THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



CANDIDATES' ITEM RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2018

141 BASIC APPLIED MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA



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141 BASIC APPLIED MATHEMATICS

Published by: The National Examinations Council of Tanzania P.O. Box 2624 Dar es Salaam, Tanzania

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FOREWORD

The National Examinations Council of Tanzania is delighted to issue this report on the Candidates' Items Response Analysis (CIRA) for the Basic Applied Mathematics Examination of Advanced Certificate for Secondary Education Examination (ACSEE) 2018. The aim of the report is to provide feedback on how the candidates responded to the questions, identifying and commenting on any difficulty areas faced by the candidates.

In general, there has been a remarkable improvement on the performance of candidates for the year 2018, which was impressive across all examined questions. The analysis shows that, the candidates performed well in questions that were set from the topics of *Matrices, Calculating Devices, Probability* and *Linear Programming* while the questions that were set from the topics of *Statistics, Functions, Algebra, Differentiation, Integration* and *Trigonometry* had an average performance.

It is the expectation of the Council that this report will be useful to students, teachers and other education stakeholders in improving the candidates' performance in future Basic Applied Mathematics examinations.

The Council would like to thank the examiners, examination officers and other personnel who participated in preparing this report. The Council will also be grateful to receive constructive comments from the education stakeholders for improving future reports.

Dr. Charles Msonde EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Basic Applied Mathematics examination paper had a total of 10 questions, each carrying 10 marks. All the questions were compulsory. This report is on the Candidates' Items Response Analysis for the 2018 Advanced Certificate of Secondary Education Examination.

In 2018, a total of 33,175 candidates sat for the Basic Applied Mathematics examination of which 55.32 percent passed. In comparison with the 2017 results, where a total of 29,204 candidates sat for Basic Applied Mathematics, and out of which 49.40 percent passed, the performance in 2018 has improved by 5.92 percent.

The analysis of the candidates' responses for each question is presented in section two of this report. For each question, the description of the question's requirements and the performance of the candidates are presented. The performance of the candidates in each question was based on the percentage of candidates who in each question scored 3.5 marks or more out of the available 10 marks as indicated below.

S/N	Range (%)	Remarks
1.	0 - 34	Weak
2.	35 - 59	Average
3.	60 - 100	Good

The third section presents the analysis of candidates' responses for each particular topic examined. Furthermore, the factors which have contributed to good and average performance in the topics examined are highlighted and the recommendations to improve the candidates' performance in this subject have been suggested.

2.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION

2.1 Question 1: Calculating Devices

This question had parts (a), (b), (c) and (d) on which the candidates were required to use a non- programmable calculator in computing the answers. In part (a), the candidates were required to compute the value of

 $\sqrt[3]{\frac{\log 122 \times \ln 315}{e^{0.9} + \cos^{-1} 0.5487}}$ correct to 6 significant figures. In part (b), they were

asked to find the mean and standard deviation of the data given in the following table, correct to 4 decimal places.

Length (cm)	110	130	150	170	190	
Frequency	12	35	24	5	3	

In part (c), they were asked to find the determinant of the following matrix:

$$A = \begin{pmatrix} -1 & 3 & 1 \\ 2 & 4 & 0 \\ 0 & 5 & -3 \end{pmatrix}.$$

In part (d), the candidates were required to solve the quadratic equation $t^2 - 5t + 3.31414 = 0$, giving the answer in 3 decimal places.

The question was attempted by 33,019 (99%) candidates, of which 11,398 (34.5%) scored from 0 to 3 marks, 7,602 (23%) scored 3.5 to 5.5 and 14,019 (42.5%) scored 6 to 10 marks. The summary of candidates' performance is represented in Figure 1. It shows that 21,621 (65.5%) candidates scored above 3 marks. Therefore the question had a good performance.



Figure 1: Shows the Summary of Candidates' Performance in Question 1.

The candidates who were able to answer this question correctly managed to use a non-programmable calculator and apply the pre-requisite knowledge mainly: to change the mode of their calculators from degree to radian in part (a), to use correctly the statistical functional keys in part (b) and in parts (c) and (d) to enter correctly the elements of the given matrix and the coefficients of the quadratic equation respectively as well as computing the required answers. Extract 1.1 is a sample solution which indicates a candidate who answered this question correctly.

Extract 1.1

1	a) 1.51529	
	b) Mean is 137.8481	
	standard deviation is 18: 9383	
	c) The determinant of matrix A, A = 40	
	d) The values of t are:	
	t = 4.213 and $t = 0.787$	

Extract 1.1 shows a solution of a candidate who was able to use a non-programmable calculator correctly in performing the computations.

However, 3,706 (11.2%) candidates who attempted this question scored zero. These candidates were not acquaintance with the use of calculators to perform computations. In part (a), some of them could not differentiate between the functional key for square root and cube root while others failed to change the angle in the expression, $\cos^{-1} 0.5487$ into radians, as a result ended up with incorrect answer, 0.587518.

In part (b), some of the candidates provided answers that were quite different from the expected mean and variance of 137.8481 and 18.9383 respectively. The analysis of these kinds of responses showed that some either entered the data wrongly using incorrect statistical function keys or failed to provide the answers correct to 4 decimal places.

It was noted that several candidates prepared tables which had values of f, x, x^2 , fx, fx^2 and $(x - \overline{x})^2$ in computing the mean and standard deviation without using the calculators' statistical function keys. On the other hand, it was also observed that those candidates were unable to get the correct answers either because they wrongly computed the values, used incorrect formulae or failed to provide the answers correct to 4 decimal places.

The analysis of responses revealed that in part (c), some candidates failed to compute the determinant for the matrix A as they either incorrectly entered the elements of the matrix into the calculator or used inappropriate function keys. Further analysis revealed that, other candidates used the method of cofactors expansion instead of calculators' functional keys in finding the determinant. Most of them ended up with incorrect answers due to poor understanding of cofactors and arithmetic errors.

In part (d), a good number of candidates failed to use the appropriate mode setting and functional keys in solving the given quadratic equation as their answers deviated much from the correct answers. It was observed that few candidates completely lacked skills of solving quadratic equations using calculators. Extract 1.2 shows a sample answer of one of the candidates.

Extract 1.2

1.	(a) solo.						
	3 log 122 x ln 315						
	$\sqrt{e^{2^{4}} + (cs^{-1} + cs^{-1})} = 54.87$						
	= 12.001	9364 8					
	√2.4596	03111 + 56.7	122 12 6.23				
	= / 12	0019364	S				
	↓ 5	9.1817299	4				
	31		1				
	= -1	0.202798					
			n an				
	2	0.5875180	062.				
	- In 6 sign	ificant fig	ure = 0.587518				
	b) solo						
	Length (cm)	Frequency.					
	110	13					
C	130	35					
	150	24					
	170	5					
	190	3.					
	$= 150 \text{ cm}^{-1}$	21=79	-				
	M	- 111	lon all				
	1 /ean (x)	Intal "					
-		10121	requerry				
		= 7500	$m = 9.49367 \approx 9.4937$				
		79					
	. Mean (x)	= 9.493	37.				
			And and a second se				

1.	c) soln
	A = (-1 3 1)
	240
	L D 5 -3]
	$= -1 \left[4 \right] \left[0 \right] + 3 \left[2 \right] \left[0 \right] - 1 \left[2 \right] \left[4 \right] \left[0 \right] $
	= -1 [4x-3] - (0x5) + 3 (2x-3) - (0x6) - 1 (2x5) - (4x0)
	$=(-1 \times -12) + (3 \times -6) - (1 \times 10)$
	= 12 + -18 - 10 = 12 + -28
	= -16
-	, <u>, </u>
	d) sola
	$t^2 - 5t + 3.31414 = 0$
	$t^{2} - 5t + 3.31414 = 0$
-	$t^2 - 5t = 3.31414$
	$t - 5 = 3 \cdot 31414$
	t = 3.31414 + 5
	$\frac{1}{1} = 8.314.$
	N x
	d) soln
	$t^{*} - 5t + 3 \cdot 31414 = 0$
	$t^{\alpha} - 5t + 3 \cdot 3 \cdot 4 \cdot 4 = 0$
	$t^* - 5t = 3.31414$
1	$t - 5 = 3 \cdot 31414$
	t = 3.31414 + 5
	$t = g \cdot 31414.$
	$\frac{1}{12} = 8.314$

Extract 1.2 shows a solution from a candidate who could not change the angle in the expression in part (a) from degrees to radians; used incorrect formula for finding the mean in part (b); incorrectly applied the definition of cofactors in part (c) and in part (d) could not solve the quadratic equation.

