix}. \quad |DC|=\sqrt{26}.$$

$$\overrightarrow{AD}=\begin{pmatrix}5\\0\\6\end{pmatrix}-\begin{pmatrix}5\\2\\0\end{pmatrix}=\begin{pmatrix}0\\-2\\6\end{pmatrix}. \quad |AD|=\sqrt{40}.$$

condition for parallel vector is. angle between them is zero.

$$\overrightarrow{AB}\cdot\overrightarrow{DC}=|AB||DC|\cos\theta.$$

$$\cos\theta=\frac{\overrightarrow{AD}\cdot\overrightarrow{DC}}{|AB||DC|}.$$

$$\cos\theta = \frac{\begin{pmatrix}-3\\4\\1\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}}{(\sqrt{26})\sqrt{26})} \Rightarrow \frac{9+16+1}{26}$$

$$\cos\theta = 1.$$

$$\theta = \cos^{-1} 1 = 0.$$

for $\vec{BC}\cdot\vec{AD} = |BC||AD| \cos\theta$.

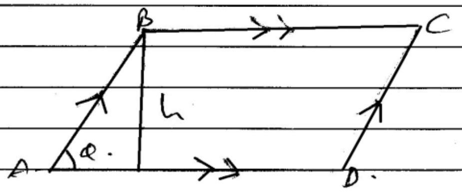
$$\cos\theta = \frac{\vec{BC}\cdot\vec{AD}}{|BC||AD|} \Rightarrow \frac{\begin{pmatrix}0\\2\\6\end{pmatrix}\cdot\begin{pmatrix}0\\2\\6\end{pmatrix}}{(\sqrt{40})(\sqrt{40})}$$

$$\cos\theta = \frac{0+4+36}{40} = \frac{40}{40} = 1.$$

$$\theta = \cos^{-1} 1 = 0^\circ.$$

since ~~their~~ the magnitudes are equal and angle between the ~~sides~~ parallel sides is zero it is a parallelogram.

3(b) (ii).



Area $= |\vec{AD}| \times h$. $\sin\theta = h / |\vec{AD}|$.

$h = |\vec{AB}| \sin\theta$.

Area $= |\vec{AD}|\,|\vec{AB}| \sin\theta$.

but $|AD \times AB| = |\vec{AD}||\vec{AB}| \sin\theta$.

Area $= |\vec{AD} \times \vec{AB}|$

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} i & j & k \\ -3 & 4 & 1 \\ 0 & -2 & 6 \end{vmatrix}$$

$$= i(24+2) - j(-18-0) + k(6-0).$$

$$\vec{AB} \times \vec{AD} = 26i + 18j + 6k.$$

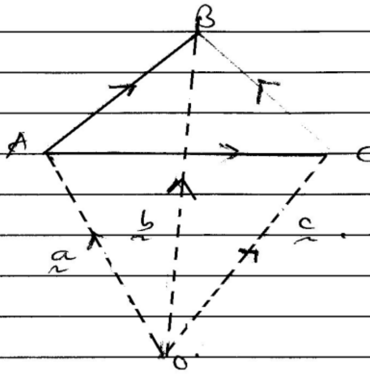
$$\text{area} = |AB \times AD| = \sqrt{(26)^2 + (18)^2 + (6)^2}$$

$$= \sqrt{1036}$$

$$= 32.186 \text{ square units.}$$

Area $= \sqrt{1036}$ or 32.186 square units.

3(c).



$$\text{Area} = \tfrac{1}{2}\left(\vec{AB} \times \vec{AC}\right) \sin\theta.$$

$$\vec{OA} + \vec{AB} = \vec{OB}.$$
$$\vec{AB} = \vec{OB} - \vec{OA}.$$
$$\vec{AB} = \underset{\sim}{b} - \underset{\sim}{a}.$$

$$\vec{OA} + \vec{AC} = \vec{OC}.$$
$$\vec{AC} = \vec{OC} - \vec{OA}.$$
$$\vec{AC} = \underset{\sim}{c} - \underset{\sim}{a}.$$

$$\text{Area} = \tfrac{1}{2}(b-a) \times (c-a) \sin\theta.$$

$$\Rightarrow \tfrac{1}{2}\left((b \times c) - (b \times a) - (a \times c) + (a \times a)\right) \sin\theta.$$

$$= \tfrac{1}{2}(b \times c) - (b \times a) - (a \times c) \sin\theta..$$

	since $-(b\times a) = (a\times b)$.
	$-(a\times c) = (c\times a)$.
	Area $= \frac{1}{2}\left|a\times b + b\times c + c\times a\right|$.

Extract 13.1: *A correct response from one of the candidates.*

On the other hand, a considerable number of candidates (43.4%) got low marks. The candidates who got such marks in part (a) (i) used inappropriate formulae, such as $Work\ done = \underline{F}\times\underline{d}$ and $\underline{a}\times\underline{b} = 0$. Thus, they got vector quantities instead of scalar quantities. Other candidates failed to get $t = -\frac{4}{3}$ or $t = -2$ from $3t^2 + 10t + 8 = 0$ although they had started off well. In part (b), the majority of the candidates were inspired by (b) (ii) , although it was directly connected to (b) (i). A significant number of the candidates found displacement vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{CB}$ and $\overrightarrow{CD}$ but failed to compute the vectors $\overrightarrow{AB}\times\overrightarrow{CD}$ and $\overrightarrow{BC}\times\overrightarrow{AD}$ to get zero vector. Hence, they provided a conclusion that is not convincing. In part (c), some of the candidates could not add the position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ to determine the resultant vectors; for example, they wrote $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OA}} - \overrightarrow{\mathrm{OB}} = \underline{\mathrm{a}} - \underline{\mathrm{b}}$ instead of $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \underline{\mathrm{b}} - \underline{\mathrm{a}}$. Others used an incorrect formula for finding the area of a triangle, commonly $Area = \frac{1}{2}\left|\overrightarrow{\mathrm{AB}}\cdot\overrightarrow{\mathrm{AC}}\right|$, to prove that $Area = \frac{1}{2}\left|\underline{\mathrm{a}}\times\underline{\mathrm{b}} + \underline{\mathrm{b}}\times\underline{\mathrm{c}} + \underline{\mathrm{c}}\times\underline{\mathrm{a}}\right|$. Furthermore, the majority of the candidates who did part (c) badly were poor in using the properties of a cross product of vectors, especially $-\underline{a}\times\underline{b} = \underline{b}\times\underline{a}$ and $\underline{a}\times\underline{a} = 0$, to manipulate $Area = \frac{1}{2}\left|(b-a)\times(c-a)\right|$ to get $Area = \frac{1}{2}\left|\underline{\mathrm{a}}\times\underline{\mathrm{b}} + \underline{\mathrm{b}}\times\underline{\mathrm{c}} + \underline{\mathrm{c}}\times\underline{\mathrm{a}}\right|$. Extract 13.2 is an incorrect response.

Extract 13.2

3(a) ii/

$$|a| = |b|$$

$$a = (3t+1)i - j - k.$$

$$|a| = \sqrt{(3t+1)^2 + (-1)^2 + (-1)^2}$$

$$= \sqrt{9t^2+6t+1+1+1}$$

$$|a| = \sqrt{9t^2+6t+3}$$

$$b = (t+3)i - 3j - 2k.$$

$$|b| = \sqrt{(t+3)^2 + (-3)^2 + (-2)^2}.$$

$$= \sqrt{t^2+6t+9+9+4}$$

$$|b| = \sqrt{t^2+6t+22}$$

$$|a| = |b|$$

$$\sqrt{9t^2+6t+3} = \sqrt{t^2+6t+22}$$

$$9t^2+\cancel{6t}+3 = t^2+\cancel{6t}+22$$

$$9t^2 - t^2 = 22 - 3.$$

$$8t^2 = 19$$

Extract 13.2: *An incorrect response from one of the candidates.*

2.2.4 Question 4: Algebra

This question comprised parts (a), (b) and (c). In part (a), the candidates were required to express $\frac{3x+1}{(x+1)(x^2+2x+3)}$ in partial fractions. In part (b), they were required to (i) show that $y = \frac{mx}{m-x}$ if $a^x = \left(\frac{a}{k}\right)^y = k^m$ where $a \neq 1$, and (ii) solve for x and y if $x^y = y^{2x}$ and $y^2 = x^3$. In part (c), the candidates were required to expand $\sqrt{1+x}$ as far as the term in x^3 and use the result to obtain the value of $\sqrt{16.08}$ correct to six decimal places.

The analysis shows that 10,836 candidates (89.8%) attempted the question. Of these candidates, 49.6 percent scored from 0 to 5 marks, 27 percent from 5.5 to 8.5 marks and 23.4 percent from 9 to 15 marks. The analysis also indicates that 212 candidates (1.8%) scored all 15 marks, while 1760 candidates (16.2%) got zero. Therefore, the candidates' general performance in this question was average, as 50.4 percent of the candidates scored from 5.5 to 15 marks. Figure 14 is a summary of the candidates' performance in this question.

