THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

CANDIDATES' ITEMS RESPONSE ANALYSIS REPORT FOR THE ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION (ACSEE) 2017

142 ADVANCED MATHEMATICS

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

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FOREWORD

The National Examinations Council of Tanzania has prepared this report on the analysis of the candidates' responses for ACSEE 2017 Advanced Mathematics items in order to show how the candidates answered the questions. The report provides the feedback for educational stakeholders to improve the standards of the candidates' performance.

The analysis of the data and the candidates' responses were done to identify the strength and weaknesses of the candidates on how they answered the questions. Principally, the report identified the main areas where the candidates had good, average or poor performance.

The analysis revealed that, the candidates performed well in questions set from topics such as Statistics, Linear Programming, Logic, Sets, Differential Equations, Numerical Methods, Hyperbolic Functions and Vectors. They did averagely in questions from topics such as Functions, Differentiation, Calculating devices and Complex Numbers and performed poorly on the questions from the topics of Coordinate Geometry I, Coordinate Geometry II, Trigonometry, Algebra, Integration and Probability.

It was observed that the following factors contributed to the candidates' good performance: the ability to perform computations, sketch graphs and show relationship of formulas correctly. Moreover, the ability to state, recall, apply and use correct formulas, techniques, laws, and identities enhanced their performance. In addition, the candidates had skills to formulate models, draw figures and define terms. However, the candidates' poor performance was caused by the lack of knowledge and skills on how to find the unknown, define terms, apply laws, sketch graphs, solve equations, and formulate required equations or expressions.

Finally, the Council would like to thank examinations officers and all people who participated in making this report a reality. The Council will be glad to receive remarks from education stakeholders to improve future reports.

Dr. Charles E. Msonde
EXECUTIVE SECRETARY

1.0 INTRODUCTION

The Candidates Item Response Analysis (CIRA) report for ACSEE 2017 provides feedback to educational stakeholders on how the candidates performed in the examination items. The report was based on the analysis of the data and the candidates' responses.

Particularly, the report analyzed the candidates' performances in all examined topics for Advanced Mathematics. The Advanced Mathematics examination had two papers: Paper 1, 142/1 Advanced Mathematics 1 which had ten (10) compulsory questions where each question carried ten (10) marks. Paper 2, 142/2 Advanced Mathematics 2, consisted of four (4) compulsory questions in section A where each question carried fifteen (15) marks. Four (4) were optional questions. In section B the candidates were requested to choose any two questions. In section B, each question carried twenty (20) marks.

A total of 10,610 candidates sat for the Advanced Mathematics examination in 2017 out of which 74.78 percent passed. However, in 2016 the number of candidates who sat for the Examination was 12,798 out of which 76.35 percent passed. The data therefore shows a decrease of the pass rate by 1.6 percent.

The analysis of each question is presented in the next section. The focus was on the question requirements, analysis of the data and analysis of the candidates' responses. Figures are used in this report to show the performance using candidates' percentages and score categories 0 – 3, 3.5 – 5.5 and 6.0 – 10 in each question. Extracts showing good or weak performance are included to illustrate specific response of candidates to an item. The percent of the candidates' performance in each question are grouped as good, average or weak for the performance that lies in the interval 60 – 100, 35 – 59 and 0 – 34 respectively.

The topics that had good, average and weak performance and the factors that caused the good or the poor performance were shown. Finally, the recommendations for improving candidates' performance are given.

2.0 ANALYSIS OF THE CANDIDATES' PERFORMANCE

2.1 142/1 ADVANCED MATHEMATICS 1

2.1.1 Question 1: Calculating Devices

This question had parts (a) and (b). In part (a), the candidates were required to use the scientific calculator to compute (i) $\frac{\sqrt{240}\times e^{\ln\frac{1}{3}}\sin 22^\circ}{\sqrt{\tan 17^\circ}\times 3^{4\ln 11}}$ correct to 3 significant figures, (ii) $\ln\frac{\sqrt{98.2}\times(0.0076)^{-1}\times 10^{7}}{\tan\frac{\pi}{3}\times\cos^{3}\frac{\pi}{4}}$ correct to 6 significant figures and (iii) $\sqrt{\frac{(0.485)^{6}+\tan^{-1}(1.54)e}{(62.54)^{4}\sin^{-1}(0.4561)}}$ correct to 4 decimal places. In part (b), the candidates were given that $M^{d}=\frac{p}{\pi t^{2}}\left[\frac{4}{3}\ln\left(\frac{D}{d}\right)+\sqrt{\log p}\right]^{\frac{1}{3}}$, $P=1.6\times10^{3}$, $t=56\times10^{-2}$, $M=50.6\times10^{2}$ and $d=\lim_{x\to\infty}\left(\frac{\cosh x}{e^{x}}\right)$; they were required to evaluate D correct to four decimal places with the aid of a non-programmable calculator.

The analysis of the data shows that 96.8 percent of the candidates attempted this question, out of which 42.4 percent scored above 3 out of 10 marks. Further, the analysis of the data revealed that 57.6 percent of the candidates scored from 0 to 3 marks, 28.2 percent scored from 3.5 to 5.5 marks and only 14.2 percent scored from 6 to 10 marks. The candidates who scored all the 10 marks were 0.5 percent while those who scored a zero mark were 26.4 percent. Therefore, the question was averagely performed. Some of these statistics are well illustrated in Figure 1.

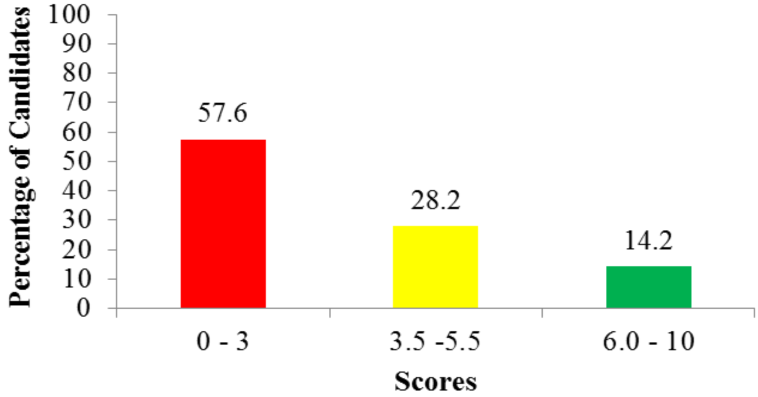


Figure 1: The candidates’ performance in question 1.

The analysis of candidates’ responses shows that 4.1 percent of the candidates did well in this question. The majority were able to compute it and managed to write correct answers according to the given instructions. Extract 1.1 depicts the work of one of the candidates who answered the question.

Extract 1.1

1 a) i) 0.0000928

ii) 23.7816.

iii) 0.0006

b). $M^d = \frac{P}{\pi t^2}\left(\frac{4}{3}\ln\left(\frac{D}{d}\right) + \sqrt{\log P}\right)^{1/3}$.

$P = 1.6 \times 10^3$

$t = 56 \times 10^{-2}$

$M = 50.6 \times 10^2$.

$d = \lim_{x\to\infty}\left(\frac{\cosh x}{e^x}\right)$.

~~ch~~ $\cosh x = \frac{e^x + e^{-x}}{2}$

$\cosh x = \frac{e^{2x} + 1}{2e^x}$

$\frac{\cosh x}{e^x} = \frac{e^{2x}+1}{2e^x} \times \frac{1}{e^x} = \frac{e^{2x}+1}{2e^{2x}} = \frac{1}{2} + \frac{1}{2e^{2x}}$

$d = \lim_{x\to\infty} \quad \frac{1}{2} + \frac{1}{2e^{2(\infty)}}$

$= \frac{1}{2} + 0$

$d. = \frac{1}{2}$.

$$\left(\frac{M^d \pi t^2}{p}\right)^3 - \sqrt{\log P'} = \frac{4}{3} \ln\left(\frac{D}{d}\right)$$

$$\frac{3}{4}\left[\left(\frac{M^d \pi t^2}{p}\right)^3 - \sqrt{\log P}\right) = \ln\left(\frac{D}{d}\right)$$

$$\frac{D}{d} = e^{\frac{3}{4}\left[\left(\frac{M^d \pi t^2}{p}\right)^3 - \sqrt{\log P'}\right]}$$

$$D = d \times \left(e^{\frac{3}{4}\left[\left(\frac{M^d \pi t^2}{p}\right)^3 - \sqrt{\log P'}\right]}\right)$$

$$D = 0.1306$$

In Extract 1.1, the candidate's work demonstrates a good understanding of how to use a calculator.

The candidates who scored low marks in this question had difficulties to answer parts (a) (i) and (ii). It was noted that many candidates did not show understanding of the significant figures and confused the term significant with decimal. For example, one of the candidates' responses showed that he/she expressed 23.7816042 as 23.781604 instead of 23.7816. In part (a) (ii), the candidates did not also recognise the meaning of the natural logarithm of $\dfrac{\sqrt{98.2}\times(0.0076)^{-1}\times 10^7}{\tan\frac{\pi}{3}\times\cos^3\frac{\pi}{4}}$ that it works on both the numerator $\sqrt{98.2}\times(0.0076)^{-1}\times 10^7$ and denominator $\tan\frac{\pi}{3}\times\cos^3\frac{\pi}{4}$. Such candidates obtained 38.0344 instead of 23.7816. In part1 (b), they could not define $\cosh x$ which was a necessary step in obtaining the correct value of d from $d = \lim_{x\to\infty}\left(\frac{\cosh x}{e^x}\right)$. A sample response from one of the candidates who performed poorly in this question is shown in Extract 1.2.

Extract 1.2

1. a (i) $$\frac{\sqrt{240} \times e^{\ln \frac{1}{3}} \sin 22^\circ}{\sqrt{\tan 17^\circ} \times 3^{4 \ln 11}}$$

$$= \frac{8.099269781}{20845.71}$$

Answer $= 3.89 \times 10^{-4}$.

(ii) $$\ln \frac{\sqrt{98.2} \times (0.0076)^{-1} \times 10^7}{\tan \frac{\pi}{3} \times \cos^3 \frac{\pi}{4}}$$

$$= \frac{3017767247}{0.6124}$$

$$= 4.92799 \times 10^9$$

(iii) $$\sqrt{\frac{(0.485)^6 + \tan^{-1}(1.54)e}{(62.54)^4 \sin^{-1}(0.4561)}}$$

01 b). $M^d = \frac{P}{\pi t^2}\left[\frac{4}{3}\ln\left(\frac{D}{d}\right) + \sqrt{\log P}\right]^{1/3}$.

$d = \lim_{\Delta x \to 1}\left(\frac{\cosh x}{e^x}\right)$

$\left(\frac{e^x + e^{-x}}{2} \cdot e^x\right)$.

$d = \lim_{\Delta x \to \infty}\left(\frac{e^{2x} + e^0}{2}\right) = \frac{e^{2x}+1}{2} = 1$.

$\sqrt[3]{\frac{M^d \times \pi t^2}{P}} = \sqrt[3]{\left(\frac{4}{3}\ln\left(\frac{D}{d}\right) + \sqrt{\log P}\right)^{1/3}}$.

$1.4605558 = \frac{4}{3}\ln D/d + 1.7900$.

$-0.329 = 4/3 \ln D/d$.

$\ln D/d = -0.247$.

$\frac{D}{d} = 0.7810763$

$D = 0.7811$.

$\therefore D = 0.7811$.

In Extract 1.2, the candidate obtained the value of $d = 1$ instead of $d = \frac{1}{2}$, eventually; this affected the procedure because he/she obtained a wrong answer for D.

2.1.2 Question 2: Hyperbolic Functions

This question had three parts (a), (b) and (c). In part (a), the candidates were given a condition that $x = \ln\left\{\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right\}$ and they were required to find e^x and e^{-x} hence to show that $\sinh x = \tan\theta$. In part (b), it was given that $a\cosh x + b\sinh x = c$, and they were required to show that the value of $x = \ln\left(\frac{c \pm \sqrt{c^2 + b^2 - a^2}}{a+b}\right)$. In part (c), they were asked to use the appropriate hyperbolic substitution to evaluate $\int_{0.1}^{0.8} \sqrt{x^2 + 4x + 3}\, dx$.

This question was attempted by 9,439 (89%) candidates. The candidates who scored from 3.5 to 10 marks were 62.9 percent. Also, the analysis showed that 37.1 percent of the candidates scored from 0 to 3 marks, 37.4 percent scored from 3.5 to 5.5 marks and 25.5 percent scored from 6 to 10 marks. Moreover, the candidates who scored all the 10 marks were 1.5 percent while 5.0 percent of the candidates scored a zero mark. Generally, the performance of this question was good as shown in Figure 2.

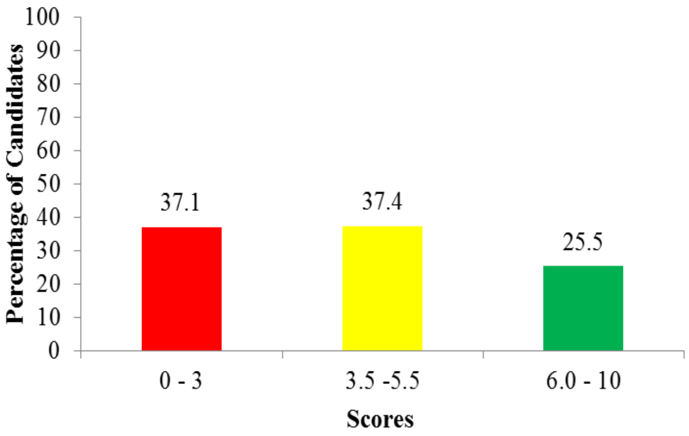


Figure 2: The candidates' performance in question 2.

The analysis of the candidates' responses shows that the candidates who performed well in part (a) and (b) were able to convert natural logarithm into exponent form, define both the concepts of compound angles of trigonometric ratios and the hyperbolic identities ($\sinh x$ and $\cosh x$) in the equation and hence solved for x. In part (c), they were able to complete the square and hence made an appropriate hyperbolic substitution and integrated it correctly. Extract 2.1 is a sample response from a candidate who performed well in part (b).

Extract 2.1

2. (b) Given:

$$a\cosh x + b\sinh x = c$$

Then:

$$\frac{a}{2}\left(e^{x}+e^{-x}\right)+\frac{b}{2}\left(e^{x}-e^{-x}\right)=c$$

$$ae^{x}+ae^{-x}+be^{x}-be^{-x}=2c$$

$$(a+b)e^{x}+(a-b)e^{-x}=2c$$

$$(a+b)e^{2x}+(a-b)=2ce^{x}$$

$$(a+b)e^{2x}-2ce^{x}+(a-b)=0$$

Quadratic in e^{x}

$$e^{x}=\frac{2c\pm\sqrt{4c^{2}-4(a+b)(a-b)}}{2(a+b)}$$

$$e^{x}=\frac{2c\pm\sqrt{4c^{2}-4(a^{2}-b^{2})}}{2(a+b)}$$

$$e^{x}=\frac{2c\pm2\sqrt{c^{2}-a^{2}+b^{2}}}{2(a+b)}$$

$$e^{x}=\frac{c\pm\sqrt{c^{2}+b^{2}-a^{2}}}{a+b}$$

$$\therefore\quad x=\ln\left(\frac{c\pm\sqrt{c^{2}+b^{2}-a^{2}}}{a+b}\right)$$ Hence shown.

Extract 2.1 shows that candidate was able to define the hyperbolic function correctly and expressed x as required.

In this question, 37.1 percent of the candidates had poor performance. The poor performance in parts (a) and (b) was due to candidates' lack of knowledge of converting logarithms to exponents. In part (b), they failed to define $\cosh x$ and $\sinh x$ which led them getting a wrong answer. In addition, in part (c), some of the candidates failed to complete the square of expression in the radical which was a necessary step in using the appropriate hyperbolic substitution for $\int_{0.1}^{0.8}\sqrt{x^2+4x+3}\,dx.$ Others, had difficulties in how to change the limits from expression $\int_{0.1}^{0.8}\sqrt{(x+2)^2-1}\,dx$ so as to match with the substitutions $x+2=\sinh u$ or $x+2=\cosh u.$ For instance, those who used the integration as;

$\int_{0.1}^{0.8}\sqrt{(x+2)^2-1}\,dx=\int_{0.1}^{0.8}\sqrt{(\cosh u)^2-1}\,\sinh u\,du,$ used a wrong procedure in changing the limits. Apart from the challenges mentioned above, other candidates failed to follow instructions. That is, instead of using hyperbolic substitution they used trigonometric substitution. The Extract 2.2 is a sample response from a candidate who performed poorly in this question.

Extract 2.2

2. (a) $\int_{0.1}^{0.8}\sqrt{(x+2)^2-1}\quad dx$

$x+2=\cosh t$

$dx=\sinh t\,dt$

$\int_{0.1}^{0.8}\sqrt{\cosh^2 t-1}\quad \sinh t\,dt$

$\int_{0.1}^{0.8}\sinh^2 t\,dt$

$\int_{0.1}^{0.8}\frac{1}{2}(1-\cosh 2t)\,dt$

$\frac{1}{2}\int(1-\cosh 2t)\,dt$

$\frac{1}{2}\left[t-\frac{1}{2}\sinh 2t\right]_{0.1}^{0.8}$

$=\frac{1}{2}\left[t-\frac{1}{2}2\sinh t\cosh t\right]_{0.1}^{0.8}$

$=\frac{1}{2}\left[\cosh(x+2)-\sinh t\cosh t\right]_{0.1}^{0.8}$

$=\frac{1}{2}\left[\cosh(x+2)-\sqrt{\cosh^2 t-x}\ \cosh t\right]_{0.1}^{0.8}$

$=\frac{1}{2}\left[\cosh(x+2)-\sqrt{(x+2)^2-1}\cdot(x+2)\right]_{0.1}^{0.8}$

$=\frac{1}{2}\Big[\left(\cosh(0.8+2)-\sqrt{(0.8+2)^2-1}\,.\,(0.8+2)\right)-$

$\cosh(0.1+2)-\sqrt{(0.1+2)^2-1}\ (0.1+2)\Big]$

$=0.33168$ (5 d.p.)