2.2 Question 2: Functions

This question had parts (a), (b) and (c). In part (a), the candidates were required to find $f(-\frac{1}{8})$, f(2) and f(-3) from the following step function:

$$f(x) = \begin{cases} 12x+5 & \text{if } x > 1\\ x-4 & \text{if } x \le 1 \end{cases}$$

In part (b), they were required to sketch the graph of the function $f(x) = \frac{1}{2-x}$, then use the sketched graph to state the domain and range of f(x). In part (c), they were required to draw the graph of the line which passes through point A(-4, 6) with the slope of -1 in the interval $-4 \le x \le 4$.

This question was attempted by 29,528 (88.5%) candidates, out of which 14,693 (50%) scored from 0 to 3 marks, 8,686 (29%) scored 3.5 to 5.5 marks and 6,149 (21%) scored 6 to 10 marks. This question was averagely performed.

The candidates who performed well in this question were able to substitute the given values of x in the given function in part (a) to get the required answers, that is, $f(-\frac{1}{8}) = -\frac{33}{8}$ or -4.125, f(2) = 29 and f(-3) = -7. In part (b), the candidates were able to correctly determine the vertical asymptote x = 2, the horizontal asymptote y = 2 and the y-intercept $y = \frac{1}{2}$ and consequently sketched the graph of the given function and stated the domain and range as required.

In part (c), the candidates were able to use the general equation $y = m(x - x_0) + y_0$, the given point and the slope to obtain the equation of the line y = -x + 2 and drew its graph as illustrated in a sample answer shown in Extract 2.1.

Extract 2.1.

2(a)	$f(x) = \begin{cases} 12x+5 & if x > 1 \end{cases}$
	$l x - 4 i p x \leq 1$
	f(-1/8) = x - 4
	$f(-\sqrt{s}) = -\sqrt{s} - t$
	$t(-\sqrt{8}) = -33\sqrt{8}$
	f(2) = 12x + 5
	f(2) = 12x2 + 5
	f(2) = 24+5
	f(2) = 29
	f(-3) = x - 4
	F(-3) = -3 - 4
	F(-3) = -7
	$f(-1/s) = -\frac{33}{s}, f(2) = 29, f(-3) = -7$
2(6)	Let f(x) = y
2(6)	Let f(x) = y
2(6)	Let $f(x) = y$ y = 1 2 - x
2(6)	Let $f(x) = y$ y = 1 2 - x. For x-intercept, $y = 0$.
2(6)	Let $f(x) = y$ y = 1 $\lambda - x$. For x-intercept, $y = 0$. $\delta = 1$
2(6)	Let $f(x) = y$ y = 1 $2 - x \cdot$ For x-intercept, $y = 0 \cdot$ 0 = 1 $2 - x \cdot$
2(6)	Let $f(x) = y$ y = 1 2 - x. For x-intercept, $y = c$. 0 = 1 2 - x. $(2 - x) \circ = 1$.
2(6)	Let $f(x) = y$ y = 1 $2 - x \cdot$ For x-intercept, $y = c \cdot$ 0 = 1 $2 - x \cdot$ $(2 - x) \circ = 1$. No x-intercept.
2(6)	Let $f(x) = y$ y = 1 2 - x. For x-intercept, $y = c$. 0 = 1 2 - x. $(2 - x) \circ = 1$. No x-intercept. For y - intercept. For y - intercept, $x = o$
2(6)	Let $f(x) = y$ y = 1 2 - x. For x-intercept, $y = c$. 0 = 1 2 - x. $(2 - x) \circ = 1$. No x-intercept. For $y - intercept$, $x = o$ y = 1
2(6)	Let $f(x) = y$ y = 1 2 - x. For x-intercept, $y = c$. 0 = 1 2 - x. $(2 - x) \circ = 1$. No x-intercept. For $y - intercept$. y = 1 2 - 0
2(6)	Let $f(x) = y$ y = 1 2 - x. For x-intercept, $y = c$. 0 = 1 2 - x. $(2 - x) \circ = 1$. No x-intercept. For $y - intercept$, $x = c$ y = 1 2 - c y = 1/2.
2(6)	Let $f(x) = y$ y = 1 2 - x. For x-intercept, $y = c$. 0 = 1 2 - x. $(2 - x) \circ = 1$. No x-intercept For $y - intercept$. y = 1 2 - 0 y = 1/2. For V.A, $2 - x = c$
2(6)	Let $f(x) = y$ y = 1 2-x. For x-intercept, $y = 0$. 0 = 1 2-x. (2-x) = 1. No x-intercept For y-intercept, $x = 0$ y = 1 2-0 y = 1/2. For V.A, $2-x = 0$ \sqrt{A} , $2-x = 0$

•												
২(৮)	Doma	$in = {$	x: x	≠ 23						-		
-	Range	=' {	S :	5 7 0	> 3							
2 (c)	A(- 4	, (,).									 	
	m =	- 1										
	Tale	(x, 5))								 	
	slope,	m =	44									
			۵×			-						
	m	= '	5-6									
			× ·	4								
	<u>ب</u>	· (=	- 1	-								tel construction of the later o
	x	+4									 	
	بر	- 6 = -	τ(x	+4)							 	
	5	- 6 =	- x -	4 .								
	ب) = -×	(- 4 -	+6								
	S		x + 2								 	
	Table	v₽ v	alue	2							 	
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	×	~4	-3	- 2	-1	0	1	2	3	4	 	
	5	6	5	4	3	2	١	0	-1	-2		
	For H.	A, M	ake ;	K, th	e su	Sject						
	ų)(2-x) =	1								
		25	- x y	= 1								
		ર	5-	1 = .	хŊ							
			хv	= 2	5-1							
			Χ-	<u>2</u>	5-1							
					У							
	Inter	chang	bing	vari	able	2					 -	
			¥ =	২৬	-1						 	
			<u>A</u>		5							
			H-A	ļ) = C)					 	



Extract 2.1 shows a solution of a candidate who had an adequate knowledge on the tested concepts of functions and was able to apply it correctly.

On the other hand, 8,834 (29.9%) candidates scored from 0 to 1.5 out of 10 marks. In part (a), most of these candidates substituted the values to incorrect functions since they were not aware of the restriction of the domain on each part of the function; for example evaluating f(2) from x-4 instead of 12x+5. Others failed due to arithmetic errors.

In part (b), some candidates drew incorrect graphs because they lacked the knowledge and skills to find the asymptotes and intercepts which were necessary in sketching the graph. Most of them were drawing the graph without finding the asymptotes and intercepts while others used incorrect

table of values. This led to failure in obtaining the domain and range. Extract 2.2 indicates a sample answer from one of the candidates.

In part (c), the candidates lacked facts and skills involved in determining the equation of the line, as a result could not draw the required graph.

Extract 2.2

20	folution =
~0	
- <u>- 1</u>	-P1-1 = (10x+5 if x 71
	F(x) = 12x 10 1011 =
	$(\lambda - \gamma, \overline{\gamma}, \overline{\gamma} = 1)$
	12X + 5 = 1
	X = 2, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12,
	$x - 4 + x \leq 1.$
	x 1 - 0 - 1 - 4 ·
	<u> </u>
	n+
	thin
	$D \neq f = 1$.
	8
	lef
	T = 1
	10 x + 5
	127710
	P 1
	721
*	42 X+5
	f(4/g) = -1
	2. 7. 4 56 2. 8. 9. 10, 11, 12 + 5.
	$-e^{-1/2}$
	7 (18) - 1+
	-11 4-3
	·: 120 7 (2/8) = 1/22 Answer.
1	

20 fotution : But (A) 7 (-3) · folution. 7 F3) X = ~ - 1)\$ $= x - 4 \quad if \quad x \leq 1.$ Nature $\chi = -1, -2, -3, -4$ $\pi = -1, -2, -3, -4$ $\pi = -1, -2, -3, -4$ $\pi = -1, -2, -3, -4$ = -1, -2, -3, -4 = -1, -2, -3, -4 = -1, -2, -3, -4 $\frac{bolution:}{f(x) = 1}$ $\frac{2 - x}{x}$ (b)Lef-2-x = U U = 2 - x U = 2. The graph fer the f X = 1 toons 2-X dravon on the brach fide of the Boofselet -



Extract 2.2 shows a solution of a candidate who lacked knowledge on the topic of functions.