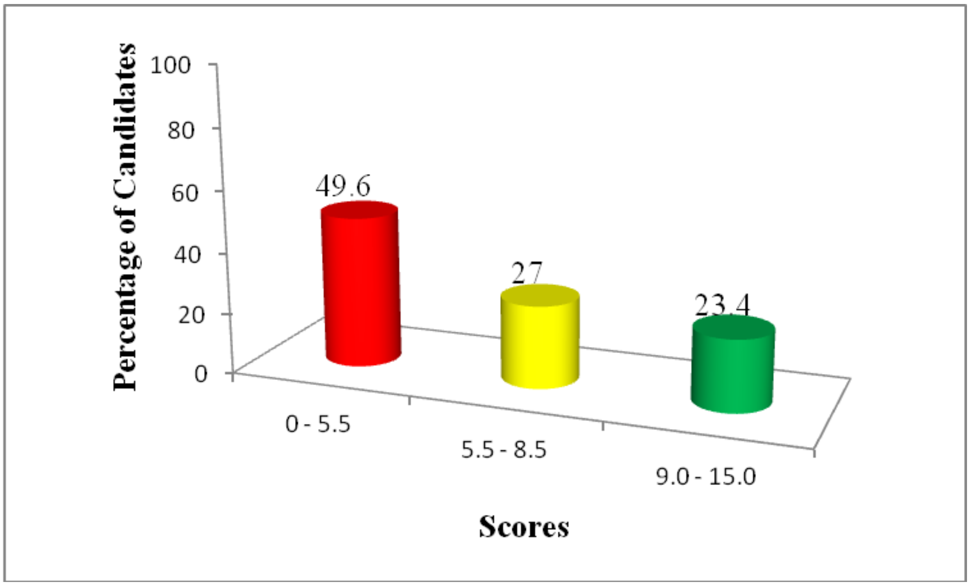


Figure 14: *Candidates' performance in question 4.*

As seen in Figure 14, the majority of the candidates had average performance. The candidates who scored all 15 marks knew that the decomposition for the given rational function in part (a) was of the form $\frac{3x+1}{(x+1)(x^2+2x+3)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+3}$. The candidates used the method of undetermined coefficients or the substitution method to establish that $A=-1$, $B=1$ and $C=4$. The majority of the candidates correctly inserted the coefficients that they had obtained previously into $\frac{A}{x+1} + \frac{Bx+C}{x^2+2x+3}$ to obtain $\frac{3x+1}{(x+1)(x^2+2x+3)} = \frac{-1}{x+1} + \frac{x+4}{x^2+2x+3}$. In part (b) (i), most of the candidates eliminated k and a from the equation $a^x = \left(\frac{a}{k}\right)^y = k^m$, and hence were able to prove that $y = \frac{mx}{m-x}$. In part (b) (ii), the candidates applied logarithms on both sides of the equations $x^y = y^{2x}$ and $y^2 = x^3$. Then they solved the resulting equations to get $x = 9$ and $y = 27$. In part (c), the majority of the candidates applied binomial theorem with $n = \frac{1}{2}$, $a = 1$ and $b = x$ to expand $\sqrt{1+x}$ as $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$. Also, they expressed $\sqrt{16.08}$ as $4(1+0.005)^{\frac{1}{2}}$ to get $x = 0.005$. Then they substituted the value of x into the expansion for $\sqrt{1+x}$ and obtained the final value of $\sqrt{16.08}$ correct to six decimal values as 4.009988. Extract 14.1 is a sample solution from a candidate who answered the question correctly.

Extract 14.1

4(a)	$\frac{3x+1}{(x+1)(x^2+2x+3)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+3}$
	$3x+1 = A(x^2+2x+3) + (Bx+C)(x+1)$
	$3x+1 = Ax^2+2Ax+3A + Bx^2+Cx+Bx+C$
	$3x+1 = (A+B)x^2 + (2A+B+C)x + (3A+C)$
	Then on comparing
	$A+B = 0$
	$2A+B+C = 3$
	$3A+C = 1$
	On solving
	$A = -1$ $B = 1$ and $C = 4$
	$\therefore \frac{3x+1}{(x+1)(x^2+2x+3)} = \frac{x+4}{x^2+2x+3} + \frac{-1}{x+1}$
	$\therefore \frac{3x+1}{(x+1)(x^2+2x+3)} = \frac{x+4}{x^2+2x+3} - \frac{1}{x+1}$ in partial fraction

(b) $a^x = \left(\frac{a}{k}\right)^y$. also $a^x = k^m$ $\left(\frac{a}{k}\right)^y = k^m$.

Taking $a^x = \frac{a^y}{k^y}$ $k^y = a^{y-x}$

$\ln(k)^y = \ln(a^{y-x})$

$y\ln k = (y-x)\ln a$ —— (I)

Again:

$a^y = k^m . k^y$.

$a^y = k^{m+y}$.

$\ln a^y = m+y \ln k$.

$y\ln a = (m+y)\ln k$. —— (II)

Considering eqn (1) $\ln k = \frac{(y-x)\ln a}{y}$; $\ln a = \frac{y\ln k}{y-x}$

Using eqn (II) $y\ln a = (m+y)\ln k$

$y\ln a = (m+y)\ln k$ but $\ln a = \frac{y\ln k}{y-x}$

$y\left[\frac{y\ln k}{y-x}\right] = (m+y)\ln k$.

$y^2\ln k = (y-x)(m+y)\ln k$ divide by $\ln k$.

$y^2 = (y-x)(m+y)$

$y^2 = y^2 - yx - mx + my$.

$y^2 - y^2 = my - yx - mx$.

$mx = my - yx$

$mx = y[m-x]$

$y = \frac{mx}{m-x}$ shown.

c (ii) $x^y = y^{2x}$ and $y^2 = x^3$.

$x^y = (y^2)^x$ —— (i)

but $y^2 = x^3$.

$x^y = (x^3)^x$

$x^y = x^{3x}$

$y = 3x$ —— (II)

~~ha~~

$x^y = y^{2x}$ and $y^2 = x^3$

Introducing natural logarithm throughout

$\ln x^y = \ln y^{2x}$

$y\ln x = 2x\ln y$.

$y\ln x = 2x\ln y$ —— (III)

and $y^2 = x^3$

$\ln y^2 = \ln x^3$

$$2\ln y = 3\ln x,$$

$$\ln x = \frac{2}{3}\ln y. \quad \text{—(IV)}$$

Subtituting eqn (III) in eqn (IV)

$$y\left[\frac{2}{3}\ln y\right] = 2x\ln y.$$

$$\frac{2y}{3}\ln y = 2x\ln y.$$

$$\frac{y}{3} = x.$$

$$3x = y,$$

$$\ln y = \frac{3}{2}\ln x.$$

$$y\ln x = \frac{3(x)\cdot 2}{2}\ln x.$$

$$y = 3x,$$

Taking $y^2 = x^3$.

but $y = 3x$,

$$(3x)^2 = x^3,$$

$$\frac{9x^2}{x^2} = \frac{x^3}{x^2}.$$

$$x = 9,$$

Since $x = 9$

$$y^2 = x^3.$$

$$y^2 = (9)^3.$$

$$\sqrt{y^2} = \sqrt{3^6}$$

$$y = 27$$

$\therefore$ $x = 9$ and $y = 27$.

The value of x is 9 and the value of y is 27.

(c) $\sqrt{1+x} = (1+x)^{1/2}$

from $(1+x)^n = 1 + nx + \frac{(n)(n-1)}{2!}x^2 + \frac{(n)(n-1)(n-2)}{3!}x^3$

but $n = \frac{1}{2}$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x + \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\frac{x^2}{2} + \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\frac{x^3}{6}.$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{x^2}{8} + \frac{x^3}{16} + \dots$$

Required $\sqrt{16.08}$

$$\sqrt{16.08} = \sqrt{16 + 0.08} = \sqrt{16\left(1 + \frac{0.08}{16}\right)}$$

$$= 4\left[1 + \frac{1}{200}\right]^{1/2}.$$

$$\therefore \sqrt{16.08} = 4[1+x]^{1/2} \text{ when } x = 1/200.$$

$$\sqrt{16.08} = 4\left[1 + \frac{1}{2}\left(\frac{1}{200}\right) + \left[\left(\frac{1}{200}\right)^2 \cdot \frac{-1}{8}\right] + \left[\frac{1}{16}\left(\frac{1}{200}\right)^3\right]\right]$$

$$\sqrt{16.08} = 4\left[1 + \frac{1}{400} - \frac{1}{320\,000} + \frac{1}{16(200)^3}\right]$$

$$= 4[1.002496883)$$

$$= 4.009987531$$

Correct to six decimal places $= 4.009988$

$$\sqrt{16.08} = 4.009988$$

Extract 14.1: *A correct response from one of the candidates.*

In spite of the candidates' average performance, there were candidates with very poor performance. In part (a), they had difficulty in expressing $\frac{3x+1}{(x+1)(x^2+2x+3)}$ as a sum of simpler rational functions. They regarded it as either a rational function with a denominator that can be factorized completely or a rational function having a denominator with repeated factors. The common mistakes were expressing $\frac{3x+1}{(x+1)(x^2+2x+3)}$ as $\frac{A}{x+1}+\frac{Bx}{x^2+2x+3}$ or $\frac{A}{x+1}+\frac{B}{x^2+2x+3}$ or $\frac{A}{x+1}+\frac{Bx^2+Cx+D}{x^2+2x+3}$. In part (b) (i), most of the candidates had difficulty using the laws of exponents and logarithms to prove that $\mathrm{y} = \frac{\mathrm{mx}}{\mathrm{m}-\mathrm{x}}$ from $\mathrm{a}^{\mathrm{x}} = \left(\frac{\mathrm{a}}{\mathrm{k}}\right)^{\mathrm{y}} = \mathrm{k}^{\mathrm{m}}$.

Also, several candidates did not draw out the equations $\mathrm{a}^{\mathrm{x}} = \left(\frac{\mathrm{a}}{\mathrm{k}}\right)^{\mathrm{y}}$ and $\mathrm{a}^{\mathrm{x}} = \mathrm{k}^{\mathrm{m}}$ from the given equation which could be solved to obtain *x* and *y* values. Most of such candidates wrote $\mathrm{a}^{\mathrm{x}} = \mathrm{a}^{\mathrm{y}}\mathrm{k}^{\mathrm{y}}$. As a result, they got an incorrect solution. Likewise, in part (b) (ii), the candidates failed to solve $\mathrm{x}^{\mathrm{y}} = \mathrm{y}^{2\mathrm{x}}$ and $y^2 = x^3$

simultaneously by the method of substitution. This indicates that they could not apply both laws of exponents and logarithms. In part (c), some of the candidates completely failed to expand $\sqrt{1+x}$. The candidates did not know the Binomial theorem. Several candidates presented the binomial expansion of $\sqrt{1+x}$ with some mistakes in terms of signs they used. The most common error was expressing $\sqrt{1+x}$ as $1+\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3$. Hence, they got $\sqrt{16.08}=4.009987$ instead of 4.009988. A significant number of the candidates expanded $\sqrt{1+x}$ correctly but could not express $\sqrt{16.08}$ in the form $4\left(1+\frac{1}{200}\right)^{\frac{1}{2}}$. Extract 14.2 shows a sample answer from a candidate who failed to answer the question correctly.

Extract 14.2

4a) $\dfrac{3x+1}{(x+1)(x^2+2x+3)}$

$\dfrac{3x+1}{(x+1)(x^2+2x+3)} = \dfrac{A}{x+1} + \dfrac{B}{x^2+2x+3}.$

$\dfrac{3x+1}{(x+1)(x^2+2x+3)} = \dfrac{Ax^2+2Ax+3A+Bx+B}{(x+1)(x^2+2x+3)}$

Comparing equations.