$\therefore \int_{0.1}^{0.8}\sqrt{x^2+4x+3}\,dx=0.33168$ (5 d.p.)

In Extract 2.2, the candidate lacked knowledge of changing limits by using hyperbolic substitution to evaluate the definite integral.

2.1.3 Question 3: Linear Programming

In this question, the candidates were required to find the cheapest way to prescribe the pills and the cost for an illness under the conditions that the daily prescription contains *x* Feelgood pills and *y* Getbetta pills and that a patient is required to take pills containing minerals and vitamins. The contents and costs of two types of pills; Feelgood and Getbetta, together with the patient's daily requirement were shown in the following table:

	Mineral	Vitamin	cost
Feelgood	80 mg	4 mg	3,000/=
Getbetta	20 mg	3 mg	1,500/=
Daily requirement	420 mg	31 mg	

This question was answered by 10,331 candidates, of whom, 88 percent scored above 3 marks. It was the second among the questions which had good performance in this examination. The analysis showed that 12 percent of the candidates scored from 0 to 3 marks, 31.4 percent scored from 3.5 to 5.5 marks and 56.6 percent scored from 6 to 10 marks. Further, the analysis of the data indicates that 27.3 percent of the candidates scored all the 10 marks, while 0.9 percent scored a zero mark. Figure 3 summarised the data.

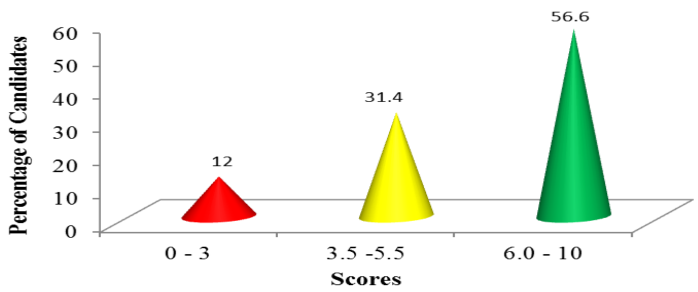


Figure 3: The candidates' performance in question 3.

The analysis of the candidates' responses shows that the candidates who performed well were able to correctly formulate the constraints as well as the objective function. Moreover, they correctly presented the constraints graphically and managed to obtain correct corner points and feasible region, which helped them to obtain the minimum cost as required. The good performance in this question was an indication that the candidates had adequate knowledge of linear programming. Extract 3.1 is a sample response from a candidate who performed well in the question.

Extract 3.1

3. Programming summary.

	Mineral	Vitamin	Cost
Feel good	80	4	3000/=
Get betta	20	3	1500/=
Daily requirment	420	31	

Given:

x — feel good pills

y — Get betta pills.

Constraints

$80x + 20y \geq 420$ — (i)

$4x + 3y \geq 31$ — (ii)

$x \geq 0$ — (iii)

$y \geq 0$ — (iv)

3. Objective function:

$f(x, y) = 3000x + 1500y$

Minimizing cost.

Linear equations:

$80x + 20y = 420$ —(i)

x	0	5.25
y	21	0

$4x + 3y = 31$ —(ii)

x	0	7.75
y	10.3	0

$x = 0$ —(iii)

$y = 0$ —(iv)

Check on the graph:

Then: From the graph:

Corner points	$f(x,y) = 3000x + 1500y$
A(7.75, 0)	23250/=
B(4, 5)	19500/=
C(0, 21)	31500/=

Optimal solution 19500/=

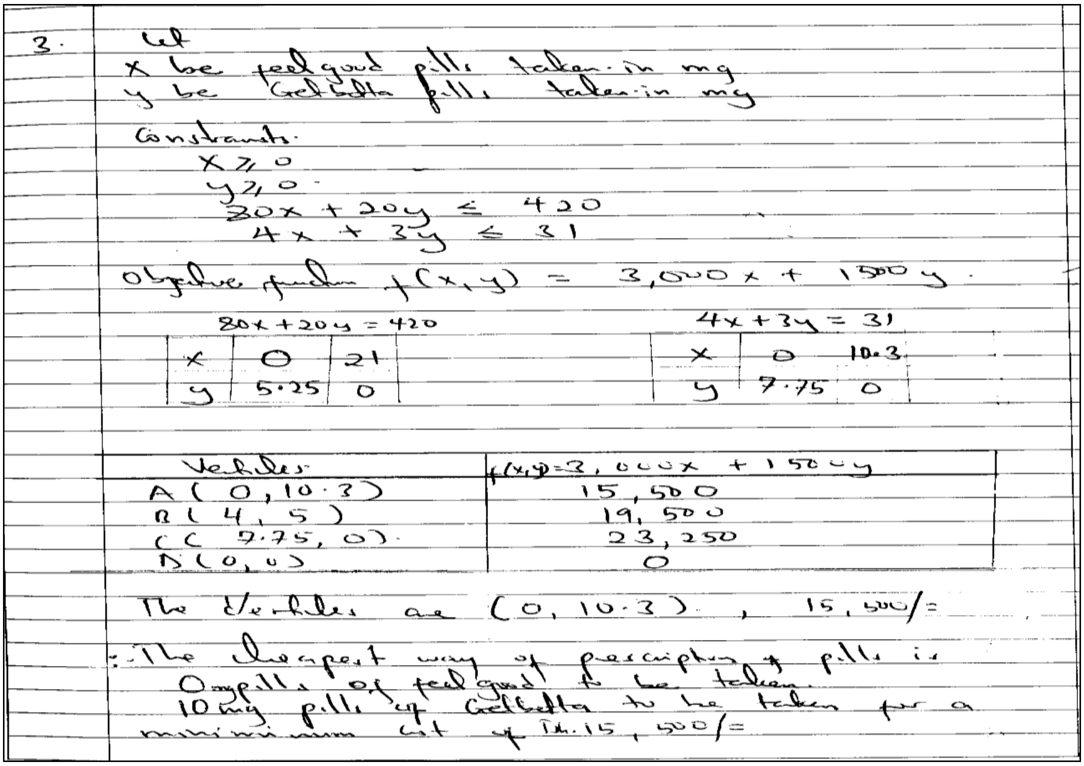
optimal point B(4, 5)

∴ The daily prescription should contain 4 feel good pills and 5 Get betta pills at a cheapest cost of 19,500/=

In Extract 3.1, the candidate was able to formulate the correct constraints as well as the objective function.

Despite that many candidates performed well in this question, a small number (12%) performed poorly. The reason for the poor performance was associated with mistakes they made as well as misconceptions of the terms used in the question. Candidates treated the term ' daily requirements' as if it was a maximum requirement problem, whereas the word 'cheapest' qualified the question to be the minimum problem. This led them to formulation of wrong inequalities. For example, they formulated the inequalities $80x+20y \le 420 \to 4x+y \le 21$, $4x+3y \le 31$ and $x, y \ge 0$ instead of $80x+20y \ge 420 \to 4x+y \ge 21$; $4x+3y \ge 31$ and $x, y \ge 0.$ Furthermore, some of the candidates failed to formulate correct objective function. For instance, they wrote the function $f(x, y) = 300x + 150y$ instead of $f(x, y) = 3{,}000x + 1{,}500y,$ as a result, they obtained incorrect cost of pills. In addition, other candidates had the problem of obtaining the corner points from the drawn inequalities because they shaded the wrong region. Extract 3.2 is a sample of a candidate's work showing incorrect inequalities.

Extract 3.2



3. Let
x be feel good pills taken in mg
y be Getbetta pills taken in mg

Constraints.
$x \geq 0$
$y \geq 0$
$80x + 20y \leq 420$
$4x + 3y \leq 31$

Objective function $f(x, y) = 3,000x + 1500y$.

$80x + 20y = 420$

x	0	21
y	5.25	0

$4x + 3y = 31$

x	0	10.3
y	7.75	0

Vertices	$f(x,y) = 3,000x + 1500y$
A (0, 10.3)	15,500
B (4, 5)	19,500
C (7.75, 0)	23,250
D (0, 0)	0

The Vertices are (0, 10.3), 15,500/=
∴ The cheapest way of prescription of pills is
0 mg pills of feel good to be taken
10 mg pills of Getbetta to be taken for a minimum cost of Tsh.15,500/=

Extract 3.2 shows a sample of the candidates who formulated the wrong constraints and thus obtained incorrect answer.

2.1.4 Question 4: Statistics

This question had parts (i), (ii) and (iii). The candidates were given the frequency distribution table which has the marks in matriculation examination of communication skills as shown below;

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	8	12	18	25	40	28	31	30	8

Then, they were required to (i) use the coding method to find the average marks given that the assumed mean is 75.5, (ii) determine the lower quartile of the distribution and (iii) calculate the 75^{th} percentile to four significant figures correctly.

The analysis of the data shows that the question was attempted by 10,394 (98%) candidates, out of which 93.3 percent of the candidates scored above 3 marks.

Further, the analysis reveals that only 6.7 percent of the candidates scored from 0 to 3 marks, 12.9 percent scored from 3.5 to 5.5 marks and 80.4 percent scored from 6 to 10 marks. The data also showed that 31.3 percent of the candidates scored all the 10 marks while 0.1 percent scored zero. It was the best performed questions in this examination. Some of these statistics are displayed in Figure 4.

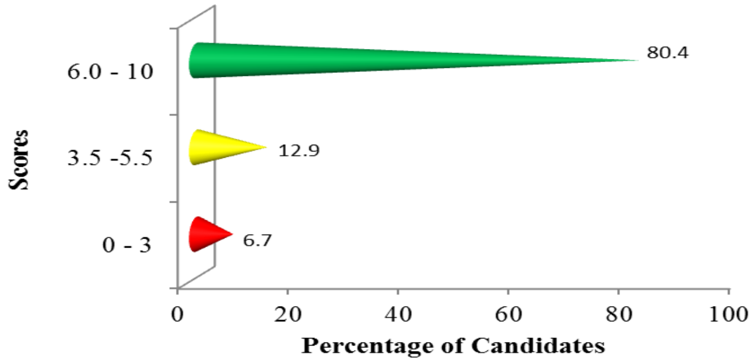


Figure 4: The candidates' performance in question 4.

The analysis of the candidates' responses reveal that a good performance in this question was due to the fact that most of candidates were able to recall and apply the formulae of coding method, quartiles and percentiles correctly. Moreover, they managed to obtain the class mark, coded value and cumulative frequency correctly. These values helped them to obtain the correct mean, quartiles as well as the percentiles. A sample response of the candidate who performed well in this question is shown in Extract 4.1.

Extract 4.1

4\. Frequency distribution table

Marks	f	$u = \left(\frac{x - 75.5}{10}\right)$	fu	x
11 – 20	8	–6	–48	15.5
21 – 30	12	–5	–60	25.5
31 – 40	18	–4	–72	35.5
41 – 50	25	–3	–75	45.5
51 – 60	40	–2	–80	55.5
61 – 70	28	–1	–28	65.5
71 – 80	31	0	0	75.5
81 – 90	30	1	30	85.5
91 – 100	8	2	16	95.5
	$\Sigma f = 200$		$\Sigma fu = -317$	

4\. (i) Average Marks, $\bar{x} = A + \frac{c\Sigma fu}{\Sigma f}$.

$A = 75.5$, $c = 10$, $\Sigma fu = -317$, $\Sigma f = 200$

$\bar{x} = 75.5 - \left(\frac{10 \times 317}{200}\right)$

$= 59.65$

$\therefore$ Average Marks $= 59.65$

4\. (ii) Lower Quartile, $Q_1 = L_{Q_1} + \left(\frac{N/4 - f_{bQ_1}}{f_{Q_1}}\right)c$

$N/4 = \frac{200}{4} = 50$

Class of Lower Quartile $= 41 - 50$.

$L_{Q_1} = 40.5$, $f_{bQ_1} = 38$, $f_{Q_1} = 25$, $c = 10$.

$Q_1 = 40.5 + \left(\frac{50 - 38}{25}\right)10$

$= 45.3$

$\therefore$ Lower Quartile $= 45.3$

(iii) 75th Percentile $= L_{P_{75}} + \left(\dfrac{\frac{75}{100}N - f_{bP_{75}}}{f_{P_{75}}}\right) \times c$

$\frac{75}{100}N = \frac{75}{100} \times 200 = 150$

Class containing $P_{75} = 71 - 80$

$L_{P_{75}} = 70.5$, $f_{bP_{75}} = 131$, $f_{P_{75}} = 31$, $c = 10$

$P_{75} = 70.5 + \left(\dfrac{150 - 131}{31}\right) 10$

$= 76.62903226$

≈ 76.63

$\therefore$ 75th Percentile $= 76.63$ (4 sign. figures).

Extract 4.1, shows that the candidate was systematic in applying the required formulae.

On the other hand, 6.7 percent of the candidates who attempted this question performed poorly. The poor performance was caused by failure to memorize a formula for finding the mean by coding method. For example, in part (i), most of the candidates used one among the following incorrect varieties of formula for finding average mark by coding method:

$\bar{x} = A + \dfrac{\sum fu}{\sum f}$, $\bar{x} = A + \left(\dfrac{\sum fd}{\sum f}\right)$, $\bar{x} = A + \left(\dfrac{\sum fu^2}{\sum f}\right)$ or $\bar{x} = \dfrac{\sum f\left[\frac{x-A}{C}\right]}{\sum f}$ instead of $\bar{x} = A + \left(\dfrac{\sum fu}{\sum f}\right) \times c$.

It was also noted that some of them used the incorrect formula $Q_1 = L + \left(\dfrac{\frac{N}{4} + n_b}{n_q}\right) \times c$ or $Q_1 = L + \left(\dfrac{\frac{N}{2} - n_b}{n_q}\right) \times c$ instead of $Q_1 = L + \left(\dfrac{\frac{N}{4} - n_b}{n_q}\right) \times c$ to find the first quartile in part (ii).

Likewise, in part (iii) a few candidates failed to write the correct formula for finding the 75^{th} percentile. They used incorrect formulae such as

$p_n = L + \left(\frac{\frac{nN}{100} + \sum f_b}{f_w} \right) \times c.$ A sample answer from a candidate who performed poorly in this question is shown in Extract 4.2.

Extract 4.2

4 (ii) From

Quartile $(Q) = \frac{1}{4}N$

$= \frac{1}{4} \times 200$

$= 50$

from

~~Quartile Median~~ $= L + \left(\frac{\frac{1}{4}N + cf Q}{fw} \right) i$

$\Rightarrow$ where $C \cdot F(Q) = 38$

$fw = 25$

$L = 40.5$

$= 40.5 + \left(\frac{50 + 38}{25} \right) 10$

$= 40.5 + 35.2$

$= 75.7$

$\therefore$ The lower quartile is 75.7

(iii) 75th percentile

from ; ~~Median~~ Percentile $= L + \left(\frac{\frac{P}{100}N + C \cdot F_{(P)}}{f_{(P)}} \right) i$

$\frac{P}{100} \cdot N = \frac{75}{100} \times 200$

$= 150$

$C \cdot F_{(P)} = 131$

$f_{(P)} = 31$

$L = 70.5$

In Extract 4.2, the candidate failed to recall the correct formulae for finding the lower quartile and the 75th percentile of the distribution.

2.1.5 Question 5: Sets

This question had parts (a) and (b). In part (a), the candidates were required to use the laws of algebra to simplify; (i) $\left[A \cap (B \cap C')\right] \cup C$ and (ii) $(X \cap Y') \cup (X \cap Y) \cup (Y \cap X')$. In part (b), the candidates were given the information, "out of the group of 17 girl guides and 15 boy scouts, 22 play handball, 16 play basketball, 12 of the boy scouts play handball, 11 of the boy scouts play basketball, 10 of the boy scouts play both and 3 of the girl play neither of the two". Next, they were required to find the number (i) of girls who play both handball and basketball and (ii) the group which play handball only and the one which plays basketball only.

The analysis of the data shows that out of 10,455 (98.5%) candidates who attempted this question, 77.5 percent scored above 3 marks. The analysis has also shown that 22.5 percent of the candidates scored from 0 to 3 marks, 23.1 percent scored from 3.5 to 5.5 marks and 54.4 percent scored from 6 to 10 marks. It was also noted that 8.3 percent of the candidates scored all the 10 marks, while 5.3 percent scored a zero mark. Generally, this question was performed well as shown in Figure 5.

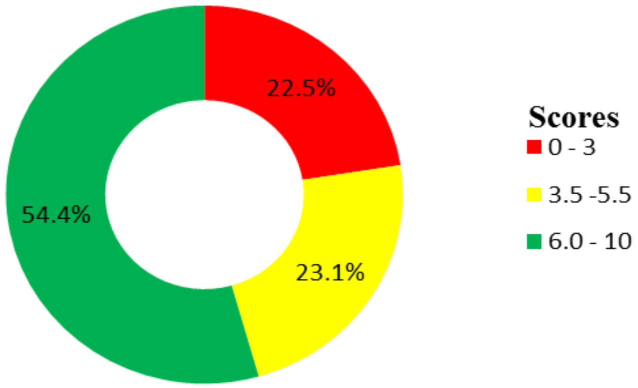


Figure 5: The candidates' performance in question 5.

The good performance was due to the fact that, in part (a), most candidates were able to state and correctly apply laws of algebra of set. In part (b), candidates were able to transform the word problem mathematically and presented the information in Venn diagram correctly. This implies that the candidates had a good knowledge of the topic of sets, as shown in Extract 5.1.

Extract 5.1

5. (a) Using laws of Algebra to simplify

(i) $[A\cap(B\cap C')]\cup C$

$(A\cup C)\cap[(B\cap C')\cup C]$ .. distributive law.