2.3 Question 3: Algebra

This question had three parts (a), (b) and (c). In part (a), the candidates were required to solve a given system of simultaneous equations $\begin{cases} x^2 - 2y = 7 \\ x + y = 4 \end{cases}$ by the substitution method. In part (b), the candidates were asked to find the sum of the series $\sum_{r=3}^{5} (-1)^{r+1} r^{-1}$. In part (c), the candidates were required to find the third term of the Arithmetic Progression (A.P), where the second and fifth terms were x - y and x + y respectively.

The question was attempted by 30,131 (90.3%) candidates, out of which 16,610 (51.1%) scored from 0 to 3 marks, 8,179 (27.1%) scored 3.5 to 5.5 marks and 5,342 (17.8%) scored 6 to 10 marks. This question was therefore averagely performed because 13,521 (44.9%) candidates scored from 3.5 to 10 marks.

The candidates who did well in this question were able to express the simultaneous equations in part (a) as the quadratic equation $x^2 + 2x - 15 = 0$ and solved it correctly to obtain the values of x and y as (3, 1) and (-5, 9).

In part (b), the candidates were able to correctly generate the terms of the series to obtain $\frac{(-1)^4}{3} - \frac{(-1)^5}{4} + \frac{(-1)^6}{5}$ by replacing the index of summation with the consecutive integers r = 3,4 and 5. They were then able to sum up the terms in the numerical expression to $\frac{17}{60}$ or 0.28⁵.

In part (c), few candidates were able to express the second term (A_2) and the fifth term (A_5) as a system of simultaneous equations: $\begin{cases} A_1 + d = x - y \\ A_1 + 4d = x + y \end{cases}$ and solved it in order to get the first term (A_1) , the common difference (d) and finally the third term (A_3) of the A.P as shown in Extract 3.1.

2	
3.	a) given the equation X2-2027 (1)
	$\times + 5 = 4 - 0)$
	from eqn(i)) $x + y = 4$
	$\mathbf{y} = 4 - \mathbf{x} - \mathbf{(11)}$
	Substitute equation (11) into eqn(i)
	$x^2 - 2y = 7$
	$x^{2} - 2(4 - x) = 1$
	$\times^2 - 8 + 2x = 1$
	×2+2×-8-7=0'
	\times^2 tax - 45 = 0
	×2-3×+5×-15=0
	\times (\times -3) + 5(\times -3)=0
	$(\times +5)(\times -3) = 0$
	X = -5 or 3

Extract 3.1

3 a) To eqn (4)

$$y = 4 - x.$$

$$h Hen x = -5, y = 4 - 5 = q$$

$$h Hen x = p, y = 4 - 1 = 1.$$

$$(x, y) = (3, 1)$$

$$\frac{r}{r} (x, y) = (1, 1)$$

$$\frac{r}{r} (x,$$

3	e) $A_1 + d = X - y$
	Ay + 4d = X+14
	$(A_1+d) - (A_1+d) = \times - Y - \times - Y$
	-34 = -29
	d= 24
	'3
	from eqn (i) , , +d = X-y
	A1 = x-y-d = x-y-3y = X-5y
	3 3
	$A_1 = x - 5/y$
	` کی
	But the third tom, Az = A1+2d.
	$a_{3} = A_{1} + 24 = (X - 5/y) + 2(3/y)$
	= × - 5/4 + 4,4
	3 /3
	= × - У
	3
	- The third term is X- 4/
	<u></u>

Extract 3.1 shows a solution from a candidate with competence in applying the necessary skills in algebra.

However, 4,834 (16%) candidates scored zero in this question. In part (a), some of the candidates could not use the substitution method to solve the given equations simultaneously while others used the elimination method contrary to the instructions of the question. Most of them also failed due to lack of skills in algebra, for example some were unable to expand the brackets in $x^2 - 2(4-x) = 7$ or $(4-y)^2 - 2y = 7$ after doing the substitution. In part (b), the majority failed to express the given series in expanded form mainly due to lack of knowledge on the sigma notation, see Extract 3.2. Other candidates were able to expand the series but failed to calculate the required sum because of arithmetic errors.

In part (c), the candidates were unable to relate the nth term formula: $A_n = A_1 + (n-1)d$ with the second and fifth terms that were provided in order to find the third term for the A.P. Most of them failed because of poor understanding of series and inability to interpret the given information.

Extract 3.2

3a.	X + (4 - X) = 4
	X + -X = 4 - 4
	X = 0
	To find y
	X - y = 4
	y = 4 - 0
	$y = \varphi$
	(x, y) = (0, 4)
36.	$\gamma = 3$
	$(-1)^{++} \Gamma^{-+}$
	$(-1)^{31} \times 3^{3} = \frac{1}{3}, \text{ or } -0.3$
	$(-1)^{+1} X 4' = -0.75$
	* (~ p
	1 = 5
	(-1) $X J = 0.2$
	$\overline{G_3} = 0.3$
	$(\tau_{1}, \tau_{2}, \tau_{2})$
	$\alpha_4 = 0.15$
	(rc = 0.2)
	41 - 02
	To find up of Gra la finding company - + 2.
<u> </u>	(+-= (T (T)
	$\frac{d_1}{d_1}$ $\frac{d_2}{d_1}$ $\frac{d_4}{d_1}$
	0:26 0:275
	There is no common ration.

30	Data airen
	$A_2 = A_1 + d - l \qquad X - y$
	$A_{5} = A_{1} + (5-1)d = X + y$
	<u> </u>
	$A_2 = A_1 + d = x - y$
	$A5 = A_1 + 4d = Xty$
	By elimination method
	$\int A_{1} d = X - Y$
	A +4d =xty
	Of 3d = O
	d = 2d = 0
	3 1
	d=0
1	

 from (ii) equation
 $A_1 + 4d = X - y$
 $A_1 + \Psi(0) = \mathbf{X} + \mathbf{Y}$
 $A_1 + 0 = X + y$
AT = Xty
 Su, third term
$A_3 = A_1 + 2d$
where A1 = Xty and d=0
$A_2 = Xty + 0$
$A_{3} = Xty$

Extract 3.2 shows a sample solution of a candidate who did incorrect substitution in part (a), lacked understanding of sigma notation in part (b) and had poor arithmetic skills.

2.4 Question 4: Differentiation

The question had parts (a) and (b). In part (a), the candidates were given the function f(x) = x and were required to find $\frac{dy}{dx}$ from the first principles. In part (b), the candidates were required to (i) find x and y intercepts, (ii) determine the maximum and minimum points of f(x) and (iii) sketch the graph of f(x) given the curve f(x) = (x+1)(x-1)(2-x).

This question was attempted by 26,052 (78.1%) candidates, out of which 14,407 (55.3%) scored from 0 to 3 marks, 7,488 (28.7%) scored 3.5 to 5.5 marks and 4,157 (16%) scored 6 to 10 marks. Figure 2 represents the summary of candidates' performance in this question. The figure shows that 11,645 (44.7%) candidates scored from 3.5 to 10 marks. Therefore, the question was averagely performed.



Figure 2: Shows the Summary of Candidates' Performance in Question 4.

The candidates who performed well in part (a), had a good understanding of the definition of differentiation of functions by the first principles that is, $\frac{dy}{dx} = f'(x) = \lim_{\delta x \to o} \left\{ \frac{f(x + \delta x) - f(x)}{\delta x} \right\}$ and correctly applied it to obtain $\frac{dy}{dx} = 1.$

In part (b) (i), the candidates correctly substituted x = 0 in the given function to obtain y = -2 as the y - intercept. They also managed to find the x - interceptby simply equating $f(\mathbf{x})$ to zero, that is. (x+1)(x-1)(2-x) = 0 and then solved this equation to obtain the required intercepts. In part (b) (ii), the candidates were able to apply the knowledge of differentiation in finding the maximum and minimum values correctly as well as managing to correctly sketch the graph in part (b) (iii) as indicated in a sample answer in Extract 4.1.