$Ax^2+2Ax+3A+Bx+B.$

$3x+1.$

$Ax^2=0$

$2Ax+Bx=3x.$

$2A+B=3.$

$3A+B=1.$

$A=-2.$

$B=7.$

$\dfrac{3x+1}{(x+1)(x^2+2x+3)} = \dfrac{-2}{x+1} + \dfrac{7}{x^2+2x+3}.$

Extract 14.2: *An incorrect response from one of the candidates.*

2.2.5 Question 5: Trigonometry

This question had four parts, (a), (b), (c) and (d). The candidates were required to: (a) (i) prove that $2\cos 3\theta = x^3 + \frac{1}{x^3}$ when $2\cos\theta = \frac{1}{x} + x$; and (ii) solve the equation $5\cos\alpha - 2\sin\alpha = 2$ for $-180^\circ \le \alpha \le 180^\circ$ using t-formula. (b) show that (i) $\tan\left(\frac{1}{8}\pi\right) = \sqrt{2} - 1$ if $\theta = \frac{\pi}{8}$ and $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$; and (ii) $a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$ given that $a\sin\theta + b\cos\theta = c$. (c) (i) simplify the expression $\tan^{-1} x + \tan^{-1}\left(\frac{1-x}{1+x}\right)$; and (ii) find all the values of θ which satisfy the equation $\cos x\theta + \cos(x+2)\theta = \cos\theta$. (d) show that $\frac{a+b+c}{abc} = 1$ if $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$.

The question was opted by 6389 candidates (53.0%). Among them 19.1 percent scored from 12 and 20 marks, 34.7 percent from 7.0 to 11.5 marks and 46.2 percent from 0 to 6.5. Moreover, the analysis shows that 14 candidates (0.2%) scored all 20 marks and 245 candidates (3.8%) got 0. The candidates' performance in the question was average, as 53.4 percent scored 7.0 marks or more. Figure 15 represents a summary of the candidates' performance in this question.

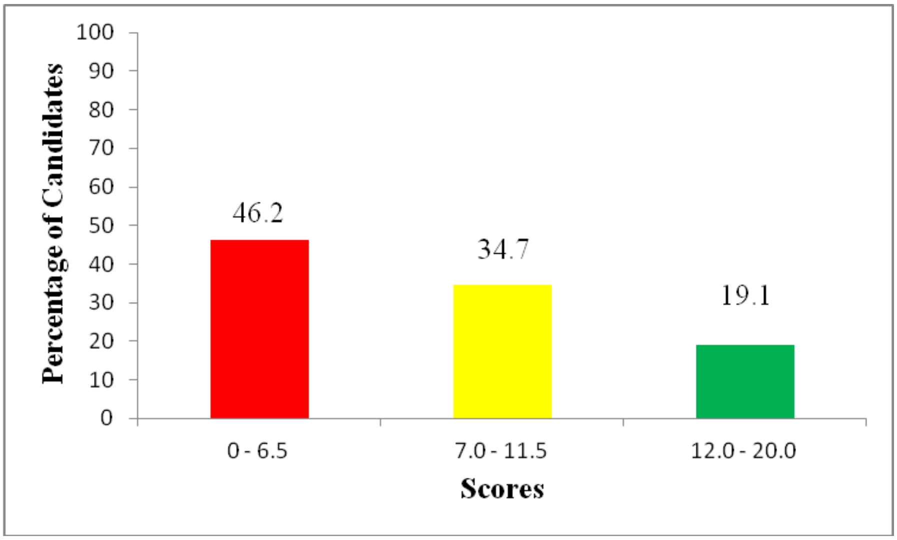


Figure 15: *Candidates' performance in question 5.*

The candidates who did part (a) (i) well used an identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ to obtain $2\cos 3\theta = (2\cos\theta)^3 - 3(2\cos\theta)$ and then replaced $2\cos\theta$ with $\frac{1}{x} + x$ to obtain $x^3 + \frac{1}{x^3}$ after careful simplification; (ii) they replaced $\cos\alpha$ and $\sin\alpha$ in $5\cos\alpha - 2\sin\alpha = 2$ with $\frac{1-t^2}{1+t^2}$ and $\frac{2t}{1+t^2}$, where $t = \tan\frac{\alpha}{2}$ and obtained an equation $7t^2 + 4t - 3 = 0$, which gives $t = -1$ and $t = \frac{3}{7}$. Then they solved $\tan\frac{\alpha}{2} = -1$ and $\tan\frac{\alpha}{2} = \frac{3}{7}$ and got the values $\alpha = -90^0$ and $\alpha = 46.4^0$ as per the given interval for α. In part (b) (i), the candidates who scored high marks obtained $\tan\theta = \frac{-2 \pm \sqrt{4 + 4\tan 2\theta \tan 2\theta}}{2\tan 2\theta}$ from $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, through which they substituted $\theta = \frac{\pi}{8}$ to arrive at $\tan\frac{1}{8}\pi = \sqrt{2} - 1$ as required. The candidates who correctly attempted part (b) (ii) squared both sides of $a\sin\theta + b\cos\theta = c$ and replaced $\sin^2\theta$ and $\cos^2\theta$ with $1 - \cos^2\theta$ and $1 - \sin^2\theta$ to obtain $a^2 - a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2 - b^2\sin^2\theta = c^2$. They then re-arranged it to get $a^2 + b^2 - c^2 = (a\cos\theta - b\sin\theta)^2$ and $a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$ as required. The analysis indicates that the good responses in part (c) (i) clearly showed the necessary steps in assigning letters to trigonometric inverses, for example $A = \tan^{-1} x$ and $B = \tan^{-1}\left(\frac{1-x}{1+x}\right)$, in writing an equation $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ and in replacing $\tan A$ and $\tan B$ with x and $\frac{1-x}{1+x}$ to obtain $\tan(A+B) = 1 \Rightarrow A + B = \frac{\pi}{4}$. After doing the necessary simplifications, they got $\tan^{-1} x + \tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{\pi}{4}$.

The candidates who did part (c) (ii) well simplified $\cos x\theta + \cos(x+2)\theta = \cos\theta$ with the aid of the factor formula to get $(2\cos(x+1)\theta - 1)\cos\theta = 0$ and solved it to obtain $\theta = \frac{1}{x+1}\left(2n \pm \frac{1}{3}\right)\pi$ and $\theta = \left(2n \pm \frac{1}{2}\right)\pi$ for $n = 0, 1, 2, \ldots$.

The candidates who did part (d) well assigned letters to trigonometric inverses, for example $A = \tan^{-1} a$, $B = \tan^{-1} b$ and $C = \tan^{-1} c$ and wrote an equation $\tan(A+B+C) = \tan\pi$. After further simplifications, they obtained $\frac{\tan A + \tan B}{1 - \tan A \tan B} + \tan C = 0$. Then they replaced $\tan A$, $\tan B$ and $\tan C$ with a, b and c, and got $\frac{a+b+c}{abc} = 1$. Extract 15.1 shows a sample response from a candidate who attempted the question well.

Extract 15.1

5 a/ i/ $2\cos\theta = \frac{1}{x} + x.$

$\cos 3\theta = \cos(2\theta + \theta)$

$\cos 3\theta = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$= (\cos^2\theta + \cos^2\theta - 1)\cos\theta - 2\sin^2\theta\cos\theta$

$= (2\cos^2\theta - 1)\cos\theta - 2\cos\theta(1 - \cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$2\cos 3\theta = 8\cos^3\theta - 6\cos\theta$

$2\cos 3\theta = (2\cos\theta)^3 - 3(2\cos\theta).$

$= \left(\frac{1}{x} + x\right)^3 - 3\left(x + \frac{1}{x}\right).$

$= \left(\frac{1}{x}\right)^3 + 3\left(\frac{1}{x}\right)^2 x + 3x^2\left(\frac{1}{x}\right) + x^3 - 3\left(x + \frac{1}{x}\right)$

$= \frac{1}{x^3} + \frac{3}{x} + 3x + x^3 - 3x - \frac{3}{x}.$

$2\cos 3\theta = \frac{1}{x^3} + x^3.$

Hence proved.

ii) From t formula.

$$\cos\alpha = \frac{1-t^2}{1+t^2} \quad \text{where } t = \tan\alpha/_2$$

$$\sin\alpha = \frac{2t}{1+t^2}$$

$$5\left(\frac{1-t^2}{1+t^2}\right) - 2\left(\frac{2t}{1+t^2}\right) = 2$$

$$5(1-t^2) - 4t = 2(1+t^2).$$
$$5 - 5t^2 - 4t = 2 + 2t^2$$
$$7t^2 + 4t - 3 = 0$$

Solving quadratically

$t = 0.428571428$ or $t = -1.$

$$\tan\alpha/_2 = 0.428571428.$$

$$\alpha/_2 = \tan^{-1}(0.428571428) = 23.2^\circ$$

from General soln

$$\theta = 180n + \alpha.$$

$$\frac{\alpha}{2} = 180n + 23.2$$

$$\alpha = 360n + 46.4^\circ \quad \text{where } n = 0, 1, 2 \ldots$$

$$\alpha = 46.4^\circ, 406.4^\circ$$

Also $\frac{\alpha}{2} = \tan^{-1}(-1). = -45$

$$\frac{\alpha}{2} = 180n + (-45)$$

$$\alpha = 360n - 90$$

$$\alpha = -90, 270$$

$$\therefore \alpha = ~~46~~ -90^\circ, 46.4^\circ.$$

5 b/. y

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}.$$

But $\theta = \pi/8$.

$$\tan(\pi/4) = \frac{2\tan(\pi/8)}{1-\tan^2(\pi/8)} = 1$$

$$1-\tan^2(\pi/8) = 2\tan(\pi/8)$$

$$\tan^2(\pi/8) + 2\tan(\pi/8) - 1 = 0$$

$$\tan(\pi/8) = \frac{-2 \pm \sqrt{4-4(1)(-1)}}{2}$$

$$\tan(\pi/8) = \frac{-2 \pm \sqrt{8}}{2}$$

$$\tan(\pi/8) = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$\tan(\pi/8) = -1 \pm \sqrt{2}.$$

But since $\pi/8$ is in first quadrant, Hence has two values

Then

$$\tan(\pi/8) = \sqrt{2} - 1.$$

Hence proved

5 b/ ii

$$(a\sin\theta + b\cos\theta)^2 = c^2$$

$$a^2\sin^2\theta + 2ab\sin\theta\cos\theta + b^2\cos^2\theta = c^2$$

$$a^2(1-\cos^2\theta) + 2ab\sin\theta\cos\theta + b^2(1-\sin^2\theta) = c^2$$

$$a^2 - a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2 - b^2\sin^2\theta = c^2$$

$$a^2\cos^2\theta + 2ab\cos\theta\sin\theta + b^2\sin^2\theta = a^2 + b^2 - c^2$$

$a\cos\theta(a\cos\theta -$

$$a^2\cos^2\theta - ab\cos\theta\sin\theta - ab\sin\theta\cos\theta + b^2\sin^2\theta = a^2 + b^2 - c^2$$

$$a\cos\theta(a\cos\theta - b\sin\theta) - b\sin\theta(a\cos\theta - b\sin\theta) = a^2 + b^2 - c^2$$

$$(a\cos\theta - b\sin\theta)(a\cos\theta - b\sin\theta) = a^2 + b^2 - c^2$$

$$(a\cos\theta - b\sin\theta)^2 = a^2 + b^2 - c^2$$

Hence

$$a\cos\theta - b\sin\theta = \pm\sqrt{a^2+b^2-c^2}$$

Proved

c. Let $\tan^{-1}(x) + \tan^{-1}\left(\frac{1-x}{1+x}\right) = c$.