$A\cup C\cap[(B\cup C)\cap(C'\cup C)]$ distributive law

$A\cup C\cap[(B\cup C)\cap U]$ ... Complement law

$(A\cup C)\cap(B\cup C)$ .... Identity law.

$(A\cap B)\cup C$ .... distributive law.

5. (a)(ii) $(X\cap Y')\cup(X\cap Y)\cup(Y\cap X')$

$[(X\cap Y')\cup(X\cap Y)]\cup(Y\cap X')$ ... Associative law

$[X\cap(Y'\cup Y)]\cup(Y\cap X')$ ... distributive law

$(X\cap U)\cup(Y\cap X')$ ... complement law

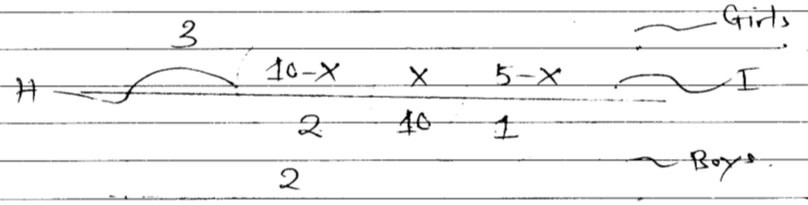
$X\cup(Y\cap X')$ ... Identity law

$(X\cup Y)\cap(X\cup X')$ ... distributive law

$(X\cup Y)\cap U$ .... complement law

$X\cup Y$ .... Identity law.

5. (b) $n(G) = 17$ ---- Girls
$n(B) = 15$ ---- Boys.
Let H be set of Handball game.
I be set of basketball game.
into Venn diagram.



5. (b)(i) Girls play both Handball and basket ball.
$n(I \cup H) = n(G) - 3$
$(10-x) + x + (5-x) = 17 - 3$
$15 - x = 14$
$x = 1.$
$\therefore$ Number of Girls $= 1.$

5. (b)(ii) · Hand ball only.
$(10-x) + 2 = 10 - 1 + 2$
$= 11.$
$\therefore$ 11 plays Handball only.

· Basketball only
$(5-x) + 1 = (5-1) + 1$
$= 5$
$\therefore$ 5 play basketball only.

Extract 5.1, illustrates a correct solution by one of the candidates who applied laws of algebra of sets correctly.

In spite of the good performance in this question, there were few candidates who performed poorly. The reasons for poor performance were; in part (a), some of the candidates failed to differentiate the algebraic laws of sets such as the associative from the commutative law, the De Morgan's' Law from the Complement Law, Identity Law from Complement Law and Identity Law from Idempotent Law. For example, some treated $\left[A\cap\left(B\cap C'\right)\right]\cup C=\left[\left(A\cap\mu\right)\cap\left(B\cap C'\right)\right]\cup C$ and

$(A \cap \mu) = A$ as Idempotent Law while it was an Identity Property. In part (b), some of the candidates failed to present the given information in Venn diagram correctly. A sample answer from one of the candidate who had low marks in this part is shown in Extract 5.2.

Extract 5.2.

5(a)(i) Solution

Given $[A \cap (B \cap C')] \cup C$

Required to simplify

$= [A \cap (B \cap C')] \cup C$ given that

$= (A \cap B) \cap (A \cap C') \cup C$ by associative law

$= (A \cap B) \cap (A \cap C') \cup C)$ by associative law

$= A \cap ((B \cap C') \cup C)$ by distributive laws

$= A \cap ((B \cup C) \cap (C' \cup C))$ by distributive law

$= A \cap ((B \cup C) \cap (\mu))$ by identity law

$= A \cap (B \cup C)$ By identity law.

$= A \cap (B \cup C)$

$= A \cap (B \cup C)$ simplified

$\therefore [A \cap (B \cap C')] \cup C = A \cap (B \cup C)$. simplified

(ii) Solution

$= (X \cap Y') \cup (X \cap Y) \cup (Y \cap X')$ Given

$= (X \cap Y') \cup (Y \cap X') \cup (X \cap Y)$ by commutat

$= ((X \cap Y') \cup (Y \cap X')) \cup (X \cap Y)$ by distribu

$=$

Solution

5(a)(ii) $= (X \cap Y') \cup (X \cap Y) \cup (Y \cap X')$ Given

$= ((X \cap Y') \cup (X \cap Y)) \cup (Y \cap X')$ by associative law

$= (X \cap (Y' \cup Y)) \cup (Y \cap X')$ by distributive law

$= (X \cap \mu) \cup (Y \cap X')$ by complement law

$= X \cup (Y \cap X')$ by identity law.

$= (X \cup Y) \cap (X \cup X')$ by distributive law

$= (X \cup Y) \cap \mu$ by complement.

$= (X \cup Y)$ by identity law.

$\therefore (X \cap Y') \cup (X \cap Y) \cup (Y \cap X') = (X \cup Y)$ simplified

In Extract 5.2, the candidate used incorrect laws in part (a). For instance, step $\left[A \cap (B \cup C')\right] \cup C$ in part (a) was justified as $(A \cap B) \cap (A \cap C') \cup C$ by associative law while it was distributive law of sets.

2.1.6 Question 6: Functions

This question had parts (a) and (b). In part (a), the candidates were required to draw the graph of $f(x) = x^3 - 3x^2 - 6x + 8$ in the interval $[-5, 6]$ and to explain how $f(x)$ behaves for positive and negative large values of *x*. In part (b), the candidates were asked to find $f\,o\,g(x)$ given that $f(x) = 2x^2 + 1$ and $g(x) = \frac{4x}{x^2 - 2}$ hence they were required to (i) determine the vertical and horizontal asymptotes of $fog(x)$, (ii) draw the graph of $fog(x)$ and (iii) state the domain and range of $fog(x)$.

The question was attempted by 10,437 (98.4 %) candidates of whom 48.1 percent scored above 3 marks. The analysis of the data in this question shows that 51.9 percent of the candidates scored from 0 to 3 marks, 32.4 percent scored from 3.5 to 5.5 marks and 15.7 percent scored from 6 to 10 marks. The analysis also shows that, 0.3 percent scored all the 10 marks while 2.9 percent scored zero. This question was averagely performed as indicated in figure 6.

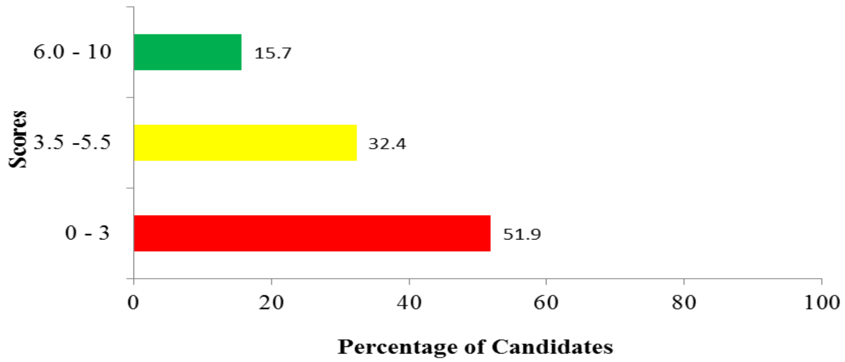


Figure 6: The candidates' performance in question 6.

The analysis of the candidates' responses in part (a) revealed that the candidates who did well were able to prepare a table showing the values of x and y, which helped them to draw the graph of the polynomial function correctly. Moreover, in part (b), candidates were able to find the composite function correctly; use the obtained function to determine x and y intercepts, vertical and horizontal asymptotes. They transferred them correctly in the x and y axes and hence managed to draw the graph of rational function. Furthermore, they were able to state the domain and range of the composite function. A sample answer for part (a) from one of the candidates is given in Extract 6.1.

Extract 6.1

6. (a) Given:

$$f(x) = x^3 - 3x^2 - 6x + 8$$

Interval $[-5, 6]$

Table of Value.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	-162	-80	-28	0	10	8	0	-8	-10	0	28	80

Check on the graph:

Then;

as $x \to +\infty$

$f(x) \to +\infty$

also:

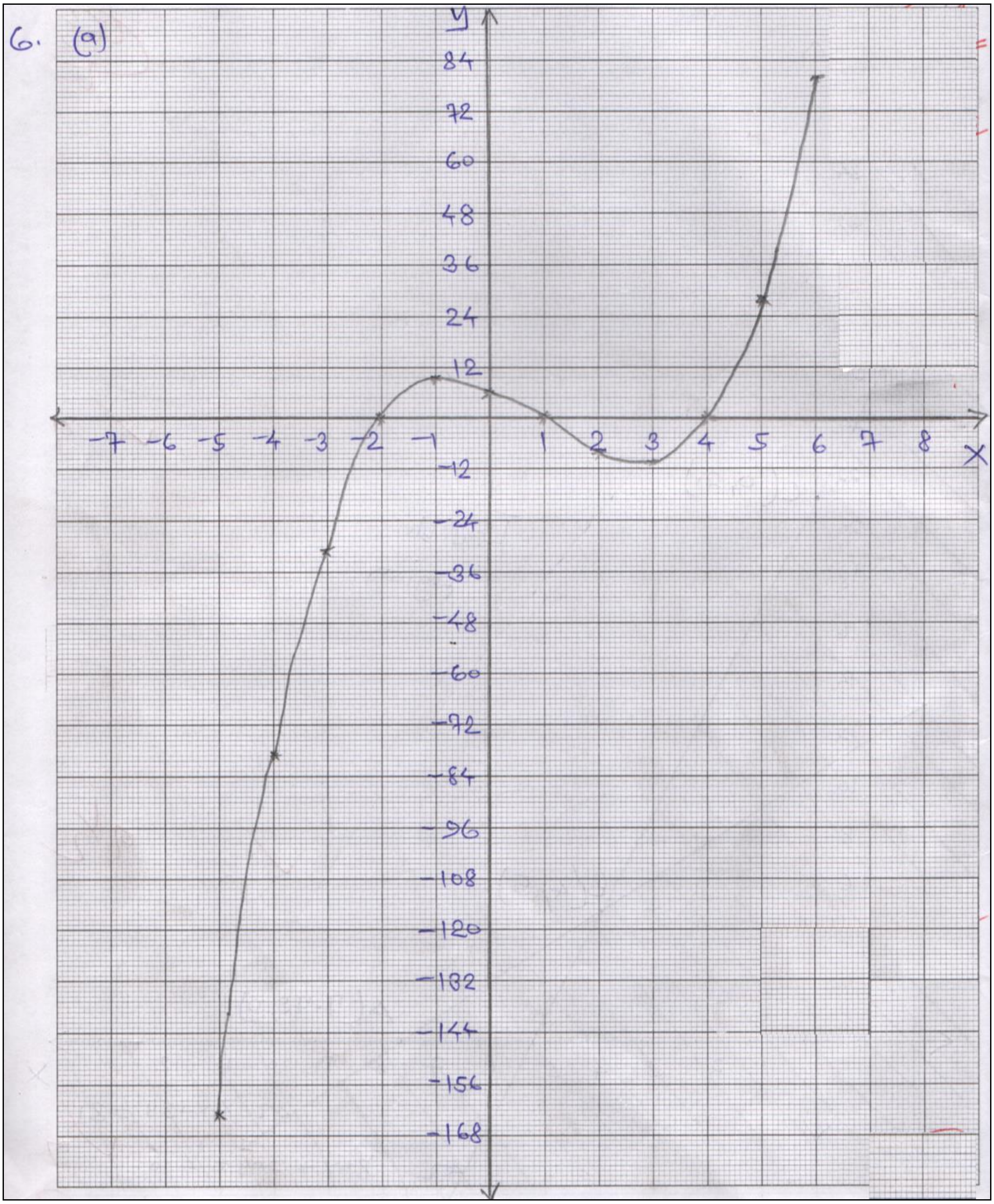
as $x \to -\infty$

$f(x) \to -\infty$

$\therefore$ for positively large value of X $f(x)$ approaches to positively large value

also

for negatively large value of X $f(x)$ approaches to negatively large value.



In Extract 6.1, the candidate successfully prepared a table of values and managed to draw the correct graph.

In this question, 51.9 percent of the candidates performed poorly. The poor performance was caused by their failure to follow instructions given in the question. For example, in part (a), the candidates drew the open graph instead of drawing it in closed interval $[-5,6]$. Likewise, in part (b) (ii), some determined $gof(x)$ instead of $fog(x)$. For example, one of the candidates wrote

$fog(x)=4\left(\frac{(2x^2+1)^2}{(2x^2+1)^2-2}\right)$ while the required was $fog(x)=2\left(\frac{4x}{x^2-2}\right)^2+1$. Moreover, in part (b) (ii), some candidates failed to find the vertical asymptotes because they expressed the denominator as follows: $x^4-4x^2+4=0$ or $x^2\left(x^2-4\right)+4=0$ and $x=2\ or\ x=-2$. However, they were supposed to follow the steps, $x^4-4x^2+4=0 \Rightarrow \left(x^2-2\right)^2=0$ where the vertical asymptotes are $x=\sqrt{2}$ and $x=-\sqrt{2}$.

In part (b) (iii), the candidates stated, domain as $\{x\in\Re : x\neq\pm2\}$ and $Range=\{y\in\Re : y>1\}$ incorrectly. The required are: domain $=\left\{x\in\Re : x\neq\pm\sqrt{2}\right\}$ and $range=\{y\in\Re : y\geq1\}$. Extract 6.2 is a sample answer showing this case.

Extract 6.2

6. (b) $f(x) = 2x^2+1$

$g(x) = \frac{4x}{x^2-2}$

$fog(x) = 2\left(\frac{4x}{x^2-2}\right)+1$

$= \frac{8x}{x^2-2}+1$

$= \frac{8x+(x^2-2)}{x^2-2}$

$= \frac{8x+(x^2-2)}{x^2-4}$

$fog(x) = \frac{x^2+8x-2}{x^2-4}$

6 b)

$f(x) = 2x^2 + 1$

$g(x) = \frac{4x}{x^2 - 2}$

Soln

$fog = f(g(x))$

$2x^2 + 1$

$fog = 2\left(\frac{4x}{x^2-2}\right)^2 + 1$

$fog(x) \quad 2\left(\frac{(4x)^2}{(x^2-2)^2}\right) + 1$

$x^2(x^2-2) - 2(x^2-2)$

$x^4 - 2x^2 - 2x^2 + 4$

$x^4 - 4x^2 + 4$

$fog(x) \quad 2\left(\frac{16x^2}{x^4 - 4x^2 + 4}\right) + 1$

$fog(x) = \frac{32x^2}{x^4 - 4x + 4} + 1$

$fog(x) = \frac{32x^2 + x^4 - 4x + 4}{x^4 - 4x + 4}$

$fog(x) = \frac{x^4 + 32x^2 - 4x + 4}{x^4 - 4x + 4}$

$fog(x) = \frac{x^3 + 32x - 4 + \frac{4}{x}}{x^3 - 4 + 4/x}$

In Extract 6.2, the candidate did a wrong substitution to form the composite function in part (b); indicating that he/she lacked knowledge and skills in the topic of functions.

2.1.7 Question 7: Numerical Methods

This question consisted of three parts (a), (b) and (c). In part (a), the candidates were required to show that the Newton Raphson formula of finding the roots of the equation $12x^3 + 4x^2 - 15x - 4 = 0$ is $x_{n+1} = \frac{(24x_n + 4)x_n^{\ 2} + 4}{(36x_n + 8)x_n - 15}$ and use the formula to find the roots of $12x^3 + 4x^2 - 15x - 4 = 0$ to three decimal places correctly. In part (b), they were supposed to approximate the area under the curve $y = \frac{1}{x-2}$ between $x = 2$ and $x = 3$ with six ordinates by (i) Trapezoidal rule, (ii) Simpson rule. In part (c), they were required to state the rule in part (b) which gives a better approximation to the area.

A total of 10,088 (95.1%) candidates answered this question, whereas, 65.1 percent scored above 3 marks. The analysis of the data indicates that 34.9 percent of the candidates scored from 0 to 3 marks, 56.3 percent scored from 3.5 to 5.5 marks and 8.8 percent scored from 6 to 10 marks. Based on the data, the question was well performed. Figure 7 represents the percentage of candidates' performance.

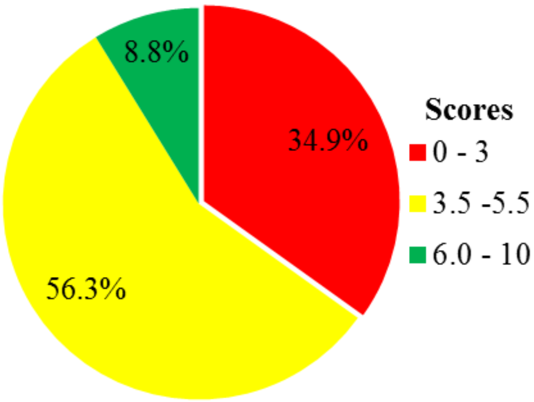


Figure 7: The candidates' performance in question 7.

The analysis of the candidates' responses shows that the good performance in this question was due to their ability to recall the Newton Raphson formula, Trapezoidal rule, and Simpson rule, also the skills to apply them correctly.

A sample answer from one of the candidates who performed part (a) well is given in Extract 7.1.

Extract 7.1

7(a) Given $12x^3+4x^2-15x-4=0$

let $f(x_n) = 12x_n^3+4x_n^2-15x_n-4$

$f'(x_n) = 36x_n^2+8x_n-15$

from Newton Raphson Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \left(\frac{12x_n^3+4x_n^2-15x_n-4}{36x_n^2+8x_n-15}\right)$$

7(a) Again let $x_0 = -0.25$

From $x_{n+1} = \frac{(24x_n+4)x_n^2+4}{(36x_n+8)x_n-15}$

1st iteration

$$x_1 = \frac{(24(-0.25)+4)(-0.25)^2+4}{(36(-0.25)+8)(-0.25)-15}$$

$x_1 = -0.263$

2nd iteration

$$x_2 = \frac{24(-0.263)+4)(-0.263)^2+4}{(36(-0.263)+8)(-0.263)-15}$$

$x_2 = -0.263$

$\therefore$ The roots of $12x^3+4x^2-15x-4=0$ are 1.19, -1.162 and -0.263

In Extract 7.1, the candidate demonstrated a good understanding of how to find the roots of a polynomial by using the Newton-Raphson formula.