Extract 4.1

4(a)	From $dy = \lim_{x \to 0} f(x+h) - f(x)$						
	dx, h->0 h.						
	dy = linx + h - x						
	dx, h->0 h						
	$dy = \lim_{x \to 0} x + h - x$						
	dx hao h.						
	$dy = \lim h$						
	dx noon						
	dy = 1						
	dx						
	dy = 1						
	dx						
4 (b)	(1) Maximum and Minimum points						
	5 = (x+1)(x-1)(2-x)						
	y = [(x+1) (x-1)] (2-x)						
	$5 = \int x^2 - x + x - 1 \int (2 - x)$						
	$5 = (x^2 - 1)(2 - x)$						
	$y = 2x^2 - x^3 - 2 + x$						
	$y = -x^3 + 2x^2 + x - 2$						
	For critical points dy = 0						
	dx						
	$dy = -3x^2 + 4x + 1$.						
	dx						
	$-3x^{2}+4x+1=0$						
	on solving						
	x = - 0.215° or x = 1.549 to 3 decimal places						
	but $\omega = (x+1)(x-1)(2-x)$						
	For $x = -0.215$						
	y = (-0.215 + 1) (-0.215 - 1) (20.215)						
	5 = - 2.113 to 3 decimal places						
	For x = 1.549						
	u = (1.549 + 1) (1.549 - 1) (2 - 1.549)						
	5 = 0.631 to 3 decimal places						
	Therefore the critical points are						
	(-0.215, -2.113) and (1.549, 0.631)						

a^2
for x = -0.215
$a^2y = -6(-0.215) + 4.$
dx ²
$d^2y = 1.29 + 4$
cl x ²
$d^2y = 5.29$
clx ²
4(b) (1) since d2y is coreater than c, then
dx2
The point (-0215, -2113) is a Minimum point
Fer X= 1.649
$d^2y = -6(1.549) + 4$
dx ²
$d^{2}y = -9.294 + 4$
dx ²
$d^2y = -5.294$
$\overline{dx^2}$
since d ² y is less than 0, then
dx ²
the point (1.549, 0.631) is a Maximum point
The Minimum point is (- 0.215, -2.113)
The Maximum point is (1549, 0.631)

4(b)	(i) $f(x) = (x+1)(x-1)(2-x)$, let $f(x) = y$
	For X-intercept, w=0
	0 = (x+1) (x-1) (2-x)
	X+1= 0, X-1=0 01 2-X=0
	x- 0-1, x= 0+1 01 2= x.
	x = -1, $x = 1$ or $x = 2$.
	x-intercepts = -1, 1 and 2.
	For y-intercept, x=0
	S = (0+1)(0-1)(2-0)
	$\omega = (1) (-1) (2)$
	y = (-1)(2)
	$\mathcal{G} = -2$
	9 - intercept = -2.



Extract 4.1 shows how a candidate correctly applied knowledge of differentiation in answering question 4.

Nevertheless, 14,407 (55.4%) candidates who attempted the question scored low marks from 0 to 3 and among them 2,431 (9.3%) scored zero. In part (a), some of the candidates failed to correctly differentiate from the first principles while others substituted wrong expressions or considered δx to be zero.

In part (b) (i), many candidates failed to get the required x-intercepts because they did not realize that equating the function to zero would lead to determination of the required x – intercepts. Most of them expanded the given expression, which was of no use, because it was not leading them to obtain the required solution. In part (b) (ii), most of the candidates could not correctly determine the derivative of the given function which led to incorrect turning points and also failure in sketching the graph in part (b) (iii). Extract 4.2 indicates a sample answer showing some of the notable candidates' weakness while answering this question.

Extract 4.2

(49) $F(x) = x$
dy = F(x+b) - (Fx)
dx, h, h
$\lim_{x \to x^2} = x^2 + hx - x$
$\lim_{x \to \infty} = x^2 + hx - x$
h=0, h ,
2
= x + o(x) - x
h
$f(x) = x^2 - x$
hzu: Gi
$= \chi^2 - \chi$
0
$\simeq 0$

$$\begin{array}{c} 45 \\ f(x) = (x + y) (x - 1) (2 - x), \\ f(x) = 2x^{2} - x^{3} - x + 2 \\ y = -x^{3} + 2x^{2} - x + 2, \\ (i) c(-x^{3} + 2x^{2} - x + 2) \\ dx, \\ z = -2x^{2} + 2x - 1, \\ x volues \\ = (0 + 5, 0 + 5) \\ \hline \\ 45) ii) dy = -4x + 2 \\ \hline \\ d''x \\ \hline \\ (then) \\ -4(0 + 5) + 2, \\ z = 0 + \\ y = -x^{3} + 2x^{2} - x + 2, \\ y = -x^{3} + 2x^{3} + 2x^{3}$$

Extract 4.2 shows a sample work of a candidate who had partial understanding of differentiation which led to incorrect solutions in parts (a) and (b) (ii). The candidate did not answer part (b) (i).

2.5 Question 5: Integration

This question had parts (a), (b) and (c). In part (a), the candidates were asked to integrate $\int 2x\sqrt{x^2+3} dx$, in part (b), they were required to find the area of the region enclosed by the curve $y = x^2$ and the line y = x. In part (c), the candidates were instructed to find the volume of a solid of revolution which is obtained when the area bounded by the line y=2x, x-axis, x=1 and x=h is rotated about the x-axis.

The question was attempted by 19,227 (57.6%) candidates, out of which 11,514 (59.9%) scored from 0 to 3 out of 10 marks, 2,619 (18.8%) scored 3.5 to 5.5 marks and 4,094 (21.4%) scored 6 to 10 marks. The question was averagely performed since 7,713 (40.2%) candidates scored above 3 marks. Further analysis revealed that 14,134 (42.4%) candidates did not attempt this question.

In part (a), the candidates who attempted this part of the question were able to apply the substitution technique to obtain $\int 2x\sqrt{x^2+3} \, dx = \frac{2}{3} (x^2+3)^{\frac{3}{2}} + c$. In part (b), several candidates were able to find the limits of integration, either by solving simultaneously the equations $y = x^2$ and y = x or through sketching the graphs, which enabled them to find the area as required. In part (c), few candidates were able to apply the formula $V = \int_{a}^{b} \pi y^2 dx$ to obtain $V = \frac{4\pi}{3} (h^3 - 1)$ for h > 1 or $v = \frac{4\pi}{3} (1 - h^3)$ for 0 < h < 1 as required. A

sample answer from one of the candidates is shown in Extract 5.1.

Extract 5.1

05(a)	(2x x2+3 dx.
	$\left(2 \times \left(\times_{j} + 1 \right)_{j} \right)_{j} q_{x}$
	$let \qquad U \ge \chi^2 + 3.$
	dydr= 2x.
	dx = du/2x
	then,
	2x u2 du = (u2 du.
	$z = y^{2t} + c$
	1/2+1
)/ 3/
	$= \frac{y}{t} + c = \frac{2}{10} \frac{y}{t} + c$
AL	×2.
	buy 42 X2 B
	then 31
	$\int 2x \left[x^{2} + 3^{2} \right] dx = 2 \left(\left(x^{2} + 3 \right)^{2} + C \right)$
	Jentie (2 (3
05(6)	Then y=x2, and y=x.
	Ten, 14 / 42x
	UCX - airea
	they x
	Arecz Jydx
	brdz XI
	X ² zX
	x ² -x=c
	X(x-1) = 0
	X20, X21

$$\frac{cqb}{tin}$$

$$\frac{(\pi o \quad y = \chi^{2}, \quad y = \chi}{1.}$$

$$\frac{1.}{\chi + \kappa c = \int_{0}^{\chi} (\chi - \chi^{2}) d\chi$$

$$= \frac{\chi^{2} - \chi^{2} + e}{2} \int_{0}^{1} \frac{1}{2} \left(-\frac{\sigma^{2}}{2} + \frac{\mu^{2}}{3} - e \right)$$

$$= \frac{\chi^{2} - \chi}{2} + e \int_{0}^{1} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left(-\frac{\sigma^{2}}{2} + \frac{\mu^{2}}{3} - e \right)$$

$$= \frac{\chi^{2} - \chi}{2} + \frac{1}{2} \frac{$$

05(1)	Whyne = TT (4x2 dx
	h
	$= \pi \left[\frac{4x_{+}^{2}}{4x_{+}^{2}} \right]^{n}$
	$= \pi \left[\frac{4h^2}{2} - \frac{4(0)^2}{2} \right]$
	$2 \Pi \left(\frac{4}{3} - \frac{7}{3} \right)$
	$z \operatorname{tr}(4)(h^{2}-1)$
	$= \frac{411}{7} (h - d)$ cubi unib
	The Willine will be 41/2 (h-1) Cubic units
	When his greater than 1.
	Ŷ
	Where 2 4Th (1-h) cubic rivers
	when h is less 1

Extract 5.1 indicates a sample work of a candidate who applied correctly the tested skills of integration in question 5.