Let $\tan^{-1} x = A \qquad \tan A = x.$

$\tan^{-1}\left(\frac{1-x}{1+x}\right) = B. \qquad \tan B = \frac{1-x}{1+x}$

$$A + B = c$$

$$\tan(A+B) = \tan c$$

$$\tan c = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan c = \frac{x + \left(\frac{1-x}{1+x}\right)}{1 - x\left(\frac{1-x}{1+x}\right)}.$$

$$\tan c = \frac{x + x^2 + 1 - x}{1 + x - x + x^2}$$

5 c. i.

$$\tan c = \frac{1+x^2}{1+x^2}.$$

$$\tan c = 1.$$

$$c = \tan^{-1}(1).$$

$$c = \pi/4.$$

Hence

$$\tan^{-1}(x) + \tan^{-1}\left(\frac{1-x}{1+x}\right) = \pi/4.$$

5 c. ii.

$$\cos x\theta + \cos(x+2)\theta = \cos\theta$$

By factor formula.

$$2\cos\left(\frac{x\theta + x\theta + 2\theta}{2}\right)\cos\left(\frac{x\theta - x\theta - 2\theta}{2}\right) = \cos\theta.$$

$$2\cos(x\theta + \theta)\cos\theta = \cos\theta$$

$$2\cos(x\theta+\theta)\cos\theta - \cos\theta = 0$$

$$\cos\theta\left(2\cos(x+1)\theta - 1\right) = 0$$

Hence

$$\cos\theta = 0. \quad \text{and} \quad \cos(x+1)\theta = \tfrac{1}{2}$$

5 c/

$$\theta = \cos^{-1}(0) = \underline{90}$$

$$\theta = 360^\circ n \pm 90$$

$$\theta = 360^\circ n \pm 90 \text{ ——— i}$$

Also $\cos(x+1)\theta = \tfrac{1}{2}$

$$(x+1)\theta = \cos^{-1}(\tfrac{1}{2}).$$

$$(x+1)\theta = 60^\circ$$

$$(x+1)\theta = 360^\circ n \pm 60^\circ$$

$$\theta = \frac{1}{(x+1)}\left(360^\circ n \pm 60^\circ\right).$$

$\therefore$ All values of θ are

$$\theta = 360^\circ n \pm 90^\circ \text{ and } \theta = \frac{1}{(x+1)}(360^\circ n \pm 60^\circ).$$

where $n = 0, 1, 2, 3, 4 \ldots$

5 d/

$$\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi.$$

$$\tan^{-1}a + \tan^{-1}b = \pi - \tan^{-1}c.$$

Apply tan both sides

$$\tan(\tan^{-1}a + \tan^{-1}b) = \tan(\pi - \tan^{-1}c)$$

$$\frac{\tan(\tan^{-1}a) + \tan(\tan^{-1}b)}{1 - \tan(\tan^{-1}(a))\tan(\tan^{-1}b)} = \frac{\tan\pi - \tan(\tan^{-1}c)}{1 - \tan\pi\tan(\tan^{-1}c)}$$

$$\frac{a+b}{1-ab} = \frac{0-c}{1-0(c)}$$

$$\frac{a+b}{1-ab} = \frac{-c}{1.}$$

$$a+b=-c(1-ab)$$
$$a+b=-c+abc$$
$$a+b+c=abc$$
$$\frac{a+b+c}{abc}=1$$
$$\therefore \frac{a+b+c}{abc}=1 \quad \text{Hence shown.}$$

Extract 15.1: *A correct response from one of the candidates.*

Further analysis shows that nearly half of the candidates who attempted the question (46.6%) had poor performance, as they scored from 0 to 6.5 marks. In part (a) (i), some of the candidates made certain mistakes at the beginning by expressing $8\cos 3\theta = (2\cos\theta)^3$, while other candidates could not write $2\cos 3\theta$ in the form of $2\cos\theta$ although they knew that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. Hence, the candidates got expressions different from $x^3 + \frac{1}{x^3}$. The candidates' poor performance in part (a) (ii) was due to the use of incorrect formulae such as $\cos\alpha = \frac{1+t^2}{1-t^2}$. In part (b), most of the candidates failed to formulate an equation $\tan^2\left(\frac{\pi}{8}\right) + 2\tan\left(\frac{\pi}{8}\right) - 1 = 0$ from $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ by substituting $\theta = \frac{\pi}{8}$. Others did not apply the quadratic formula to $\tan^2\left(\frac{\pi}{8}\right) + 2\tan\left(\frac{\pi}{8}\right) - 1 = 0$. As a result, they could not show $\tan\left(\frac{1}{8}\pi\right) = \sqrt{2} - 1$. Moreover, a few candidates who reached at $\tan\left(\frac{1}{8}\pi\right) = -1 \pm \sqrt{2}$ did not know that $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$. These candidates were not aware of the fact that $\frac{\pi}{8}$ is an acute angle. Most of the candidates skipped part (b) (ii) or did start off in the right way by comparing $a\sin\theta + b\cos\theta$ with

$R\sin(\theta+\alpha)$. In (c) (i), the candidates did very well. In part (c) (ii), they could not obtain two factors $(2\cos(x+1)\theta-1)$ and $\cos\theta$ from $\cos x\theta+\cos(x+2)\theta=\cos\theta$. Hence, they got an incorrect solution. Furthermore, some of the candidates failed to express solutions in general form. The vast majority of the candidates who attempted part (d) assigned letters to three trigonometric inverses, but most of them failed to express $\tan(A+B+C)$ in terms of $\tan A$, $\tan B$ and $\tan C$. Extract 15.1 is a sample response from a candidate who got the question wrong.

Extract 15.2

Qn 5(b) Solution

Given

$\theta = \pi/8$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

from

$\theta = \pi/8$

$\tan 2(\pi/8) = \frac{2(\tan(\pi/8))}{1-\tan^2(\pi/8)^2}$

$\tan 2\pi/8 = \frac{2\tan \pi/8}{1-(\tan \pi/8)^2}$

$\tan 45 = \frac{0.828}{1-0.17}$

$\tan 45 = 0.9975$

$1 =$

Qn 5 (c) (i) Solution

Extract 15.2: *An incorrect response from one of the candidates.*

2.2.6 Question 6: Probability

This question had parts (a), (b), (c) and (d). In part (a), the candidates were instructed that the independent events A and B were such that $P(A)=2$ and $P(B)=0.4$ and were required to determine (i) $P(\text{not A and B})$ and (ii) $\text{P}(\text{A or B})$. In part (b), the candidates were instructed that "Kalihose's family consists of mother, father and their ten children and it is invited to send a group of 4 representatives to a wedding". They were required to find the number of the ways in which the group could be formed so that it could contain (i) both parents, (ii) one parent only and (iii) none of the parents. In part (c), the candidates were given that a continuous random variable X had the probability density function given by $f(x)=\begin{cases} kx & 0\le x\le 1 \\ k(2-x) & 1\le x\le 2 \\ 0 & \text{otherwise} \end{cases}$ and were required to find (i) the value of the constant k, (ii) $E(X)$ and (iii) $P\left(\frac{1}{2}\le x\le 1\frac{1}{2}\right)$. In part (d), the candidates were given the information that a random variable X with mean 4 and variance 2 follows a Binomial distribution and were required to find $P(|x-4|\le 2)$ in four significant figures.

The analysis shows that the question was opted by 1939 candidates (16.1%), out of whom, 23.9 percent passed. Further analysis shows that 76.1 percent of the candidates scored from 0 to 6.5 marks, 16.3 percent from 7 to 11.5 marks and 7.6 percent from 12 to 20 marks. The analysis also indicates that only one candidate scored all 20 marks and that 331 candidates (17.1%) got zero. Therefore, the performance was very poor, as shown in Figure 16.

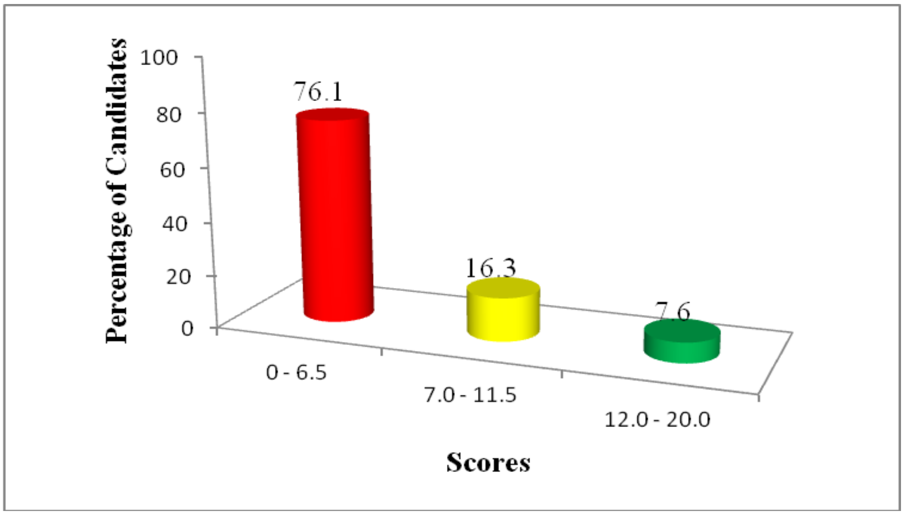


Figure 16*: Candidates' performance in question 6.*

The analysis reveals that those who failed part (a) of the question could not apply the right formula to determine the probability of $P(\text{not A and B})$. For example, they wrote $P(\text{not A and B}) = P(A') + P(B)$, instead of writing $P(\text{not A and B}) = P(A') \times P(B)$. This means that they were not aware of the probability of independent events. Other candidates used the formulae correctly but could not comment that $P(\text{not A and B})$ and $\text{P}(\text{A or B})$ does not exist because $P(A) = 2$ is not in the interval $[0, 1]$. This means that the candidates did not know the axioms and laws of probability. In part (b), several candidates could not comprehend the given word problem so as to realize that they should use the principle of combination in answering this question. Some of them employed the principle of permutations, while others interpreted the given word problem in the perception of the Sets' concept and hence used Venn diagrams contrary to the requirements of the question. In part (c) (i), most the candidates could not compute the value of *k*. Such candidates evaluated the integral $\int_0^2 \left(kx + k\left(2 - x\right)\right)dx = 1$ rather than $\int_0^1 \text{kxdx} + \int_1^2 \text{k}(2 - \text{x})\text{dx} = 1$. As a result, they got $k = \frac{1}{4}$ instead of $k = 1$. Part (c) (ii) and (iii), which required the candidates to find $E\left(X\right)$ and $P\left(\frac{1}{2} \le x \le 1\frac{1}{2}\right)$, were also badly done for the same reasons as those in part (c) (i). In part (d), some the candidates did not use a Binomial distribution as instructed. Instead, they used a Poisson distribution and formulae, such as $z = \frac{x - \mu}{\sigma}$, to find $P(|x - 4| \le 2)$. Additionally, several candidates could not use the formulae $\text{variance} = npq$ and $\text{mean} = np$ which would enable them to find the value of *n* as $\frac{npq}{np} = \frac{2}{4} = \frac{1}{2}$. As a result, they unsuccessfully computed $P(|x - 4| \le 2)$. Surprisingly, a significant number of the candidates failed to express $P(|x - 4| \le 2)$ as

$P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)$. This implies that they had not understood the concept of absolute value. Extract 16.1 is a sample answer from one of the candidates who got part (d) wrong.