However, 34.9 percent of candidates who had poor performance in this question scored below 3.5 marks. The poor performance was attributed to the lack of knowledge and skills to identify the initial roots of the polynomial function. Another factor was inadequate knowledge to sketch the graph. A sample answer from one of the candidates who performed poorly in part (b) is given in Extract 7.2.

Extract 7.2.

7b (i) $\int_2^3 \frac{1}{x-2}\,dx = \frac{0.2}{2}\left[1+2(16.4467)\right]$

$\therefore \int_2^3 \frac{1}{x-2}\,dx = (\cancel{54.7335})\ \underline{\underline{2.18934}}$

(ii) Simpson's rule $= \frac{h}{3}\left[\text{Sum of first \& last ordinates} + 4\left(\text{Sum of odd ordinates}\right) + 2\left(\text{Sum of even ordinates}\right)\right]$

$\int_2^3 \frac{dx}{x-2} = \frac{0.2}{3}\left[1+4[6.6667]+2(3.78)\right]$

$\therefore \int_2^3 \frac{dx}{x-2} = \underline{\underline{2.34845}}$

c The Simpson's rule gives a better approximation to the area.

Extract 7.2 shows the candidate's work in which the incorrect values of y were used and thus led to incorrect solution.

2.1.8 Question 8: Coordinate Geometry I

This question had three parts (a), (b) and (c). In part (a), the candidates were required to find the value of k so that the given equation $k\left(x^2+y^2\right)+\left(y-2x+1\right)\left(y+2x+3\right)=0$ is a circle and to obtain the centre and radius of the circle. In part (b), the candidates were given the circle as $x^2+y^2-2x-4y-5=0$ whereby centre C is cut by the line $y=2x+5$ at A and B. Next, they were required to show that BC is perpendicular to AC, hence to find the area of triangle ABC. In part (c), the candidates were asked to find the equation of a straight line which goes through the intersection of $3x+2y+4=0$ and $x-y=2$; and forms the triangle with the axes whose area is 8 square units.

The analysis of the data shows that 6,271 (59.1%) candidates attempted the question. On this attempt, 15.8 percent of the candidates scored from 3.5 to 10 marks. Moreover, the analysis indicates that 84.2 percent of the candidates scored from 0 to 3 marks, 9.6 percent scored from 3.5 to 5.5 marks and only 6.2 percent scored from 6 to 10 marks. Further, 22 percent of the candidates scored zero, while 0.2 percent scored all the 10 marks. This data implies that the performance in this question was generally poor as shown in Figure 8.

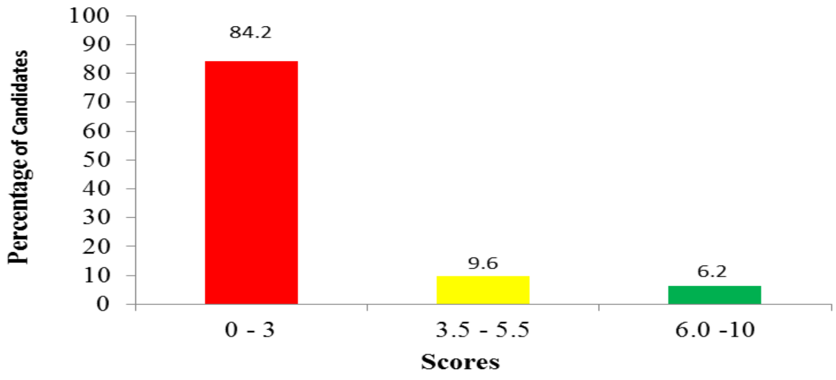


Figure 8: The candidates' performance in question 8.

The analysis of the candidates' responses revealed that many candidates had low marks, likely because they failed to apply the fundamental concepts of the circle, which means that for the equation to be a circle, the coefficients of x^2 and y^2 must be equal. In addition, in part (b), they failed to recall the general formula of a circle, which is $x^2+y^2+2gx+2fy+c=0$. Nevertheless, some candidates in part (c), managed to calculate the points of intersection of the lines $3x+2y+4=0$ and $x-y=2$. They however, had no knowledge of calculating the slopes. Hence, they failed to show that the two lines were perpendicular. Extract 8.1 is a sample answer from a candidate who performed poorly in the question.

Extract 8.1

8b $y = 2x + 5$

$C = (1, 2)$

From,

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|2(1) + 0(2) + 0|}{\sqrt{2^2 + 0^2}}$$

$$= \frac{2}{2} = 1$$

Hence BC is perpendicular to AC.

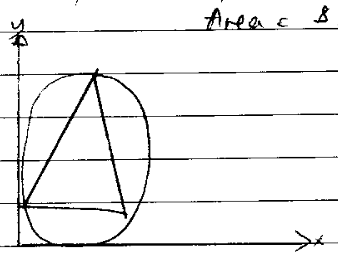
8c $L_1 = \begin{cases} 2x + 2y + 4 = 0 \\ x - y = 2 \end{cases}$ solve simulteneosly

$(x, y) = (0, 2)$

The points of intersection are $(0, 2)$

Area = 8 sq units

$(x - r)^2 + (y - r)^2 = r^2$



In Extract 8.1, the candidate used wrong concepts in answering parts (b) and (c). This shows that he/she lacked knowledge and skills of the topic of coordinate geometry.

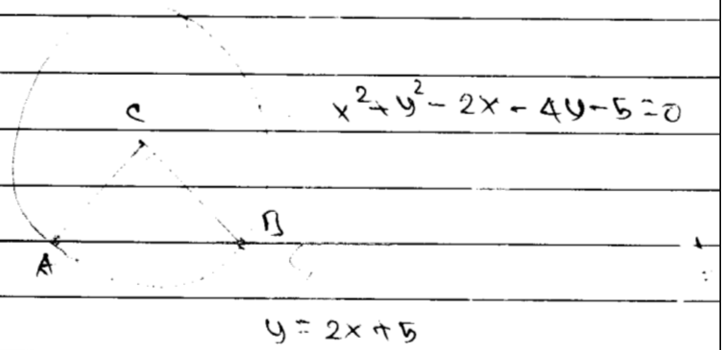
However, there were few candidates (15.8%) who did the question well. They obtained the value of k by comparing the coefficients of x^2 and y^2. Moreover, they demonstrated the ability to complete the square to obtain the centre of the circle. They were also able to solve the equations of the lines simultaneously to obtain, $(0, -2)$ as a point of intersection which helped them to find the slopes of the lines BC and AC; hence they proved that BC and AC are perpendicular. Finally, this step enabled them to obtain the area of triangle ABC. Extract 8.2 is a sample answer from a candidate who performed the question well.

Extract 8.2

b) Given the circle $x^2+y^2-2x-4y-5=0$

line $y=2x+5$

i.e



Soln:

$x^2+y^2-2x-4y-5=0 \quad —(i)$

$y=2x+5 \quad —(ii)$

Solving —(i) and —(ii) simultaneously

$x^2+(2x+5)^2-2x-4(2x+5)-5=0$

$x^2+4x^2+20x+25-2x-8x-20-5=0$

$5x^2+10x=0$

$5x(x+2)=0$

$x_1 = 0$ or $x_2 = -2$

Hence

$y = 2x + 5$

$y_1 = 2(0) + 5 = 5$

$y_2 = 2(-2) + 5 = 1$

Hence $A(0, 5)$ $B(-2, 1)$

~~For $\bar{AC}$~~ For C

$C = (-g, -f)$

From $x^2 + y^2 - 2x - 4y - 5 = 0$

$-2 = 2g$

$g = -1$

$-4 = 2f$

$f = -2$

i.e $C = (-g, -f)$

$= (1, 2)$

For $\bar{AC}$

Slope, $m_1 = \frac{\Delta y}{\Delta x}$

$= \frac{5-2}{0-1} = \frac{3}{-1}$

$m_1 = -3$

For $\bar{BC}$

Slope, $m_2 = \frac{\Delta y}{\Delta x}$

$= \frac{1-2}{-2-1} = -1/-3$

$m_2 = 1/3$

Hence $m_1 m_2 = -3 \times 1/3$

$m_1 m_2 = -1$

since slope of $\bar{AC}$ $(m_1) \times$ slope $(\bar{BC})(m_2) = 1$

Hence the lines are perpendicular

for Area

From

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} 0 & 5 & 1 \\ -2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\left[0(1-1) - 5(-2-1) + 1(-4-1)\right]$$

$$= \frac{1}{2}(15-5) = \frac{10}{2}$$

$A = 5$ square units

$\therefore$ The area of ΔABC is 5 square units.

In Extract 8.2, the candidate showed a good understanding of the concepts of coordinate geometry in answering part (b) of the question.

2.1.9 Question 9: Integration

This question had parts (a) and (b). In part (a), the candidates were required to evaluate $I_{ab} = \int \sin ax \cos bx \, dx$ for $a \neq b$ and use it to find the value of n in $\int_0^n \sin 3x \cos 2x \, dx = \frac{3-\sqrt{3}}{5}$. Part (b) required the candidates to find the length of an arc of the semi-cubical parabola, $y^2 = x^3$ between the points $(1,1)$ and $(4,8)$.

A total of 5,209 candidates, which is equivalent to 49.1 percent, attempted this question. This means, more than 50 percent of the candidates did not answer the question. The analysis of the data shows that 88.9 percent scored from 0 to 3 marks, 9 percent scored from 3.5 to 5.5 marks and 2.1 percent scored from 6 to 10 marks. Further, the analysis has revealed that 32.1 percent of the candidates scored a zero mark, while none of the candidates scored all the 10 marks. Therefore, the question was poorly performed as indicated in Figure 9.

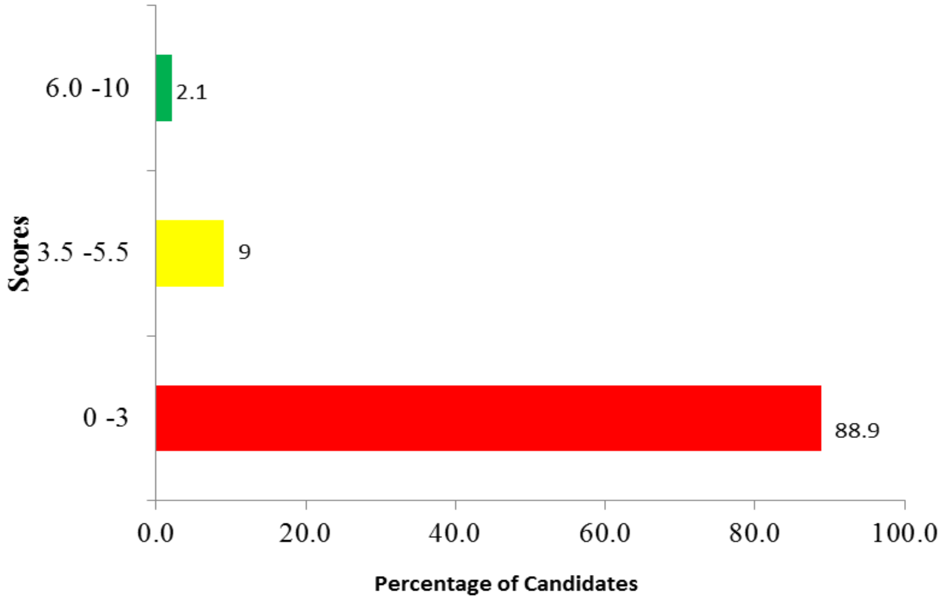


Figure 9: The candidates' performance in question 9.

The factors that contributed to the poor performance of the question were; candidates' inability to apply either the technique of integration by parts or use factor formula. Moreover, they lacked skills to express $\cos 5n$ as $\cos(3n+2n)$, that is, $\cos 3n \cos 2n + \sin 3n \sin 2n$ and finally as $16\cos^5 n - 20\cos^3 n + 5\cos n$, which hindered them to compute the value of n .

Likewise, inability to correctly recall the formula for an arc length, $L = \int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ caused the candidates to use incomplete formulas $L = \int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)}dx$, $L = \int_1^4 \sqrt{1-\left(\frac{dy}{dx}\right)^2}dx$ and $L = \int 1+\left(\frac{dy}{dx}\right)^2 dx$. A sample answer from a candidate who performed poorly in the question is given in Extract 9.1.

Extract 9.1

9a $\cdot\ I_{ab} = \int \sin ax \cos bx \, dx$

From factor formulae

$$2\sin ax \cos bx = \sin\left(\frac{a+b}{2}\right)x + \sin\left(\frac{a-b}{2}\right)$$

$$\therefore \sin ax \cos bx = \frac{1}{2}\left[\sin\left(\frac{a+b}{2}\right)x + \sin\left(\frac{a-b}{2}\right)\right]$$

$$I_{ab} = \frac{1}{2}\int \sin\left(\frac{a+b}{2}\right)x \, dx + \frac{1}{2}\int \sin\left(\frac{a-b}{2}\right)x \, dx$$

$$I_{ab} = \frac{-2\times 1}{2(a+b)}\cos\left(\frac{a+b}{2}\right)x - \frac{2\times 1}{2(a-b)}\cos\left(\frac{a-b}{2}\right)$$

$$I_{ab} = \frac{-1}{a+b}\cos\left(\frac{a+b}{2}\right)x - \frac{1}{a-b}\cos\left(\frac{a-b}{2}\right)$$

Given: $\int_0^n \sin 3x \cos 2x \, dx = \frac{3-\sqrt{3}}{5}$

$$\frac{3-\sqrt{3}}{5} = \left[\frac{-1}{3+2}\cos\left(\frac{3+2}{2}\right)x - \frac{1}{3-2}\cos\left(\frac{3-2}{2}\right)x\right]_0^n$$

$$\frac{3-\sqrt{3}}{5} = \left[\frac{-1}{5}\cos 2.5x - \cos 0.5x\right]_0^n$$

$$\frac{3-\sqrt{3}}{5} = \left(\frac{-1}{5}\cos 2.5n - \cos 0.5n\right) - \left(\frac{-1}{5}\cos 2.5(0) - \cos 0.5(0)\right)$$

$$\frac{3-\sqrt{3}}{5} = \frac{-1}{5}\cos 2.5n - \cos 0.5n + \frac{1}{5} + 1$$

$$\frac{3-\sqrt{3}}{5} - \frac{6}{5} = \frac{-1}{5}\cos 2.5n - \cos 0.5n$$

In Extract 9.1, the candidate could not recall the right factor formula in part (a). Hence, he/she ended up with incorrect answer.

Despite the fact that the majority of the candidates performed poorly in this question, there were few candidates (11.1%) who performed well. These candidates scored from 3.5 to 10 marks and some showed to have adequate knowledge and skills in the topic. They were able to apply factor formula in splitting the product of two trigonometric functions to the sum. They were also able to use the binomial theorem and De Moivres' theorem to expand $\cos 5n$ to get the value of n as required. In addition, they were able to use the formula for the length of the arc. A sample answer from a candidate who performed well in part (b) is given in Extract 9.2.

Extract 9.2

9 (b) Soln

$y^2 = x^3.$

$y = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$ $\quad \left(\frac{dy}{dx}\right)^2 = \frac{9}{4}x$

$S = \int_1^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$

$S = \int_1^4 \sqrt{1 + \frac{9}{4}x}\, dx.$

let $u = 1 + \frac{9}{4}x$

$\frac{du}{dx} = \frac{9}{4}.$ $\quad \frac{4du}{9} = dx.$

$S = \int_1^4 \frac{4}{9} \cdot u^{1/2} du$

$S = \int_1^4 \frac{4}{9} u^{1/2} du = \frac{4}{9} \cdot \frac{2}{3} \left[u^{3/2} \right]_1^4$

$S = \frac{8}{27} \left[\sqrt{\left(1 + \frac{9}{4}x\right)^3} \right]_1^4$

$S = \frac{8}{27} \left[\sqrt{1000} - \sqrt{2197/64} \right]$

$S = 8/27$ $\quad S = 7.634$ units

$\therefore$ The Length of an arc is 7.634 units

In Extract 9.2 a candidate was able to find the length of the arc as required.

2.1.10 Question 10: Differentiation

This question had three parts (a), (b) and (c), In part (a), the candidates were given $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and were required to prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$. In part (b), they were given that $f = \sin xy$ and were required to find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. In part (c), the candidates were asked to use the Taylor's theorem to expand $\sin\left(\frac{\pi}{6} + h\right)$ in ascending power of h as far as the term in h^4 and to evaluate $\sin 31°$ to three decimal places correctly.

This question was answered by 8,107 (76.4%) candidates, of whom, 43.5 percent of the candidates scored above 3 marks. The analysis of the data indicates that 56.5 percent of the candidates scored from 0 to 3 marks, 25.7 percent scored from 3.5 to 5.5 marks and 17.8 percent scored from 6.5 to 10 marks. The data has also revealed that 20.9 percent of the candidates scored zero, while 0.1 percent of the candidates scored all the 10 marks. Figure 10 illustrates the average performance of this question.

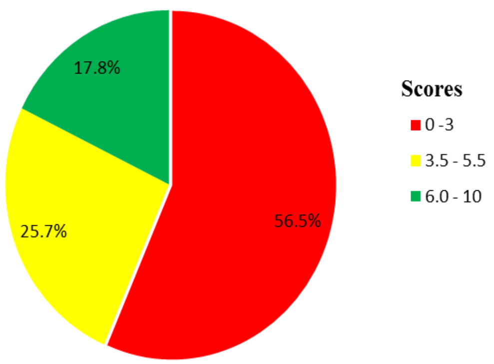


Figure 10: The candidates' performance in question 10.