On the other hand, 3,500(18.2%) candidates scored zero in this question. In part (a), some of the candidates used incorrect substitutions, for example $u = x^2 + 3dx$ or $u = \sqrt{x^2} + 3dx$ instead of $u = x^2 + 3$ while others had no idea on the techniques of integration, as illustrated in Extract 5.2.

In part (b), the candidates were unable to integrate a definite integral of the form, $I = \int_{a}^{b} f(x)dx$ to find the area enclosed by the given curves. They could not identify the integrand and the limits of integration.

In part (c), the candidates were incapable of applying the formula; $V = \int_{a}^{b} \pi y^{2} dx$ to find the required volume of a solid of revolution. Also the candidates failed to identify the limits of integration to be used in the formula of computing the volume of revolution. In this case the candidates were to find the volume as follows; for h > 1, $V = \int_{1}^{h} \pi (2x)^2 dx$ and for 0 < h < 1, $v = \int_{h}^{1} \pi (2x)^2 dx$. Extract 5.2 is a sample work of a candidate who did not do well in this question.

Extract 5.2





Extract 5.2 shows a sample solution of a candidate who had inadequate knowledge in integration.

2.6 Question 6: Statistics

This question consisted of parts (a), (b) and (c). The candidates were given a list of masses of 50 apples measured to the nearest grams where in part (a) were required to prepare a frequency distribution table using equal class interval widths of 5 grams, taking the lower class boundary of the first interval as 84.5. In part (b), the candidates were asked to draw the histogram to illustrate the data and in part (c) to calculate the mode using the appropriate formula.

The question was attempted by 29,319 (87.9%) candidates, of which 14,556 (49.6%) scored from 0 to 3 marks, 4,163 (19.9%) scored 3.5 to 5.5 and 10,600 (30.5%) scored 6 to 10 marks. In this question 13.9 percent scored all the 10 marks. The performance of the candidates is also shown in Figure 3.



Figure 3: Shows the Summary of Candidates' Performance in Question 6.

Figure 3 shows that 14,763 (50.4%) candidates scored above 3 marks, therefore the question had an average performance.

In part (a), some candidates were able to prepare the frequency distribution table as it was demanded by the question. This frequency distribution table then made it possible for them in part (b) to correctly draw the histogram. In

part (c), they correctly applied the formula; Mode = $L + \left(\frac{t_1}{t_1 + t_2}\right)c$ as

illustrated in a sample answer from one of the candidates in Extract 6.1.

6	0)	FREQUE	NCY P	ISTR(BU	TION	TABLE		
		class Interval	class mark (X)	Frequency (F)				
		85-89	87	4				
		90-94	92	6				
		95-99	97	7				
		(00 - (04	(02	3				
		105-109	107	(0				
		110-114	112	5				
		115-119	(17.	Ľ)				
	6)	he Histog	ram's Qr	the GN	aph	Papers		
		0			,			

Extract 6.1



Extract 6.1 shows a sample solution of a candidate who correctly prepared the frequency distribution table; drew the histogram and applied the formula to find the mode.

On the other hand, about half of the candidates 14,556 (49.6%) scored from 0 to 3 marks and among them 1,962 (6.7%) scored zero. In part (a), the candidates failed to use 84.5 - 89.5 or 85 - 89 as the first class interval and as a result prepared incorrect frequency distribution tables. This failure led to incorrect histograms and modes in parts (b) and (c) respectively. Extract 6.2 is a sample answer showing how a candidate failed to answer this question.

Extract 6.2

\sim	D						
	12) Atreaman	Disti Supon	table.				
- O ,	(1					
	CIGH In Ford.	frennen an	14				
	ar a	p. contract,	<u>X</u>				
	85-90	4	87.5				
	91 - 96	9	9.3, J.				
	97 - 102	14	99.5				
	103 - 108	13	105.5				
	109 - 114	5	111.5				
	115-120	5	117.5				
	() Mode						
	Fron						
	Mode = C + F Fr. 7C						
	$L + t_2$						
	hlere L = 97-05=96.5						
	$f_{1} = 14 - 2 = 5$						
	Z, - 14-13-11.						
	C = 96 - 90 = 6,						
	Mode = 96:5+ 1 1.						
	$5 + a \int b$.						
	= 96.7 + (5, 76.						
	~ 96:5+1.						
	- 101.5.						
	. Mode	is 10	1.5.				





2.7 Question 7: Probability

This question consisted of three parts. In part (a), the candidates were required to verify that ${}^{8}C_{3} + {}^{8}C_{2} = {}^{9}C_{3}$, whilst in part (b), the candidates were given events *A* and *B* which are such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{7}$ thus required to find: (i) $P(A \cup B)$ when *A* and *B* are mutually exclusive events and (ii) $P(A \cap B)$ when A and B are independent events. In part (c), the question was as follows:

Two students are chosen at random from a class containing 20 girls and 15 boys to form a student welfare committee. If replacement is allowed, find the probability that: (i) both are girls (ii) one is a girl and the other is a boy.

The question was attempted by 27,754 (83.2%) candidates, of which 9,800 (35.3%) scored from 0 to 3 marks, 17,954 (64.7%) scored 3.5 to 10 marks and 897 (3.2%) candidates scored full marks. Generally, the question had a good performance.

In part (a), most of the candidates were able to verify the given expression by either using the formula of combination or calculator. In part (b) (i), candidates succeeded to apply the formula for mutually exclusive events and in part (b) (ii) were able to apply the correct formula for the probability of independent events, hence obtained the required answers. In part (c), the candidates used the method of tree diagram which enabled them to obtain the required solution. Extract 7.1 shows a sample response of a candidate who used the correct approach to answer question 7.

Extract 7.1

·7(5)	Required to varies;
	363 + 362 = 963
	From; Littis;
	ncr = nb
	(n-1) $[r]$
	$2C_2 + 8C_2 = 8! + 8!$
	$\overline{(3-3)}$
	= 81 + 81
	5132 62x21
	$8C_3 + 8C_2 = 84$
	from, Rithis;
	$9C_3 = 9!$
	(<u>(-3)</u>] 3]
	a(z = 41
	6 [X2]
	9(3 = 84
	Since, L.H.S = R.H.S; Hence of Versted.
コル	$G_{1,ven}$; $P(A) = 1$, $P(B) = 2/2$
	\$ · · · ·
	(1) For Mutually Exclusive Events;
	P(AUR) = P(A) + P(B)
	$P(AUB) = \frac{1}{2} + \frac{2}{2}$
	5 7
	$-1, P(AUB) = \frac{13}{21}$

Extract 7.1 illustrates a sample solution of a candidate who performed well in this question.

However, 9,800 (35.3%) candidates who attempted this question scored low marks (from 0 to 3) and among them 2,190 (7.9%) scored zero mark. The analysis revealed that in part (a), candidates were not able to distinguish between combinations and permutations. For example some candidates expressed the formula of combination as ${}^{n}C_{r} = \frac{n!}{(n-r)!}$ which is permutation

 $\binom{n}{r}P_r$ instead of ${}^nC_r = \frac{n!}{r!(n-r)!}$. Also few candidates had a misconception

on the definition of factorial (n-r)! whereby they perceived it as n!-r!. In part (b) (i) and (ii) some candidates applied incorrect formulae, for example $P(A \cup B) = P(A) \times P(B)$ instead of $P(A \cup B) = P(A) + P(B)$ where A and B are mutually exclusive events and $P(A \cap B) = P(A) + P(B)$ instead of $P(A \cap B) = P(A) + P(B)$ instead of $P(A \cap B) = P(A) + P(B)$ instead of $P(A \cap B) = P(A) \times P(B)$ where A and B are independent events. In part (c), most of the candidates failed to prepare a tree diagram which was essential in answering this part. It was observed that the candidates had inadequate knowledge in this topic, as illustrated in Extract 7.2.