Extract 16.1

6 (ii) $P\left(\frac{1}{2} \leq x \leq \frac{3}{2}\right)$.

From $\int f(x)\,dx$

$$\int_{1/2}^{3/2} x\,dx + \int_{1/2}^{3/2} (2-x)\,dx.$$

$$P(x) = \left[\frac{x^2}{2}\right]_{1/2}^{3/2} + \left[2x\right]_{1/2}^{3/2} - \left[\frac{x^2}{2}\right]_{1/2}^{3/2}$$

$$P(x) = [1.125 - 0.125] + (3 - 1) - [1.125 - 0.125]$$

$$P(x) = 1.$$

Extract 16.1: *An incorrect response from one of the candidates.*

Only 29, out of the 1939 candidates who opted the question, produced good responses. A sample answer from one of such candidates is shown in Extract 16.2

Extract 16.2

6a Given $P(A) = 2$ and $P(B) = 0.4$

(i) If two events are independent, means, $P(A \cap B) = P(A) \times P(B)$

then, $P(\text{not } A \text{ and } B) = P(A' \cap B)$

$= P(A') \times P(B)$

$= [1 - P(A)] \times P(B)$

$= [1 - 2] \times 0.4$

$= -0.4$

(ii) $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 2 + 0.4 - (2 \times 0.4)$

$= 1.6$

∴ The probability in (i) is negative because we are given $P(A) = 2$.

but from axioms of probability it say that Probability of any event is in interval $0 \leq P(E) \leq 1$

and for (ii), The probability is greater than 1 because $P(A) = 2$ which is not possible.

b Total number of parents is 2.

and total number of children is 10

(i) For both parents, we use the concept of combination,

$2C2 \times 10C_2 = 45$ ways

∴ 45 ways for both parents.

6b (ii) For one parent only,

$$2C_1 \times 10C_3 = 240 \text{ ways}$$

∴ There are 240 ways for one parent only

(iii) For none of the parents,

$$2C_0 \times 10C_4 = 210 \text{ ways.}$$

∴ There 210 ways for none of the parents.

c Given,

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k(2-x), & 1 \le x \le 2 \\ 0, & \text{other wise} \end{cases}$$

(i) From $\int_{-\infty}^{\infty} f(x)dx = 1$

then, $\int_0^1 kx\,dx + \int_1^2 k(2-x)dx = 1$

$$\left(\frac{kx^2}{2}\right)\Big|_0^1 + \left(k\left(2x - \frac{x^2}{2}\right)\right)\Big|_1^2 = 1$$

$$\frac{k}{2} + k(4-2) - k\left(2 - \frac{1}{2}\right) = 1$$

$$\frac{k}{2} + 2k - \frac{3}{2}k = 1$$

$$2k - 1 = 1$$

$$2k = 2$$

$$k = 1$$

∴ The value of the k is 1

6c (ii) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

then,

$$E(X) = \int_0^1 x^2\,dx + \int_1^2 (2x - x^2)\,dx$$

$$E(X) = \left(\frac{x^3}{3}\right)\Big|_0^1 + \left(x^2 - \frac{x^3}{3}\right)\Big|_1^2$$

$$E(X) = \frac{1}{3} + \left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)$$

$$E(X) = \frac{1}{3} + \frac{4}{3} - \frac{2}{3}$$

$$E(X) = 1$$

(iii) $$P\left(\frac{1}{2} \le x \le 1\frac{1}{2}\right) = \int_{1/2}^{1} x\,dx + \int_1^{3/2} (2 - x)\,dx$$

$$= \left(\frac{x^2}{2}\right)\Big|_{1/2}^{1} + \left(2x - \frac{x^2}{2}\right)\Big|_1^{3/2}$$

$$= \left(\frac{1}{2} - \frac{1}{8}\right) + \left(3 - \frac{9}{8}\right) - \left(2 - \frac{1}{2}\right)$$

$$= \frac{3}{4}$$

d Given, Mean $= np = 4$

and Variance $= npq = 2$

then make the two equations,

$np = 4$ —(i)

$npq = 2$ —(ii)

Make subject n in the equation (i)

$n = \frac{4}{p}$, then substitute to the equation

6d $\frac{4}{p} \times p \times q = 2$

$4q = 2$

$q = \frac{2}{4}$

$q = 0 \cdot 5$

then, $p = 0 \cdot 5$

Now, $P(|X-4| \leq 2) = P(-2 \leq X-4 \leq 2)$

$= P(-2+4 \leq X-4+4 \leq 2+4)$

$= P(2 \leq x \leq 6)$

$= P(2) + P(3) + P(4) + P(5) + P(6)$

From formular of probability in binomial distribution,

$P(X=x) = \binom{n}{x} p^x q^{n-x}$

but $np = 4$

$n = \frac{4}{0 \cdot 5}$

$n = 8$

$P(X=2) = \binom{8}{2} (0 \cdot 5)^2 (0 \cdot 5)^{8-2} = 0 \cdot 1094$

$P(X=3) = \binom{8}{3} (0 \cdot 5)^3 (0 \cdot 5)^5 = 0 \cdot 2188$

$P(X=4) = \binom{8}{4} (0 \cdot 5)^4 (0 \cdot 5)^4 = 0 \cdot 2734$

$P(X=5) = \binom{8}{5} (0 \cdot 5)^5 (0 \cdot 5)^3 = 0 \cdot 2188$

$P(X=6) = \binom{8}{6} (0 \cdot 5)^6 (0 \cdot 5)^2 = 0 \cdot 1094$

If we add them,

$P(|X-4| \leq 2) = 0 \cdot 9297$ to 4 significant figures.

Extract: 16.2 *A correct response from one of the candidates.*

2.2.7 Question 7: Differential Equations

This question comprised parts (a), (b), (c) and (d). In part (a), the candidates were required to form a differential equation by eliminating arbitrary constants in the equation $Ax^2 + By^2 = 1$. In part (b), they were required to solve $(1+y^2)dx = (\tan^{-1} y - x)dy$. In part (c), the candidates were instructed that "the rate of decrease of the temperature of a body is proportional to the difference between the temperature of the body and that of the surrounding air. If water at temperature 100^{o}C cools in 20 minutes to 78^{o}C in a room temperature 25^{o}C". They were asked to find the temperature of water after 30 minutes and write in two decimal places. In part (d), the candidates were given the condition that $y = 1$, $\frac{dy}{dx} = 0$, where $x = 0$, and required to find the general solution of $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2\sin 3x$.

The statistical analysis shows that 7275 candidates (60.3%) opted the question. Out of such candidates, 43.2 percent scored from 0 to 6.5 marks, 39 percent from 6.5 to 11.5 marks and 17.8 percent from 12 to 20 marks. The candidates' performance in this question was average, since 56.8 percent scored from 7 to 20 marks. Figure 17 shows the candidates' performance in this question.

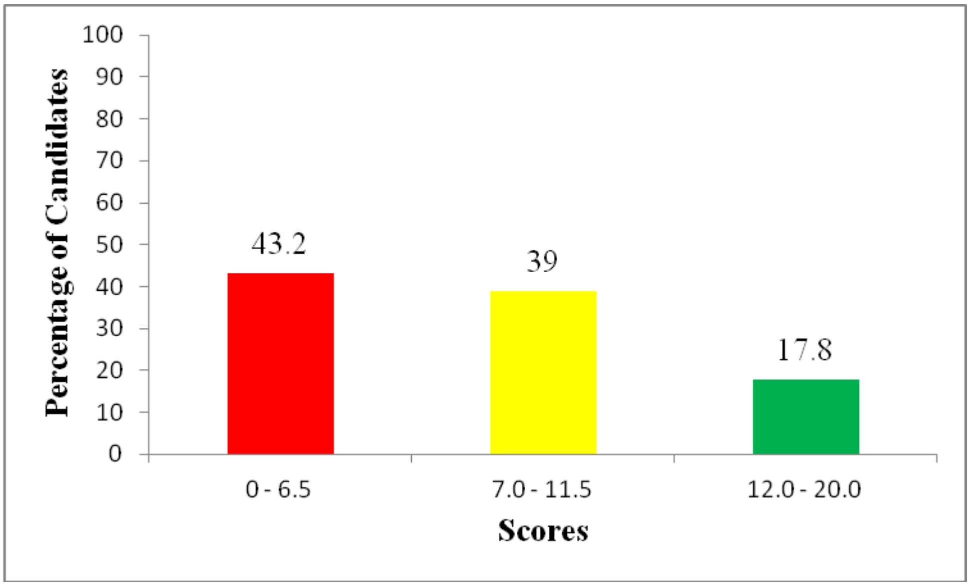


Figure 17: C*andidates' performance in question 7.*

In part (a), several candidates eliminated the constants A and B from the equation $Ax^2 + By^2 = 1$ to form the differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$. In part (b), a few candidates wrote the given differential equation in the form $\frac{dx}{dy}+Px=Q$ as $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$ that enabled them to correctly derive the integrating factor $e^{\tan^{-1}y}$. The candidates also multiplied $e^{\tan^{-1}y}$ on both sides of $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$ to produce another equation which was solved to obtain $x=\left(\tan^{-1}y-1\right)+ce^{-\tan^{-1}y}$.

In part (c), a number of candidates derived the equation $\frac{dT}{dt}=-k\left(T-T_0\right)$ from the given word problem and used it to calculate the temperature of water after 30 minutes, as shown in Extract 17.2. In part (d), the candidates determined the complementary solution $y_c=Ae^{x}+Be^{6x}$ as well as the particular solution $y_p=\frac{5}{37}\sin x+\frac{7}{37}\cos x$ of the non-homogeneous second order differential equation. The candidates also combined the expression for y_c and y_p correctly and obtained the general solutions as $y=Ae^{x}+Be^{6x}+\frac{5}{37}\sin x+\frac{7}{37}\cos x$. Finally, they used the initial conditions $y=1$, $\frac{dy}{dx}=0$, where $x=0$, to find the constants A and B and thus got the required general solution, that is, $y=e^{x}-\frac{7}{37}e^{6x}+\frac{5}{37}\sin x+\frac{7}{37}\cos x$.