The analysis of the candidates' response shows that the candidates who performed well were able to use different techniques such as expressing y in terms of x and found the first derivative as required. They were also able to find the partial derivatives, state the Taylor's theorem and expand the given expression correctly. A sample solution from a candidate who performed the question well is given in Extract 10.1.

Extract 10.1

10. (a) $x\sqrt{1+y} + y\sqrt{1+x} = 0$

Soln:

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$x\sqrt{1+y} = -y\sqrt{1+x}$$

Square both sides

$$x^2(1+y) = y^2(1+x)$$

$$x^2 + x^2y = y^2 + y^2x$$

$$x^2 - y^2 = y^2x - x^2y$$

$$(x-y)(x+y) = xy(y-x)$$

$$\cancel{(x-y)}(x+y) = -xy\cancel{(x-y)}$$

$$x+y = -xy$$

$$x = -xy - y$$

$$x = -y(x+1)$$

$$-y = \frac{x}{1+x}$$

$$y = \frac{-x}{1+x}$$

$$y = \frac{-x}{1+x}$$

$$\frac{dy}{dx} = -\left[\frac{(1+x)(1) - x(1)}{(1+x)^2}\right]$$

$$\frac{dy}{dx} = -\left[\frac{1+x-x}{(1+x)^2}\right]$$

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

Hence proved !!

In Extract 10.1, the candidate showed correct steps of finding the derivative of the given equation.

On the other hand, the candidates who performed poorly in the question had inadequate knowledge and skills in the topic. For instance, in part (a), they failed to express y in terms of x before differentiating. It was also noted that some of them differentiated the equation $x\sqrt{1+y}+y\sqrt{1+x}=0$ implicitly, yet obtained the wrong answer $\frac{dy}{dx}=\frac{1}{(1+x)^2}$ because they failed to remove the radicals which still had x and y in their expressions.

The candidates had also inadequate knowledge of how to find the partial derivatives in part (b). For instance, they calculated $\frac{df}{dx}=\cos xy\left(x\frac{dy}{dx}+y\right)$, instead of $\frac{\partial f}{\partial x}=y\cos xy$ and $\frac{\partial f}{\partial y}=x\cos xy$.

In part (c), the candidates failed to state the Taylor's theorem. As a result, they were not able to find the expansion of $\sin\left(\frac{\pi}{6}+h\right)$. Those who managed to find the expansion, still faced difficulties in converting degrees into radians. For instance, they wrote $\sin 31^\circ=\sin\left(\frac{\pi}{6}+\frac{\pi}{180}\right)$, then expanded it wrongly as, $\sin 31^0=\frac{1}{2}+\frac{\sqrt{3}}{2}\left(1^0\right)-\frac{1}{4}\left(1^0\right)^2-\frac{\sqrt{3}}{12}\left(1^0\right)^3+\frac{1}{48}\left(1^0\right)^4$. In obtaining the correct answer the candidates were supposed to expand $\sin 31^\circ$ as follows:
$\sin\left(31^\circ\right)=\sin\left(\frac{\pi}{6}+\frac{\pi}{180}\right)=\frac{1}{2}+\frac{\sqrt{3}}{2}\left(\frac{\pi}{180}\right)-\frac{1}{4}\left(\frac{\pi}{180}\right)^2-\frac{\sqrt{3}}{12}\left(\frac{\pi}{180}\right)^3+\frac{1}{48}\left(\frac{\pi}{180}\right)^4$. A sample answer from a candidate who performed poorly in the question is given in Extract 10.2.

Extract 10.2

10 (b)

$f = \sin u$

$\frac{df}{du} = \cos u.$

$\frac{df}{du} \times \frac{du}{dx} = \left(y + x\frac{dy}{dx}\right)\cos u$

$\frac{df}{dx} = y\cos u + x\frac{dy}{dx}\cos u.$

but $u = xy.$

Hence

$\frac{df}{dx} = y\cos xy + x\frac{dy}{dx}\cos xy.$

$\therefore \frac{df}{dx} = \left(y + x\frac{dy}{dx}\right)\cos xy.$

Also Required $\frac{df}{dy}$

$f = \sin xy$

Let $u = xy.$

$\frac{du}{dy} = y\frac{dx}{dy} + x$

$\frac{df}{du} = \cos u$

$\frac{df}{du} \times \frac{du}{dy} = \left(y\frac{dx}{dy} + x\right)\cos u$

$\frac{df}{dy} = \left(y\frac{dx}{dy} + x\right)\cos u$

where $u = xy$

$\therefore \frac{df}{dy} = \left(x + y\frac{dx}{dy}\right)\cos xy.$

10. c) $\sin\left(\frac{\pi}{6}+h\right) = \sin(30+h)$

Then $\sin 31^\circ = \sin(30+1)$

thus $h = 1$.

from.

$$\sin\left(\frac{\pi}{6}+h\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}h - \frac{1}{4}h^2 - \frac{\sqrt{3}}{12}h^3 + \frac{1}{48}h^4$$

but $h = 1$.

$$\sin 31 = \frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{1}{4} + -\frac{\sqrt{3}}{12} + \frac{1}{48}$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{1}{4} - \frac{\sqrt{3}}{12} + \frac{1}{48}$$

$$= 0.992521$$

$$\approx 0.993$$

$\therefore \sin 31^\circ = 0.993$

a) Given

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\frac{x}{2\sqrt{1+y}}\frac{dy}{dx} + \sqrt{1+y} + \frac{y}{2\sqrt{1+x}} + \sqrt{1+x}\frac{dy}{dx} = 0$$

$$\frac{x}{2\sqrt{1+y}}\frac{dy}{dx} + \sqrt{1+x}\frac{dy}{dx} = -\sqrt{1+y} - \frac{y}{2\sqrt{1+x}}$$

$$\frac{dy}{dx}\left(\frac{x\,\cancel{\sqrt{1+x}}}{2\sqrt{1+y}} + \sqrt{1+x}\right) = -\sqrt{1+y} - \frac{y}{2\sqrt{1+x}}$$

Extract 10.2 shows that the candidate applied wrong concepts of differentiation in part (a) and (b), while in part (c) he/she failed to convert degree into radian.

2.2 142/2 - ADVANCED MATHEMATICS 2

2.2.1 Question 1: Complex Numbers

The question had three parts (a), (b) and (c). The candidates were required to: (a) use the De Moivre's theorem to find the value of $\left(\frac{1}{2}+\frac{1}{2}i\right)^{10}$, (b) show that $\left[r(\cos\theta + i\sin\theta)\right]^n = r^n e^{in\theta}$ and to write all complex numbers z in form of $re^{i\theta}$ such that $z^3 = \dfrac{5+i}{2+3i}$ and (c) (i) solve the equation $x^4 + 1 = 0$ and leave the roots in radical form. In part (c) (ii) the candidates were given that $w = \dfrac{z+2}{2}$ and $|z| = 4$, they were required to find the locus of w.

The analysis of the data shows that 10,020 candidates attempted this question, out of which, 41.4 percent of the candidates scored from 3.5 to 10 marks. The analysis has also indicated that 58.6 percent of the candidates scored from 0 to 5 marks, 30.7 percent scored from 5.5 to 8.5 marks and 10.7 percent scored from 9 to 15 marks. Furthermore, 0.2 percent of the candidates scored all the 15 mark while 9 percent of the candidates scored a zero mark. Basing on the data, the question was averagely performed. These statistics are visually shown in figure 11.

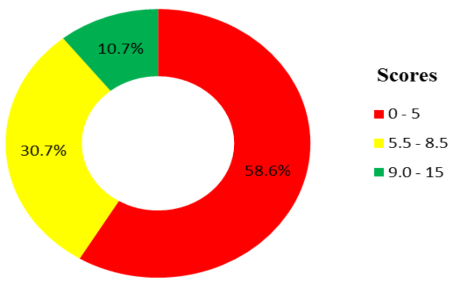


Figure 11: The candidates' performance in question 1.

The analysis of the responses of the candidates shows that a good number of candidates had adequate knowledge of the topic. Those who did well in part (a) were able to find the modulus and principal argument of $\dfrac{1}{2}+\dfrac{1}{2}i$ and correctly applied De Moivres theorem to find the value of $\left(\frac{1}{2}+\frac{1}{2}i\right)^{10}$. In part (b), the candidates were able to show that $\left[r\left(\cos\theta + i\sin\theta\right)\right]^n = r^n e^{in\theta}$ and had the ability to find in form of $re^{i\theta}$ all complex numbers, such that $z^3 = \frac{5+i}{2+3i}$. Moreover, in part (c), the candidates managed to obtain the four roots of $x^4+1=0$ in radical form as $x_1 = \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, $x_2 = -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, $x_3 = -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ and $x_4 = \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$. The candidates had the ability to use the conditions $w = \frac{z+2}{2}$ and the modulus of $z=4$ to obtain the circle $(x-1)^2 + y^2 = 2^2$ centered at (1, 0) with radius 2. Extract 11.1 depicts a response from candidates who did well in this question.

Extract 11.1

11(c) (i) Soln

$x^4 + 1 = 0$

$x^4 = -1$

$x^4 = -1 + 0i$

$x^4 = \cos\pi + i\sin\pi$

$x^4 = \cos(\pi + 2\pi k) + i\sin(\pi + 2\pi k)$

$x = \cos\left(\frac{\pi + 2\pi k}{4}\right) + i\sin\left(\frac{\pi + 2\pi k}{4}\right)$

If $k = 0$

$x = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(1+i)$

If $k = 1$

$x = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

If $k = 2$

$x = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

If $k = 3$

$x = \cos 7\pi/4 + i\sin 7\pi/4 = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

In Extract 11.1, the candidate was able to evaluate the modulus and argument of the complex number and used the formula to get the roots.

However, there were a few candidates who had poor performance in this question. As a result, they failed to find modulus of $\frac{1}{2}+\frac{1}{2}i$ in part (a). For example, they wrote $r=\sqrt{\left(\frac{1}{4}+\frac{1}{4}\right)}=\frac{1}{2}$ instead of $\sqrt{\frac{1}{2}}$ which led to obtaining a wrong value of $\left(\frac{1}{2}+\frac{1}{2}i\right)^{10}$. Moreover, they were unable to get the principal argument of $x^4=-1$, which led to the failure to solve the equation $x^4+1=0$. For example, in solving the equation $x^4+1=0$ they obtained $\tan^{-1}\left(\frac{0}{-1}\right)=0$ instead of π. They also failed to divide complex numbers. For instance, they wrote $\frac{5+i}{2+3i}=\frac{10-3i}{13}$ instead of $\frac{5+3i}{2+3i}=\frac{13-13i}{13}=1-i$. These mistakes indicated that there were few candidates who lacked knowledge and skills on complex numbers (see Extract 11.2).

Extract 11.2

1	(c)(i) $x^4+1=0$
	when $x=1$
	$x^4+1=0$
	$\sqrt[4]{x^4}=\sqrt{-1}$
	$x^4=\pm\sqrt{1}$
	$x=\pm\sqrt{-1}$
	$x=+\sqrt{-1}$ or $-\sqrt{1}$

In Extract 11.2, the candidate failed to solve the equation with complex roots. He/she treated the equation as a normal algebraic equation with real distinct roots or double roots.

2.2.2 Question 2: Logic

The question consisted of three parts (a), (b) and (c). In part (a), the candidates were required to (i) write the contrapositive of the inverse $p \to q$ and (ii) use the truth table to verify that the statement $(p \vee q) \wedge ((\sim p) \wedge (\sim q))$ is a contradiction. In part (b), the question demanded the candidates to (i) use the laws of algebra of propositions to simplify the statement $q \vee (p \wedge \sim q) \vee (r \wedge q)$ and to draw the corresponding simple electrical network and (ii) use the truth table to show that $p \leftrightarrow q$ logically implies $p \to q$. Lastly, in part (c), the candidates were required to prove that the proposition $[(p \to q) \wedge (\sim q)] \to \sim p$ is tautology without using the truth tables.

The analysis of data shows that this question was attempted by 10,540 candidates, out of which, 72.3 percent of the candidates scored above 5 marks. The detail analysis of the data revealed that 27.7 percent of the candidates scored from 0 to 5 marks, 25.5 percent scored from 5.5 to 8.5 and 46.8 percent scored from 9 to 15 marks. Further, 1.5 percent of the candidates scored all the 15 marks while 0.6 percent scored a zero mark. In this case, the general performance of this question was good. Figure 12 summarizes some of the statistics for good performance in this question.

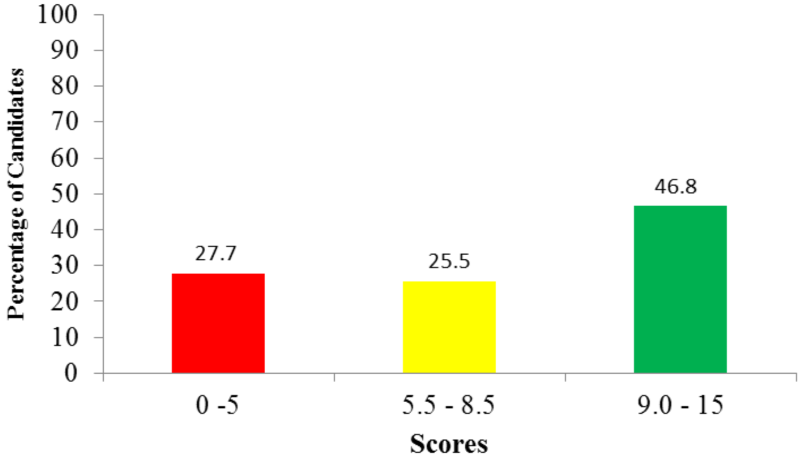


Figure 12: The candidates' performance in question 2.

In the analysis of responses, it was observed that the candidates had adequate knowledge and skills in the topic of logic. In part (a), (i) and (ii), the candidates were able to write the contrapositive of the inverse $p \rightarrow q$ as $q \rightarrow p$ and verified that the statement $(p \vee q) \wedge ((\sim p) \wedge (\sim q))$ is a contradiction using the truth table as follows:

p	q	$\sim p$	$\sim q$	$(p \vee q)$	$(\sim p) \wedge (\sim q)$	$(p \vee q) \wedge ((\sim p) \wedge (\sim q))$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

In part (b) (ii); the candidates' were able to use the truth table to show that $p \leftrightarrow q$ logically implies $p \rightarrow q$. The following table shows the case:

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$(p \leftrightarrow q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	T	T	T

Therefore, the candidates succeeded to use the laws of algebra of proposition to prove that the proposition $[(p \rightarrow q) \wedge (\sim q)] \rightarrow \sim p$ is tautology as it was instructed. Extract 12.1, demonstrates the work of one of the candidates who had solution.

Extract 12.1

(i) $p \rightarrow q$.

its converse   inverse

$= q \rightarrow p$, $\sim p \rightarrow \sim q$.

contrapositive of the inverse.

$q \rightarrow p$.

(ii): $(p \vee q) \wedge [(\sim p) \wedge (\sim q)]$.

let $p \vee q$ be A.

and $\sim p \wedge \sim q$ be B.

$(p \vee q) \wedge [\sim p \wedge \sim q]$ be C.

Truth table:

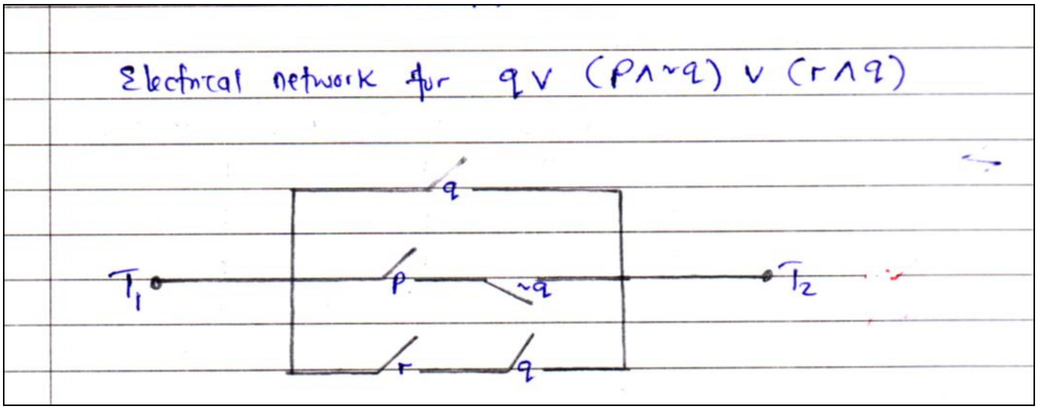
p	q	~p	~q	A	B	C
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

∴ since the last column of the truth table contain of only truth value false F then ~~the~~ the statement is contradiction

Extract 12.1, shows the answer from the candidate who drew the truth table using truth values correctly.

Nevertheless, few candidates scored below 5.5 out of 15 marks because they were unable to use the laws of algebra of proposition to simplify the given statements in (a) (i), (iii) and (c). Moreover, they failed to identify the truth value of $p \rightarrow q$ and $p \leftrightarrow q$ which led them to getting a wrong conclusion in part (b) (ii). Some of them drew the electrical network before simplifying the expression as instructed in the question. As a result, this kind of response affected the candidates' performance and they lost all the marks. Extract 12.2 indicates how the candidates lacked the basic concepts in logic.

Extract 12.2



Extract 12.2 shows that the candidates drew the electrical network without simplifying the given statement as instructed in the question.

2.2.3 Question 3: Vectors

This question comprised of four parts (a), (b), (c) and (d). In part (a) (i), the candidates were given that $\underset{\sim}{a} = 3\underset{\sim}{i} - 5\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 7\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ are non-zero vectors. Next, they were required to find the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. In (a) (ii), they were instructed to use vectors to prove the sine rule. Part (b) required the candidates to show that $\frac{1}{2}\left|\underset{\sim}{a} + \underset{\sim}{b}\right| = \cos\left(\frac{\theta}{2}\right)$, where θ is the angle between two unit vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. In part (c), the candidates were supposed to (i) find $\frac{d}{dt}\left[(\sin t)G(t)\right]$ if $G(t) = e^t\underset{\sim}{i} + \cos t\underset{\sim}{j} + t\underset{\sim}{k}$ and (ii) integrate the vector $e^t\underset{\sim}{i} + 2t\underset{\sim}{j} + \ln t\underset{\sim}{k}$ with respect to t. Finally, in part (d) the candidates were informed that two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ have the same magnitudes, an angle between them is 60° and their scalar product is $\frac{1}{2}$. From this information, they were required to find the magnitudes of these vectors.