7	$(a) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$
	for m
	$C_{-} = 0!$
	(n-r)!
	- 8! - 81
	(8-3)! $(8-2)!$
	8×7 8! + 8!
	81-31 81-21
	1 1 -2 + -6 = -2
	-31 -21 47 3
7	b) ii) From .
	P(AUB) = P(A) + P(B) - P(ADB)
	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
	$P(A \cap B) = \frac{1}{2} + \frac{3}{2} - \frac{13}{2}$
	P(ADB) = O
	- 8
	$C) \mathbb{P}(G) = 20$
	P(B) = 15
	$\frac{1}{P(t_{r})}$
	$P(G \cup B) = P(G) + P(B)$
	P(GUB) = 20 + 15.

Extract 7.2

Extract 7.2 shows a sample solution of a candidate who generally lacked knowledge of probability.

2.8 Question 8: Trigonometry

This question had parts (a) and (b) whereby, the candidates were required in part (a) (i) to find the value of $\cos 15^{\circ}$ without using a calculator; in part (a) (ii) to prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ while in part (a) (iii) to sketch the graph of $f(x) = \sin x$, where $-2\pi \le x \le 2\pi$. In part (b), the candidates were required to solve the equation $\cos 2x + \sin^2 x = 0$, where $0^{\circ} \le x \le 360^{\circ}$.

This question was attempted by 15,613 (46.8%) candidates, out of which 5,624 (36%) scored from 3.5 to 10 marks and among them 248 (1.6%) scored all the 10 marks. This question was averagely performed and notable of lower performance than other questions.

The candidates, who did well in this question, were able to apply the compound angle formula and other appropriate trigonometric identities.

In part (a) (i), the candidates were able to express 15° as a difference of special angles: $15^{\circ} = (45^{\circ} - 30^{\circ})$ or $15^{\circ} = (60^{\circ} - 45^{\circ})$ and then applied the compound angle formula $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ as well as substituting the correct values of special angles to obtain the required answer.

In part (a) (ii), the candidates correctly applied the trigonometric identities $sin(A \pm B) = sinAcosB \pm cosAsinB$ to carry out the proof, in part (a) (iii), they were able to make a table of values based on the given range in order to sketch the graph of the provided function.

In part (b), the candidates were able to correctly apply the trigonometric identity: $\cos^2 x + \sin^2 x = 1$ and the double angle formula: $\cos 2x = \cos^2 x - \sin^2 x$ to solve the given equation in obtaining the required values of x as 90° and 270°. Extract 8.1 is a sample answer from one of the candidates who responded to the question correctly.

Extract 8.1

8 (4)	(1) Reguired;					
	(os 15°					
	$Fron;$ $Cos(5^\circ = Cos(45^\circ - 3c^\circ)$					
	Buti as per compound Angle formularei					
	COS(A-B) = CUSALOSB + SINASINB					
	let: A = 45°, B = 30°					
	$(os(45^{\circ} - 30^{\circ}) = cos(45) cos(3) + sin(45) sin(3)$					
	but from special angles;					
	$\cos 45^\circ = \sqrt{2}$, $\cos 30 = \sqrt{3}$					
	2' 2					
	$Sin 45^\circ = \sqrt{2}$ $Sin 3\circ = 1$					
	2					
	Henco; $\cos_{15} = (\sqrt{2} \times \sqrt{3}) + (\sqrt{2} \times 1)$					
	(2 2) (2 2)					
	Cos15° = V6 + 52					
	4 4					
	$\int (0515^\circ) = \sqrt{6} + \sqrt{2}$					
	4					

3(9)	Tix Regured to prove;							
	$\sin((A + B) \sin(A - R) = 3u^2 A - \sin^2 B$							
	fime Littisi							
	Su(A+R) Su(A-R) = (SmAcus + SmBlosA) (Sin Alus B-SmBlosA)							
	$\sin(A+B)\sin(A-R) = (\sin A\cos B)^2 - \sin B\cos A^2$							
	$Sin(A+B)(Sin(A-0)) = Sin^2 A cos^2 B - Sin^2 B cos^2 A$							
	$Sin(A-B) = Sin^2 A (I-Sin^2 B) - Sin^2 B (I-Sin^2 A)$							
	Sin(A+B) Sin(A-B) = Sin2A - Sin2Asin2B - Sin2B + Sin2Asin2B							
	$Sin(A+B)(Sin(A-B) = Sin^2A - Sin^2B = Rith.S$							
	Hence provod!.							
8(4)	11/ Required, to sketch;							
	$f(x) = \sin x$ from $-2\pi + x \neq 2\pi$							
	Tuble & values;							
	$x - 2\pi - \frac{3}{2\pi} - \frac{1}{12} - $							
	Persesinx 0 1 0 -1 0 1 0 -1 0							
	Refu to Graph paper;							
8(6)	Griven; COJ RX + JURX=D X=? at 0° ≤ X ≤ 360"							
	$F_{\text{DM}} = 1 - 2 \pi R x$							
	$= p \qquad 1 - 2 \sin^2 x + \sin^2 x = 0$							
	$0 = \chi^{S_{u12}} - 1$							
	$S_{1n} X = 1$							
	$\delta m \chi = \pm \sqrt{2}$							
	Sink = +1 & Sin X = -1							
	$x = s_{m-1}(4)$; $c_{m} = x = \delta_{m-1}(-4)$							
	$x = 90^{\circ}, \qquad \qquad$							
	Generally, $X = 90^\circ$, 270°							



Extract 8.1 shows a sample work of a candidate who answered all parts of this question correctly.

On the other hand, there were 3,370 (28.6%) candidates who did not manage to answer this question correctly as per demand of the marking scheme and scored zero. These candidates lacked the basic trigonometric knowledge.

In part (a) (i) and (ii), the candidates incorrectly applied the compound angle formula. For example, some expressed $\cos(45^\circ - 30^\circ)$ as $\cos 45^\circ - \cos 30^\circ$ while others as $\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$. In part (a) (iii), some candidates used incorrect table of values while others lacked skills in sketching the graph. Extract 8.2 indicates a sample answer of a candidate who failed to respond correctly to part (b).

Extract 8.2

8	6/ (0) 2x + sim x = were O'CK < 360.						
	٥يارير						
	Ces 2x + sin x to						
	$\cos 2x + C(-\cos^2 x) = 0$						
	$\cos ix \neq i - \cos^2 x = c$						
-	CUSZX - CUSZX +(20 '						
	let cos x = x						
	2x-x2+1=0						
	-21-x =-1						
	x (2 - x)=-1						
	x=+1 or x=2.						
	Alione bur						
	Cou x =-1						
	K = (0) = [-]						
	x = (80						
	Ger prom the porneclas						
	COS x = 2 Tin t x.						
	(W) K - 2 (120x) 1 (183						
	CONX = 360× +130						
	2. CON K = 180.						
	·. (05) = (180 (80)						

Extract 8.2 shows a sample solution of a candidate who was unable to apply the compound angle formula in part (b).

2.9 Question 9: Matrices

This question had parts (a), (b) and (c). In part (a), the candidates were given two matrices S and P, which were defined as follows:

 $A \quad B$ $R1 \begin{bmatrix} 12 & 13 \\ 8 & 5 \\ R3 \begin{bmatrix} 16 & 9 \end{bmatrix}$, represents the sales per month for the three entrepreneurs:

 R_1 , R_2 and R_3 . Matrix $P = \frac{A \begin{bmatrix} 2500 \\ 3500 \end{bmatrix}}{B \begin{bmatrix} 3500 \end{bmatrix}}$ represents the price paid (in Tsh) for

two types of seedlings species A and B.

The candidates were required to find the total sales for each of the three entrepreneurs.

In part (b), the candidates were given the matrix $A = \begin{bmatrix} 3 & -5 \\ 7 & -11 \end{bmatrix}$ and were

required to verify that $A^{-1}A = I$ where *I* is an identity matrix. In part (c), they were required to use Cramer's rule to solve the system of equations:

 $\begin{cases} x + y + z = 6\\ 2x + y - z = 1\\ x - y + z = 2 \end{cases}$

This was the best performed question in this examination. It was attempted by 30,659 (91.9%) candidates, out of which 13,641 (44.5%) scored from 6 to 10 marks and 8,705 (28.4%) scored 3.5 to 5.5 marks. The performance of the candidates is represented in Figure 4.