Extract 17.1

7(c). Given that:

Rate of temperature fall α excess temp.

or $-\frac{d\theta}{dt} \cancel{\alpha} \cdot (\theta - \theta_s)$

or $-\frac{d\theta}{dt} = k(\theta - \theta_s)$

$\theta_s =$ Surrounding air temperature

$k =$ constant.

$$\frac{1\, d\theta}{\theta - \theta_s} = -k\,dt$$

$$\int_{\theta_0}^{\theta} \frac{d\theta}{\theta - \theta_s} = -k\int_0^t dt$$

$$\ln\left(\frac{\theta - \theta_s}{\theta_0 - \theta_s}\right) = -kt$$

$$k = -\frac{1}{t}\ln\left(\frac{\theta - \theta_s}{\theta_0 - \theta_s}\right).$$

Given that, $\theta_0 = 100°c$, $t = 20$ min
$\theta = 78°$, $\theta_s = 25°c$

$$k = -\frac{1}{20}\ln\left(\frac{78 - 25}{100 - 25}\right)$$

$$k = -\frac{1}{20}\ln\left(\frac{53}{75}\right) \text{ ——— (1)}$$

7(c). Use $\ln\left(\frac{\theta - \theta_s}{\theta_0 - \theta}\right) = -kt$

when, $t = 30$ min $\theta = ?$,

$$\theta = \theta_s + (\theta_0 - \theta_s)e^{-kt}.$$

$$\theta = 25 + (\theta_0 - 25)e^{-kt}.$$

30 minutes after the water was originally at $\theta_0 = 100^\circ C$,

$$\theta = 25 + (100 - 25)e^{+\left(\frac{1}{20}\ln\left(\frac{50}{77}\right) \times 30\right)}.$$

$$\theta = \underline{69.55^\circ C}.$$

Therefore, 30 minutes after the water was originally at $100^\circ C$, it cooled to $\underline{\underline{69.55^\circ C}}$.

7(d) Given $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2\sin x$

AUXILLIARY QUADRATIC EQUATION

A.Q.E,

$m^2 - 7m + 6 = 0$

$m = 6$ or $m = 1$

Hence the Complementary function solution is

$$y_c = Ae^{6x} + Be^{x} \quad \text{——— (i)}$$

For particular integral,

let $y_p = A\sin x + B\cos x$

A and B = constants,

$y_p' = A\cos x - B\sin x$

$y_p'' = -A\sin x - B\cos x$.

Insertions into D.E

$-A\sin x - B\cos x - 7A\cos x + 7B\sin x$

$+ 6A\sin x + 6B\cos x = 2\sin x$.

$(-A + 7B + 6A)\sin x + (-B - 7A + 6B)\cos x = 2\sin x$

$(5A + 7B)\sin x + (-7A + 5B)\cos x = 2\sin$.

Equating corresponding coefficients

$5A + 7B = 2$

$-7A + 5B = 0$.

Hence $A = \frac{5}{37}$ $B = \frac{7}{37}$,

$y_p = \frac{5}{37}\sin x + \frac{7}{37}\cos x$ ——— (ii)

Hence the General solution is

$y = y_c + y_p$

$y = Ae^{6x} + Be^{x} + \frac{5}{37}\sin x + \frac{7}{37}\cos x$.

7(d) When $x = 0$, $y = 1$, $\frac{dy}{dx} = 0$

$1 = A + B + \frac{7}{37}$.

$A + B = \frac{30}{37}$ ——— (iii).

$\frac{dy}{dx} = 6Ae^{6x} + Be^{x} + \frac{5}{37}\cos x - \frac{7}{37}\sin x$

Eqtn x 2

$0 = 6A + B + 5/37$

$6A + B = -5/37$ — (iv)

solving (iii) and (iv)

$A = -7/37 \quad B = 1.$

Hence the solution is.

$Y = -\frac{7}{37}e^{6x} + e^{x} + \frac{5}{37}\sin x + \frac{7}{37}\cos x$

solution is

$Y = -\frac{7}{37}e^{6x} + e^{x} + \frac{5}{37}\sin x + \frac{7}{37}\cos x.$

Extract 17.1: *A correct response from one of the candidates.*

On the other hand, the candidates' inability to answer the question correctly can be attributed to various factors. In part (a), the candidates failed to differentiate the equation $Ax^2 + By^2 = 1$ twice in order to eliminate the constants A and B. In part (b), some of the candidates failed to rearrange the terms of the equation $(1 + y^2)dx = (\tan^{-1} y - x)dy$ into an exact differential equation of the form $\frac{dx}{dy} = f(y)g(x)$ using the appropriate integrating factor $e^{\tan^{-1} y}$. As a result, they did not solve it. In part (c), the candidates did not transform the given word problem mathematically as $\frac{dT}{dt} = -k(T - T_0)$. Most of them ignored the negative sign in the formula expressing the rate of the decrease in temperature as

$\frac{dT}{dt} = k(T - T_0)$. However, the error did not affect the final answer. Other candidates did not know that, at time $t = 0$, the temperature of water is 100°C. Hence, they failed to compute the constant of proportionality k in the equation $\frac{dT}{dt} = -k(T - T_0)$. In part (d), some of the candidates failed to find the solution of the complementary and particular integral, while others did not apply the initial conditions $y = 1$, $\frac{dy}{dx} = 0$, where $x = 0$, to eliminate the arbitrary constants A and B from the equation $y = Ae^x + Be^{6x} + \frac{5}{37}\sin x + \frac{7}{37}\cos x$. Extract 17.2 is a sample answer from a candidate who got the question wrong.

Extract 17.2

7 (a) Soln.

$Ax^2 + By^2 = 1$

diff w.r.t x

$\frac{d}{dx}\left(Ax^2 + By^2\right) = \frac{d}{dx}(1)$

$2Ax + 2yB\frac{dy}{dx} = 0$

diff w.r.t x

$2A + 2B\frac{d^2y}{dx^2} = 0$

$A + B\frac{d^2y}{dx^2} = 0$

$B\frac{d^2y}{dx^2} = -A$

$\cancel{B}y\frac{dy}{dx} = \cancel{B}\frac{d^2y}{dx^2}x$

$y\frac{dy}{dx} - x\frac{d^2y}{dx^2} = 0$

$x\frac{d^2y}{dx^2} - y\frac{dy}{dx} = 0$

Extract 17: *An incorrect response from one of the candidates.*

2.2.8 Question 8: Coordinate Geometry II

The question had parts (a), (b) and (c). In part (a), the candidates were required to show that the point $B(5,-5)$ lies on the parabola $y^2 = 5x$ and find the equation of the normal to the parabola at the point B in the form $y = mx + c$. In part (b), the candidates were given that the equation $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and were required to express c in terms of a, b and m. Part (c) required the candidates to (i) find the rectangular equation of $r = 12(1+\sin\theta)$ and (ii) sketch the graph of $r = \sin 2\theta$ for $0 \le \theta \le \pi$.

The analysis shows that the question was opted by 7818 candidates, (64.8%) out of whom, 48.3 percent scored from 12 to 20 marks, 28.1 percent scored from 7.0 to 11.5 marks and 23.6 percent scored from 0 to 6.5 marks. Further analysis indicates that 91 candidates (0.8%) did the question well and scored all 20 marks, and that 223 candidates (2.9%) scored 0. The candidates' performance in this question was good, as a substantial number of the candidates (76.3%) scored 7 marks or more. Figure 18 shows the candidates' performance in this question.

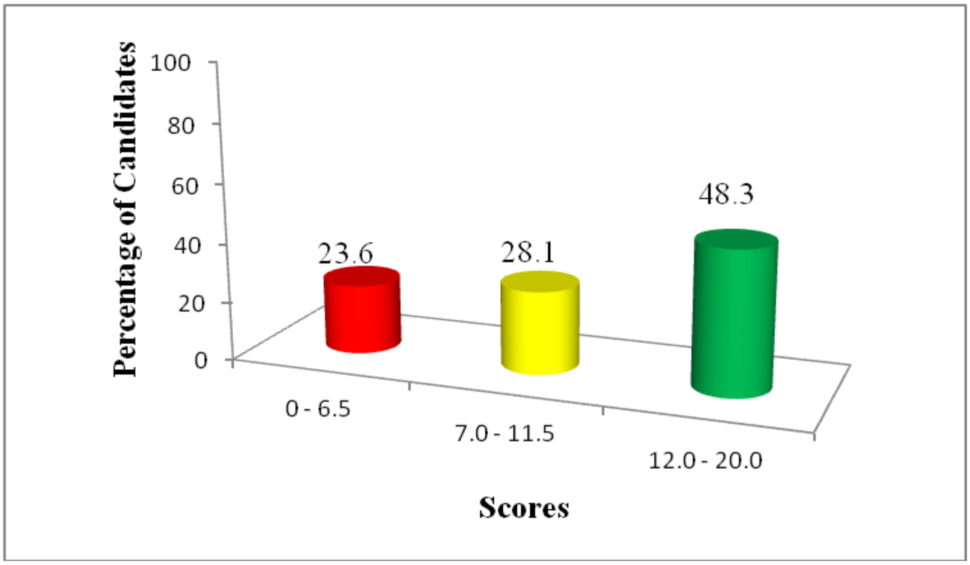


Figure: 18 *Candidates' performance in question 8.*

The candidates who did part (a) well substituted $x=5$ and $y=-5$ into $y^2=5x$ and got $25=25$, which confirms that the point $B(5,-5)$ lies on the parabola $y^2=5x$. Furthermore, they differentiated $y^2=5x$ implicitly to obtain $\frac{dy}{dx}=\frac{5}{2y}$ and substituted a point $B(5,-5)$ into $\frac{5}{2y}$ to get $-\frac{1}{2}$, which is the slope of the tangent to the parabola. Since the normal is always perpendicular to the tangent, they used the formula $m_1m_2=-1$ to get 2, which is the gradient of the normal to the parabola. The candidates then determined the equation of the normal to the parabola as being $y=2x-15$. The candidates who did part (b) well substituted $y=mx+c$ into $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and formulated a quadratic equation in x as $\left(b^2+a^2m^2\right)x^2+2mca^2x+a^2c^2-a^2b^2=0$. Consequently, they recalled that the condition for a quadratic equation had equal roots and upon necessary manipulation and simplification of $\left(2mca^2\right)^2=4\left(a^2c^2-a^2b^2\right)\left(b^2+a^2m^2\right)$ they arrived at $c=\pm\sqrt{a^2m^2+b^2}$. The candidates' good performance in part (c) (i) can be attributed to the candidates' ability to correctly substitute $r=\sqrt{x^2+y^2}$ and $\sin\theta=\frac{y}{\sqrt{x^2+y^2}}$ into $r=12(1+\sin\theta)$ so as to eliminate r and θ and to perform the associated algebra to get an equation $\left(x^2+y^2\right)^2=144\left(y+\sqrt{x^2+y^2}\right)^2$. In addition, the candidates created a table of values relating r and θ in the interval $0\le\theta\le\pi$, plotted the points (r,θ) on a system of polar coordinates and drew a smooth curve through the points in part (c) (ii). Extract 18.1 is a sample response from a candidate who got the question right.