The question was answered by 95.4 percent of the candidates, of which, 59.6 percent of the candidates scored above 5 out of 15 marks, 40.4 percent of the candidates scored from 0 to 5 marks, 34.5 percent scored from 5.5 to 8.5 marks and 25.1 percent scored from 9 to 15 marks. Further analysis of the data indicated that 1.3 percent of the candidates scored all the 15 marks while 3.6 percent scored a zero mark. The good performance in this question is shown in Figure 13.

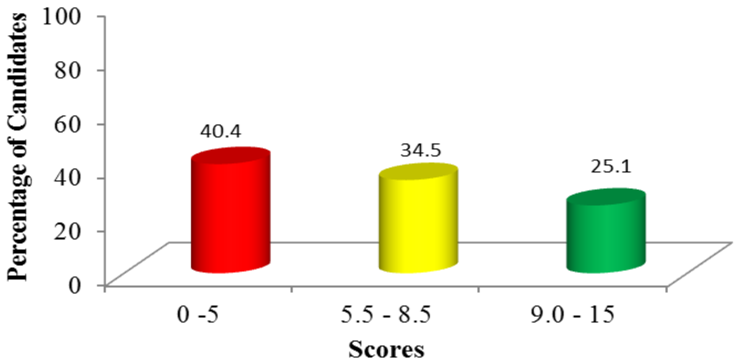


Figure 13: The candidates' performance in question 3.

The responses for those who performed well show that they were able to find the projection of a vector **a** onto vector **b** by using the formula $\text{Proj}_{\underline{b}}\ \underline{a} = \frac{\underline{a}.\underline{b}}{|\underline{b}|}$. Further, they managed to prove the sine rule using vector method with the aid of the diagram. In addition, they applied dot product of two vectors to show that $\frac{1}{2}|\underline{a}+\underline{b}| = \cos\left(\frac{\theta}{2}\right)$. In part (c), they showed their ability to use the concept of differentiation of the exponential and trigonometric function to find $\frac{d}{dt}\left[(\sin t)G(t)\right]$. This proved that they had sufficient knowledge and skills on vector differentiation. It was also realized that, candidates were able to integrate the given vector function and got $e^t\underline{i} + t^2\underline{j} + (t\ln t - t)\underline{k} + c$ as required in part (c) (ii). Finally, in part (d), the candidates used the given angle (60^0) and scalar product ($\frac{1}{2}$) of vectors $\underline{a}$ and $\underline{b}$ to obtain 1 unit which is the magnitude of both vectors successfully. A vivid example from one of the candidate who answered this question according to the instruction is shown by Extract 13.1.

Extract 13.1.

3(b) $|\underline{a}+\underline{b}|^2 = |\underline{a}|^2 + |\underline{b}|^2 + 2|\underline{a}||\underline{b}|\cos\theta$

For unit vectors

$|\underline{a}| = |\underline{b}| = 1$.

$|\underline{a}+\underline{b}|^2 = 1 + 1 + 2\cos\theta$

$|\underline{a}+\underline{b}|^2 = 2 + 2\cos\theta$

But $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$|\underline{a}+\underline{b}|^2 = 2 + 2\cos^2\frac{\theta}{2} - 2\sin^2\frac{\theta}{2}$

$|\underline{a}+\underline{b}|^2 = 2 - 2\sin^2\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}$

$|\underline{a}+\underline{b}|^2 = 2\left(1-\sin^2\frac{\theta}{2}\right) + 2\cos^2\frac{\theta}{2}$

But $\cos^2\frac{\theta}{2} = 1 - \sin^2\frac{\theta}{2}$.

$|\underline{a}+\underline{b}|^2 = 2\cos^2\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}$

$|\underline{a}+\underline{b}|^2 = 4\cos^2\frac{\theta}{2}$

$|\underline{a}+\underline{b}| = 2\cos\frac{\theta}{2}$

$\frac{1}{2}|\underline{a}+\underline{b}| = \cos\left(\frac{\theta}{2}\right)$

$\therefore$ Hence shown

Extract 13.1 shows that the candidate proved the given equation by using the dot product and trigonometric identities.

On the other hand, the candidates who scored less than 5 out of 15 marks lacked the knowledge of dot product, competence in differentiation and integration. As a result, they scored low marks. The analysis of the candidates' responses revealed that they had difficulties in finding the projection of a vector $\underline{a}$ onto vector $\underline{b}$. For instance, one candidate wrote projection of **a** onto **b** = $\frac{|a||b|}{a}$ that resulted in getting -11.32 instead of 2.72 units. Others failed to derive the sine rule by using vectors. Nevertheless, they managed to score some marks in other parts of the question. Moreover, in part (c) (i) and (ii) some candidates got the marks because they were able to perform initial steps $\frac{d}{dt}[(\sin t)G(t)] = (\sin t)G'(t) + \frac{d}{dt}(\sin t)G(t)$ and

$\int\left(e^t \underline{i} + 2t\underline{j} + \ln t\underline{k}\right)dt = \int e^t \underline{i}dt + 2\int t\underline{j}dt + \int \ln t\underline{k}dt$ respectively. Finally, the candidates ended with wrong answers such $e^t i\cos t + \sin tk + \cos 2tj + tk\cos t$ and $e^t i + t^2 j + \frac{t^2}{2}\ln k + c$ respectively. Extract 13.2 is a sample of one of the worst solution from one of the candidates.

Extract 13.2

3(a) i) given

$\underline{a} = 3i - 5j - 2k$

$\underline{b} = 7i + j - 2k$

Projection of $\underline{a}$ onto $\underline{b}$ is given by

$\text{proj}_{\underline{b}}\underline{a} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|}$

$|\underline{b}| = \sqrt{(7)^2 + (1)^2 + (-2)^2}$

$|\underline{b}| = \sqrt{49 + 1 + 4}$

$|\underline{b}| = \sqrt{54}$

$\underline{a}\cdot\underline{b} = \begin{pmatrix}3\\-5\\-2\end{pmatrix}\cdot\begin{pmatrix}7\\1\\-2\end{pmatrix}$

$\underline{a}\cdot\underline{b} = 21 - 5 + 4$

$\underline{a}\cdot\underline{b} = 20$ units

$\therefore \text{Proj}_{\underline{b}}\underline{a} = 20$ units

Hence projection of $\underline{a}$ onto $\underline{b}$ is 20 units

b) Angle $\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|\,|\underline{b}|}$

but $\theta = |\underline{a} + \underline{b}|$

$\cos(\underline{a} + \underline{b}| = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|\,|\underline{b}|}$

$\underline{a} + \underline{b} = \cos^{-1} \frac{a \cdot b}{|\underline{a}||\underline{b}|}$

let $\theta = a \cos^{-1}\left(\frac{\underline{a} \cdot \underline{b}}{|\underline{a}|\,|\underline{b}|}\right)$

$\underline{a} + \underline{b} = \cos\theta$ divede by 2

$\frac{1}{2}(\underline{a} + \underline{b}| = \cos(\theta/_2)$

Extract 13.2, indicates a sample answer from the candidate who started with the correct steps in part (a) but ended with a wrong answer. The candidate had the incorrect steps going by how he/she struggled to show that $\frac{1}{2}|\underline{a} + \underline{b}| = \cos\left(\frac{\theta}{2}\right)$ in part (b).

2.2.4 Question 4: Algebra

The question had three parts (a), (b) and (c). In part (a), the candidates were required to (i) solve the equation $\log_3 x - 3 + \log_x 9 = 0$ and (ii) find the quadratic equation giving the two actual possible values of k, if the equations $x^2 + 9x + 2 = 0$ and $x^2 + kx + 5 = 0$ have a common root. In part (b), the candidates were supposed to find the sum of the series; $\frac{5}{1\times2\times3} + \frac{8}{2\times3\times4} + \frac{11}{3\times4\times5} + \ldots + \frac{3n+2}{n(n+1)(n+2)}$, hence find $\sum_{r=1}^{\infty} \frac{3r+2}{r(r+1)(r+2)}$.

Further, in part (c), the candidates were given the matrices $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 5 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 3 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ and were required to find the value of $A^{-1}B$.

This question was attempted by 96.3 percent of the candidates, out of which, 71.1 percent scored below 5.5 marks. The analysis of the data shows that 20.5 percent of the candidates scored from 5.5 to 8.5 marks, 8.4 percent scored from 9 to 15 marks. It was also noted that 0.3 percent of the candidates scored all the 15 marks, while, 7.8 percent scored a zero mark. Figure 14 exhibits poor performance in the question.

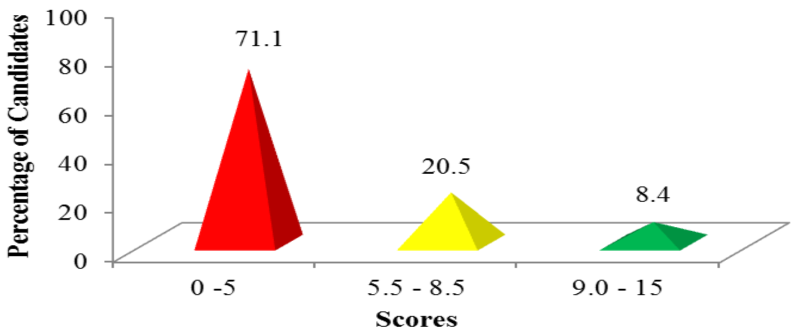


Figure 14: The candidates' performance in question 4.

The poor performance in this question was caused by the misinterpretation of the given logarithmic equation. It was observed that many candidates considered $\log_3 x - 3 + \log_x 9 = 0$ as $\log_3(x-3) + \log_x 9 = 0$. Additionally, the candidates had difficulties in finding the quadratic equation and determining the possible value of k in (a) (ii). For instance, some of them assumed that a letter z was a root for both equations yet they substituted it in $x^2 + 9x + 2 = 0$ and thus got $z^2 + 9z + 2 = 0$. Next, they solved the equation to obtain $z_{1,2} = \frac{-9 \pm \sqrt{73}}{2}$ hence incorrectly concluded that $z_{1,2} = \frac{-9 \pm \sqrt{73}}{2}$ are the possible values of k.

Likewise, in part (b), they were proving the given nth term $\frac{3n+2}{n(n+1)(n+2)}$ of the series using arithmetical progression instead of using the technique of partial fractions of which they get $\frac{3n+2}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2}$.

Lastly, in part (c) the candidates failed to find the minors and cofactors of the given matrix A, which led them to getting incorrect A^{-1} and $A^{-1}B$. For instance, they computed $A_{21} = -2$ instead of $A_{21} = -1$. This shows that they lacked knowledge and skills in algebra. Extract 14.1 is a sample answer from a candidate who poorly answered part (a) (i).

Extract 14.1

4 (a) (i) Soln

$\log_3 x - 3 + \log_x 9 = 0$

[illegible]

$\log_3 x - 3 + 3\log_x 3 = 0$

$\log_3 3(x-3) = 0$

$3x - 3 = 0$

$3x = 3$ $\quad 3x = 3/3$

$x = 1$

$x = 1$

$x = 1$

Extract 14.1 represents the worst solution from the candidate who failed to solve logarithmic equation because of the lack in knowledge of logarithmic laws.

On the other hand, there were a few candidates (7 %) who performed well. They applied the laws of logarithms to solve the given logarithmic equation and got the values of $x = 3$ and $x = 9$ as required in part (a) (i). Moreover, they were capable of distinguishing between common roots from equal roots. In this case, they correctly formed the quadratic equation of k and solved for k as $k = 22.158$ or 9.342. In part (b), they were able to write $\frac{3n+2}{n(n+1)(n+2)}$ in partial fractions and as a sum of series, whereby, they found that the value of s_{∞} is approximately equal to 2.

In part (c), they succeeded in finding the minors of matrix $\boldsymbol{A}$ which were the necessary step to obtain the A^{-1}. Hence, they evaluated $A^{-1}B$. Extract 14.2 shows a sample answer from a candidate who correctly answered part (a) (i).

Extract 14.2

4 (a) (i) required to solve equation

$$\log_3 x - 3 + \log_x 9 = 0$$

then

$$\log_3 x + \log_x 9 = 3.$$

$$\frac{\log x}{\log 3} + \frac{\log 9}{\log x} = 3.$$

$$\frac{\log x}{\log 3} + \frac{2\log 3.}{\log x} = 3.$$

$$\frac{\log x . \log x + \log 3 . 2\log 3}{\log 3 . \log x} = 3.$$

$$(\text{Log}\, x)^2 + 2(\log 3)^2 = 3\log x \log 3.$$

$$(\text{Log}\, x)^2 - 3\log x \log 3 + 2(\log 3)^2 = 0$$

$$(\text{Log}\, x)^2 - (3\log 3)\log x + 2(\log 3)^2 = 0$$

Let $\log x = y.$

$$y^2 - 3\log 3\, y + 2(\log 3)^2.$$

on solving we get

$$y = \frac{3\log 3 \pm \sqrt{(3\log 3)^2 - 4(2(\log 3)^2)}}{2}$$

$$y = \frac{3\log 3 \pm \sqrt{9(\log 3)^2 - 8(\log 3)^2}}{2}$$

$$y = \frac{3\log 3 \pm \sqrt{(\text{Log}\, 3)^2}}{2}$$

$$y = \frac{3\log 3 \pm \log 3}{2.}$$

4(a)(i) then $y = \frac{3\log 3 \pm \log 3}{2}$

for +ve

$y = \frac{3\log 3 + \log 3}{2} = 2\log 3.$

for −ve

$y = 3\log 3 - \log 3$

$y = \log 3^2$

but $y = \log x$.

$y = \log x = 2\log 3$ or $y = \log x = \log 3$.

$\log x = \log 3^2$ or $y = \log x = \log 3$

$x = 9$ $x = 3$.

$\therefore$ the value of x is 3 or 9.

In Extract 14.2, the candidate showed the important steps to solve the logarithmic equation and successfully arrived to the final correct answer.

2.2.5 Question 5: Trigonometry

This question had four parts (a), (b), (c) and (d). In part (a), the candidates were required to (i) use trigonometric identities to prove that $16\sin^5\theta - 21\sin^3\theta + 5\sin\theta = \sin 5\theta$ and (ii) eliminate θ from the equations $x\sec\theta + y\tan\theta = 3$ and $x\tan\theta + y\sec\theta = 2$. In part (b) (i), the candidates were given $\tan\theta = \frac{4}{3}$ and $0° \le \theta \le 360°$ and were required to find without using tables the value of $\tan\left(\frac{1}{2}\theta\right)$. In part (b) (ii), the candidates were required to show that $\frac{\cos 3x - \cos 5x}{4\sin 2x\cos 2x} = \sin x$. In part (c), they were required to verify that $A + B + C = ABC$ when $\tan^{-1}A + \tan^{-1}B + \tan^{-1}C = \pi$. Finally, part (d) required the candidates to express the sum of $\sec x$ and $\tan x$ as the tangent of $\left(\frac{\pi}{4} + \frac{x}{2}\right)$ and to evaluate $\tan\frac{\pi}{12}$ giving the answer in surd form.

The analysis of the data shows that only 56.7 percent of the candidates responded to this question. It was further observed that only 32.1 percent of the candidates scored from 7 marks and above. Moreover, the data show that 67.9 percent of the candidates scored from 0 to 6.5 marks, 23.9 percent scored from 7 to 11.5 marks and only 8.2 percent scored from 12 to 20 marks. In general this question was poorly performed as illustrated in figure 15.

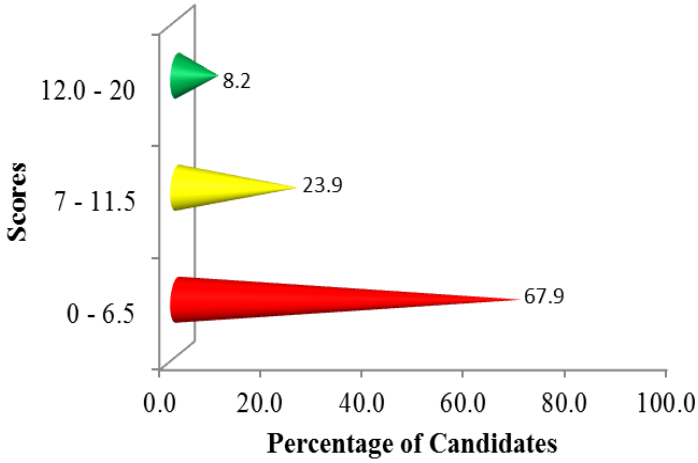


Figure 15: The candidates' performance in question 5.

In part (a) (i), the candidates experienced difficulties in using the trigonometric identities to prove the equation $16\sin^5\theta - 21\sin^3\theta + 5\sin\theta = \sin 5\theta$. Some of them wrote $\sin 5\theta = \sin 2\theta\cos 3\theta + \cos 2\theta\sin 3\theta$, but failed to simplify and rearrange $\sin 2\theta\cos 3\theta + \cos 2\theta\sin 3\theta$ to get the final expression $16\sin^5\theta - 21\sin^3\theta + 5\sin\theta$. Moreover, in (b) (i) some candidates did not use half angle formula of tangent to find $\tan\left(\frac{1}{2}\theta\right)$. Instead, they got $\theta = \tan^{-1}\left(\frac{4}{3}\right)$ which was incorrect step. Moreover, they lacked the knowledge of factor formulae in (b) (ii), as they wrote $\cos 3x - \cos 5x = \sin(3x - 5x)$. In part (c), the candidates were unable to verify the given equation due to insufficient knowledge on how to use the compound angle formulae for sine and cosine to deduce $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ as a basic concept to verify that $A + B + C = ABC$ when $\tan^{-1}A + \tan^{-1}B + \tan^{-1}C = \pi$. Furthermore, in part (d), the candidates failed to express the sum of $\sec x$ and $\tan x$ as the tangent of $\left(\frac{\pi}{4} + \frac{x}{2}\right)$: many of them did not substitute

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos x = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$ which were the important step to end up with the correct answer. Extract 15.1 illustrates a wrong answer in part (a) (i) from one candidate who lacked knowledge of trigonometry.