Figure 4: Shows the Summary of Candidates' Performance in Question 9.

This figure shows that 22,346 (72.9%) candidates scored above 3 marks. Therefore, the question had a good performance. This shows that, the majority of candidates seemed to have an adequate knowledge and skills on the tested concepts of multiplying matrices, verifying the properties related to identity matrices and applying Cramer's rule in solving systems of linear equations as illustrated in a sample answer in Extract 9.1.

Extract 9.1.

On 9.	Soln					
(a).	Given					
	Sales of Enhanceurs A B					
	R1 12 13					
	$S = R_2$ 8 5 and					
	R3 16 9					
	Price paid (in Tsh)					
	A 2500					
	$P = B \begin{bmatrix} 3500 \end{bmatrix}$					
	Required total sales for each RI, RI and RJ					
	which would be given as					
	12 13 2500					
	SP = 852500					
	16 9					
	$= k_1 [30000 + 45500]$					
	R2 20000 + 17500					
	R3 40000 + 31500					
	$= k_1 75500$					
	R1 37500					
	R_{1} 71500					
	Hence					
	lotal sales for K1, K2 and K3 ahe					
	75500, 37500 and 11500 respectively.					
	``````````````````````````````````````					
9(b)	Son					
	Tiven					
	(Matrix 3 -5)					
	A - 7 -11					
	Premius to leave that A A = T					
	Leguineo vo veripy incer () is - 1					
	where I is Identify Matrix					

Son 96 Given (Matrix -5 3 7 A -// : Required to Venjey that A A = I I is Identify Matrix where 96) I = TO 01 lets Find A-1 -start with (A) = (-33) - (-35) = 2 -5 -11 3 2 -11/2 1 -11 5 5/2 3/2 Ľ 3 -1/2 AI -7 -1/2____ 5/2 -5 3 <u>-1</u> 3/2 7 - A = -7/2 -11 -33/ + 31/2 53/ - 55/ -21/+21/ 35/ - 33/ -3/2 92 Ξ 2/2 %2 1 -T Ο -0  $A^{-1}A =$ Hence = I and 1 0 0 1 Verfied

9 c)	Soln
	Tiven
	X + y + z = G
	92x+y-z=1 Leguired to solve by
	X-y+2=2 Crameis rule
	In Matrix form and let it be G-
	1 1 1 X 7 6
	G = 2 -1 -1 -1 = 1
	lets Find (G)
	a  =   1 -   2 -   +   2
	-0-3-3
	G  = -G
	loget X Interchange 1 with 6.
	2 1 find
	173 aletominant divide by (G)
	$X = \{\varphi \mid \varphi \mid \varphi \in \varphi \}$
	$\frac{1}{1} = \frac{1}{1} = \frac{1}$
	x = (0 - 3 - 3) = (-6) = -6 = -6 = 1
	X = 1

Extract 9.1 shows the responses from one of the candidates who did well in question 9.

However, 8,313 candidates equivalent to 27.1 percent scored from 0 to 3 and among them 2002 (6.5%) scored zero. In part (a), most of the candidates were not able to correctly multiply the corresponding elements in

 $SP = \begin{bmatrix} 12 & 13 \\ 5 & 8 \\ 16 & 9 \end{bmatrix} \begin{bmatrix} 2500 \\ 3500 \end{bmatrix}$  to obtain the required total sales. Others did not

understand the theory of matrix multiplication, hence expressed the total

sales 
$$PS = \begin{bmatrix} 2500\\ 3500 \end{bmatrix} \begin{bmatrix} 12 & 13\\ 8 & 5\\ 16 & 9 \end{bmatrix}$$
 which is not possible.

In part (b), some of the candidates failed to understand the correct procedures of finding the inverse of matrix A as well as the product of  $A^{-1}$  and A. These candidates failed to correctly find the determinant and the cofactors of matrix A. Likewise, in part (c), most candidates failed due to lack of skills in finding determinants of 3 by 3 matrices, as indicated in Extract 9.2.

9a)	Soln	RI RZ RZ	
	P =	A [ 2500 ] , s = [ 12 8 16 ]	
	50 M. D	B 3500 [13 5 9]	
		For species A	
		R1 = 2500 × 12 + 2500 × 13 = 62500	
		R2 = 2'500 × 138 + 2500 × 5 = 32500	
		R3 = 2500 × 16 + 2500 × 9 = 62500	
	01010	For species B	
		R1 = 3500 × 12 + 3500 × 13 = 87500	
		R2 = 3600 × 8 + 3500 × 5 = 42500	
		R3 z 3500 x 16 + 3500 x 9 = 87500.	

Extract 9.2

96)	Liven								
	$A = \begin{bmatrix} 3 & -5 \end{bmatrix}$								
	$i = A - A^{-1}$								
	$A^{-1} = \begin{bmatrix} -11 & 5 \end{bmatrix} \begin{bmatrix} 3 & -5 \end{bmatrix}$								
	$I = [-11 \times 33 + 5 \times 7 - 11 \times -5 + 5 \times -11]$								
	$2 - 7 \times 3 + 3 \times 7$ $- 7 \times 5 + 3 \times -11$								
	I= 2 O hence shown-								
()									
	$1 \times + 9 + 7 = 6$								
	2x + y - z = 1								
	x - y + z = ?								
	lot the Matix be A								
	$ A  = 1 \times (1-1) - 1 (2+1) + 1 (+2+1)$								
	$ A  = -3 \pm 1$								
	A  = -4								
	For $X = ?$								
	ſ· Ģ								
	$      -    z \times mshx$								
	2 -1 1								

90	G(1-1) - 1(1+2) + 1(-1+2)							
	0 - 3 + 1							
	X = -2							
	X =  X  mgmx							
	X = -4 = 2 , $X = 2$							
	-2							
	Y = ?							
1. V.S.								
	0   -1 = y  matrix							
-								
	1(1+2) - 6(2+1) + 1(4-1)							
	3 - 18 + 3							
	- 15 +3							
	(y) = -12							
	y =  y  = -12 = 3, $y = 3$ .							
	IAI - 4							
	For Z =?							
	$\left[ 1 \\ 6 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7$							
	2 1 1 2 2							
	2  =  (2+1) - 1(4-1) + 6(-2-1)							
	= 3 - 3 + 16							
	z  = -18							
	z  =  z  = - 8  = 9/							
	1×1 -4 2							
	z = 9/2							
	$\therefore$ X = Q, y = 3 and z = 4.5							

Extract 9.2 shows a sample answer of a candidate who performed calculations that were not related to the demand of question in part (a) and lacked knowledge to find the determinants in part (b) and (c).

#### 2.10 Question 10: Linear Programming

This question had parts (a), (b) and (c). The candidates were required, in part (a), to mention any four applications of linear programming, in part (b), to define the terms (i) objective function, (ii) constraints and (iii) feasible region. In part (c), the candidates were provided with a linear programming problem and were required to find the amount of units of food and drinks in order to meet daily needs and minimize the cost.

The question was attempted by 31,176 (93.5%) candidates; out of which 11,171 (35.7%) scored from 0 to 3 marks, 10,396 (33.3%) scored 3.5 to 5.5 marks, 9,609 (31%) scored 6 to 10 marks. This question had a good performance.

Most of the candidates were able to list down the applications of linear programming in part (a) and in part (b), to define the given terms correctly. In part (c), the candidates were able to correctly formulate the constraints and objective function from the given problem. They also managed to use the obtained constraints to draw the graph. From the graph, the candidates correctly located corner points of the feasible region which were needed in order to find the optimal solution. Extract 10.1 is a sample answer from one of the candidates.

Extract 10.1

10_(e)	The four applications include,
	(1) In hospitalsi
	- During the prescription of Medicane, purse may give minimum
	among a desape to be given to patients the such caroon lineas
	programming is used
	(") In schuds;
	- when the school budget is limited, and the school wants to make new
	furnitures and tables at minimal cut, we allop linear programming

to (iii) At home; - The organization of family budgets may utilize the concept of linear programming to that needs of family may be met. (10) In industrios; - During Manufacture of products, linear programming is -used to idouting what kind of products to be produced willyield Maximum amount of Ropol. 10(b) 1/ Objective functioni - Is that linear function that gives the desiredgoal to be achieved either maximization minimization 10 f (x, y) = ax+by "/ Constraints; These are linear inequalities which describe the condition of the problem; 14/ Fearble Regioni - A region bounded by inequalities and all possible-point that will lead to solution pertaining the problem. Let; Number of foods be X 10 (0) Number of drinks be Y JUMMARY! DRINKS total FOOD 4 units 8 units 2 units VITAMIN B 60 undi 2 unds 5 units TRON 1600Tshs 2000 Tsh Inst

10.0	objectue function;						
	$\mathcal{M}(x_1, x_2) = \frac{2000 \times f(x_0)}{1000 \times f(x_0)} = \frac{f(x_0)}{1000 \times f(x_0)}$						
	Subject to Constrainte;						
	$2x + 4\mu \geq 8$ (1)						
	$5^{+} + 2_{+} = 6^{-}$						
	$4 _{(1)}$ $\times \geq 0$ $(11)$						
	y 20 - (v)						
	The Equalities that anse;						
	x + 2y = 4						
	5×72y = 60						
	X = 6						
	9 = 0						
	Table q values for, X+2y= 4						
	X   0   4						
	5 2 0						
	Table of values fai 5x+24 = 60						