Extract 18.1

08 Soln

$y^2 = 5x$ ——— (i)

point $= B(5, -5)$

If point B satisfy equation (i) on subustuting it's value on it then lies on that parabola.

Consider L.H.S

$= y^2$ but $B(5, -5)$

$= (-5)^2$

$= 25$

Consider R.H.S

$= 5x$ $B(+5, -5)$

$= 5 \times 5$

$= 25$

(a)

Since R.H.S = L.H.S = 25 then point B lies on the parabola $y^2 = 5x$

<u>gradient of tangent</u> at B.

$y^2 = 5x$

$2y \frac{dy}{dx} = 5.$

$\frac{dy}{dx} = \frac{5}{2y}$ at $(5, -5)$

$\frac{dy}{dx} = \frac{5}{2 \times -5}$

$\frac{dy}{dx} = -1/2$

But tangent is perpendicular to normal then m_2 (slope of a normal)

$m_1 m_2 = -1$

$m_2 = \frac{-1}{-1/2} = 2.$

then. <u>equation of a normal</u>

From

$(y - y_0) = m(x - x_0)$

$(y + 5) = 2(x - 5)$

$y + 5 = 2x - 10$

$y = 2x - 10 - 5$

$y = 2x - 15.$

$\therefore$ equation of the normal to the parabola is given as $y = 2x - 15$

02 Soln.

(b)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ——— (i) ellipse}$$

$y = mx + c$ — — (2) tangent to ellipse

Substitute (2) into (i)

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

$b^2x^2 + a^2(mx+c)^2 = a^2b^2$

$b^2x^2 + a^2(m^2x^2 + 2mcx + c^2) = a^2b^2$

$b^2x^2 + a^2m^2x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$

$(b^2 + a^2m^2)x^2 + 2a^2mcx + a^2(c^2 - b^2) = 0$

But equation (2) is a tangent then it touches ellipse at one point, hence equation above is perfect square

$b^2 = 4ac$

$(2a^2mc)^2 = 4a^2(b^2 + a^2m^2)(c^2 - b^2)$

$4a^4m^2c^2 = 4a^2(b^2 + a^2m^2)(c^2 - b^2)$

$a^2m^2c^2 = (b^2c^2 - b^4 + a^2m^2c^2 - a^2m^2b^2)$

$b^2c^2 = b^4 + a^2m^2b^2$

$c^2 = b^2 + a^2m^2$.

$c = \pm\sqrt{b^2 + a^2m^2}$

∴ the value of c in terms of a, b, m is given as $c = \pm\sqrt{b^2 + a^2m^2}$.

$$\sqrt{x^2+y^2} = \frac{12(\sqrt{x^2+y^2} + y)}{\sqrt{x^2+y^2}}$$

$(x^2+y^2) = 12(\sqrt{x^2+y^2} + y)$.

$[(x^2+y^2) - 12y]^2 = [12\sqrt{x^2+y^2}]^2$

$(x^2+y^2)^2 - 24(x^2+y^2)y + 144y^2 = 144(x^2+y^2)$

$(x^2+y^2)^2 - 24(x^2+y^2)y = 144x^2$.

$x^4 + 2x^2y^2 + y^4 - 24x^2y - 24y^3 - 144x^2 = 0$.

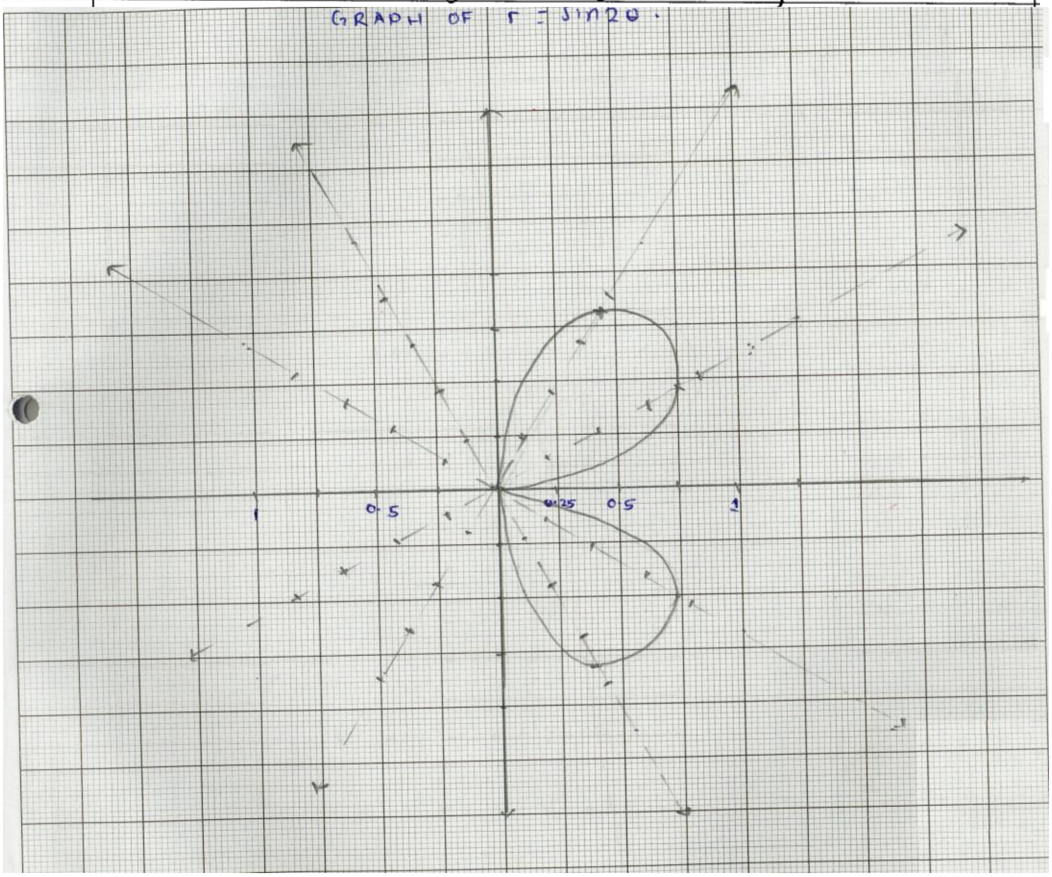
$(x^2+y^2)^2 - 24(x^2+y^2)y = 144x^2.$

$x^4+2x^2y^2+y^4-24x^2y-24y^3-144x^2=0.$

$x^4+y^4-24y^3+2x^2y^2-24x^2y-144x^2=0.$

$\therefore$ rectangular equation is given as

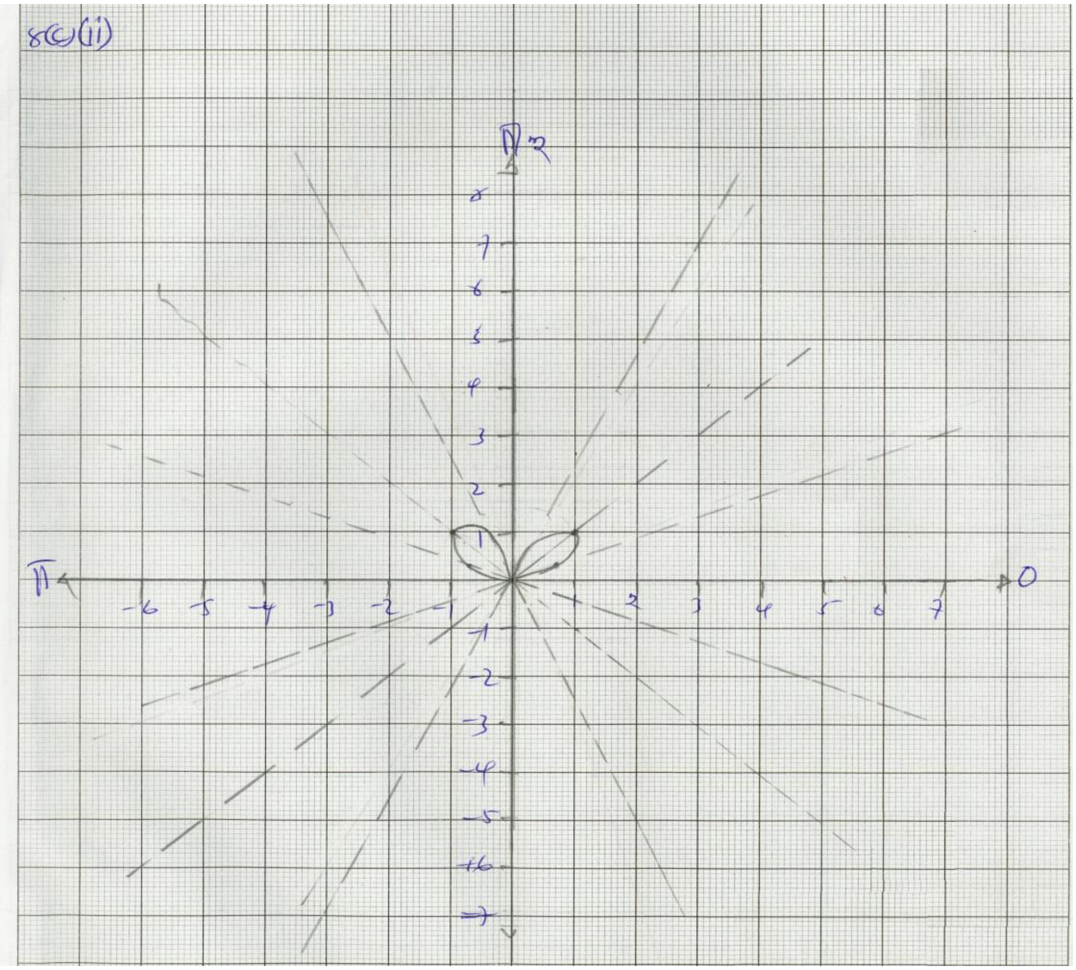
$x^4+y^4-24y^3+2x^2y^2-24x^2y-144x^2=0$



Extract 18.1: *A correct response from one of the candidates.*

Although the performance was good, 1843 candidates (23.7%) did not answer the question well. In part (a), a few candidates did not understand the question, as they did not substitute $x=5$ and $y=-5$ into $y^2=5x$. Similarly, in part (b), a few candidates did not understand the question, as they did not substitute $y=mx+c$ into $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, while others made the right substitution but failed to simplify it to get $\left(b^2+a^2m^2\right)x^2+2mca^2x+a^2c^2-a^2b^2=0$. Moreover, the analysis shows that many candidates did not know the tangency condition to a curve i.e. $b^2 = 4ac$. In part (c) (i), some of the candidates used an incorrect substitution of $\sin\theta = \frac{x}{\sqrt{x^2+y^2}}$ instead of $\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$. Thus, they got an incorrect rectangular equation. In part (c) (ii), the candidates could not plot the points (r,θ) on an xy plane. The analysis indicates that the majority of the candidates prepared a correct table of values for $r = \sin 2\theta$ using the interval $0 \le \theta \le \pi$, but plotted loops on a system of polar coordinates which were not in the first and fourth quadrants. Extract 18.2 is a sample response from a candidate who got the question wrong.