Extract 15.1

5. (a) (i) $16\sin^5\theta - 21\sin^3\theta + 5\sin\theta = \sin 5\theta$.

Consider:

LHS:

$16\sin^5\theta - 21\sin^3\theta + 5\sin\theta$.

$16\sin^5\theta + 5\sin\theta - 21\sin^3\theta$.

$\sin\theta\left(16\sin^4\theta + 5\right) - 21\sin^3\theta$.

$\sin\theta\left(16\sin^2\theta(\sin^2\theta) + 5\right) - 21\sin^2\theta(\sin\theta)$

Divide by $\sin\theta$ both each:

$\frac{\cancel{\sin\theta}\left(16\sin^2\theta(\sin^2\theta) + 5\right)}{\cancel{\sin\theta}} - \frac{21\sin^2\theta\cancel{\sin\theta}}{\cancel{\sin\theta}}$

$16\sin^2\theta(\sin^2\theta) + 5 - 21\sin^2\theta$

$\cancel{16\sin^2}$

$16\sin^2\theta(\sin^2\theta) - 21\sin^2\theta + 5$.

$\sin^2\theta\left(16\sin^2\theta - 21 + 5\right)$.

$\sin(2\theta + 2\theta + \theta)\left[16 - 21 + 5\right]$

$\sin 5\theta$. hence proved.

Extract 15.1 shows that the candidate completely lacked knowledge and skills in compound angle formulae for sine and cosine.

However, a few candidates answered this question correctly. In part (a) (i), those candidates were able to use compound angle formulae and trigonometric identities for sine and cosine to prove the given equation. For instance, $\sin 2\theta$ was substituted by $2\sin\theta\cos\theta$, $\cos 2\theta$ by $1 - 2\sin^2\theta$ and $\sin 3\theta$ was substituted by $\sin\theta\cos 2\theta + \cos\theta\sin 2\theta$. In part (a) (ii), those candidates eliminated θ from the given equations using the identity $\sec^2\theta - \tan^2\theta = 1$ to obtain $x^2 - y^2 = 5$.

Additionally, in (b) (i), they were able to find the value of $\tan\left(\frac{1}{2}\theta\right)$ using half angle formula for tangent of an angle.

Furthermore, they used the factor formulae and properties of even or odd function to show that the Left Hand Side (LHS) and the Right Hand Side (RHS) of the given equation are equal. In part (c), the candidates were able to verify that $A + B + C = ABC$ using the given information of the compound angle formulae for tangent of an angle. In part (d), they obtained the sum of $\sec x$ and $\tan x$ as instructed and were able to obtain the value of $\tan\frac{\pi}{12}$ as $2-\sqrt{3}$. Extract 15.2 is a sample solution from a candidates who answered part (b) (i) correctly.

Extract 15.2

5. (b) (i) Given:

$\tan \alpha = 4/3$

$0^\circ \le \alpha \le 360^\circ$

Then:

$\tan(\frac{1}{2}\alpha)$

from:

$\tan\alpha = \tan(\alpha/2 + \alpha/2)$

$\tan\alpha = \dfrac{\tan\alpha/2 + \tan\alpha/2}{1 - \tan^2\alpha/2}$

$\tan\alpha = \dfrac{2\tan\alpha/2}{1-\tan^2\alpha/2}$

Then:

$\dfrac{4}{3} = \dfrac{2\tan\alpha/2}{1-\tan^2\alpha/2}$

$4(1-\tan^2\alpha/2) = 6\tan\alpha/2$

$4 - 4\tan^2\alpha/2 = 6\tan\alpha/2$

$4\tan^2\alpha/2 + 6\tan\alpha/2 - 4 = 0$

Quadratic in $\tan\alpha/2$.

$\tan\alpha/2 = \dfrac{-6 \pm \sqrt{36-4(4)(-4)}}{2(4)}$

$\tan\alpha/2 = \dfrac{-6 \pm 10}{8}$

$\tan\alpha/2 = \frac{1}{2}$ or -2

In Extract 15.2 the candidate correctly evaluated the value of $\tan\left(\frac{1}{2}\theta\right)$ without using tables.

2.2.6 Question 6: Probability

The question consisted of four parts (a), (b), (c) and (d). In part (a), the candidates were required to define the terms (i) Continuous random variable, (ii) discrete random variable, (iii) probability density function and to write one example for each term. In part (b) (i), the candidates were asked to find the number of ways in which a team of 5 members can be selected from a group of students consisting of 4 girls and 7 boys if the team has at least a boy and a girl. In part (b) (ii), the candidates were given $P(A)=\frac{1}{4}$, $P(A/B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{3}$ and were required to verify whether A and B are independent or mutually exclusive events. In part (c), the candidates were given information that Rehema and Seni play a game in which Rehema should win 8 games for every 7 games won by Seni; then, they were required to prove that if they played three games, the probability that Rehema won at least two games was approximately to 0.55. Finally, in part (d), the candidates were given information that in a family, the boy tells a lie in 30 percent cases and the girl in 35 percent cases; then, they were required to evaluate the probability that both contradict each other on the same fact.

The analysis of the data shows that only 12.9 percent of the candidates attempted this question out of which only 12.4 percent of the candidates passed. The analysis revealed that 87.6 percent of the candidates scored from 0 to 6.5 marks, 9.2 percent scored from 7 to 11.5 marks and only 3.2 percent scored from 12 to 20 marks. The data also indicate that none of the candidates scored all the 20 marks, 31.7 percent scored a zero mark. Therefore, the question was poorly done as shown in Figure 16.

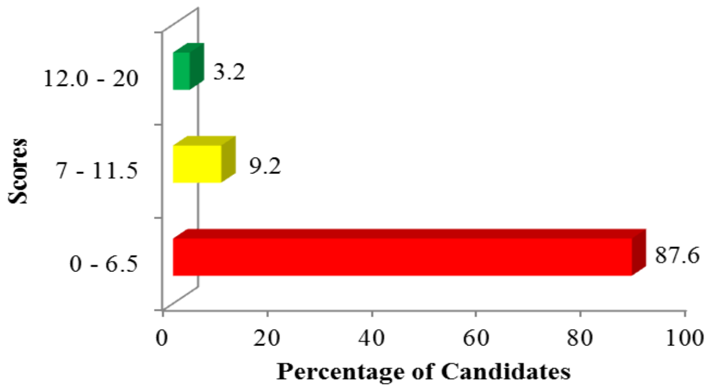


Figure 16: The candidates' performance in question 6.

The analysis shows that the candidates lacked knowledge and skills in the topic of probability. In part (a) (i), they were neither able to define the three terms nor able to write relevant examples on each term. In part (b) (i), the candidates were unable to apply the combination formula in order to find the number of ways a team of 5 members would be selected. For example, some candidates wrote $\frac{11!}{5!}$ whereas others wrote ${}^{7}c_{5} = 21$ of which all were incorrect. The correct answer was supposed to be 441 different ways obtained by $\left({}^{7}C_{4}\right)\left({}^{4}C_{1}\right)+\left({}^{7}C_{3}\right)\left({}^{4}C_{2}\right)+\left({}^{7}C_{2}\right)\left({}^{4}C_{3}\right)+\left({}^{7}C_{1}\right)\left({}^{4}C_{4}\right)$.

In part (b) (ii), they failed to distinguish between the two formulas; $P(A\cap B)=P(A)\times P(B)$ for independent events and $P(A\cup B)=P(A)+P(B)$ when $P(A\cap B)=0$ for mutually exclusive events which were necessary in answering this part. The lack of knowledge on tree diagram method affected the candidates' performance in part (c). For instance, the candidates who were able to get the probability of Rehema winning the game wrote $P(R)=\frac{8}{8+7}=\frac{8}{15}$, but they were unable to find the probability of Seni which was supposed to be

$P(S)=\dfrac{7}{7+8}=\dfrac{7}{15}$. Extract 16.1 is a sample answer from one of the candidates who failed part (a) (i).

Extract 16.1

> 6. (a i) continaus random variable
> → Is the variable in which the element of either A or B are incluse in the event.
>
> ii) Descrete random variable
> - Refer to the variable ~~which~~ in a given set either A and B and A and B are independent variable.
>
> (iii) Probability density function:
> Is the probability that including the function on the number of events and sample space.

In Extract 16.1, the candidate failed to define and provide one example for each of the given terms in part (a) (i).

Despite the fact that many candidates performed poorly, a few demonstrated knowledge and skills in solving a probability problem. For example, in part (a), they were able to define and give relevant examples on each of the given terms. For example, one of the candidates defined the discrete random variable as a variable which cannot be expressed in decimal or fraction; giving an example of the number of patients in a certain hospital. The candidates attempted (b) (i) correctly by using combination to find a number of ways of selecting 5 members of a team. Further, they verified that A and B are neither independent nor mutually exclusive events by using the proper formulae. They found $P(B)=\dfrac{1}{3}$ and $P(A\cap B)=\dfrac{1}{6}$ which enabled them to get the correct answer in (b) (ii). Finally, they found probability of Rehema winning at least two games, by first calculating the $P(R)=\dfrac{8}{15}$ and $P(S)=\dfrac{7}{15}$; where p(R) is for probability of Rehema and p(S) is probability of Seni. The final solution was obtained by using the following procedure; $\frac{8}{15}\times\frac{8}{15}\times\frac{8}{15}+\frac{8}{15}\times\frac{8}{15}\times\frac{7}{15}+\frac{8}{15}\times\frac{7}{15}\times\frac{8}{15}+\frac{7}{15}\times\frac{8}{15}\times\frac{8}{15}\cong 0.55$. In part (d), they calculated the Probability of both contradict each other by using a tree diagram and obtained 0.44. Extract 16.2, shows a sample response from one of the candidates who had a correct solution.

Extract 16.2

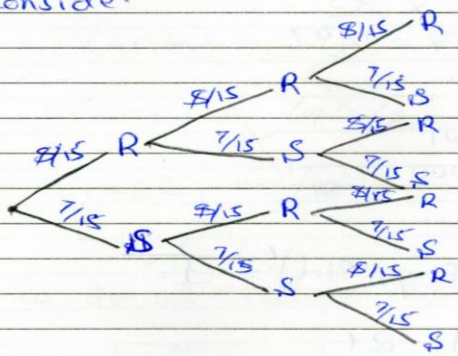
6. c) Soln.

$n(S) = 8+7 = 15$. games.

$P(\text{Rehema}) = 8/15$

$P(\text{Seni}) = 7/15$.

Conside.



$n(e) = \{RRR, RRS, RSR, SRR,\}$

$= \left(\frac{8}{15}\times\frac{8}{15}\times\frac{8}{15}\right)+\left(\frac{8}{15}\times\frac{8}{15}\times\frac{7}{15}\right)+\left(\frac{8}{15}\times\frac{7}{15}\times\frac{8}{15}\right)+\left(\frac{7}{15}\times\frac{8}{15}\times\frac{8}{15}\right)$

$\frac{512}{3375}+\frac{448}{3375}+\frac{448}{3375}+\frac{448}{3375}$

$= 0.549925$

$\simeq 0.55$

Hence, proved.

In Extract 16.2, the candidate was able to use a tree diagram in answering part (c) correctly.

2.2.7 Question 7: Differential Equations

The question had parts (a), (b), (c) and (d). In part (a), the candidates were required to (i) solve the differential equation $\frac{r\tan\theta}{a^2-r^2}\frac{dr}{d\theta}=1$ with the initial condition $r=0$ and $\theta=\frac{\pi}{4}$ and (ii) verify that $y=10\sin 3x+9\cos 3x$ is a solution of the differential equation $\frac{d^2y}{dx^2}+9y=0$. In part (b), the candidates were provided with the information, "The population of a certain country doubles in 15 years. If the rate of increase is proportional to the number of inhabitants, the candidates were required to find the numbers of years that the population would be six times". Part (c) required the candidates to find the particular solution of the differential equation $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}+3\frac{dy}{dx}+2y=\cos x$. Finally, part (d) required the candidates to form a differential equation which has a general solution $\mathrm{y}=\mathrm{Ae}^{\mathrm{mx}}+\mathrm{Be}^{-\mathrm{mx}}$ where A, B and m are constants.

This question was attempted by 6,787 (64%) candidates, out of which, 79.7 percent of the candidates passed. Further analysis indicated that 20.3 percent of the candidates scored from 0 to 6.5 marks, 26.2 percent scored from 7 to 11.5 marks and 53.5 percent scored from 12 to 20 marks. Moreover, the analysis revealed that 276 (4.1%) candidates scored all the 20 marks and 1.9 percent of the candidates scored a zero mark. Generally, this question was well performed as summarized in Figure17.

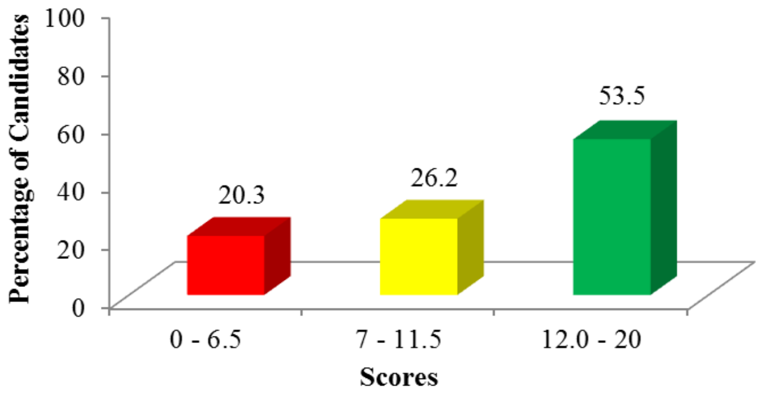


Figure 17: The candidates' performance in question 7.

The good performance in this question was due to the fact that candidates had adequate knowledge and skills in the topic of differential equations. In part (a) (i), candidates were able to solve the first order separable differential equations and used the condition to get the particular solution. They used the substitution technique to integrate $\int \frac{1}{\tan\theta} d\theta$ and obtained $\ln|\sin\theta| + c$. In part (a) (ii), the candidates had skills and knowledge of eliminating the cosine and sine terms using differentiation to arrive to the required solution. For instance, they differentiated $y = 10\sin 3x + 9\cos 3x$ to obtain the derivatives $y' = 30\cos 3x - 27\sin 3x$ and $y'' = -90\sin 3x - 81\cos 3x$, which were required for the substitution and verify the problem.

Likewise, for part (b), they were able to form the first order separable differential equation $\frac{dp}{p} = kdt$, of which after integration, they got the required answer. Furthermore, in part (c), they firstly calculated the auxiliary quadratic equation $\lambda^2 + 3\lambda + 2 = 0$ which was the necessary step to get the complementary solution $y_c = Ae^{-x} + Be^{-2x}$. Then, they assigned the particular integral as $y_p = \frac{3}{10}\sin x + \frac{1}{10}\cos x$ and finally combined the complementary and particular integral to form the general solution.

In part (d), the candidates differentiated the equation $y = Ae^{mx} + Be^{-mx}$ to get the first and second derivatives; and successfully eliminated the constants A and B to arrive to the required answer. Extract 17.1, illustrates a sample of the work from one of the candidates for part (c).

Extract 17.1

7 c $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = \cos x$

$y_{C.F} = m^2 + 3m + 2 = 0$

$m_1 = -1$

$m_2 = -2$

$y_{C.F} = Ae^{m_1x} + Be^{m_2x}$

$y_{C.F} = Ae^{-x} + Be^{-2x}$

$y_{P.I} = A\cos x + B\sin x$

$y' = -A\sin x + B\cos x$

$y'' = -A\cos x - B\sin x$

$-A\cos x - B\sin x + 3(-A\sin x + B\cos x) + 2(A\cos x + B\sin x) = \cos x$

$-A\cos x - B\sin x - 3A\sin x + 3B\cos x + 2A\cos x + 2B\sin x = \cos x$

$-A + 3B + 2A = 1.$

$-B - 3A + 2B = 0$

$A + 3B = 1.$

$-3A + B = 0$

$A = 0.1$

$B = 0.3$

$y_{P.I} = \frac{1}{10}\cos x + \frac{3}{10}\sin x$

$y = y_{C.F} + y_{P.I}$

$y = Ae^{-x} + Be^{-2x} + \frac{1}{10}\cos x + \frac{3}{10}\sin x$

Extract 17.1, shows how one of the candidates' solved the differential equation in part (c) correctly.

Despite the good performance, there were few candidates who performed poorly. The analysis of the candidates' responses shows that they lacked knowledge and skills in differentiation and integration techniques. For instance, in part (a) (i) they wrote $\int \frac{r}{a^2 - r^2} dr = -2\ln(a^2 - r^2)$ instead of $-\frac{1}{2}\ln\left|\left(a^2 - r^2\right)\right|$ hence ended with incorrect solution in part (a)(i). Moreover, they failed to differentiate $\sin 3x$ and $\cos 3x$ by writing $\frac{d}{dx}(\sin 3x) = -3\cos 3x$ and $\frac{d}{dx}(\cos 3x) = 3\sin 3x$ which were wrong. They therefore failed to verify that the given equation was a solution for the differential equation as demanded in part (a) (ii). In part (b), many candidates did not manage to transform the given information into a relevant equation which they can manipulate to obtain $p = p_o e^{kt}$.