	TX 10 112						
	y 30 D						
	- The Resa to forced prover:						
	- was well to make to take						
	points are; (12,0) and (0,30)						
	(X19) 2000x+16004 Values						
	A (12,0) 2000(17) + 0(1600) 24,000						
	B 0,30 0(2000) + 30(1600) 48,000						
	optimalipti funco Minimum valuos are (12,0) which give 24,000 =						
	Hence, It is Aduced that the person should take						
	12 padeages of food with no drinks so						
	as to most daily Resurrements and minimize the						
	costs.						
	-> The optimal point is (120)						



Extract 10.1 shows a sample solution of a candidate who answered this question as required.

The analysis shows that, 1,293 (4.1%) candidates failed to answer this question correctly hence scored zero. These candidates did not understand the requirements of the question and lacked knowledge and skills in linear programming. In part (a), they provided incorrect applications of linear programming and definitions of the terms in part (b). In part (c), the candidates were unable to identify the decision variables, formulate the required constraints and objective function, hence ended with incorrect graph. Extract 10.2 is a sample response from a candidate who did not do well in this question.

Extract 10.2

Ne 08 linear DGAMMINS Corner pomts Ti 10 1 de Ram 10 Tú 1 Dentis simum Mim imum and Coct. 10 determine the 70 O breathe 106 ohe ettir ten. to determine the Maximum an uminum 1/alues' Signs That 106 11. to the Thes anni. Chen tran ad the Values m less 01 inear programmin 04 example 07X オムの *>0 106 This m that Gre a la the Corner



Extract 10.2 shows that in part (a), a candidate mentioned some of the steps of solving a linear programming problem instead of the applications; in part (b), gave incorrect definitions and in part (c) could not formulate the constraints and objective function.

# 3.0 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH TOPIC

The Basic Applied Mathematics Examination had ten questions which were set from ten topics. The analysis shows that, the candidates had a good performance in four questions which were set from the topics of *Matrices, Calculating Devices, Probability* and *Linear Programming*. It also shows that, the candidates had an average performance in six questions that were set from the topics of *Statistics, Functions, Algebra, Differentiation, Integration* and *Trigonometry*.

As introduced earlier, the performance has improved remarkably, with an increase of 5.92 percent of the candidates who passed this examination in 2018. The average performance for the year 2018 and 2017 were 53.4 percent and 31.6 percent respectively. This means, candidates were likely to score 3.5 or more marks in the year 2018 than in 2017, see Figure 5 and Appendix I.



**Figure 5**: Shows the Comparison of Candidates' Performance per Topic in Basic Applied Mathematics in 2018 and 2017.

The comparison of the 2018 and 2017 candidates' performance per topic showed that;

- Two topics of *Matrices* and *Calculating Devices* had a good performance and were the best performed topics in 2018. These topics were averagely performed in 2017.
- One topic of *Linear Programming* has consistently remained with good performance in both years, although the 2018 performance is 3% less than 2017 performance.
- One topic of *Probability* had a good performance in 2018 whereas this topic had a weak performance in 2017.
- Five topics of *Functions*, *Algebra*, *Differentiation*, *Integration* and *Trigonometry* had an average performance in 2018 whereas all these topics had a weak performance in 2017.
- Four topics of *Integration*, *Differentiation*, *Probability* and *Functions* portrayed significant improvement in performance of the candidates.
- Lastly, one topic of *Statistics* had an average performance in 2018 whereas this topic had a good performance in 2017.

The analysis indicated that, there are several reasons which contributed to the candidate's average performance in 2018 examination including;

- Inability to understand and comprehend the requirements of the questions;
- Insufficient skills to draw correct graphs and diagrams;
- Poor algebraic and computation skills which affect the quality of the responses;
- Lack of understanding of the sigma notation;
- Inability to apply differentiation and integration concepts in solving problems;
- inadequate knowledge in trigonometric identities/formulae;
- lack of understanding on the basic rules of probability and their applications;
- Lack of understanding of the properties of matrices and inability to apply Cramer's rule to solve system of linear equations.

#### 4.0 CONCLUSION AND RECOMMENDATIONS

#### 4.1 Conclusion

The conclusion was made on the basis of analysis of the candidates' performance in each question. Generally the analysis indicated that, the performance of the candidates for the questions of the Basic Applied Mathematics ACSEE 2018 was average. This performance was impressive across all questions and a notable significant increase as compared to the performance of 2017.

In 2018, there were four topics with a good performance, which were *Matrices, Calculating Devices, Probability, Linear Programming* and six topics with an average performance which were *Statistics, Function, Algebra, Differentiation, Integration* and *Trigonometry*. The best performed question was from the topic of *Matrices* (72.9%). On the other hand, the worst performed question was from the topic of *Trigonometry* (36%), see Appendices I and II.

There were several reasons observed by examiners for the candidates' average performance in the 2018 examination. These reasons were mainly due to lack of understanding on the tested topics and inability to identify and respond appropriately to the requirements of the questions.

#### 4.2 Recommendations

It is recommended that both teachers and students should strive to understand all the topics in the syllabus, in order to improve future candidates' performance in Basic Applied Mathematics. However, based on the performance of candidates in 2018, it is further recommended that more effort should be made in the topics of *Statistics, Function, Algebra, Differentiation, Integration* and *Trigonometry* that had an average performance.

In addition, the candidates are advised to do various exercises in order to be able to apply theories, facts and formulae in solving questions.

The teachers are also advised to motivate students and provide more support to candidates in order to help them to achieve better in this subject. Furthermore, the factors which have contributed to candidates' low scores should be notified and fully addressed to the future candidates in order to improve the performance in this subject.

The Government through, the Ministry of Education, Science and Technology is advised to use this report to influence/establish policies and operations that necessitate effective follow up on the teaching and learning process in order to raise the standard of performance in this subject.

# APPENDICES

# Appendix I

			2018		2017	
S/N	Торіс	Question Number	Percentage of Candidates who Passed (3.5 marks and above)	Remarks	Percentage of Candidates who Passed (3.5 marks and above)	Remarks
1	Matrices	9	72.9	Good	49.4	Average
2	Calculating Devices	1	65.5	Good	39	Average
3	Probability	7	64.7	Good	13.8	Weak
4	Linear Programming	10	64.2	Good	66.4	Good
5	Statistics	6	50.4	Average	77.3	Good
6	Functions	2	50.2	Average	19.3	Weak
7	Algebra	3	44.9	Average	24	Weak
8	Differentiation	4	44.7	Average	8.3	Weak
9	Integration	5	40.1	Average	3	Weak
10	Trigonometry	8	36	Average	15.6	Weak
Average Performance per Topic			53.36	Average	31.61	Weak

## Analysis of Candidates' Performance per Topic in Basic Applied Mathematics

In this Appendix, green, yellow and red colors represent good, average and weak performance respectively.

## **Appendix II**



## Analysis of Candidates' Performance in each Topic for ACSEE 2018