Extract 18.2



Extract 18.2: *An incorrect response from one of the candidates.*

3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Advanced Mathematics Examination consisted of two question papers, Advanced Mathematics 1 and Advanced Mathematics 2 with a total of eighteen (18) questions based on eighteen (18) topics. In both papers, the candidates were required to attempt a total of sixteen (16) questions.

The topics on which paper 1 was based are: *Calculating Devices*, *Hyperbolic Functions*, *Linear Programming*, *Statistics*, *Sets*, *Functions*, *Numerical Methods*, *Coordinate Geometry* I, *Integration* and *Differentiation*. The topics on which paper 2 was based are: *Complex Numbers*, *Logic*, *Vectors*, *Algebra*, *Trigonometry*, *Probability*, *Differential Equations* and *Coordinate Geometry* II.

The analysis indicates that the candidates' performance was good in eleven (11) topics. These topics are: *Linear Programming*, *Hyperbolic Functions*, *Statistics*, *Sets*, *Functions*, *Numerical Methods*, *Calculating Devices*, *Coordinate Geometry I*, *Logic*, *Coordinate Geometry* II and *Differentiation*. The candidates' good performance in those topics was due to candidates' ability to apply the formulae, definitions, laws, theorems and techniques correctly. Further analysis shows that the candidates' performance in six (6) topics was average. The topics are *Differential Equations*, *Vectors*, *Trigonometry*, *Algebra*, *Complex Numbers* and *Integration*. These topics were performed averagely because of the presence of (30 - 49) percent of the candidates who got more than 35 percent of the marks which were allocated for the questions**.**

On the other hand, the poorest or worst candidates' performance was in the topic on Probability, as only 23.9 percent of the candidates passed. This performance is due to the candidates' inability to apply the formulae, theorems, axioms, principles and special statistical distributions to answer the question based on this topic.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

This report is specifically intended to make stakeholders aware of the candidates' performance in the 2018 ACSEE. Therefore, it presents the strengths and weaknesses in relation to the responses. The Candidates' Items-Response Analysis (CIRA) in 142-Advanced Mathematics 2018 has shown that 83.74 percent of the candidates passed the examination, in comparison to the 74.78 percent of the candidates who passed the 2017 examination.

The analysis shows that the candidates had good performance in eleven (11) topics, had good performance, average performance in six (6) topics and poor performance in one (1) topic. The factors for good or poor performance are given in section 3.0.

Finally, it is the hope of the Council that this report will be a useful special guide to teachers, students, policy makers, curriculum developers and other education stakeholders in their endeavours to improve the candidates' performance in future Advanced Mathematics examinations.

4.2 Recommendations

In order to improve the candidates' performance in similar future examinations, the following should be done:

1. Teachers should teach all the topics set out in the syllabus.
2. Teachers should thoroughly teach their students how to use non-programmable scientific calculators and computer packages.
3. Teachers should regularly assign their students adequate exercises, tests and assessments to do.
4. Projects and excursions should be incorporated into teaching and learning activities.
5. Teachers should mark students' exercises, tests and examinations on time and give feedback to their students so that the difficult concepts are elaborated before the students sit for national examinations.
6. Teachers should identify learners with learning difficulties and assist them through remedial classes and consultations on difficult areas.
7. Teachers should encourage students to work in groups so that they share knowledge.
8. Teachers should use appropriate teaching resources during the teaching and learning process to enhance understanding of concepts by students.
9. Teachers should always encourage their students to study and understand the facts and theories pertaining to a particular topic. Memorization of any facts and theories should be discouraged.

10. Teachers should strategize in order to improve candidates' performance in poorly performed topics (see Appendix III).
11. School authorities should ensure that teachers and students are linked to various mathematical associations, such as the Mathematical Association of Tanzania (MAT).
12. The curriculum developers should think of integrating some topics with ICT. Integrating them will help students to "visualize" abstract concepts. Moreover, the application of Advanced Mathematics in real life may easily be realized through the use of ICT.
13. Students should thoroughly revise the topics that are difficult to them and, where necessary, consult their teachers for assistance.
14. Candidates should read questions carefully in order to understand all questions and then attempt them as expected.
15. Candidates should draw proper and neat sketches and diagrams when deriving formulae for showing the actual facts needed to achieve the purposes of deriving a particular formula.
16. Candidates should not make any mistakes in the addition and subtraction of numbers or expressions because mistakes make subsequent calculations very difficult and, in some cases, impossible.

Appendix I

Analysis of Candidates' Performance in each Topic in the 2018 Advanced Mathematics Examination

S/N	Topic	Number of Question	The % of Candidates who Scored 35 %or more	Remarks
1	Linear Programming	1	93.6	Good
2	Hyperbolic Functions	1	93.5	Good
3	Statistics	1	92.1	Good
4	Sets	1	91.5	Good
5	Functions	1	85.5	Good
6	Numerical Methods	1	81.2	Good
7	Calculating Devices	1	80.1	Good
8	Coordinate Geometry I	1	79.7	Good
9	Logic	1	78.0	Good
10	Coordinate Geometry II	1	76.4	Good
11	Differentiation	1	65.8	Good
12	Differential Equations	1	56.8	Average
13	Vectors	1	56.6	Average
14	Trigonometry	1	53.8	Average
15	Algebra	1	50.4	Average
16	Complex Numbers	1	47.8	Average
17	Integration	1	47.4	Average
18	Probability	1	23.9	Weak

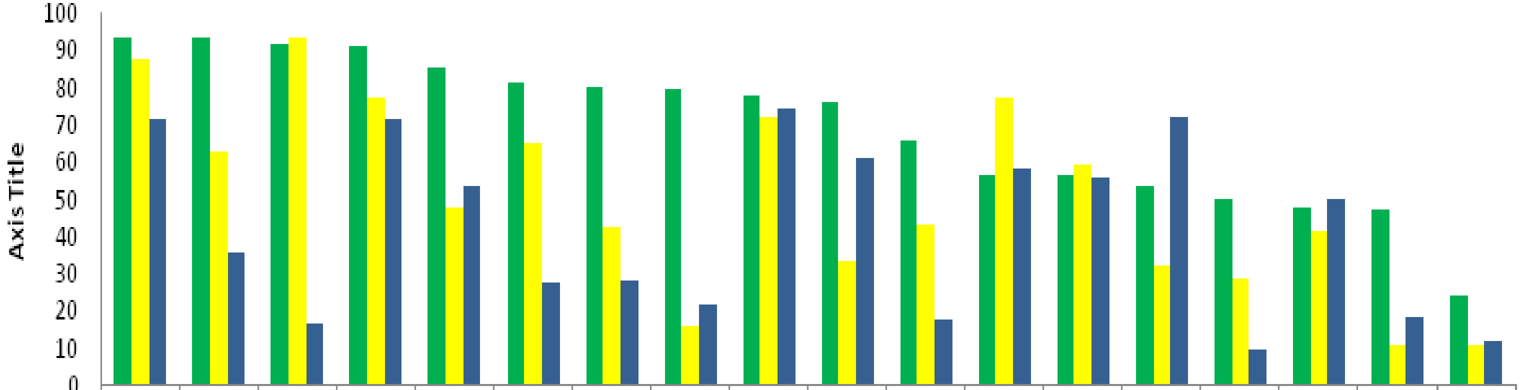
Appendix II

Analysis of Candidates' Performance in each Topic in the 2017 & 2018 Advanced Mathematics Examinations

S/N	Topic	Number of Questions	The % of Candidates who Scored 35 %or more	Remarks	Number of Questions	Percentage of Candidates who Scored 35 or more 2016	Remarks
1	Linear Programming	1	93.6	Good	1	88	Good
2	Hyperbolic Functions	1	93.5	Good	1	62.9	Good
3	Statistics	1	92.1	Good	1	93.3	Good
4	Sets	1	91.5	Good	1	77.5	Good
5	Functions	1	85.5	Good	1	48.1	Average
6	Numerical Methods	1	81.2	Good	1	65.1	Good
7	Calculating Devices	1	80.1	Good	1	42.4	Average
8	Coordinate Geometry I	1	79.7	Good	1	15.8	Weak
9	Logic	1	78.0	Good	1	72.3	Good
10	Coordinate Geometry II	1	76.4	Good	1	33.5	Weak
11	Differentiation	1	65.8	Good	1	43.5	Average
12	Differential Equations	1	56.8	Average	1	77.4	Good
13	Vectors	1	56.6	Average	1	59.6	Good
14	Trigonometry	1	53.8	Average	1	32.1	Weak
15	Algebra	1	50.4	Average	1	28.9	Weak
16	Complex Numbers	1	47.8	Average	1	41.4	Average
17	Integration	1	47.4	Average	1	11.1	Weak
18	Probability	1	23.9	Weak	1	10.8	Weak

Appendix III

The Percentage of Candidates' Performance and the Topics Tested in 2016, 2017 and 2018



	Linear Programming	Hyperbolic Functions	Statistics	Sets	Functions	Numerical Methods	Calculating Devices	Coordinate Geometry I	Logic	Coordinate Geometry II	Differentiation	Differential Equations	Vectors	Trigonometry	Algebra	Complex Numbers	Integration	Probability
2018	93.6	93.5	92.1	91.5	85.5	81.2	80.1	79.7	78.0	76.4	65.8	56.8	56.6	53.8	50.4	47.8	47.4	23.9
2017	88	62.9	93.3	77.5	48.1	65.1	42.4	15.8	72.3	33.5	43.5	77.4	59.6	32.1	28.9	41.4	11.1	10.8
2016	71.7	35.6	16.6	71.8	53.9	27.8	27.9	21.8	74.2	61.4	17.9	58	56.1	72.3	9.5	50	18.4	12.1