In part (c), the candidates failed to define the particular integral and hence ended up with a wrong general solution as illustrated in Extract 17.2. In part (d), the candidates lacked knowledge and skills for formulating the differential equation. As a result, they failed to obtain the first and second derivatives of $y = Ae^{mx} + Be^{-mx}$. Further, the lack of that knowledge prevented them from eliminating the constants A and B; hence, they ended up with incorrect answer. For example, there were candidates who wrote $\frac{d}{dx}\left(e^{mx}\right) = e^{mx}$ instead of $\frac{d}{dx}\left(e^{mx}\right) = me^{mx}$.

Extract 17.2

$y_p = A\cos x$

$\frac{dy}{dx} = -A\sin x$

$\frac{d^2y}{dx^2} = -A\cos x$.

$-A\cos x + 3(-A\sin x) + 2(A\cos x) = \cos x$

$-A\cos x - 3A\sin x + 2A\cos x = \cos x$

$(-A+2A)\cos x = \cos x$.

$-A + 2A = 1$.

$A = 1$.

$y_p = \cos x$.

$\therefore$ $y = y_p + y_c$

$y = Ae^{-x} + Be^{-2x} + \cos x$

In Extract 17.2, a candidate defined a particular integral wrongly as $y_p = A\cos x$ instead of $y_p = A\cos x + B\sin x$.

2.2.8 Question 8: Coordinate Geometry II

The question had four parts (a), (b), (c) and (d). In part (a) (i), the candidates were supposed to find the Cartesian equation of the ellipse having foci at the points $(-1, 0)$ and $(7, 0)$ when the eccentricity is $\frac{1}{2}$. In part (a) (ii), the candidates were required to convert $y^2 = 4a(a-x)$ into a polar equation. In part (iii), they were required to use the equation $y = 2x^2 - 6x + 4$ to determine the directrix and the focus. In part (b), the candidates were provided with the information: "A cable used to support a swinging bridge approximates the shape of a parabola. If the length of the bridge is 100m and the vertical distance from where the cable is attached to the bridge to the lowest point of the cable is 20". Next, the candidates were required to determine the equation of the parabola. Part (c) required the candidates to (i) define the term hyperbola and (ii) to show that the latus rectum of the equation $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ is $\frac{2b^2}{a}$. Finally, part (d) required the candidates to sketch the graph of $r=2+4\cos t$.

The analysis of the data shows that 6,134 (57.8%) candidates answered this question, out of which, 33.5 percent of the candidates scored above 6.5 marks. Further analysis revealed that 66.5 percent scored from 0 to 6.5 marks, 28.3 percent scored from 7 to 11.5 marks and 5.2 percent scored from 12 to 20 marks. The analysis has also indicated that 0.1 percent of the candidates scored all the 20 marks, while 3.8 percent scored a zero. Therefore, the general performance in the question was poor. Figure 18 displays the performance in the question.

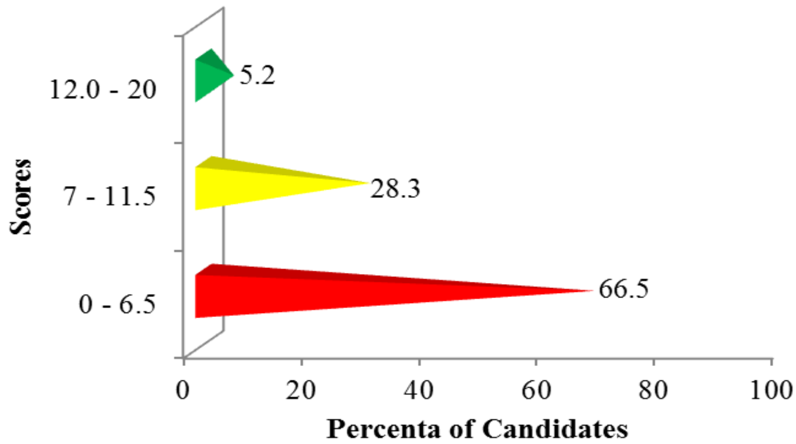


Figure 18: The candidates' performance in question 8.

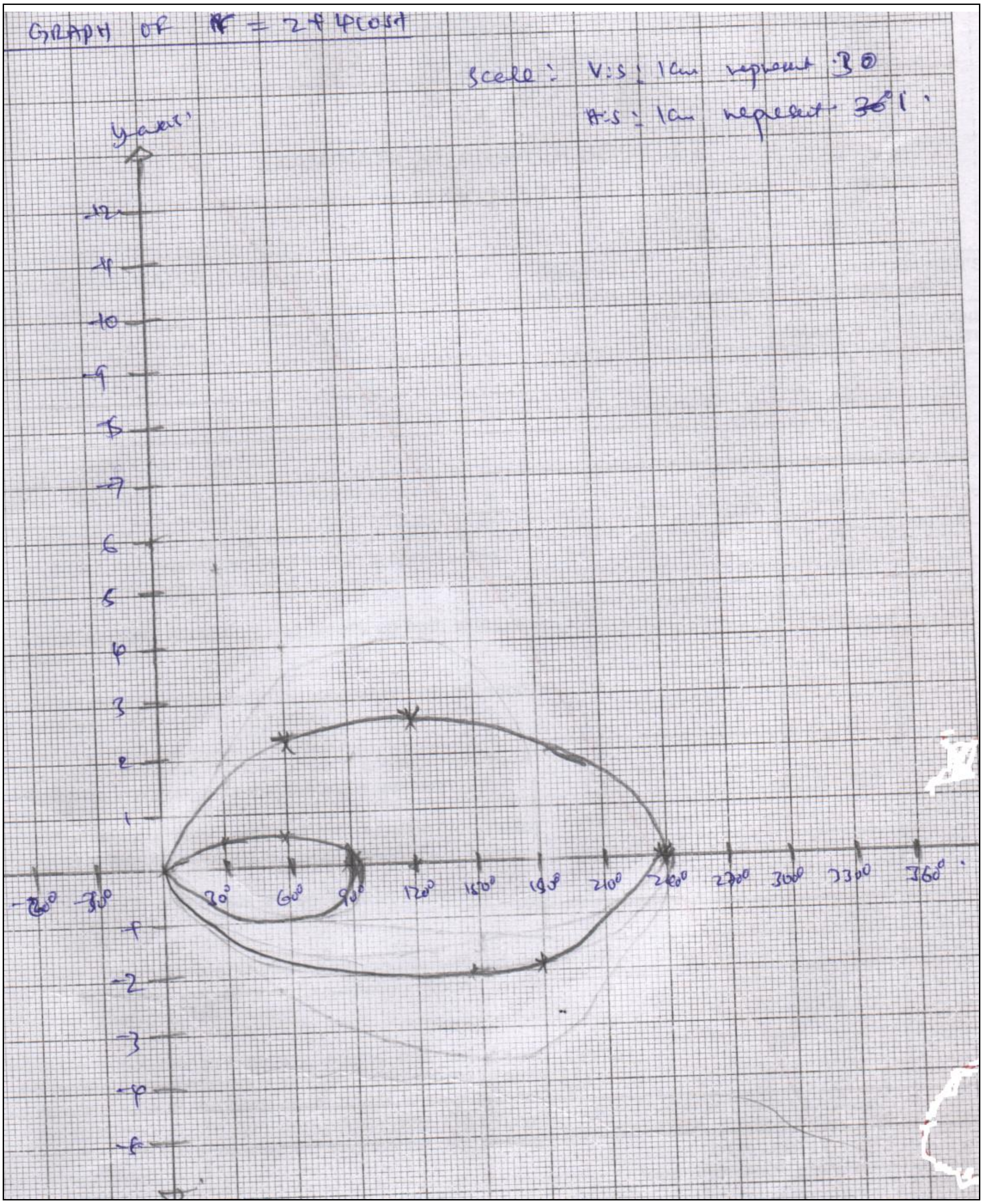
The analysis indicates that many of them performed poorly because they had inadequate knowledge and skills in the topic. In part (a) (i), they failed to find the correct point of the centre of the ellipse from the given foci, which led them getting a wrong equation. For instance, they treated (-1, 7) as (h, k) and plugged in $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ to get $\frac{(x+1)^2}{a^2}+\frac{(y-7)^2}{b^2}=1$ which was wrong. The candidates who failed in part (ii) did not know the relationship between the polar and Cartesian coordinates. They considered $y^2=4a(a-x)$ the same as $y^2=4x$

and obtained $a=1$, which resulted in getting y^2+4x-4. Moreover, in part (iii) they failed to express the equation $y=2x^2-6x+4$ into standard translated form $y+k=a(x+h)^2$ so as to get (h,k). Eventually, those who struggled obtained the equation $\left(x-\frac{3}{2}\right)^2=\frac{1}{2}\left(y+\frac{7}{2}\right)$ instead of getting $\left(y+\frac{1}{2}\right)=2\left(x-\frac{3}{2}\right)^2$.

In part (b), the candidates were unable to transform the word problem into mathematical model or diagram; as a result they did not understand where the parabola opens up. Many candidates assumed that the equation would be a translated parabola and just substituted h and k to obtain $y-100=4a(x-20)$. However; they had a non-parabola at the end.

Further in (c) (i), some candidates defined the term hyperbola as a conic section whose eccentricity is less than unity, which is incorrect. Such candidates had difficulties in using the property of hyperbola to show that $\frac{2b^2}{a}$ was the latus rectum of $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$. In addition, in part (d), the lack of knowledge of preparing a table of value of polar equations correctly affected the candidates' performance. They sketched a wrong polar graph for the equation $r=2+4\cos t$ as illustrated in Extract 18.1.

Extract 18.1.



In Extract 18.1, the candidate sketched a wrong polar graph of $r = 2 + 4\cos t$ indicating the lack of knowledge and skills in the topic.

Conversely, those who performed well, were able to calculate the center of the ellipse and used the properties of ellipse to arrive at $\frac{(x-3)^2}{64}+\frac{y^2}{48}=1$ as demanded in part (a) (i). Moreover, they had sufficient knowledge and skills in using $y = r\sin\theta$ and $x = r\cos\theta$ to transform the Cartesian equation to polar equation as required in part (ii). In part (iii), the candidates were able to determine directrix and focus of the parabola by first expressing $y = 2x^2 - 6x + 4$ in the form of $\mathrm{y + k = p(x + h)^2}$ and stating the values of k, p and h to be $-\frac{1}{2}$, $\frac{3}{2}$ and $\frac{1}{8}$ respectively.

Furthermore, in part (b), the candidates were able to translate the word problem diagrammatically, as $x^2 = 4ay$ and used the result to determine the focus $(0, a)$ and vertex $(0, 0)$. At last, they substituted $x = 50$ and $y = 20$ to get $a = 31.25$, which eventually led them to getting the required answer, $x^2 = 125y$.

The candidates were also able to define the term hyperbola as demanded in part (c) and proved that $\frac{2b^2}{a}$ is the length of the latus rectum of $\mathrm{\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1}$.

For instance, they defined the latus rectum as a focal chord perpendicular to the axis of the parabola. Finally, they prepared the appropriate table of values for the equation $r = 2 + 4\cos t$ and correctly used the ordered pairs (r, t) to sketch the polar graph in part (d). Extract 18.2 is a sample solution for part (c) (ii) from one of the candidates.

Extract 18.2

8 a (ii)

$$y = 2x^2 - 6x + 4$$
$$y = 2[x^2 - 3x] + 4$$
$$y = 2(x - 3x + (3/2)^2] + 4$$
$$y - 4 = 2(x - 3/2)^2$$
$$y - 4 + 9/2$$
$$y + 1/2 = 2(x - 3/2)^2$$
$$(x - 3/2)^2 = \frac{1}{2}[y + 1/2]$$

8 a (iii)

$$(x - 3/2)^2 = \frac{1}{2}[y + 1/2]$$

Compare $(x - p)^2 = 4a(y - q)$

$$p = 3/2 \quad q = -1/2$$
$$4a = 1/2$$
$$a = 1/8$$

directric $y = (-a + q)$

$$y = (-1/8 + -1/2)$$
$$y = -5/8$$

directrix $y = \frac{-5}{8}$

Focus $(0, a) + (p, q)$

$$p, (a + q)$$
$$3/2, (1/8 + -1/2)$$

Focus $(3/2, -3/8)$

$\therefore$ Focus is $(3/2, -3/8)$ directry $y = (-5/8)$

Extract 18.2, a sample of the best solution from one of the candidates who determined the directrix and the focus from the equation $y = 2x^2 - 6x + 4$.

3.0 A SUMMARY OF THE CANDIDATES' PERFORMANCE

The Advanced Mathematics examination had two papers namely 142/1 Advanced Mathematics 1 and 142/2 Advanced Mathematics 2. The examination tested 18 topics out of which 10 topics were in paper one and eight topics in paper two. The topics which were tested in paper one were *Calculating Devices, Hyperbolic Function, Linear Programming, Statistics, Sets, Functions, Numerical methods, Coordinate geometry I, Integration and Differentiation.* The topics which were tested in paper two were *Complex Numbers, Logic, Vectors, Algebra, Trigonometry, Probability, Differential Equations* and *Coordinate Geometry II.*

The eight topics which had good performance in the examination were *Statistics, Linear programming, Sets, Differential Equations, Logic, Numerical methods, Hyperbolic Functions and Vectors.* Four topics which had average performance were *Functions, Differentiation, Calculating Devices and Complex Numbers.* Moreover, six topics which had poor performance were *Probability, Integration, Coordinate geometry I, Algebra, Trigonometry and Coordinate Geometry II,* see appendix 1.

The good performance in those topics was attributed to the candidates' understanding of the computations. Moreover, the ability to state, recall, apply and use correct formulae, techniques, laws, and identities.

The factors that contributed to poor performance included: the lack of knowledge and skills in computations and inability to define terms, apply laws, sketch graphs, and solve equations. Further, the inability of the candidates to remember required formulae, laws, techniques, concepts and identities affected the performance. Others were failure to follow instructions and misconceptions.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

The CIRA report has been specifically designed to provide the awareness to stakeholders about the candidates' responses in ACSEE 2017. Therefore, the focus of the analysis was therefore to identify the strength and weaknesses of the candidates' responses in various items. The Candidates' Items Response Analysis (CIRA) in 142-Advanced Mathematics 2017 revealed that 74.78 percent of the candidates passed the examination, in comparison to 76.35 percent who passed the examination in 2016.

The analysis for 2017 shows that 5 topics were well performed, 4 topics were averagely done and 6 topics were poorly performed. The analysis has shown some factors that caused good or poor performance. The details of the performances including the factors for the performances are indicated in section 3.0.

Primarily, the report has revealed the main areas where the candidates had good, average or poor performance. It is expected that the stakeholders will use the recommendation of this report to improve the performance in Advanced Mathematics in future.

4.2 Recommendations

For the purpose of improving future candidates' performance in this subject, it is suggested that:

1. Students should put more effort in learning so as to improve their performance in all topics. They should do as many exercises as possible (on variety of questions) for mastery of computations, formulas, laws and techniques.

2. Students should consistently follow instructions outlined in the examinations and show all necessary steps in arriving to a final solution.

3. Teachers should train students on how to identify requirements of questions and how to make learning environment conducive.

4. Parents, guardians and other education stakeholders should ensure that supporting materials such books, calculators and other mathematical instruments are provided to students to enhance learning of Mathematics.

5. The Ministry of Education, Science and Technology, should make sure that every school has enough manpower, infrastructure, supervision and inspection to improve and control the quality of education.

Appendix I

Analysis of the Candidates' Performance per topic in Advanced Mathematics 2017

S/N	Topic	Number of Question	The % of Candidates who Scored at 35 or above	Remarks
1	Statistics	1	93.3	Good
2	Linear Programming	1	88	Good
3	Differantial Equations	1	79.7	Good
4	Sets	1	77.5	Good
5	Logic	1	72.3	Good
6	Numerical methods	1	65.1	Good
7	Hyperbolic Functions	1	62.9	Good
8	Vectors	1	59.6	Good
9	Functions	1	48.1	Average
10	Differentiation	1	43.5	Average
11	Calculating Devices	1	42.4	Average
12	Complex Numbers	1	41.4	Average
13	Coordinate Geometry II	1	33.5	Weak
14	Trigonometry	1	32.1	Weak
15	Algebra	1	28.9	Weak
16	Coordinate Geometry I	1	15.8	Weak
17	Probability	1	12.4	Weak
18	Integration	1	11.1	Weak

Appendix II

Analysis of the Candidates' Performance per topic in Advanced Mathematics 2016 & 2017

S/N	Topic	Number of Question	The % of Candidates who Scored at 35 or above 2017	Remarks	Number of Question	The % of Candidates who Scored at 35 or above 2016	Remarks
1	Statistics	1	93.3	Good	1	16.6	Weak
2	Linear Programming	1	88	Good	1	71.7	Good
3	Differantial Equations	1	77.4	Good	1	58.0	Average
4	Sets	1	77.5	Good	1	71.8	Good
5	Logic	1	72.3	Good	1	74.2	Good
6	Numerical methods	1	65.1	Good	1	27.8	Weak
7	Hyperbolic Functions	1	62.9	Good	1	35.6	Average
8	Vectors	1	59.6	Good	1	56.1	Average
9	Functions	1	48.1	Average	1	53.9	Average
10	Differentiation	1	43.5	Average	1	17.9	Weak
11	Calculating Devices	1	42.4	Average	1	27.9	Weak
12	Complex Numbers	1	41.4	Average	1	50.0	Average
13	Coordinate Geometry II	1	33.5	Weak	1	61.4	Good
14	Trigonometry	1	32.1	Weak	1	72.3	Good
15	Algebra	1	28.9	Weak	1	9.5	Weak

16	Coordinate Geometry I	1	15.8	Weak	1	21.8	Weak
17	Integration	1	11.1	Weak	1	18.4	Weak
18	Probability	1	10.8	Weak	1	12.1	Weak

The percentage of the Candidates' Performance and Topics Tested in 2016 and 2017